The natural vibrations of shell structures taking into account dissipative properties and structural heterogeneity

Sherzod Khudainazarov[1*], *Tulkin* Mavlanov[1], *Feruza* Umarova[1], and *Talibjan* Sabirjanov[2]

[1]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, National Research University, 100000 Tashkent, Uzbekistan
[2]Fergana Polytechnical Institute, Fergana, Uzbekistan

Abstract. The paper considers the natural oscillations of shell structures. In general, these structures are a set of deformable elements with different rheological properties. An algorithm for solving the viscoelastic dynamic problems has been developed for the complex axisymmetric structures. The physical properties of the viscoelastic structural elements are described by linear Boltzmann-Voltaire relations with integral difference cores. The three-parameter core of Rzhanitsyn-Koltunov was used as the relaxation core. In general, the problem is reduced to solving the systems of first-order ordinary differential equations in the complex variables. A frequency equation is obtained, for the solution where the Muller method is applied. The calculated values of the natural frequencies of oscillations with a given degree of accuracy are given.

1 Introduction

In industry, shell systems can be used to protect pipelines, buildings and other structures from corrosion and abrasion. They can be made of various materials, including the polymers, metals and ceramics.

The shell systems are widely used in the construction, especially in the construction of buildings and structures, and in particular hydraulic structures. For example:

1. The shell systems for protection against moisture: the moisture-proof shells are used to protect building structures from the moisture. They can be made of various materials, such as polyethylene, PVC, rubber and others. These shells are installed at the construction site to protect walls, the floors and ceilings from water ingress.

2. The shell systems for fire protection: the shells made of the special materials are used to protect buildings and structures from the fires. These materials can be made of drywall, fiberglass and other fire-resistant materials.

3. The shell systems for the thermal insulation: the insulation shells are used for the thermal insulation of walls, ceilings and floors. They can be made of various materials, such as the mineral wool, polystyrene, polyurethane foam and others. These shells help to reduce heating and air conditioning costs.

* Corresponding author: scherzodshox77@mail.ru

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4. The shell systems for the sound insulation: the shells made of sound insulation materials are used to reduce the noise levels in buildings. These materials can be made of the drywall, mineral wool, polyurethane and other sound insulation materials.

The shell systems are also widely used in the aircraft construction. They can be used for various purposes, such as the corrosion protection, increasing the strength, the stability and rigidity of structures, as well as for the heat and sound insulation.

Some examples of using the shell systems in the aircraft construction:

1. The shell panels: the shell panels are made of various materials such as aluminum, the composite materials or fiberglass. They are used to create the skin of wings, fuselage, rudders and other parts of the aircraft.

2. The shell systems for corrosion protection: the aircraft industry uses the shell systems for corrosion protection, such as anti-corrosion coatings or composite materials that are not susceptible to corrosion.

3. The shell systems to increase the strength, stability and rigidity: the shell systems can be used to increase the strength, stability and rigidity of the aircraft structures. For example, a special rigid shell panels, frames can be used to strengthen the wing of the aircraft.

4. The shell systems for heat and the sound insulation: the shell systems are used for heat and the sound insulation in the aircraft. For example, special insulation materials can be installed between the shell and the cabin of the aircraft to reduce a noise and improve the passenger comfort.

The shell systems are also widely used in space technology. Their main task is to protect spacecraft and satellites from the effects of the space environment, such as vacuum, cosmic radiation, extreme temperatures and micrometeoroids. Some examples of the use of shell systems in space technology:

1. The shell systems for protection against the thermal loads: the spacecraft and satellites are subject to extreme the temperatures in space. The shell systems are used to protect the spacecraft from overheating and hypothermia by regulating a heat loss.

2. The shell systems for protection against cosmic radiation: the cosmic radiation can cause a serious damage to the electronics and other systems of the spacecraft and satellites. The shell systems are used to protect these systems from the effects of cosmic radiation.

3. The shell systems for protection against the micrometeorites: the micrometeoroids are the small particles of cosmic origin that can damage the spacecraft. The shell systems can be used to protect against the micrometeorites by creating a stronger shell.

4. The shell systems for a vacuum protection: the spacecraft and satellites must withstand extremely a low pressure in the vacuum of space. The shell systems can be used to create a sealed shell that will protect internal systems from vacuum.

The shell systems can be used in pumping stations to protect the pumps and other equipment from the environmental influences and to reduce fluid leaks. Such shells for the pumps, also called the pump housings, are special shells that cover the pumps and protect them from damage, corrosion and contamination. They can also be used to reduce a noise and vibrations caused by the operation of pumps.

In mechanical engineering, the shell systems can be used to create the lightweight and durable structures. For example, the shells can be used to create car bodies, cabinets for computers and equipment, enclosures for electronic equipment, production equipment, etc.

The shell systems are also used for the transportation of various types of liquid cargo, such as oil, petroleum products, chemical reagents, aqueous solutions, etc. One of the most common examples of the shell systems for the transportation of liquid cargo are the tanks.

The tanks can be made of various materials, including steel, aluminum, plastic and composites. However, for the transportation of dangerous or highly toxic substances, special tanks with a double shell or a shell with a special coating can be used to prevent leaks.

Thus, there are many types of shell systems, each of which has its own characteristics and is used in various fields of engineering and industry.

The study of the proper vibrations of the shells began in the XIX century, when the French mathematician Jean Coulomb began working on this problem. Subsequently, at the beginning of the XX century, the German mathematician Theodor von Karman conducted a series of experiments to study the intrinsic vibrations of shells. In the 1930s, this topic was investigated in detail by the Russian mathematician Leonid Sedov, who developed a number of new methods and approaches to the study of this issue.

In the 1950s, the methods that are more accurate were developed for calculating the intrinsic vibrations of shells, based on model equations and the principle of minimum energy. During this period, numerical methods for solving equations were also developed, which make it possible to calculate the proper vibrations of shells on a computer. Among the most famous scientists who have worked in the field of natural oscillations of the shell systems are Yevgeny Paton, Nikolai Minkovsky, Joseph Keller, Leonid Sedov and Donald Wright. They conducted the experiments, developed the theoretical models and calculation methods, which led to a significant development of the theory of natural oscillations of the shell systems.

The mathematical theory of viscoelastic bodies was intensively developed in the 60s. Here, first of all, it should be noted the fundamental works of domestic scientists such as A.A.Ilyushin, B.E. Pobedrya, N.H. Arutyunyan, P. M. Ogibalov, V. V. Moskvitin, M. A. Koltunov, Yu. N. Rabotnov, as well as fundamental research by I.I.Bulgakov, V. G. Gromov, V. V. Kolokolchikov, A. S. Kravchuk, V. P. Mayborody, L. E. Maltsev, V. P. Matveenko, M.I. Rozovsky, N.A.Trufanov, I. E. Troyanovsky.

The fundamental research on viscoelasticity is contained in the works of foreign authors such as T. Alfrey, D. Bland, R. Christensen, B. Coleman, T. Ferry, D. Fitzgerald, A. Green, B. Gross, M. Gurtin, R. Rivlin, A. Reddy, R. Shepery and others.

These scientists made a significant contribution to the study of natural oscillations of the shell systems and helped to develop the methods for calculating and designing such systems. Today, the researches of natural oscillations of the shell systems continue, and this area remains active and important for many industries and science.

The physical basis of the systems under consideration, bearing all other subsystems, is a structure composed of rod, thin-walled or other elements made of materials that, in the presence of sufficiently small deformations, can be considered elastic or viscoelastic. The result of the interaction of structural elements is its periodic oscillations. The parameters of these oscillations determine the suitability of the structure for operation according to the strength criteria, the amplitude values of the displacements.

An important stage in the study of the dynamic behavior of shell structures is the determination of dynamic characteristics, which include the natural frequencies and modes of oscillation, amplitude-phase frequency characteristics. In general, these structures are a set of deformable elements with different rheological properties. Such mechanical systems are called structurally heterogeneous.

An algorithm for solving the viscoelastic dynamic problems has been developed for complex axisymmetric structures [1]. The similar problems and geometric relations for elastic shell structures are given in [2].

In the work [3] a nonlinear static and dynamic analysis of the shell was carried out taking into account a homogeneous, isotropic and linearly elastic material. The critical load and natural frequency with their corresponding bending and vibration modes were determined. In linear dynamic analysis, it was noted that the natural frequency increases with the height of the curved edge.

In the papers [4-6] the problem of forced vibrations of a shell structure with free boundary conditions at the ends is considered. Examples of calculation of amplitude-frequency characteristics and vibration modes of a shell structure are given.

In the papers [7-14] forced vibrations of multilayer cylindrical, spherical and conical shells under non-stationary loading were studied. Within the framework of the Timoshenko-type shell model, the equations of motion of multilayer shells of revolution are derived and numerical simulation of the dynamic behavior of multilayer shells under axisymmetric impulsive loading is carried out. Numerical examples of the dynamic behavior of shells are given and the results obtained are analyzed.

In the papers [15-18], the dynamic stability of a cylindrical orthotropic shell was studied. Using the Bubnov–Galerkin method, the problem was reduced to an infinite system of ordinary differential equations, which is reduced to an infinite system of homogeneous algebraic equations in the form of time trigonometric series. By reducing the resulting system and equating the reduced determinant of the matrix to zero, a characteristic equation was obtained for determining the critical frequencies of external pressure pulsations. The developed mathematical model makes it possible to calculate the dynamic stability of orthotropic cylindrical shells of linearly variable thickness.

The papers [19-21] present methods and results of experimental studies of the stress-strain state and strength of multilayer thick-walled anisotropic cylinders under dynamic loading. An algorithm and a program for calculating the stress-strain state and strength of multilayer composite cylinders under internal impulse loading have been developed.

In articles (22-24), on the basis of experimental and theoretical studies, mathematical models of a viscoelastic shell have been developed. The main dynamic characteristics of the structure are determined taking into account the actual geometry and mechanical characteristics of the material.

In articles (25-29) the dynamics of structurally inhomogeneous, multi-connected shell structures was studied. Based on the laws of mechanics and the Lagrange principle, a mathematical model and an algorithm for solving the problem were developed.

Articles [30-31] are devoted to the theoretical calculation of the box-shaped structure of on dynamic effects, taking into account the spatial work of transverse and longitudinal walls under dynamic effects, set by the base displacement according to a sinusoidal law.

In [32], the development of a theory and method for calculating structure is considered. A theory and a method were developed to assess the stress-strain state of structure without simplifying hypotheses within the framework of the three-dimensional theory of elasticity.

In contrast to the above scientific studies, in this work, studies of the dynamics of shell structures were carried out, taking into account structural heterogeneity and rheological properties of structural elements.

2 Methods

The physical properties of viscoelastic structural elements are described by the linear Boltzmann-Voltaire relations with integral difference cores R(t-s) [33-35]. Then the following relations are characteristic for each viscoelastic structural element [36-41]:

$$\tilde{E}_{\boldsymbol{1}}^{\boldsymbol{j}} = \tilde{E}_{\boldsymbol{1R}}^{\boldsymbol{j}} + \boldsymbol{i}\tilde{E}_{\boldsymbol{1z},}^{\boldsymbol{j}} \qquad \tilde{E}_{\boldsymbol{2}}^{\boldsymbol{j}} = \tilde{E}_{2}^{\boldsymbol{j}} + \boldsymbol{i}\tilde{E}_{\boldsymbol{2z}}^{\boldsymbol{j}}, \tag{1}$$

and also the corresponding complex values of the Poisson 's ratio are:

$$\boldsymbol{\nu}_{\boldsymbol{1}}^{\boldsymbol{j}} = \boldsymbol{\nu}_{\boldsymbol{1R}}^{\boldsymbol{j}} + \boldsymbol{i}\tilde{\boldsymbol{\nu}}_{\boldsymbol{1I}}^{\boldsymbol{j}}, \qquad \boldsymbol{\nu}_{\boldsymbol{2}}^{\boldsymbol{j}} = \boldsymbol{\nu}_{\boldsymbol{2R}}^{\boldsymbol{j}} + \boldsymbol{i}\tilde{\boldsymbol{\nu}}_{\boldsymbol{2I}}^{\boldsymbol{j}}, \tag{2}$$

According to [42-43], the relationship between stresses and deformations for a shell element made of orthotropic viscoelastic material can be represented as

$$\boldsymbol{\sigma_{11}(t) = \frac{E_1}{1-\nu_1\nu_2}\left\{\varepsilon_{11}\left[1-\int_0^\infty R_{11}(\tau)e^{i\omega r\tau}d\tau + \nu_2\varepsilon_{22}\left[1-\int_0^\infty R_{21}(\tau)e^{i\omega r\tau}{}_{d\tau}\right]\right]\right\}},$$

$$\boldsymbol{\sigma_{22}(t) = \frac{E_1}{1-\nu_1\nu_2}\left\{\varepsilon_{22}\left[1-\int_0^\infty R_{22}(\tau)e^{i\omega r\tau}d\tau + \nu_1\varepsilon_{11}\left[1-\int_0^\infty R_{21}(\tau)e^{i\omega r\tau}{}_{d\tau}\right]\right]\right\}},$$

$$\boldsymbol{\sigma_{12} = G\varepsilon_{22}\left[1-\int_0^\infty R_G(\tau)^{i\omega r\tau}\ d\tau\right]\varepsilon_{12}}, \tag{3}$$

Introducing the notation

$$\boldsymbol{\widetilde{E_1} = E_1(1-\Gamma_{11c}+i\Gamma_{11s})(1 \leftrightarrows 2)}, \tag{4}$$

$$\boldsymbol{\widetilde{V_2} = \widetilde{V_2}\frac{1-\Gamma_{12c}+i\Gamma_{12c}}{1-\Gamma_{11c}+i\Gamma_{11c}}},\quad \boldsymbol{(1 \leftrightarrows 2)\widetilde{G} = G_1(1-\Gamma_{GC}+i\Gamma_{GC})}, \tag{5}$$

We get

$$\boldsymbol{\sigma_{11} = \frac{\widetilde{E_1}}{1-V_1V_2}(\varepsilon_{11}+\widetilde{\nu_2}\varepsilon_{22})}, \tag{6}$$

$$\boldsymbol{\sigma_{22} = \frac{\widetilde{E_2}}{1-V_1V_2}(\varepsilon_{22}+\widetilde{\nu_1}\varepsilon_{11});\ \sigma_{12} = \widetilde{G}\varepsilon_{11}}, \tag{7}$$

The values G_{11c}, G_{11s}, in the ratios (4-5) are the cosine and sine Fourier images of the core R(t):

$$\boldsymbol{\Gamma_{11c} = \int_D^\infty R_{11}(\tau)cos\omega_R\,\tau d\tau,\ \ \Gamma_{11s} = \int_D^\infty R_{11}(\tau)sin\omega_R}, \tag{8}$$

$$\boldsymbol{\Gamma_{GC} = \int_D^\infty R_G(\tau)cos\omega_R\,\tau d\tau,\ \ \Gamma_{GS} = \int_D^\infty R_G(\tau)sin\omega_R\,\tau d\tau}, \tag{9}$$

In the case when the shell structure contains frames of materials which are characterized by a complex modulus of elasticity E and a complex shear modulus G, then the basic physical relations can be represented as

$$\boldsymbol{Q_k = [G_3]\varepsilon_k,\ Q_k = [T\ \ M_x\ \ M_z\ \ M]^{\mathrm{T}}}, \tag{10}$$

$$[G_3] = \begin{bmatrix} \tilde{E}F & 0 & 0 & 0 \\ 0 & \tilde{E}I_z & \tilde{E}I_{xz} & 0 \\ 0 & \tilde{E}I_{xz} & \tilde{E}I_x & 0 \\ 0 & 0 & 0 & GJ \end{bmatrix}, \tag{11}$$

where **F** is the cross-sectional area of the frame; $\boldsymbol{I_z}$**,** $\boldsymbol{I_x}$ are the axial moments of inertia of the section relative to the **x** and **z** axes; $\boldsymbol{I_{zx}}$ is the centrifugal moment of inertia; **J** is the moment of inertia during torsion.

As a ratio for viscoelastic hereditary connections between the nodal structural elements, which for the simplest option under consideration will have the following form:

$$\boldsymbol{N_c} = [\boldsymbol{G_c}](\boldsymbol{v_k}\text{-}\boldsymbol{v_H}) \quad \boldsymbol{N_c} = [\,\boldsymbol{N_{c1}}\ \ \boldsymbol{N_{c3}}\ \ \boldsymbol{M_c}\ \ \boldsymbol{N_{cz}}]^{\mathrm{T}}, \tag{12}$$

$$\boldsymbol{v_H} = [\,\boldsymbol{u_H}\ \ \boldsymbol{w_H}\ \ \boldsymbol{\theta_H}\ \ \boldsymbol{v_H}]^{\mathrm{T}} \quad \boldsymbol{v_k} = [\,\boldsymbol{u_k}\ \ \boldsymbol{w_k}\ \ \boldsymbol{\theta_k}\ \ \boldsymbol{v_k}]^{\mathrm{T}}, \tag{13}$$

$$[G_c] = \begin{bmatrix} K_{1c} & 0 & 0 & 0 \\ 0 & K_{3c} & 0 & 0 \\ 0 & 0 & K_{Mc} & 0 \\ 0 & 0 & 0 & K_{2c} \end{bmatrix}, \tag{14}$$

where K_{1c} , K_{3c} , $K_{Mc,}$ K_{2c} are the complex values of the stiffness coefficients of viscoelastic bonds.

The above relations are the physical relations for all the elements of the multi-connected structurally inhomogeneous shell structures under consideration. To determine the complex frequencies and forms of natural oscillations of axisymmetric shell structures made of viscoelastic material, we use the displacement method in the form proposed in [2].

The system of differential-algebraic relations describing the behavior of the shell element, according to [44-46] has the form:

$$\boldsymbol{L_p} + \boldsymbol{\widetilde{\omega}^2}\left[\overline{\boldsymbol{\rho}_{\mathrm{p}}}\right]\boldsymbol{U_p} = \boldsymbol{0}\ \ (\boldsymbol{p} = \boldsymbol{1}, \dots, \boldsymbol{N_s}) \tag{15}$$

$$\boldsymbol{L_r^i} + \boldsymbol{\widetilde{\omega}^2}[\boldsymbol{G_\omega}]\Delta_i + = \textstyle\sum_j \sum_s \boldsymbol{\xi_{ci}^{ijs}} \left[\overline{\boldsymbol{\eta_i^{ijs}}}\right] \boldsymbol{Q_i^{ijs}} + \sum_j \sum_s \boldsymbol{\xi_{ci}^{ijs}} \left[\overline{\boldsymbol{\eta_{ci}^{ijs}}}\right] \boldsymbol{N_{ci}^{ijs}} = \boldsymbol{0} \tag{16}$$

$$(\boldsymbol{i} = \boldsymbol{1}, \dots\ \boldsymbol{N_r})$$

In this case, the solution of system (15) is sought in the form

$$X = X_0(\alpha_1) sin\alpha_2 \exp\,(i\widetilde{\omega t})\,, \tag{17}$$

Then the problem is reduced to a system of eight ordinary linear differential equations of the first order in complex variables:

$$\tilde{y}' = f(\alpha_1, \tilde{y}, h, \widetilde{\omega}), \tag{18}$$

Using the Godunov's orthogonal sweep method, it is possible to calculate the stiffness matrices of each shell element of the structure. In particular, for viscoelastic bonds, the bond stiffness matrix has the form:

$$[\mathrm{K_c}] = \begin{bmatrix} \tilde{\mathrm{C}}_{\mathrm{c}} & -\tilde{\mathrm{C}}_{\mathrm{c}} \\ -\tilde{\mathrm{C}}_{\mathrm{c}} & \tilde{\mathrm{C}}_{\mathrm{c}} \end{bmatrix}, \tag{19}$$

For the case of natural oscillations of structurally inhomogeneous shell structures, the resolving system of equations with complex coefficients regarding the displacements of nodal elements has the form:

$$[P(\widetilde{\omega})]\bar{\Delta} = 0, \tag{20}$$

The set of parameters ω̃* for which a nontrivial solution (20) exists, transformed in accordance with the restrictions imposed on the nodal elements, is the set of frequencies of the shell structure under consideration.

For the existence of a nontrivial solution of system (20), it is necessary that the determinant of this system be equal to zero. Consequently, the problem of determining the frequencies of natural oscillations of structurally inhomogeneous shell structures reduces to finding the roots of the equation with complex coefficients:

$$D(\widetilde{\omega}) = |\mathrm{P}(n, \widetilde{\omega}^*)| = 0 \,, \tag{21}$$

Next, the Muller method is used. The essence of this method is as follows.

Using three given initial approximations, we calculate the values of the determinants D_0 $(\omega_0)\tilde{}$, D_1 $(\omega_1)\tilde{}$, D_2 (ω_2).

The new approximation ω_3 is determined by the formula:

$$\widetilde{\omega_3} = \widetilde{\omega_2} + (\widetilde{\omega_2} - \widetilde{\omega_1})\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \,, \tag{22}$$

$$A = q[D_2 - (1+q)D_1 + qD_0] \,, \tag{23}$$

$$B = (2q+1)D_2 - (1+q)^2 D_1 + q^2 D_0 \,, \tag{24}$$

$$C = (1+q)D_2, \tag{25}$$

$$q = \frac{(\widetilde{\omega_2} - \widetilde{\omega_1})}{(\widetilde{\omega_1} - \widetilde{\omega_0})}, \tag{26}$$

Having chosen the most important to $(\omega_2)\tilde{}$ value of the root $(\omega_3)\tilde{}$, we form three new initial approximations $\omega_1\tilde{}, \omega_2\tilde{}, \omega_3\tilde{}$ after which we repeat the process until the inequalities are met:

$$\left|\frac{\omega_{Rm} - \omega_{Rm-1}}{\omega_{Rm}}\right| < \varepsilon; \quad \left|\frac{\omega_{Lm} - \omega_{Lm-1}}{\omega_{Lm}}\right| < \varepsilon \,, \tag{27}$$

where ω_R, ω_L are, respectively, the real and imaginary components of the complex frequency ω; ε is specified relative accuracy of calculating the roots of the equation.

As an example, consider the weakly singular Rzhanitsyn-Koltunov core:

$$R(t) = A \exp(-\beta t) \cdot t^{\alpha - 1} \tag{28}$$

The Rzhanitsyn-Koltunov core very satisfactorily reflects the quasi-static and dynamic behavior of viscoelastic materials and is the most convenient when performing the appropriate calculations.

3 Results and discussion

The real and imaginary components of the complex modulus of elasticity E in this case are determined by the formulas [36, 42**]**

$$E_R = E(1 - \Gamma_c), \;\; E_I = E\Gamma_s \,, \tag{29}$$

Here G_c G_s are the cosine and sine of Fourier transforms of the Rzhanitsyn-Koltunov core:

$$\Gamma_c(\omega_R) = \frac{A\Gamma(\alpha)}{(\beta^2+\omega_R^2)^{\frac{\alpha}{2}}}\cos(\alpha\varphi) \ , \tag{30}$$

$$\Gamma_s(\omega_R) = \frac{A\Gamma(\alpha)}{(\beta^2+\omega_R^2)^{\frac{\alpha}{2}}}\sin(\alpha\varphi) \ , \tag{31}$$

$$\varphi = arctg(\frac{\omega_R}{\beta}) \ , \tag{32}$$

Having calculated the values of G_c and G_s , we find the complex modulus of elasticity:

$$\tilde{E} = E(1 - \Gamma_c - i\Gamma_s) \ , \tag{33}$$

The complex Poisson's ratio is determined by the formula:

$$\tilde{\mu} = 0.5 - \frac{0.5(1-2\mu)\tilde{E}}{E} \ , \tag{34}$$

The complex shear modulus is determined by the formula:

$$\tilde{G} = 0.5\tilde{E} \ /(1 + \tilde{\mu}) \ , \tag{35}$$

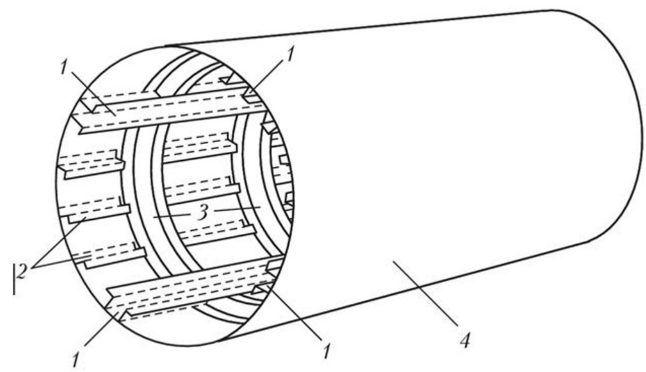

Fig.1. Calculation scheme of a complex shell structure. 1 – solid stringer; 2 – intermediate stringer ; 3 – frame; 4 – sheathing.

The calculation results are shown in Table 1.

Table 1. The calculation results.

N	QR	QI	DR	DI	IS
2	2.8474E+01	0.0000 E+00	-7.0908 E+00	4.3724 E-01	93
2	2.8517 E+01	0.0000 E+00	-7.1773 E+00	4.5420 E-01	93
2	2.8560 E+01	0.0000 E+00	-7.2633 E+00	4.7109 E-01	93
2	2.5678 E+01	-2.7600 E-01	-7.3030 E+00	-6.4527 E-01	91
2	2.5654 E+01	-2.8160 E-01	-7.2172 E+00	-1.6035 E+00	89
2	2.5654 E+01	-2.8169 E-01	-1.1318 E-01	1.5549 E+00	86

Here:

N is the number of waves in the longitudinal direction;

QR, QI are respectively real and imaginary components of the complex oscillation frequency;

DR, DI are the real and imaginary components of the mantissa of the determinant D(ω);

IS is the order of the determinant D(ω).

As a result of the calculation, we obtain the values of the oscillation frequency:

$$\omega_R^*=25.5635; \quad \omega_I^*=0.281688$$

In this case, the relative accuracy:

$$\omega_R = 4.1198610^{-8}; \omega_I = 5.2414410^{-6}$$

4 Conclusions

1. Taking into account the viscoelastic properties of the elements, the intrinsic vibrations of shell structures based on the integral Boltzmann-Voltaire relations are investigated. The three-parameter Rzhanitsyn-Koltunov core was used as the relaxation core.

2. It is established that the use of integral Boltzmann-Voltaire relations makes it possible to more accurately determine the values of eigenvalues.

3. A method for solving problems in relation to the problems of natural oscillations, taking into account the viscoelastic properties of structural materials, has been developed.

4. A frequency equation is obtained, for the solution of which the Muller method is applied.

5. The calculated values of the natural frequencies of oscillations with a given degree of accuracy are given.

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