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Non-Stationary Vibrations of a Viscoelastic Structure

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Abstract. The effect of the linear and nonlinear properties of the beam's material on the dynamic behavior of a structure under different kinematic influences is investigated. With an account for the viscoelastic properties of the material, the problem under consideration is reduced to a system of small-order linear or nonlinear integro-differential equations by the selection of coordinate functions satisfying geometric boundary conditions; the problem is solved by the averaging method or using quadrature formulas. In the article, the problems are solved using the finite element method and the Newmark method. First, a resolving matrix system of linear or nonlinear differential or integro-differential equations is obtained using the finite element method, and then it is solved by the Newmark method. The advantage of the proposed algorithm is the use in the solution of all possible modes of vibration, which are ignored in conventional methods. Comparisons of the results of the forced vibrations of a beam, taking into account the viscoelastic properties of the material under different kinematic influences, show that at the initial time, the elastic and viscoelastic solutions practically do not differ from each other. Over time, the amplitude of oscillations of the points of the beam, reaching a certain maximum value, remains constant and then begins to decrease gradually. Analysis of the presented results of forced vibrations of the beam shows that the general case, when nonlinear and viscous properties of the material are taken into account, leads to the greatest decrease in the amplitudes of displacements of the beam points compared with all other results obtained.

INTRODUCTION

The practice of modern construction in seismic regions requires improving the methods for calculating engineering structures and buildings, taking into account the geometric features of structures and the nonlinear and dissipative properties of the material.

Practical calculation methods are based on the dynamic analysis of structures as linearly elastic systems. However, instrumental data and the engineering analysis results on the structure operation during strong earthquakes indicate that the rigidity of structures does not always remain constant. Therefore, the parameters of the current response of structures must be determined by nonlinear analysis, which allows the development of more substantiated methods of design and construction, increasing the efficiency of structures while maintaining the required level of reliability.

The current stage of development of the theory of seismic resistance involves an account for the nonlinear behavior of the structure material under dynamic loads; besides, real objects possess nonlinear properties in varying degrees. When the influence of nonlinearity is negligible, linear models and corresponding linear theories are used.

As is known, when studying the dynamic behavior of structures, the problem under consideration for linear and nonlinear elastic systems is usually reduced, in some approximate way, to the Cauchy problem for the systems of ordinary linear or nonlinear differential equations, solved by the Runge-Kutta or Wilson methods [1–6].

With an account for the viscoelastic properties of the material, the problem under consideration is reduced to a system of small-order linear or nonlinear integro-differential equations by the selection of coordinate functions satisfying geometric boundary conditions and is solved by the averaging method or using quadrature formulas [7–8].

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In [9–12], a method is presented for determining the dynamic characteristics of a viscoelastic beam in the framework of the one-dimensional theory of viscoelasticity. The hereditary Boltzmann-Voltaire theory was used to describe dissipative processes in the building material. The natural vibrations of a viscoelastic beam are investigated, and the results obtained are compared with the results of field experiments.

In [13–14], forced vibration analysis of isotropic thin circular plate resting on a nonlinear viscoelastic foundation is investigated. The system coupled nonlinear partial differential equations are transformed to a system of nonlinear ordinary differential equations using the Galerkin decomposition method. The developed solutions are verified using the existing results in the literature, and good agreement is observed. Subsequently, the analytical solutions are used to investigate the effects of various parameters on the dynamic response of the plate. The results show that the nonlinear frequency ratio of vibrating circular plates increases with increased linear elastic foundation and tensile force.

In [15–18], vibrations of high buildings caused by wind and tornado waves were studied to assess the aeroelastic effects of high buildings using the wind tunnel tests. The aerodynamic damping coefficient and aerodynamic stiffness were determined by analyzing the aeroelastic force acting on the oscillating model. For a 347-meter-high building, the effect of aeroelastic parameters on wind-induced responses and equivalent static wind loads was analyzed. The results showed that during a return period of 100 years, aerodynamic damping was positive and aerodynamic stiffness was negative.

In [19–22], a statement and a method for solving the problem of axisymmetric vibrations of a physically nonlinear viscoelastic cylindrical shell with lumped masses are presented. The function characterizing the deviation of the stress intensity curve from the Hooke's straight line is taken in the form of cubic nonlinearity. A mathematical model, a solution method, and a computational algorithm for the problem of axisymmetric vibrations of a cylindrical shell with a concentrated mass taking into account the physically nonlinear deformation of the material under various boundary conditions are developed within the framework of the Kirchhoff-Love hypothesis. To solve the resulting system with the Koltunov-Rzhanitsyn weakly singular kernel, a numerical method was applied using quadrature formulas.

In this study, all the above problems are solved using the finite element and Newmark methods. First, using the finite element method, a resolving matrix system of linear or nonlinear differential or integro-differential equations is obtained, and then it is solved by the Newmark method. The advantage of the proposed algorithm is the use in the solution of all possible modes of vibration, even the ones ignored in other commonly used methods.

The issue of assessing the account for nonlinearity for real structures remains open due to several mathematical problems that arise when solving the problem and the absence of the parameters of the material nonlinearity. This dictates the relevance of the studies presented in this article, where the dynamic behavior of a beam is studied, taking into account the linear and nonlinear viscoelastic properties of the material of structures under various kinematic influences.

METHODS

Unsteady-state forced vibrations of a high axisymmetric structure are considered; the structure is represented by a one-dimensional model - a viscoelastic beam of the annular cross-section with a variable slope of the generatrix and a variable thickness. The lower end of the beam ($z = 0$) is rigidly fixed, and the kinematic effect $w_0 (t)$ is set on it; the upper end ($z = L$) is free. The beam material is a nonlinearly viscoelastic one. Bending unsteady-state forced vibrations of points located at different levels of a structure under set kinematic effects are to be determined.

The mathematical statement of the problem includes the variational equation of the principle of virtual displacements, according to which the sum of work of all active forces, including inertia forces, on a virtual displacement δw, satisfying geometrical boundary conditions is zero

$$\delta A_M + \delta A_u + \delta A_P = 0 \tag{1}$$

Here $\delta A_{м}$, $\delta A_{и}$, δA_{P} are the virtual work of the bending moment, inertial forces and external forces, respectively, calculated by the formulas:

$$\delta A_M = -\int_0^L M(z)\delta\left(\frac{\partial^2 w}{\partial z^2}\right)dz\text{, } \delta A_u = -\rho\int_0^L F(z)\left(\frac{\partial^2 w}{\partial t^2}\right)\delta w dz\text{, } \delta A_P = \int_0^L P(z,t)\delta w dz \tag{2}$$

where ρ is the beam material density, L is the beam length, $w(z)$ is the beam deflection, $M(z)$ is the bending moment; $F(z)$ is the cross-sectional area; $P(z, t)$ is the external dynamic forces.

The kinematic boundary condition at the base is

$$z = 0 : w(z,t) = w_0(t) \tag{3}$$

where $w_0(t)$ is the known time function.

Initial conditions are

$$w(z,0) = u_0, \frac{\partial w(z,0)}{\partial t} = \dot{u}_0 \tag{4}$$

where u_0, $\dot{u}_0$ are the given constants.

To describe the relationship between the stress σ_z and the strain ε_z the nonlinear theory of viscoelasticity [3-4] is used, which has the form

$$\sigma_z = E\left\{\left[\varepsilon_z(t) - \int_0^t R_1(t-\tau)\varepsilon_z(\tau)d\tau\right] - \gamma\left[\varepsilon_z^3(t) - \int_0^t R_2(t-\tau)\varepsilon_z^3(\tau)d\tau\right]\right\} \tag{5}$$

where E is the instantaneous modulus of elasticity of the material; R_1, R_2 are the relaxation kernels; γ=const>0 is the nonlinearity coefficient, depending on the beam's material.

The dependence between the deflection w and the strain ε_z is taken in the form

$$\varepsilon_z = -x\frac{\partial^2 w}{\partial z^2} \tag{6}$$

and the relationship between bending moment M_z and stress σ_z is

$$M_z = \int_F x\sigma_z dF \tag{7}$$

The problem of unsteady-state nonlinear forced vibrations of a beam consists of the following: for a given function $w_0(t)$ under initial conditions $u_0, \dot{u}_0$ - to find the deflection $w(z, t)$, strain $\varepsilon_z(z, t)$, stress $\sigma_z(z, t)$ and bending moment $M_z(z, t)$, satisfying equations (1), (2), (5) - (7) and conditions (3), (4) for any possible δw.

To reduce the variational problem posed above to a system of resolving equations, the finite element method is used [23], where a one-dimensional element is selected as the finite element, taken in the form of a truncated cone that works on bending with four degrees of freedom [25].

For the displacement function w inside the e-th element, the cubic approximation is used:

$$w = \alpha_1 + \alpha_2 z + \alpha_3 z^2 + \alpha_4 z^3 \tag{8}$$

then for the first and second derivatives, the following expressions are obtained

$$\frac{\partial w}{\partial z} = \alpha_2 + 2\alpha_3 z + 3\alpha_4 z^2 ; \ \frac{\partial^2 w}{\partial z^2} = 2\alpha_3 + 6\alpha_4 z \tag{9}$$

The dependence of the nodal displacements and rotation angles of the e-th finite element $\{w_i\}$ on the vector of arbitrary constants $\{\alpha_i\}$ in matrix form is written as

$$\{w_i\}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 1 & l & l^2 & l^3\\ 0 & 1 & 2l & 3l^2\end{bmatrix}\{\alpha_i\} \tag{10}$$

where $\{w_i\}^T=\{w_i,\beta_1,w_2,\beta_2\}$, $\beta_1=\frac{\partial w}{\partial z}\ |_{z=0}$, $\beta_2=\frac{\partial w}{\partial z}\ |_{z=l}$.

Hereinafter, the following notation is used: {} - vector, [] - matrix, ^{T} - transposition operation.

The transformation inverse to (10), i.e. the matrix dependence of $\{\alpha_i\}$ on $\{w_i\}$ is expressed as

$$\{\alpha_i\}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l}\\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2}\end{bmatrix}\{w_i\},\ \text{i.e. } \{\alpha_i\}=[A]\{w_i\} \tag{11}$$

Using the indicated transformations (11), we express the displacement function (8) and its derivatives in a matrix form in terms of nodal displacements $\{w_i\}$

$$w=\left[1\cdot\cdot z\cdot\cdot z^2\cdot\cdot z^3\right]\{\alpha_i\}=\left[1\cdot\cdot z\cdot\cdot z^2\cdot\cdot z^3\right][A]\{w_i\} \tag{12}$$

$$\frac{\partial w}{\partial z}=\left[0\cdot\cdot 1\cdot\cdot 2z\cdot\cdot 3z^2\right][A]\{w_i\} \tag{13}$$

$$\frac{\partial^2 w}{\partial z^2}=\left[0\cdot\cdot 0\cdot\cdot 2\cdot\cdot 6z\right][A]\{w_i\} \tag{14}$$

We introduce the matrix [B]

$$[B]=\left[0\cdot\cdot 0\cdot\cdot 2\cdot\cdot 6z\right][A][\ -\frac{6}{l^2}+\frac{12z}{l^3};-\frac{4}{l}+\frac{6z}{l^2};\frac{6}{l^2}-\frac{12z}{l^3};-\frac{2}{l}+\frac{6z}{l^2}\ ],$$

then

$$\frac{\partial^2 w}{\partial z^2}=[B]\{w_i\} \tag{15}$$

Substituting expression (5-7) in (2), we obtain the virtual work of the bending moment for the *e*-th element

$$\delta A_M^e=\int_0^l\left[\frac{\partial^2 w}{\partial z^2}\delta\left(\frac{\partial^2 w}{\partial z^2}\right)\int_F x^2dF\right]dz-E\int_0^t\left\{R_1(t-\tau)\int_0^l\left[\frac{\partial^2 w}{\partial z^2}\delta\left(\frac{\partial^2 w}{\partial z}\right)\int_F x^2dF\right]dz\right\}d\tau-$$
$$-E\gamma\int_0^l\left[\left(\frac{\partial^2 w}{\partial z^2}\right)^3\delta\left(\frac{\partial^2 w}{\partial z^2}\right)\int_F x^4dF\right]dz+E\gamma\int_0^t\left\{R_2(t-\tau)\int_0^l\left[\left(\frac{\partial^2 w}{\partial z^2}\right)^3\delta\left(\frac{\partial^2 w}{\partial z}\right)\int_F x^4dF\right]dz\right\}d\tau \tag{16}$$

Substitution of (15) into (16) and integration over the cross-sectional area leads each term of expression (16) to the following form:

the first term

$$\delta\{w_i\}^T E\int_0^l J^e(z)[B]^T[B]dz\{w_i\} = \delta\{w_i\}^T\left[K^e\right]\{w_i\} \quad (17)$$

where

$J^e(z) = \frac{\pi}{4}\left[(R_H(z))^4 - (R_B(z))^4\right]$ - is the moment of inertia of the cross-section of the *e*-th element; R_H, R_B are the outer and inner radii of the element, respectively;

$\left[K^e\right] = E\int_0^l J^e(z)[B]^T[B]dz$ - is the stiffness matrix of the *e*-th element.

the second term

$$E\int_0^t\left\{R_1(t-\tau)\int_0^l\left[\frac{\partial^2 w}{\partial z^2}\delta\left(\frac{\partial^2 w}{\partial z^2}\right)\int_F x^2 dF\right]dz\right\}d\tau = \delta\{w_i\}^T\int_0^t\{R_1(t-\tau)\left[K^e\right]\{w_i\}d\tau \quad (18)$$

the third term

$$E\gamma\int_0^l\left[\left(\frac{\partial^2 w}{\partial z^2}\right)^3\delta\left(\frac{\partial^2 w}{\partial z^2}\right)\int_F x^4 dF\right]dz = \delta\{w_i\}^T E\gamma\int_0^l J_1^e(z)[B]^T[B]\{w_i\}[B]\{w_i\}[B]\{w_i\}dz \quad (19)$$

Here $J^e(z) = \frac{\pi}{8}\left[(R_H(z))^6 - (R_B(z))^6\right]$.

Expanding the expression under the integral sign in (19):

$$\{V^e\} = J_1^e(z)\left[-\frac{6}{l^2}+\frac{12z}{l^3}; -\frac{4}{l}+\frac{6z}{l^2}; \frac{6}{l^2}-\frac{12z}{l^3}; -\frac{2}{l}+\frac{6z}{l^2}\right]^T\left[(-\frac{6}{l^2}+\frac{12z}{l^3})w_1 + (-\frac{4}{l}+\frac{6z}{l^2})\varphi_1 + (\frac{6}{l^2}-\frac{12z}{l^3})w_2 + (-\frac{2}{l}+\frac{6z}{l^2})\varphi_2\right]^3 \quad (20)$$

we see that it is a vector whose coordinates are cubic polynomials from nodal displacements.

As a result of integration over the length of the element, the third term (19) is

$$E\gamma\int_0^l\left[\left(\frac{\partial^2 w}{\partial z^2}\right)^3\delta\left(\frac{\partial^2 w}{\partial z^2}\right)\int_F x^4 dF\right]dz = \delta\{w_i\}^T E\gamma\{V^e\} \quad (21)$$

where the index "e" indicates that the vector {Ve} is defined for the *e*-th element.

The fourth term

$$E\gamma\int_0^t\left\{R_2(t-\tau)\int_0^l\left[\left(\frac{\partial^2 w}{\partial z^2}\right)^3\delta\left(\frac{\partial^2 w}{\partial z}\right)\int_F x^4 dF\right]dz\right\}d\tau = \\ = \delta\{w_i\}^T E\gamma\int_0^t R_2(t-\tau)\int_0^l J_1^e(z)[B]^T[B]\{w_i\}[B]\{w_i\}dzd\tau \quad (22)$$

Considering (20), we get

$$E\gamma\int_F x^5\int_0^l R_2(t-\tau)\left(\frac{\partial^2 w}{\partial z^2}\right)^3 \delta\left(\frac{\partial^2 w}{\partial z^2}\right)dzdF = \delta\{w_i\}^T E\gamma\int_0^t R_2(t-\tau)d\tau\{V^e\} \tag{23}$$

The use of the finite element method procedure leads the variational problem (1) and (3) to a nonlinear system of integro-differential equations, which has the following matrix form:

$$\begin{aligned}&[M]\{\ddot{w}(t)\}+[K]\{w(t)\}=\{P(t)\}-\int_0^t R_1(t-\tau)[K]\{w(t)d\tau+E\gamma\{V(t)\}\}-\\&-E\gamma\int_0^t R_2(t-\tau)d\tau\{V(t)\}\end{aligned} \tag{24}$$

Here [M], [K] are the matrices of mass and rigidity of the entire structure; {w} is the displacement vector of all the nodal points of the structure; {V} is a vector whose coordinates are determined by cubic polynomials of system displacements, {P} is a vector of external influences.

This equation is solved by the Newmark method [24]. Equation (24) at given initial conditions (4) is solved by direct integration using a numerical step-by-step procedure. We used the Newmark method to solve the system of equations (24), based on independent expansions of $w(t_i+\tau)$ and its derivative into the series in powers τ, while holding the terms containing the third derivative w_i. The coefficients for the residual terms α and β are selected from the condition for ensuring the unconditional convergence of the integration process:

$$\begin{aligned}\mathrm{w}(t_i+\tau)&=\mathrm{w}_i+\tau\dot{\mathrm{w}}_i+\frac{\tau^2}{2}\ddot{\mathrm{w}}_i+\alpha\tau^3\dddot{\mathrm{w}}_i\\ \dot{\mathrm{w}}(t_i+\tau)&=\dot{\mathrm{w}}_i+\tau\ddot{\mathrm{w}}_i+\beta\tau^2\dddot{\mathrm{w}}_i\end{aligned} \tag{25}$$

Substituting $\dddot{\mathrm{w}}_i=\dfrac{\ddot{\mathrm{w}}_{i+1}-\ddot{\mathrm{w}}_i}{\tau}$, expressions for displacements and velocities (25) are written as

$$\mathrm{w}_{i+1}=\mathrm{w}_i+\tau\dot{\mathrm{w}}_i+\frac{\tau^2}{2}\ddot{\mathrm{w}}_i+\alpha\tau^3(\ddot{\mathrm{w}}_{i+1}-\ddot{\mathrm{w}}_i) \tag{26}$$

$$\dot{\mathrm{w}}_{i+1}=\mathrm{w}_i+\tau\ddot{\mathrm{w}}_i+\beta\tau^2(\ddot{\mathrm{w}}_{i+1}-\ddot{\mathrm{w}}_i) \tag{27}$$

Then the acceleration obtained form (26)

$$\ddot{\mathrm{w}}_{i+1}=\frac{1}{\alpha\tau^2}(\mathrm{w}_{i+1}-\mathrm{w}_i)-\frac{1}{\alpha\tau}\dot{\mathrm{w}}_i+\left(1-\frac{1}{2\alpha}\right)\ddot{\mathrm{w}}_i \tag{28}$$

is substituted into the velocity expression (27)

$$\dot{\mathrm{w}}_{i+1}=\frac{\beta}{\alpha\tau}(\mathrm{w}_{i+1}-\mathrm{w}_i)+\left(1-\frac{\beta}{\alpha}\right)\mathrm{w}_i+\frac{\tau}{2}\left(2-\frac{\beta}{\alpha}\right)\ddot{\mathrm{w}}_i \tag{29}$$

To find a solution w_{i+1} for time t_{i+1}, the general equation of motion is written as follows:

$$[M]\ddot{w}_{i+1}+[C]\dot{w}_{i+1}+[K]w_{i+1}=\{P_{i+1}\} \tag{30}$$

After substituting expressions for accelerations (28) and velocity (29) into (30), an algebraic system of equations is obtained

$$[A]\{w_{i+1}\} = \{R_{i+1}\} \tag{31}$$

Where

$$[A] = [K] + \frac{1}{\alpha\tau^2}[M]$$

$$\{R_{i+1}\} = \{P_{i+1}\} + [M]\left(\frac{1}{\alpha\tau^2}\{w_i\} + \frac{1}{\alpha\tau}\{\dot{w}_i\} + \left(\frac{1}{2\alpha} - 1\right)\{\ddot{w}_i\}\right) + \{W_i\} \tag{32}$$

Where

$$\{W_i\} = \int_0^t R_1(t-\tau)[K]\{w_i\}d\tau + E\gamma\{V_i\} - E\gamma\int_0^t R_2(t-\tau)d\tau\{V_i\} \tag{33}$$

To solve the resulting system of equations (31), it is necessary to specify at the initial moment the values of displacements $\{w_0\}$, velocity $\{\dot{w}_0\}$, and accelerations $\{\ddot{w}_0\}$. Usually $\{\ddot{w}_0\} = 0$ is taken. The Newmark method is unconditionally stable if

$$\beta \geq 0.5, \alpha \geq 0.25(\beta + 0.5)^2 \tag{34}$$

RESULTS AND DISCUSSION

Thus, the algorithm that implements the Newmark method for solving the matrix system of nonlinear differential equations (24) obtained in the course of finite element discretization with the initial condition (4) is as follows:

1. The initial values are set $\{w_0\}$, $\{\dot{w}_0\}$.

2. A system of algebraic equations (31) is formed. The right-hand side contains nonlinear terms that determine the viscoelastic and nonlinear-viscous properties of the material, depending on the deformed state reached by the system.

When accounting for the viscoelastic properties of the material with the above formulation, terms containing cubic terms from displacements are excluded in the right-hand side of the resulting resolving algebraic system of equations (24). In this case, the equation takes the following form

$$[M]\{\ddot{w}(t)\} + [K]\{w(t)\} = \{P(t)\} + \int_0^t R_1(t-\tau)[K]\{w(t)\}d\tau \tag{35}$$

with homogeneous initial conditions:

$$w(z,0) = 0, \frac{\partial w(z,0)}{\partial t} = 0 \tag{36}$$

The task is to determine the displacements of the points of the structure at different time points. The resulting nonlinear system of integro-differential equations (35) with initial conditions (36) is solved by the Newmark method.

The algorithm that implements the Newmark method for solving the matrix system of nonlinear integro-differential equations obtained in the course of finite element discretization (35) is as follows:

1. The initial values are set $\{w_0\}$, $\{\dot{w}_0\}$.

2. A system of algebraic equations (31) with a linear right-hand side is formed, i.e., for $\{W_i(t)\}=0$.

3. The system of linear algebraic equations (31) is solved, as a result of which the current value of the displacement vector $\{w_{i+1}\}$ is determined.

4. The formulas of Newmark's method (28), (29) are used to determine the vectors of velocity and acceleration at the current time point t_{i+1}.

5. The value of integral $\int_{t_{i-1}}^{t_i} R_1(t_i-\tau)\,d\tau$ on the segment (t_{i-1}, t_i) is calculated by the formulas of numerical integration, for example, by the approximate formula of averages or by the formula of trapezoids. In the first case, the approximate value of the integral is defined as the product of the integrand at the point $t_i-\tau/2$ by the length of the segment τ. For i=1 $t_{i-1}=0$. In the second case, the value of the integral is approximately equal to

$$0.5(t_i-t_{i-1})[R_1(t_{i-1})+R_1(t_i)] \tag{37}$$

6. The resulting value of the integral is multiplied by the vector $[K]\{w_i\}$, and this vector is added to the right-hand side of the system (31).

7. Steps 3 - 6 are repeated until the end of the process.

In this formulation, with the developed methodology and the created computer program, several problems previously investigated in a linearly elastic formulation were solved [24-26]. In all the examples considered below, the values of the viscosity parameters are taken as R_1 A=0.0194; β=0.00000014; α=0.075, $R_2=2R_1$; γ = 120000 [27].

As seen from the comparison (Fig. 1) of the presented results of forced vibrations of the beams at the initial period, the elastic and viscoelastic solutions practically do not differ. Then, over time, the vibrations of viscous beams begin to differ markedly from the vibrations of elastic beams, the amplitude of which increases linearly. The amplitude of vibrations of points of beams with viscoelastic characteristics of the material, after reaching a certain maximum value, begins to decrease gradually.

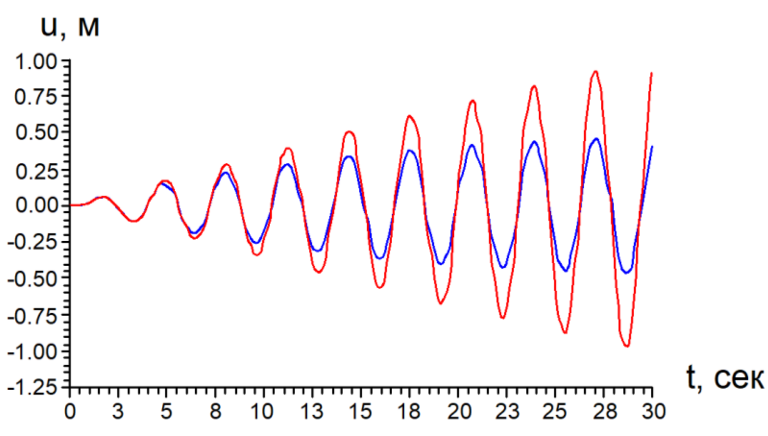

FIGURE 1. Forced vibrations of the point z = 325m of the beams at resonance mode $\ddot{w}_0 = 0.1A\sin(1.68t)$: ——— - viscoelastic solution; ——— -elastic solution.

Figure 2 shows the results of forced vibrations of the beams under the sinusoidal damping effect.

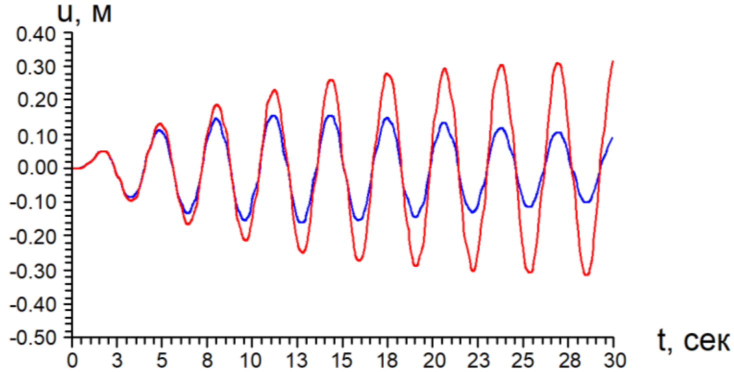

FIGURE 2. Forced vibrations of a point (z = 325m) of beams under the impact $\ddot{w}_0 = 0.1A\sin(1.68t)\exp(-0.1t)$:
——— - viscoelastic solution; ——— -elastic solution.

Let us consider the forced unsteady vibrations of high-rise beams taking into account the viscoelastic properties of the material when the horizontal component of the real accelerogram of the Gazli earthquake acts on the base of the structure [24].

The results of solving the problem (Fig. 3) show that, at the initial period, the behavior of a viscoelastic structure does not differ from the behavior of an elastic one. Subsequently, an account for the viscoelastic properties of the structure material leads to a noticeable decrease in the vibration amplitude, high frequencies damp, and the vibrations of the viscoelastic structure have a pattern of free damped vibrations with the fundamental frequency of natural vibrations. It is seen here that an account for the viscoelastic properties of the material somewhat averages the displacements of the points of the beams, leaving as significant only the oscillations of the fundamental mode.

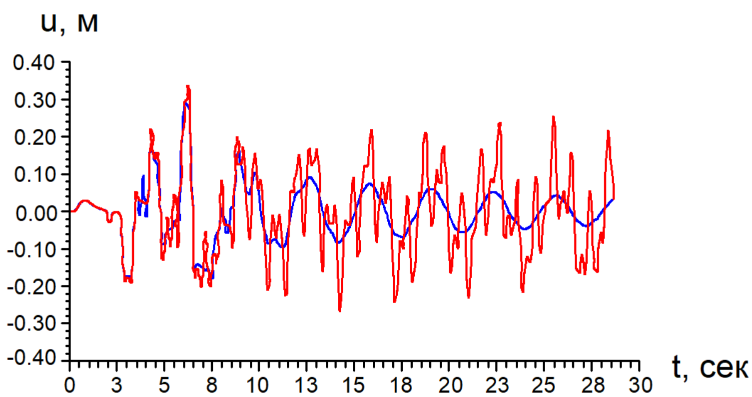

FIGURE 3. Forced vibrations of the point (z = 325m) of the beams under the effect of the accelerogram of the Gazli earthquake: ——— - viscoelastic solution;
——— -elastic solution.

Now let us consider a general case when both nonlinear and viscoelastic properties of the material are taken into account. In this case, the above formulation does not allow any simplifications on the right-hand side of the resolving system of nonlinear integro-differential equations (24). The equation has the form

$$[M]\{\ddot{w}(t)\}+[K]\{w(t)\}=\{P(t)\}+\int_0^t R_1(t-\tau)[K]\{w\}d\tau+EJ_1\gamma\{V(t)\}-$$

$$-EJ_1\gamma \int_0^t R_2(t-\tau)d\tau \{V(t)\} \quad (38)$$

with homogeneous initial conditions (36).

The task is to determine the displacements of the points of the structure at different time points. The nonlinear system of integro-differential equations (24) with initial conditions (36) is solved by the Newmark method.

The algorithm that implements the Newmark method for solving the matrix system of nonlinear integro-differential equations obtained in the course of finite element discretization (24) is as follows:

1. The initial values are set $\{w_0\}$, $\{\dot{w}_0\}$.
2. A system of algebraic equations (31) with a linear right-hand side is formed, i.e., for $\{W_i\}=0$.
3. The system of linear algebraic equations (31) is solved, as a result of which the current value of the displacement vector $\{w_{i+1}\}$ is determined.
4. The formulas of Newmark's method (28), (29) are used to determine the vectors of velocity and acceleration at the current time moment t_{i+1}.
5. The coordinates of the nonlinear vector $\{W_i\}=E\gamma\{V_i\}$, which are cubic polynomials in the found nodal displacements and rotation angles, are calculated using formulas (33).
6. The resulting vector is added to the right-hand side of the system (31).
7. By the formulas of numerical integration, for example, by the approximate formula of averages, the value of the integral $\int_{t_{i-1}}^{t_i} R_1(t_i-\tau)\,d\tau$ on the segment (t_{i-1}, t_i) is calculated as the product of the integrand at point $t_i-\tau/2$ by the length of the segment τ. For $i=1$ $t_{i-1}=0$.
8. The resulting value of the integral is multiplied by the vector [K] $\{w_i\}$, and this vector is added to the right-hand side of the system (31).
9. The product of the nonlinear vector $\{W_i\}$ by the integral $\int_{t_{i-1}}^{t_i} R_2(t_i-\tau)\,d\tau$ is found. This product is also added to the right-hand side of the system (31).
10. Steps 3 - 9 are repeated until the end of the process.

In such a general statement, with the developed methodology and the created computer program, problems are solved in linear, nonlinear, and viscoelastic formulations [25-26]. In all the examples considered below, the values of the viscosity parameters are taken as $R_1\,A=0.0194$; $\beta=0.00000014$; $\alpha=0.075$, $R_2=2R_1$; $\gamma=120000$.

According to the harmonic law, let us consider the forced vibrations of nonlinear-visco-elastic high-rise beams under kinematic excitation of the base [25]. The obtained horizontal displacements for the beams point z = 325m are shown in Fig. 4. A line with asterisks corresponds to these displacements. Here, for comparison, the solid line represents the solution for the same point of a linearly elastic structure.

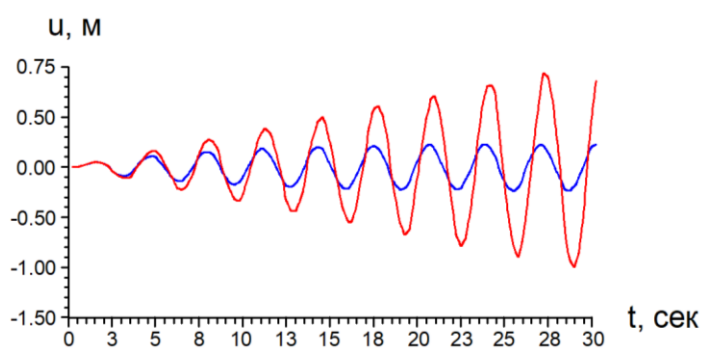

FIGURE 4. Forced vibrations of the point z = 325m of the beams taking into account the nonlinear viscoelastic properties of the material under the impact $\ddot{W}_0=0.1A\sin(1.68t)$: ——— - nonlinear-viscoelastic solution; ——— - linear-elastic solution.

The analysis of the results presented shows that the general case, when nonlinear and viscous properties of the material are taken into account, leads to the greatest decrease in the amplitudes of displacements of the points of a high-rise structure compared with all previous options. Figure 5 shows the results for sinusoidal damping of kinematic impact [24] at the base of the beams.

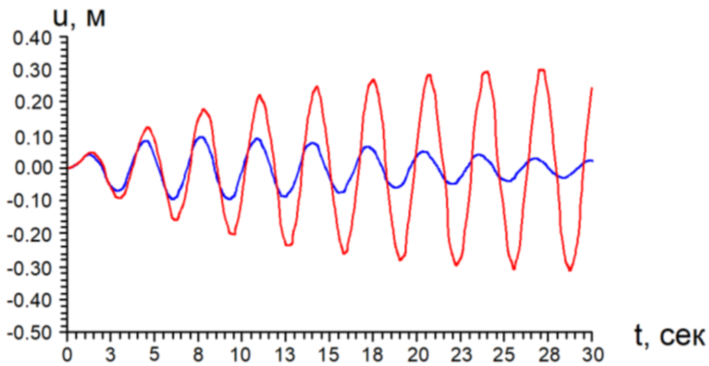

FIGURE 5. Forced vibrations of the point z = 325m of the beams, taking into account the nonlinear viscoelastic properties of the material under the impact $\ddot{W}_0$ =0.1Asin(1.68t)exp(-0,1t): ——— - nonlinear-viscoelastic solution; ——— - linear-elastic solution.

Thus, the study of the dynamic behavior of a high-rise structure, taking into account the material's nonlinear and dissipative properties (different in nature), shows that the joint consideration of all these properties brings the resulting pattern closer to the one observed in reality. That is, the oscillation amplitude of the structure does not grow infinitely. Still, it gradually decreases over time, and the maximum possible consideration of nonlinear and dissipative properties leads to the fastest damping of oscillations.

CONCLUSIONS

1. The problem under consideration is reduced to a system of small-order linear or nonlinear integro-differential equations using the selection of coordinate functions; it is solved using the finite element method and Newmark's method.
2. Investigation of forced vibrations of the beam considering the viscoelastic properties of the material under the resonance mode, sinusoidal damping, and the impact of the horizontal component of the real accelerogram of the Gazli earthquake on the structure foundation shows that at the initial time, the elastic and viscoelastic solutions practically do not differ from each other. Over time, the amplitude of vibrations of the beam points reaches its maximum value and begins to decrease gradually.
3. Analysis of the presented results of forced vibrations of the beam shows that the general case, when nonlinear and viscous properties of the material are taken into account, leads to the greatest decrease in the amplitudes of displacements of the beam points compared with other results obtained.
4. At a frequency close to the eigenfrequency of the beam, significant maximum displacements occur. If the material does not possess dissipative properties, then the structure will collapse. An account for the viscoelastic and nonlinear properties of the material significantly reduces the dynamic response of the structure and eventually takes it out of resonance.

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