# Investigation of natural vibrations of thin-walled structures interacting with fluid

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**Abstract.** The problem of studying the dynamics of elements of hydro-technical structures interacting with fluid is considered in the article. On the basis of Lagrange's variational principles, the basic equations are obtained that characterize the dynamics of complex, multiply connected structurally non-homogeneous shell systems interacting with flowing fluid. Dynamic equations of a cylindrical shell are obtained. To determine the fluid pressure on the shell surface, a boundary value problem based on the laws of hydroelasticity was used. A software package was developed for studying the dynamic characteristics of complex, multiply connected structurally non-homogeneous shell structures, as well as programs for studying the dynamic characteristics of a composite structure using the orthogonal sweep method. Dynamic characteristics are determined for different levels of water filling.

## 1 Introduction

Structurally non-homogeneous shell structures are widely used in shipbuilding, aircraft construction, the construction of large structures, and in space technology, and the study of the strain processes under external multifactorial influences is one of the most important tasks in the mechanics of a deformable rigid body.

It is known that the presence of components interacting with a liquid in a complex viscoelastic shell system significantly complicates the task of studying dynamic behavior of structures. Such problems are relevant for studying the dynamics of aircraft, hydro-technical structures, and the problems of transporting liquid cargo. As a rule, liquid-containing structures are thin-walled vessels, and their dynamic behavior is well described using shell theory relationships.

The first results in the field of the dynamics of vibrations of elastic shells interacting with fluid are associated with the names of the researchers of the late 19th - early 20th centuries. However, the period of the most intensive development of this problem dates back to the second half of the 20th century in connection with the development of rocket and aviation technology, as well as electronic computing systems. The modern stages of development of methods for solving this class of problems are developed in the works of

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such domestic researchers as Moiseev N.N. [1], Rabinovich B.I. [2], Shmakov V.P. [3], and Rapoport I.M. [4]. Among foreign experts, we can mention the studies by Abramson H.N., Kana D.D., Lindholm U.S., and Bauer H.F. A significant contribution to the solution of the problem was made by E.I. Grigolyuk, F.N. Shklyarchuk, A.G. Gorshkov, L.I. Balabukha, Yu.G., and others.

In general, the number of papers published to date, devoted to the problem of calculating the dynamic characteristics of shells containing liquid, is very large. The main part of them consists of works in which shells of a certain shape are studied and solutions are obtained, as a rule, using some variational method; the choice of coordinate functions is determined by the shape of the cavity. In these works, approximate or exact formulas were obtained for a number of shells of a simple geometric shape. However, for practice, where the complexity of design solutions often makes it difficult to obtain analytical estimates and does not always allow the use of simple models, the most valuable are universal numerical methods that do not impose strict restrictions on the shape and parameters of the structures under study. At present, solutions to problems are known for a very wide class of shells with liquid: cylindrical shells with various bottoms (flat, spherical, conical), spherical shells, conical shells, and coaxial cylindrical shells.

At present, the most common numerical methods for solving boundary value problems of shell theory are the finite difference method (FDM), the finite element method (FEM), and the numerical integration method (NIM). Publications by A.S. Volmir [5], A.F. Smirnov, D.V. Weinberg, A.P. Filin, R.E. Fulton, J.A. Stricklin, D. Bushnell, and others are devoted to the analysis of the prospects for the use of numerical methods. To solve boundary value problems of the theory of shells, when their behavior can be described by a system of first-order ordinary differential equations, the numerical integration method (NIM), based on the reduction of the boundary value problem to a number of Cauchy problems, turned out to be effective. In 1961, S.K. Godunov [6] proposed the NIM with orthonormalization of the solution at intermediate points, which subsequently made a leap forward in solving boundary value problems of the mechanics of a deformable rigid body. To solve the problems of determining the stress-strain state of shells of revolution, the method was used for the first time in the works of Ya.M. Grigorenko and his students [7]; V.P. Myachenkov and A.N. Frolov proposed to apply this method for solving problems of stability and oscillations of shells of revolution [8], and V.P. Maltsev and V.I. Myachenkov - for solving problems of dynamics of shells of revolution [9]. Numerical methods for solving problems of the dynamics of shell structures with inelastic properties were considered in the research works of Ya.M. Grigorenko, V.I. Myachenkov, A.N. Frolov, V.P. V.P.Maltsev, T.Mavlanov, and others.

Moreover, the formulation and numerical implementation of this technique were developed and generalized within the framework of the mathematical theory of viscoelasticity [11], modern methods of averaging and freezing [12], variational principles of dynamics, and were based on a unified approach that used a discrete-continuum structure model and the orthogonal sweep method, generalized in the article to solve complex boundary value problems and to calculate the stiffness matrices of shell elements.

In [13], the main relations of the theory of thin shells are given.

In [16-22], natural oscillations of high-rise stacks were studied using a viscoelastic theory of the shell and taking into account the elastic and viscoelastic properties of their material. The reliability of the results obtained was verified by comparing them with the exact solution of a number of test problems, and by comparing the obtained results with the results of field experiments.

In [23–24], nonlinear vibrations of thin shells partially filled with liquid were studied. The main patterns were considered that determine the dynamic deformation of load-bearing shell structures with a large deflection and significant fluctuations of the free surface of liquid due to natural, forced and parametrically excited vibrations of the combined system, and due to impulse loads acting on the load-bearing object. The nonlinear dynamic interaction of shells with a liquid filler is analyzed taking into account the wave motions of the free surface of liquid.

Dynamic interaction of an orthotropic cylindrical shell with a fluid medium flowing inside it was considered in [25-28]. A method was proposed for calculating the characteristics of parametric vibrations of a shell at a liquid velocity close to critical. The amplitude-frequency response of the shell-liquid system was determined.

In works [29-33], theoretical and methodological foundations were developed for calculating the dynamic characteristics of complex shell structures. The dynamic characteristics of the structure are determined and an algorithm and a program for calculating the dynamic problems of inhomogeneous shell systems are developed.

Articles [34-35] present a detailed analysis of the stress-strain state of various dams under seasonal changes, taking into account hydrostatic water pressure, environmental vibration and seismic impact. A mathematical model is proposed to determine the dynamic behavior of earth dams, taking into account the viscoelastic properties of the soil, using the hereditary Boltzmann-Volterra theory with the Yadro Rzhanitsyn model under periodic kinematic effects.

In [36], a method was proposed for analyzing unsteady oscillations of cylindrical shells interacting with external and internal fluid flows under the action of an external constant longitudinal compressive force. The method is used for the numerical analysis of transient processes in the shell-liquid system for various values of the excitation parameters.

In [37] considers free vibrations of a thin-walled cylindrical shell containing a compressible fluid. For some values of the system parameters, its natural frequencies were determined and the influence of the geometric and physical parameters of the cylindrical shell–liquid system on the free oscillations of the cylinder was studied. The oscillation frequency of a shell without liquid is expressed in explicit form in terms of the oscillation frequency of the system; this makes it possible to study the frequency spectra of the system both analytically and graphically. Graphs of oscillation frequencies for different regimes of the system as a function of the frequency of the hollow shell are plotted.

In the paper [38] a numerical-analytical method for calculating the forced vibrations of a shell structure immersed in a liquid is proposed. The design is a set of finite elastic cylindrical shells and elastic rings, to which concentrated discrete forces are applied. Examples of comparative calculation of frequency response and vibration modes of a shell structure in vacuum and in liquid are given.

In works [39-40], the dynamic behavior of an elastic cylindrical shell filled with a motionless or flowing fluid is investigated. We also study the dynamic behavior of the "shell-liquid" system under various boundary conditions for the perturbed velocity potential. To describe the fluid, the perturbed velocity potential is used, the equations of which with the appropriate boundary conditions are solved by the Bubnov-Galerkin method. To describe the shell, a variational principle is used, including the linearized Bernoulli equation for calculating the hydrodynamic pressure. The calculation results are compared with known experimental, analytical and numerical data.

In the work of [41], the nonlinear response of a thin round cylindrical shell filled with water, freely supported at the edges to multiharmonic excitation, was studied. The sheath has suitable dimensions, so that the eigenfrequencies of the two modes (driver and vane) with three circumferential waves are almost twice as high as the eigenfrequencies of the two modes (driver and camber) with two circumferential waves. This introduces a one-to-one-to-two-to-two internal resonance in the presence of harmonic excitation in the spectral neighborhood of the eigenfrequency of the two circumferential wave mode, and a very complex non-linear dynamics around the fundamental mode resonance is obtained.

Nonlinear dynamics is studied using bifurcation diagrams of Poincaré maps and time characteristics.

In [42], presents a technique for analyzing the characteristics of free oscillations and the dynamic behavior of an axisymmetric cylindrical shell of finite length, which is acted upon by a moving internal pressure. The equations of motion are derived on the basis of the classical theory of shells using the Hamilton principle. A sensitivity analysis has been performed and the influence of various parameters on the results obtained is being studied. The results obtained are compared with known data, as well as with the results of the finite element method.

In [43], provides a review of studies and analysis of the results on the influence of discrete ribs on the dynamic characteristics of cylindrical shells. The influence of the Pasternak elastic foundation on the critical velocities of a structurally orthotropic model of a ribbed cylindrical shell is determined. Non-stationary problems are solved for perforated and ribbed shells of revolution filled with liquid or resting on an elastic foundation and subjected to moving or impulsive loads. The results of studies of the behavior of multilayer shell structures under the influence of impulse loads of various types are presented.

It should be noted that the class of problems where the dynamic behavior of complex, multiply connected structurally non-homogeneous shell systems interacting with a liquid is studied, presents great practical importance for modern engineering and technology.

## 2 Methods

Let each shell element of the structure under consideration be affected by loads $q_1^P\ q_2^P\ q_3^P$ distributed over the coordinate surface. We assume that external loads reduced to the midline of this element are applied to each annular element of the structure under consideration.

To obtain the equilibrium equations for the structure, we use the variational Lagrange equation [10]:

$$\sum_{p=1}^{Ns}\delta Э_p+\sum_{i=1}^{Nr}\delta Э_i+\sum_{l=1}^{Ne}\delta Э_e-\sum_{p=1}^{Ns}\delta A_p-\sum_{i=1}^{Nr}\delta A_i=0, \quad (1)$$

where

$\delta Э_p$ is the variation of the potential strain energy of the *p*-th shell element;

$\delta Э_i$ is the variation of the potential strain energy of the *i*-th annular element;

$\delta Э_e$ is the variation of the potential strain energy of the *e*-th viscoelastic constraint; $\delta A_P$ is the elementary work of external loads applied to the *p*–th shell element;

$\delta A_i$ is the elementary work of external loads applied to the *i*-th annular element.

Then, according to the methods given in [10], the problem posed is reduced to solving a system of an integro-differential-algebraic system of equations with complex coefficients:

$$L_p+q_p=0\quad (p=1,2,\dots,N_s), L_r^i+\|\Theta_i\|f_i=\sum_j\sum_s\xi_{ci}^{ijs}[\eta_i^{ijs}]Q_i^{ijs}+$$
$$\sum_j\sum_s\xi_{ci}^{ijs}[\eta_{ci}^{ijs}]N_{ci}^{ijs}=0,\quad (i=1,2,\dots,N_r) \quad (2)$$

As an example, consider a closed circular cylindrical shell with radius $R_0$, length *l,* and thickness *δ*. We introduce a cylindrical coordinate system 0xRβ. Let us assume that at the lower end of the shell (in section x=0), there is an absolutely rigid flat bottom, filled with an ideal incompressible fluid with density $\rho_0$. We will solve the problem in a linear

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formulation, ignoring initial forces in the middle surface of the shell, which arise due to the hydrostatic pressure of liquid.

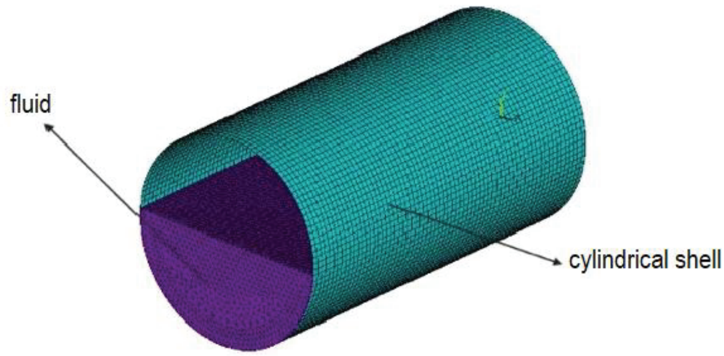


**Fig.1.** Calculation scheme of a cylindrical shell, partially filled with liquid.

From system (2), after some transformations of a mathematical nature, we obtain the following dynamic equations of a cylindrical shell [14, 15]:

$$L_{11}(u)+L_{12}(v)+L_{13}(w)=\frac{\rho(1-\mu^2)}{E}\frac{\partial^2 u}{\partial t^2}\ ;$$
$$L_{21}(u)+L_{22}(v)+L_{23}(w)=\frac{\rho(1-\mu^2)}{E}\frac{\partial^2 v}{\partial t^2}\ ;\quad (3)$$
$$L_{31}(u)+L_{32}(v)+L_{33}(w)=-\frac{\rho(1-\mu^2)}{E}\left(\frac{\partial^2 w}{\partial t^2}-\sigma_*(x)\frac{1}{\rho\delta}\Delta p\right)$$
$$N_1+(-1)^k c_u u=0\ ;\quad S+(-1)^k c_v v=0\ ;$$
$$Q_1^*+(-1)^k c_w w=0\ ;\quad M_1+(-1)^k c_\psi \frac{\partial w}{\partial x}=0\ ;\quad (4)$$

(x = 0,l; k = 1,2).

Here it is necessary to assume that k=1 for x=0 and k=2 for x=1.

Here:

$L_{ik}$ are known differential operators of the theory of shells (further, when obtaining specific results, the technical theory of shells [13] is used);

$N_1$, S, $Q_1^*$ и $M_1$ – are normal, shear, and generalized (in the sense of Kirchhoff) transverse forces and bending moment, respectively, acting in the shell section x=const and determined by formulas (9);

$c_u$, $c_v$, $c_\psi$, $c_w$ are the constant end edges of the shell in the circumferential direction, in directions u, v, w, and the angle of rotation $\psi=\partial w/\partial x$ of the normal to the middle surface (however, at x=0 and x=1, the coefficients can differ);

$\Delta p$ is the difference between the total pressure of liquid and the pressure above the free surface.

$$\sigma_*(x)=\begin{cases}1\ for\ 0\le x\le h;\\ 0\ for\ \ x>h;\end{cases}$$

To determine pressure Δp, we will use the linearized Lagrange-Cauchy integral:

$$\Delta p(x,R,\beta,t)=-\rho_0\left[\frac{\partial^2\Phi}{\partial t^2}+jx\right],\quad (5)$$

where

Φ - the potential of fluid particles displacement under its excited motion;

j - the acceleration of the field of body forces.

An arbitrary function of time x(t), entered potential Φ. The displacement potential Φ must be a solution to the following boundary value problem:

$$\frac{\partial^2\Phi}{\partial R^2}+\frac{1}{R}\frac{\partial\Phi}{\partial R}+\frac{\partial^2\Phi}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2\Phi}{\partial\beta^2}=0\ ; \qquad (6)$$

$$\frac{\partial\Phi}{\partial R}=w(x,\beta,t)\ for\ R=R_0\ \frac{\partial\Phi}{\partial x}=0\ for\ x=0 \qquad (7)$$

$$\frac{\partial^2\Phi}{\partial t^2}+j\frac{\partial\Phi}{\partial x}=0\ \ for\ \ x=h.$$

For natural oscillations with a frequency of ω, functions u, v, w and Φ, taking into account the conditions for their periodicity in β, can be represented in the following form:

$$u(x,\beta,t)=e^{iwt}\sum_{m=0}^{\infty}u_m(x)\cos m\beta$$

$$v(x,\beta,t)=e^{iwt}\sum_{m=0}^{\infty}v_m(x)\sin m\beta$$

$$w(x,\beta,t)=e^{iwt}\sum_{m=0}^{\infty}w_m(x)\cos m\beta \qquad (8)$$

$$\Phi(x,R,\beta,t)=e^{iwt}\sum_{m=0}^{\infty}\Phi_m(x)\cos m\beta$$

Let us introduce dimensionless variables and dimensionless parameters:

$$\alpha=\frac{x}{R_0};\qquad r=\frac{R}{R_0};\ \tau_1=\frac{h}{R_0};\quad \tau=\frac{l}{R_0};\quad \varepsilon=\frac{h}{l}=\frac{\tau_1}{\tau};$$

$$c^2=\frac{1}{12}\left(\frac{\delta}{R_0}\right)^2\ ;\ \lambda^2=\frac{\rho R_0^2(1-\mu^2)}{E}\omega^2\ ;\quad \eta=\frac{j\rho R_0(1-\mu^2)}{E}\ ;\ a=\frac{\rho_0}{\rho}\frac{R_0}{\delta}\ , \qquad (9)$$

and dimensionless stiffness coefficients $\beta_1$ related to the initial stiffness values $c_1$ by the following relationships:

$$c_u=\frac{E}{1-\mu^2}\frac{\delta}{R_0}\frac{\beta_u}{1-\beta_u}\ ;\quad c_v=\frac{E}{2(1+\mu)}\frac{\delta}{R_0}\frac{\beta_u}{1-\beta_v}\ ;$$

$$c_w=\frac{E}{12(1-\mu^2)}\left(\frac{\delta}{R_0}\right)^3\frac{\beta_w}{1-\beta_w}\ ;\quad c_\psi=\frac{E}{12(1-\mu^2)}\left(\frac{\delta}{R_0}\right)^3R_0^2\frac{\beta_\psi}{1-\beta_\psi} \qquad (10)$$

As follows from the above formula, the dimensionless stiffness coefficients $\beta_1$ can vary within $0\le\beta_1\le1$, which corresponds to the transition to absolutely rigid fixing in the considered direction.

After substituting (8) into (6), (7), taking into account (9), we find that functions $\Phi_m$ for $m\ge1$ must satisfy the following equations and boundary conditions:

$$\frac{\partial^2\Phi_m}{\partial\kappa^2}+\frac{1}{r}\frac{r\Phi_m}{\partial r}+\frac{\partial^2\Phi_m}{\partial x^2}-\frac{m^2}{r^2}\Phi_m=0; \qquad (11)$$

$$\frac{\partial\Phi_m}{\partial r}=R_0W_m(\alpha)\ for\ r=1;\ \frac{\partial\Phi_m}{\partial\alpha}=0\ \ for\ \ \alpha=0;$$

$$-\lambda^2\Phi_m+\eta\frac{\partial\Phi_m}{\partial\alpha}=0\ \ for\ \ \alpha=\tau_1 \qquad (12)$$

The boundary value problem (11), (12) can be directly solved by the method of separation of variables. As a result, we obtain

$$\Phi_m = R_0 \sum_{n=1}^{\infty} \frac{1}{\xi_n N_n^2} \frac{I_m(\xi_n r)}{I_m^1(\xi_n)} \cos \xi_n \alpha \int_0^{\tau_1} W_m(\zeta) \cos \xi_n \zeta d\zeta \,, \qquad (13)$$

where $I_m$ and $I_m^1$ are the Bessel functions of a purely imaginary argument and their derivatives;

$\xi_n$ are the roots of equation

$$\lambda^2 \cos \xi\tau_1 + \eta\xi \sin \xi\tau_1 = 0 \qquad (14)$$

and $N_n^2$ is the square of the norm, calculated by the following formula:

$$N_n^2 = \frac{1}{2}\tau_1 + \frac{1}{4\xi_n} \sin 2\xi_n\tau_1 \qquad (15)$$

Expression (13) for function $\Phi_m$, which is called below the displacement potential, differs from the known ones [44]; it directly considers the influence of wave motions and does not require additional determination of the generalized coordinates characterizing the fluctuations of the free surface of liquid.

The equations that must be satisfied by the dimensionless frequency λ and functions $U_m$ ,$V_m$ , $W_m$ for natural non-axisymmetric oscillations (m≥1) can be obtained based on system (1) and relations (3), (8) and (13). When calculating the liquid pressure on the shell according to formula (3), it is necessary to assume that $R = R_0$ (r=1), and, in accordance with the initial assumptions, only the hydrodynamic pressure should be left. Thus, we obtain:

$$\begin{gathered} L_{11}^m(U_m) + L_{12}^m(V_m) + L_{13}^m(W_m) + \lambda^2 U_m = 0 \\ L_{21}^m(U_m) + L_{22}^m(V_m) + L_{23}^m(W_m) + \lambda^2 V_m = 0 \\ L_{31}^m(U_m) + L_{32}^m(V_m) + L_{33}^m(W_m) + \lambda^2 W_m = \sigma_*(\alpha)\lambda^2 \int_0^{\tau_1} K(\alpha,\zeta) W_m(\zeta) d\zeta \,; \qquad (16) \\ (m = 1,2,3,\ldots), \end{gathered}$$

$$\begin{gathered} L_{11}^m = \frac{d^2}{d\alpha^2} - m^2 \frac{1-\mu}{2}; \quad L_{12}^m = -L_{21}^m = m \frac{1+\mu}{2} \frac{d}{d\alpha}; \; L_{13}^m = L_{31}^m = \mu \frac{d}{d\alpha}; \\ L_{22}^m = \frac{1-\mu}{2} \frac{d^2}{d\alpha^2} - m^2; \; L_{23}^m = -L_{32}^m = -m; L_{33}^m = c^2 \nabla m^2 \nabla m^2 + 1; \qquad (17) \\ \nabla m^2 = \frac{d^2}{d\alpha^2} - m^2; \; \mathrm{K}(\alpha,\zeta) = \sum_{n=1}^{\infty} \rho_n(\alpha)\sigma_n(\zeta); \; \rho_n(\alpha) = d_{mn} \cos \xi_n \alpha; \\ \sigma_n(\zeta) = \cos \xi_n \zeta \,; \; d_{mn} = \frac{1}{\xi_n N_n^2} \frac{I_m(\xi_n)}{I_m^1(\xi_n)} \qquad (18) \end{gathered}$$

It is easy to show that as $\gamma_1 \to 0$, the system of equations (16) describes oscillations of the shell without liquid. Boundary conditions (2) with relations (8) - (10) take the following form:

$$\begin{gathered} (1-\beta_u)N_{1m} + (-1)^k \beta_u U_m = 0 \,; \; (1-\beta_v)S_m + (-1)^k \beta_v V_m = 0; \\ (1-\beta_w)Q_{1m} + (-1)^k \beta_w W_m = 0 \,; \; (1-\beta_\psi)M_{1m} + (-1)^k \beta_\psi \psi_m = 0; \qquad (19) \\ (\alpha = 0, \tau, ; for\ \alpha = 0, \mathrm{к} = 2\ for\ \alpha = \tau; m = 1,2,3,\ldots), \end{gathered}$$

where

$$\begin{gathered} N_{1m} = \frac{dU_m}{d\alpha} + \mu(mV_m + W_m); \quad S_m = -mU_m + \frac{dV_m}{d\alpha}; \\ Q_{1m} = -\left[\frac{d^3 W_m}{d\alpha^3} - m^2(2-\mu)\frac{dW_m}{d\alpha}\right]; \qquad (20) \\ M_{1m} = \frac{d^2 W_m}{d\alpha^2} - \mu m^2 W_m, \; \psi_m = \frac{dW_m}{d\alpha} \end{gathered}$$

So, the determination of the frequencies and forms of natural non-axisymmetric oscillations of a cylindrical shell partially filled with liquid is reduced to solving a boundary value problem. Number $m \geq 1$ should be considered as a parameter. It characterizes the mode of natural vibrations of the "shell-liquid" system in the circumferential direction and is equal to half the number of nodal lines of the middle surface of the shell, parallel to the ***Ox***-axis or the number of nodal diameters of the free surface of liquid.

To solve the boundary value problem (16), (19), it is appropriate to consider the initial shell, which consists of two shells. Each shell has its own geometric and force factors $U_m^{(i)}, \dots, \psi_m^{(i)}, N_{1m}^{(i)}, \dots, M_{1m}^{(i)}$ (i=1, 2). As the first shell (i=1), we take the wetted part of the original shell, as the second - the remaining (unwetted) part. For each of these shells we introduce variables: $\alpha_1 = \alpha$, $\alpha_2 = \alpha - \gamma_1$,$(0 \leq \alpha_1 \leq \tau_1$ , $0 \leq \alpha_2 \leq \tau - \tau_1 = \tau_2)$.

In this formulation, the problem was reduced to constructing solutions to the equations of oscillations of a shell completely filled with liquid and a shell without liquid. In the first case, functions $U_m^{(1)}, V_m^{(1)}, W_m^{(1)}$ must satisfy system (16) for $\sigma_*(\alpha) = 1$, in the second case, functions $U_m^{(2)}, V_m^{(2)}, W_m^{(2)}$ are solutions to the same equations (16), but for $\sigma_*(\alpha) = 0$.

Boundary conditions (19) for functions $U_m^{(1)}, V_m^{(1)}, W_m^{(1)}$ must be satisfied for $\alpha_1 = 0$, and for functions $U_m^{(2)}, V_m^{(2)}, W_m^{(2)}$ - for $\alpha_2 = \gamma_2$. In addition, in section $\alpha = \gamma_1$, the following conditions for the continuity of the geometric and force factors for both shells must be met:

$$U_m^{(1)}(\gamma_1) = U_m^{(2)}(0)\,; \quad V_m^{(1)}(\gamma_1) = V_m^{(2)}(0)\,; W_m^{(1)}(\gamma_1) = W_m^{(2)}(0)\,;$$
$$\psi_m^{(1)}(\gamma_1) = \psi_m^{(2)}(0)\,; \quad N_{1m}^{(1)}(\gamma_1) = N_{1m}^{(2)}(0)\,; \quad S_m^{(1)}(\gamma_1) = S_m^{(2)}(0);$$
$$Q_{1m}^{(1)}(\gamma_1) = Q_{1m}^{(2)}(0)\,; \quad M_{1m}^{(1)}(\gamma_1) = M_{1m}^{(2)}(0) \qquad (21)$$

Based on the known results, functions $U_m^{(2)}, V_m^{(2)}, W_m^{(2)}$, corresponding to the unwetted part of the shell $(0 \leq \alpha_2 \leq \gamma_2)$ can be represented in the following form:

$$U_m^{(2)}(\alpha_2) = \sum_{j=1}^{\infty} C_{jm}^{(2)}\, \varphi_{jm}^{(u)}(\alpha_2)$$
$$V_m^{(2)}(\alpha_2) = \sum_{j=1}^{\infty} C_{jm}^{(2)} \varphi_{jm}^{(v)}(\alpha_2) \qquad (22)$$
$$W_m^{(2)}(\alpha_2) = \sum_{j=1}^{\infty} C_{jm}^{(2)} \varphi_{jm}^{(w)}(\alpha_2)$$

Here

$C_{jm}^{(2)}$ are the real integration constants;

$\varphi_{jm}^{(u)}, \varphi_{jm}^{(v)}, \varphi_{jm}^{(w)}$ are the real partial solutions to the equations of natural oscillations of a cylindrical shell without liquid (system (16) for $\sigma_*(\alpha) = 0$), which correspond to the roots of the characteristic equation; $v_{jm}$ can take the following forms:

$$v_{jm} = +x, \pm i\tau, \pm(\theta \pm i\zeta); \quad (j=1, 2,..\ 8)$$

## 3 Results and discussion

The following values are taken as specific parameters of the system:

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$L$ = 2.2 m, $R_0$ = 0.3 m, $h$ = 0.0015 m, $E$ = 6.8·1010 N/m$^2$, $\nu$ = 0.3, $\rho$=648 kg/m$^3$, $M_1$ = 481.264 kg, $M_2$ =326.334 kg, $l$=1.6 m, $d_0$ = 0.06 m.

Frequency characteristics are shown in Fig. 2

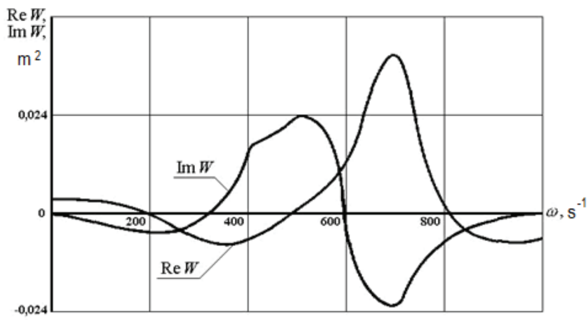


**Fig.2.** Frequency characteristics of the structure.

Dynamic characteristics of the structure are calculated for four levels of filling the shell with liquid:

$$H=L\,,\quad H=0{,}75L\,,\qquad H=0{,}5L\,,\qquad H=0{,}25L.$$

Fig. 3 show the dependences of the natural frequencies of the four lowest tones on the number of waves along the circle *m* for each of the levels.

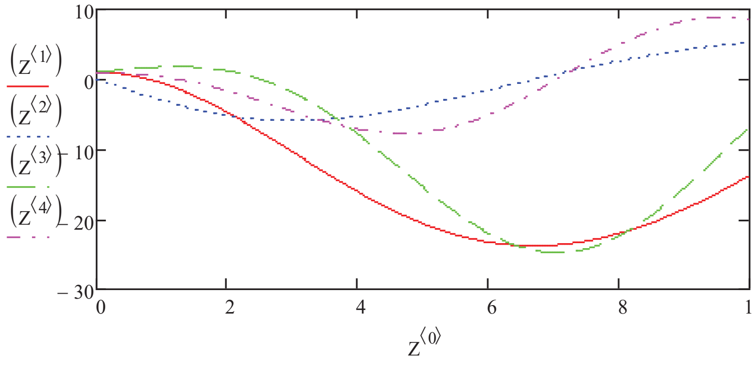


**Fig.3.** Dependence of the natural frequencies of the four lowest vibration tones for different levels of filling the shell with liquid

As seen from these dependencies, the lowest natural frequencies correspond to the modes of oscillation for m=4. The lower part of the spectrum is characterized by a high-frequency density, which contributes to the emergence of various resonant effects during vibrations due to the nonlinear behavior of the shell. In particular, among the eigenfrequencies of non-axisymmetric oscillations, there are such frequencies that their doubled values are close to the eigenfrequencies of longitudinal (longitudinal-radial) oscillations.

## 4 Conclusions

1. Based on the Lagrange variational principles and the laws of mechanics, equations of dynamics were obtained for a structurally non-homogeneous multiply connected shell system, taking into account the influence of liquid.
2. Relationships were derived and eigenfrequencies were investigated for cylindrical shells partially filled with liquid.
3. Dynamic characteristics of a structurally non-homogeneous multiply connected shell were determined for four levels of filling with fluid.
4. It was determined that the lowest natural frequencies correspond to the modes of oscillation for $m = 4$.
5. It was determined that an increase in the level of filling with water led to a decrease in the values of natural frequencies.

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