Evaluation of the dynamic characteristics of complex multiply connected, structurally inhomogeneous systems

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Abstract. In this work, its application's theoretical and methodological foundations are developed for calculating the dynamic characteristics of shell structures that function independently or are parts of a complex mechanical system as a substructure. An important stage in the study of the dynamic behavior of the complex multiply connected shell structure under consideration is the determination of the dynamic characteristics of the structure, which include natural frequencies and modes of vibration, amplitude-phase frequency characteristics, dynamic coefficients of influence, dynamic stiffness and the coefficient of dynamicity. A solution method, an algorithm, and a program for calculating dynamic problems of structurally inhomogeneous shell systems have been developed. The developed method makes it possible to determine the dynamic characteristics of structurally inhomogeneous shell systems, numerical results are obtained, and graphs of the change in vibration frequencies depending on the inhomogeneity parameter are plotted.

1 Introduction

Due to the complexity of solving problems of statics and dynamics of complex, multiconnected, structurally inhomogeneous shell structures with viscoelastic bonds, the most acceptable is the creation of numerical calculation methods. At the same time, shell elements can be isotropic, orthotropic, multilayer, and structurally heterogeneous.

It is known that there are quite a lot of published works devoted to the problem of calculating the dynamic characteristics of shells. The main part consists of works in which shells of a certain shape are studied, solutions are obtained, as a rule, using some variation method, and the shape of the cavity determines the choice of coordinate functions. In these works, approximate or exact formulas were obtained for several shells of a simple geometric shape. However, in practice, where the complexity of design solutions often makes it difficult to obtain analytical estimates and does not always allow the use of simple

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models, the most valuable are universal numerical methods that do not impose strict restrictions on the shape and parameters of the researched structures.

However, despite this, several issues remained unresolved for a long time. For example, there are unknown works in which vibrations of a complex shell structure would be studied. Only a few works deal with the oscillations of multiply connected shell systems. At the same time, in practice, for example, the shell structures of a very complex configuration are often found in products. Factors such as the variability of shell thickness, the presence of frames, and reinforcing ribs significantly complicate the development of simple models for analytical estimates and approximate calculations.

All this caused the need to develop a simple and universal algorithm that allows you quickly perform the calculation, taking into account as many design features as possible.

In this work, its application's theoretical and methodological foundations are developed for calculating the dynamic characteristics of shell structures that function independently or are parts of a complex mechanical system as a substructure.

An important stage in the study of the dynamic behavior of the complex multiply connected shell structure under consideration is the determination of the dynamic characteristics included in the structure, which are the natural frequencies and the modes of vibration, the amplitude-phase frequency characteristics, the dynamic coefficients of influence of dynamic stiffness and the coefficient of dynamicity. Since obtaining, an analytical solution is impossible, the construction of numerical and numerical-analytical algorithms to use for the dynamic calculation of complex mechanical systems using modern computers is relevant.

In article [1], the time domain has developed an efficient numerical algorithm for analyzing the dynamic response of orthotropic viscoelastic composite laminates. The integral form of the constitutive laws is exploited. The Generalized Wiechert model is adopted to simulate the viscoelasticity of the structure Mindlin-Reissner plate theory is utilized in finite element formulation employing the consistent mass matrix. The developed recurrence formula permits the new time solutions to be evaluated using only previous time values. The developed solution technique is applied to the orthotropic plate under two types of force: the step-pulse and sin-pulse force.

This paper [2] presents a finite element formulation of a reduction method for dynamic buckling analysis of imperfection-sensitive shell structures. The reduction method uses a perturbation approach, initially developed for static buckling and later extended to dynamic buckling analysis. Results of the reduction method are compared with results available in the literature. The results are also compared with full model finite element explicit dynamic analysis, and a reasonable agreement is obtained.

Articles [3-4] investigate the oscillations and stability of an ideal round cylindrical shell subjected to axial harmonic excitation in the vicinity of the lowest natural frequencies. Donnell's theory of shallow shells is used, and the spatial discretization of the shell is obtained by the Ritz method. An efficient low-dimensional model presented in previous publications is used to discretize a continuous system. This paper's main goal is to discuss a conservative system's nonlinear behavior. Then the behavior of forced vibrations of a harmonically excited shell is analyzed. The results show that analyzing the evolution of secure links and obtaining the appropriate indicators of their stability is an important step in developing procedures for the secure design of multiply connected systems.

In articles [5-6], a new algorithm is developed for enforcing constraints within the framework of a nonlinear, flexible multibody system modeled with the finite element approach. The proposed algorithm exactly satisfies the constraints at the displacement and velocity levels. Furthermore, it achieves nonlinear unconditional stability by imposing the vanishing of the work done by the constraint forces when combined with specific discretizations of the inertial and elastic forces.

The article [7] presents a vibrational analysis of a freely supported rotating multilayer reinforced cylindrical shell. The stiffeners include rings and stringers. The equations are obtained by the Rayleigh-Ritz method and the Sander relations. The results are compared to those available from other sources to validate this method. The parameters of the shell and stiffeners are optimized by the genetic algorithm method under weight and frequency constraints. Stiffener shape, material properties, and shell dimensions have been optimized.

The article [8] analyzes free vibrations of rotating functionally main cylindrical shells with rectangular ribs. Based on the theory of the first approximation lava and the theory of smoothed hard means, the basic equations of motion are obtained, which take into account the influence of the initial hinge tension and the centrifugal and Coriolis forces. To confirm this analysis, comparisons are made with known results for specific cases.

This paper [9] presents free vibrations of thick rotating reinforced composite cylindrical shells with different boundary conditions. The analysis is based on the three-dimensional theory using the layer-by-layer differential quadrature method. The equations of motion are derived using Hamilton's principle. This study demonstrates the applicability, accuracy, stability, and high convergence rate of the present method's analysis of free vibrations of rotating reinforced cylindrical shells. The presented results are compared with the results of other shell theories obtained by traditional methods. Some new results are presented that can be used as reference solutions for future research.

The article [10] investigates the characteristics of free oscillations of a prestressed connected spherical, cylindrical shell with free-free boundary conditions. The Flügge shell theory and the Rayleigh-Ritz energy method are used to analyze the characteristics of free vibrations of a coupled shell. In the modal test, the LMS software calculates the combined shell structure's mode shapes and natural frequencies. Natural frequencies and mode shapes are calculated numerically and compared with those of the FEM and modal test to confirm the robustness of the analytical solution. The effect of shallow water and the length of a cylindrical shell on the behavior of free vibrations of the connected shell structure, as well as the effect of internal pressure on modal characteristics, is studied.

The article [11] presents a general formulation for studying modal characteristics and the vibrational response of a cylindrical shell with a floor partition. The model is based on a variational formulation in which the structural connection is modeled using systems of artificial springs. To show the accuracy of the approach, the numerical results are carried out with particular attention to the characteristics of the hull-to-floor connection. Eigenfrequencies and modes of vibrations are determined. The results are compared with previous publications, as well as with the finite element method. The influence of individual artificial springs and floor movement in a plane on the modal characteristics is discussed. A forced reaction analysis illustrates the physical phenomena due to the connection between the hull and bottom. The results show the method's effectiveness when working with a wide range of structural bonds.

The article [12] analyzes free and forced vibrations of stepped cylindrical shells with several intermediate flexible supports using vector signals with expansion in the Fourier series. Flexible support includes springs with arbitrary properties in any possible direction. Based on the Flügge theory of thin shells, the reflection, propagation, and transmission matrices for a round cylindrical shell are determined. Continuous vector-matrix dependencies are established for the analysis of free and forced vibrations, considering the thickness of the shell, shell pitches, and intermediate supports lengths. The results of this study are compared with the results of famous scientists obtained using the finite element method. For example, a cylindrical shell with three flexible intermediate supports and three geometric steps is considered. The natural frequency and vibration modes of the complex shell are derived.

In the article [13], the free oscillations of cylindrical shells with circumferential stiffeners, rings with a non-uniform eccentricity of the stiffeners and unequal spacing between the stiffeners, are investigated. The study uses analytical, experimental methods, and the finite element method. The Ritz method is applied in the analytical solution, and the stiffeners are considered discrete elements. In the experimental method, modal testing obtains modal parameters, including natural frequencies, waveforms, and damping. The ANSYS software uses two types of modeling, including shell elements. Analytical and numerical results are compared with experimental ones for the reliability of the developed algorithm and methodology for solving the problem.

Articles [14-16] are devoted to studying the dynamics of structurally inhomogeneous, multiconnected shell structures, considering the influence of liquid and viscoelastic elements. A mathematical model of the structure has been developed based on the laws of mechanics and the Lagrange principle. The three-parameter Rzhanitsyn-Koltunov kernel was used as the relaxation kernel.

In the articles [17-19], the spatial natural vibrations of high-rise smoke stacks of various thermal power plants and the ventilation stack of nuclear power plants according to the theory of shells in the elastic statement and viscoelastic statement using the developed methods and PC-IBM calculation programs.

In calculations, a high-rise smokestack is modeled by an elastic axisymmetric shell of variable thickness with separate variable slopes of both internal and external surfaces that describe the real geometry of the structures.

The reliability of the developed methods and algorithms was verified by solving several test problems and comparing the results obtained by known solutions and the results of field experiments. It was revealed that not only are the frequencies of bending modes of vibration in the dangerous range of earthquake frequencies, but also some other modes of spatial vibrations of structures, determined by the theory of shells. It was found that the value of the logarithmic decrement of structure vibrations when accounting for viscoelastic properties of the structure material weakly depends on eigenfrequencies of vibrations.

Very little applied research has been conducted in this area, and some studies have been conducted on the study of oscillatory processes occurring in multilayer shell structures.

Therefore, the development of algorithms and software products for calculating complex, structurally inhomogeneous shell structures that allow optimizing physical and mechanical characteristics is an urgent task of modern mechanics. Moreover, taking into account dissipative properties allows for a decrease in amplitude characteristics. Ultimately, it will be possible to control strength properties to create more stable, reliable structures.

An important stage in studying the dynamic behavior of the complex multiconnected shell structure under consideration is the determination of the structure's dynamic characteristics, which include natural frequencies and waveforms, amplitude-phase frequency characteristics, dynamic stiffness coefficients, and dynamicity coefficients.

Since it is impossible to obtain an analytical solution, the construction of numerical and numerical-analytical algorithms for the purpose of using complex mechanical systems for dynamic calculation using modern computers is an urgent task.

2 Methods

The adequacy of the original mathematical model (system of equations, variation principle) to the physical phenomenon under study is of great importance to obtain the correct result when using the numerical method. The possibility of neglecting the influence of individual factors in the mathematical model to simplify the solution procedure should be justified and the corresponding transformations should be performed correctly, without leading to contradictions with physical laws. In the mathematical theory of viscoelasticity apparatus, variation principles of dynamics are involved in formulating the problem [20-25].

The most general linear theory, which most fully reflects almost all the features of the quasi-static and the dynamic behavior of viscoelastic materials, is the Boltzmann-Volterr's theory, according to which the relationship between stresses and strains has the form:

$$\varsigma(t) = E[\varepsilon(t) - \int_0^t R(t-\tau)\varepsilon(\tau)d\tau] \qquad (1)$$

where ς is voltage, ε is deformation, t is observation time, $0 \le \tau \le t$ is intermediate moment. Instantaneous Yung's modulus, R is memory function or relaxation core. The type of the core R largely separates both the behavior of the material model and the possibility of using certain methods for solving problems for materials modeled using correlation (1). Suppose the function R has the form of an exponent or a sum of exponents. In that case, correlations (1) are reduced to differential correlations, the order of which is equal to the number of exponents in the indicated sum. Weakly singular heredities describe the behavior of viscoelastic materials most adequately. To solve the problems posed in this work, the core of M.A. Koltunov, Rzhanitsyna A.P. was used [26-28]:

$$\tau(t-\tau) = \frac{Ae^{-\beta(t-\tau)}}{(t-\tau)^{\alpha-1}} \qquad (2)$$

which, on the one hand, very satisfactorily reflects both the quasi-static [29] and dynamic behavior of materials; on the other hand, it is most convenient when carrying out quasi-static and, on the other hand, it is most convenient when carrying out quasi-static and dynamic calculations and determining mechanical vibrations.

In this work, the physical relations for the elements of structurally inhomogeneous shell structures, given in [30-32], are used as geometric relations.

Let the shell element be composed of a certain number of orthotropic viscoelastic layers with significantly different rheological properties. The main directions of elasticity at each layer's point coincide with the directions of the coordinate lines α1, α2, and point Z, i.e., at each layer's point, one of the planes of elastic symmetry is parallel to the coordinate surface of the shelled element. The other two are perpendicular to the lines $\alpha_i = const\ (i=1,2)$ [32]. It is believed that a structurally inhomogeneous shell element experiences small deformations, and the material of each of these elements has its own significantly different rheological properties.

The physical properties of the material of each layer, following the above, are described by linear hereditary of Boltzmann-Volterr's correlations with integral difference cores, subject to the closed cycle condition [20-22]. Note that for essentially inhomogeneous shell elements, when the heredity functions (1) of each viscoelastic layer are different, the assumption of the linearity of the process is justified and ultimately leads to insignificant calculation errors [31]. In this case, the integral terms in the hereditary correlations of the form (1) are usually small in comparison with the instantaneous elastic terms, which, together with the assumption of the oscillatory nature of the motion, makes it possible to apply the well-known [29] procedure of "freezing" the integral terms, which leads to the following complex physical relations [22].

Assuming that the normal stresses σ_{zz} in areas parallel to the coordinate surface of a thin shell element can be neglected in comparison with the stresses σ_{11} and σ_{22} in areas normal to this surface, the physical ratios of the material of the j^{th} layer can be described in the direction by complex values of the modulus of elasticity [32]:

$$\tilde{E}_1^j = \tilde{E}_{1R}^j + i\tilde{E}_{1z}^j \tilde{E}_2^j = \tilde{E}_2^j + i\tilde{E}_{2z}^j, \qquad (3)$$

as well as the corresponding complex values of Poisson's coefficient:

$$\nu_1^j = \nu_{1R}^j + i\tilde{\nu}_{1I}^j \nu_2^j = \nu_{2R}^j + i\tilde{\nu}_{2I}^j, \tag{4}$$

According to [32], the relationship between stresses and strains for a shell element made of an orthotropic viscoelastic material can be represented as

$$\sigma_{11}(t) = \frac{E_1}{1-\nu_1\nu_2}\left\{\varepsilon_{11}\left[1-\int_0^\infty R_{11}(\tau)e^{i\omega r\tau}d\tau + \nu_2\varepsilon_{22}\left[1-\int_0^\infty R_{21}(\tau)e^{i\omega r\tau}{}_{d\tau}\right]\right]\right\},$$

$$\sigma_{22}(t) = \frac{E_1}{1-\nu_1\nu_2}\left\{\varepsilon_{22}\left[1-\int_0^\infty R_{22}(\tau)e^{i\omega r\tau}d\tau + \nu_1\varepsilon_{11}\left[1-\int_0^\infty R_{21}(\tau)e^{i\omega r\tau}{}_{d\tau}\right]\right]\right\},$$

$$\sigma_{12} = G\varepsilon_{22}\left[1-\int_0^\infty R_G(\tau)^{i\omega r\tau} \quad d\tau\right]\varepsilon_{12}, \tag{5}$$

Introducing the notation:

$$\widetilde{E_1} = E_1(1-\Gamma_{11c}+i\Gamma_{11s})(1 \leftrightarrows 2), \tag{6}$$

$$\widetilde{V_2} = \widetilde{V_2}\frac{1-\Gamma_{12c}+i\Gamma_{12c}}{1-\Gamma_{11c}+i\Gamma_{11c}}, (1 \leftrightarrows 2)\tilde{G} = G_1(1-\Gamma_{GC}+i\Gamma_{GC}) \tag{7}$$

We get:

$$\sigma_{11} = \frac{\widetilde{E_1}}{1-V_1V_2}(\varepsilon_{11}+\widetilde{\nu_2}\varepsilon_{22});\ \sigma_{22} = \frac{\widetilde{E_2}}{1-V_1V_2}(\varepsilon_{22}+\widetilde{\nu_1}\varepsilon_{11});\ \sigma_{12} = \tilde{G}\varepsilon_{11}. \tag{8}$$

The quantities Γ_{11c}, Γ_{11s}, in relations (6-7) are the cosine and sine of Fourier images of the core R(t):

$$\Gamma_{11c} = \int_D^\infty R_{11}(\tau)cos\omega_R\,\tau d\tau,\ \ \Gamma_{11c} = \int_D^\infty R_{11}(\tau)sin\omega_R\,\tau d\tau \tag{9}$$

$$\Gamma_{GC} = \int_D^\infty R_G(\tau)cos\omega_R\,\tau d\tau,\ \ \Gamma_{GC} = \int_D^\infty R_G(\tau)sin\omega_R\,\tau d\tau \tag{10}$$

Without imposing any restrictions on the type of cores R(t) yet, subjecting to the condition of a closed cycle, we introduce the forces and moments acting on the sites $\alpha_i =$ *const*: normal T_{11}, T_{22} and shearing T_{12}, T_{21} forces, bending M_{11}, M_{22} moments. Next, the transition to complex arithmetic is carried out. Knowing the geometric and physical correlations, we can derive the basic dynamics equations. In this work, the derivation of equations is based on [32], the approach for both an elastic structure and a viscoelastic system. As applied to the case of small oscillations, the contact correlations are linearized at approximately stationary values of the variable parameters.

The equations of natural oscillations of structures will have the form:

$$L_p + \widetilde{\omega}^2[\overline{\rho_p}]U_p = 0 \ \ (p = 1, \dots, N_s) \tag{11}$$

$$L_r^i + \widetilde{\omega}^2[G_\omega]\Delta_i += \sum_j\sum_s \xi_{ci}^{ijs}\left[\overline{\eta_i^{ijs}}\right]Q_i^{ijs} + \sum_j\sum_s \xi_{ci}^{ijs}\left[\overline{\eta_{ci}^{ijs}}\right]N_{ci}^{ijs} = 0. \tag{12}$$
$$(i = 1, \dots N_r)$$

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Values $\tilde{\omega}^*$, at which the non-trivial solution is systemic complex coefficients, are the complex values of the natural vibration frequencies of the considered structurally - inhomogeneous shell structures. In more detail, let us dwell on the mathematical meaning of (11-12). In form, equation (11) is a contact equation of motion of multilayer elastic shells of the equation and prismatic shells of the non-circular section. Each of the equations describes the behavior of an individual shell element of a wall shell structure. In our case, the difference between the elemental equations is fundamental and lies in the fact that the solutions of the equations are complex due to the complexity of the relations, describing the structural heterogeneity. The complete ensemble of equations with complex coefficients (11), (12) describes the motion of a multiply connected structurally - inhomogeneous shell structure, composed in the general case from a set of multilayer elastic and viscous-elastic shells, hereditary bonds, frames with significantly different rheological properties, taking into account the joint work of all structural elements. No restrictions are imposed on this ensemble of equations, except for the subordination of the closed-loop condition for viscous-elastic elements and structural connections.

Further, in particular, the problem of vibrations of viscoelastic systems is reduced to a system of integro-differential equations (differential in coordinates and integro-differential in time). The system of equations contains partial derivatives in the case of systems with distributed parameters and ordinary derivatives in the case of systems with a finite number of degrees of freedom. Statements of problems of the theory of viscoelasticity as quasi-static are presented with exhaustive completeness in the monographs [20-22].

The dynamic problems with an infinite number of degrees of freedom with the help of an approximate method (Bubnov-Galerkin, Ritz, of finite elements) can be reduced to a system of a finite number of differential or integro-differential equations in time. From the point of view of mechanics, this means replacing a system with an infinite number of degrees of freedom with a system with lumped parameters or, at the same time, imposing an infinite set of additional constraints on the original system. In this way, most problems of the nonlinear and linear theory of viscoelasticity have been solved.

The main difficulty along this path is the choice of the basis coordinate functions in which the desired solution is expanded. These functions are quite simple in the case of bodies of a simple shape (beams, plates, cylindrical shells, hollow cylinders of infinite length). For such objects, the vast majority of solutions known to us for dynamic viscoelasticity problems have been obtained. In the case of a body of a more or less complex shape, the choice of the system of basis coordinate functions of the projection method, which reduces the original system to a system with a finite number of degrees of freedom, is a difficult problem.

3 Results and Discussion

Based on the principle of modularity, an algorithm and a software package for determining natural oscillations have been developed. Following [31], we will compose the calculation system of a shell structure (Fig. 1), consisting of the shell (1-8), nodal (1-7), and bonds (4).

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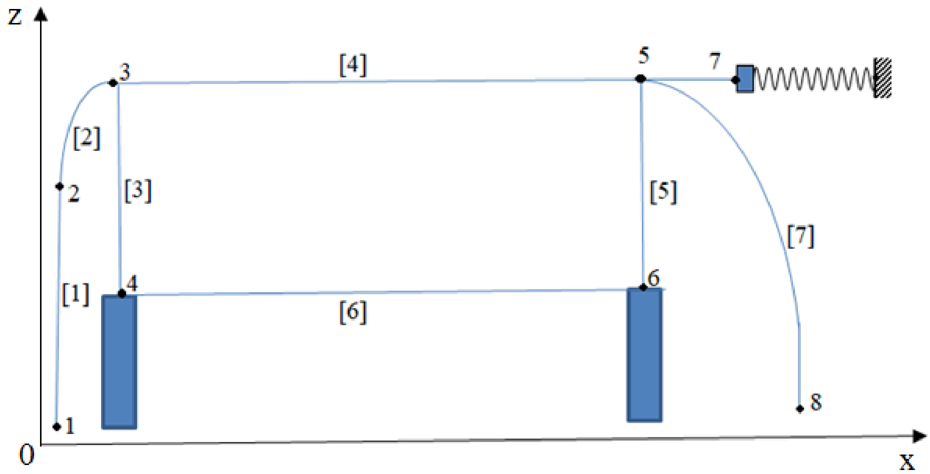

Fig.1 Calculation scheme of the shell structure

Here, Ox is axis of rotation, z is shell element coordinates.

Consider an example. The problem for the elastic case was solved in [31]. Structurally inhomogeneous axisymmetric design - a two-cavity high-pressure tank consisting of 6 shells and 6 nodal elements. Knots numbered No. 3, 5, 6 are circular frames, the cross sections of which have rectangular shapes with a size of 0.04m x 0.06m, the rest of the geometric dimensions are shown in fig.1. Node No. 6 of the structure is pinched, the shell elements No. 1, 4, 6, as well as frames are elastic ($E = 2.10^{11}$ N/m^2; ν =0.3; $\rho=7.8\cdot10^2$ $^{kg}/_{sm^3}$. The shell elements forming the inner cavity (No. 2, 3, 5) are viscous-elastic ($\rho=7.8\cdot10^2$ $^{kg}/_{sm^3}$; ν = 0.3).

Their instantaneous modulus of elasticity - 1 was determined as a parameter of structural inhomogeneity and varied from $E = 2.10^8$ N/m^2 to $E = 2.10^{11}$ N/m^2. The parameters of the core of relaxation of the material of these shell elements have the values A=0.01; α=0.1; β=0.05. The thickness of all shell elements is constant and equal to 0.01m. The work investigated the behavior of the defining damping coefficient of this design depending on the instantaneous modulus of elasticity of viscoelastic elements.

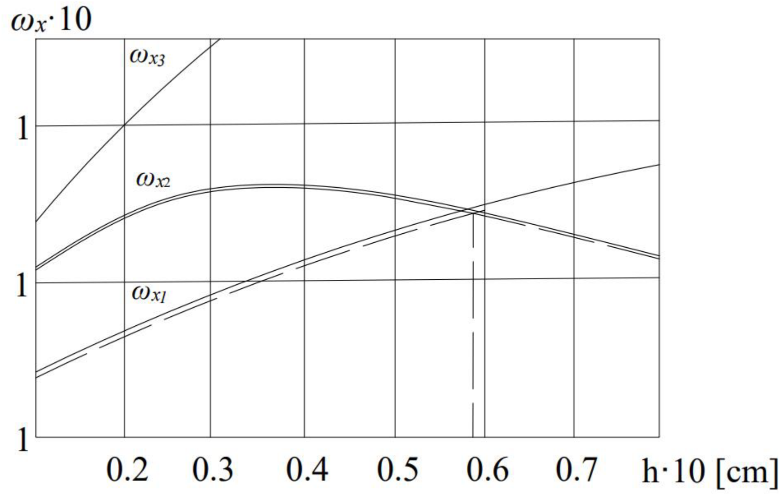

Fig.2. Plot of damping coefficients versus natural modes.

Fig. 2 shows the calculated dependences of the damping coefficients ω_{I1}, ω_{I2}, ω_{I3} of 3rd lower natural vibration modes determined by the operating conditions of the structure on the parameter of structural inhomogeneity - E. As well as in the previous case for a multiconnected shell structure, a synergism of dissipative properties was revealed.

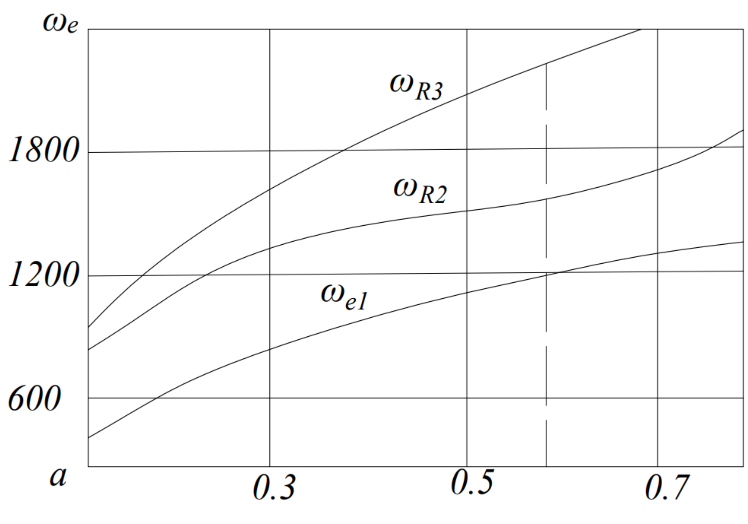

Fig.3. Plot of natural frequencies versus respective damping factors.

Figure 3 shows the calculated dependences of natural frequencies, two of which ω_{R1} and $\omega_{R2,}$ approximately the intersection point of the corresponding damping coefficients, also converge somewhat. The given calculation and analysis of the dynamics of this structure makes it possible already at the stage of preliminary design to provide the necessary characteristics of the dissipative properties of the latter required by the technical conditions and normal and the ability to dampen oscillations of the frequencies ω_{R1} and ω_{R2} specified by the operating conditions. In this case, the construction provides the greatest energy dissipation with the instantaneous modulus of elasticity of viscoelastic elements E.

4 Conclusions

1. Based on the mathematical theory of visco-elasticity, variational principles of dynamics, and modern computational and asymptotic methods, the problem of calculating the dynamic characteristics (including VAT) of multiconnected structurally inhomogeneous axisymmetric shell structures is reduced to an effectively solvable mathematical problem for complex eigenvalues.
2. With the help of the developed complex, numerical modeling and calculation of the dynamics of several shell structures with viscoelastic elements were carried out.
3. The developed engineering methods and calculation algorithms are generalized to studies of the dynamics of multiconnected structurally inhomogeneous shell structures interacting with various environments.
4. Numerical results are obtained, based on which graphs of changes in the frequencies of natural oscillations depending on the inhomogeneity parameter are constructed.
5. It is established that the nature of the frequency change is increasing.
6. Numerical experimental studies have shown satisfactory accuracy and convergence of the developed methods in solving problems of dynamics of structurally inhomogeneous shell structures.

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