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Spatial natural vibrations of viscoelastic axisymmetric structures

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Abstract. A mathematical model, methods and algorithm to assess the spatial natural vibrations of axisymmetric structures are given in the paper taking into account the variability of the slope and structure thickness in the framework of the viscoelastic theory of shells. Dissipative properties of the material are taken into account by the Boltzmann-Volterra hereditary theory of viscoelasticity. The spatial natural vibrations of high-rise ventilation pipes of the Armenian nuclear power plant (NPP) and smokestacks of the Novo-Angren, Syrdarya, Azerbaijan and Ekibastuz thermal power plants (TPP) were studied taking into account the elastic and viscoelastic properties of their material. The reliability of results was verified by comparing the results obtained with the exact solution of a number of test problems, as well as by comparing the results with the results of field experiments. It was found that when the viscoelastic properties of structure material are taken into account, the decrement of their vibrations weakly depends on the values of eigenfrequency. Along with this, the dangerous range of earthquake frequencies includes not only the bending frequencies of the structure, but the spatial ones as well.

1. *Introduction*

As a rule, high-rise smoke and ventilation pipes, cooling towers of thermal and nuclear power plants (TPP and NPP) and protective shells of NPP are considered as axisymmetric structures. They are unique structures in their design features and geometric dimensions. Today, a great number of different axisymmetric high-rise structures are being operated and erected all over the world; one of such structures is a high-rise smoke stack, the height of which reaches up to 150 m–600 m [1, 2].

If for pipes of a height of about 50 m, the ratio of the wall thickness δ to the radius R of its middle surface at the base is δ/R=1/5÷1/7, then for the pipes of a height of 250–300 m – δ/R=1/12÷1/15, and for the pipes of a height of 420 m, the ratio δ/R is - $\delta/R\approx$1/23. With increasing height H and radius R, the wall thickness δ of the pipe grows slowly [1, 2]. Besides, along the height of the pipe, its radius, cone thickness and slope change, gradually moving from a conical section to a cylindrical one.

In the existing building norms of many countries, an elastic cone-shaped console with a constant slope is used as a calculation model for such structures; it does not take into account such features as real geometry, structural features and dissipative properties of their material, which have a direct impact on the value of dynamic characteristics of structure.

The reliability of such structures is largely determined by the accuracy of dynamic calculations, which in turn depends on the correct choice of structure design model and exact determination of its dynamic characteristics.

In the dynamics of structures, the study of dynamic characteristics (i.e., eigenfrequencies, modes and decrements of vibration) of structures occupies a special place, since the dynamic characteristics are a passport of the structure and make it possible to judge the dynamic properties of the structure as a whole, without even examining its behavior under various effects.

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Determination of eigenfrequencies and vibration modes even for elastic structures is a self-contained and rather difficult task. When accounting for dissipative properties of the material, the determination of structure dynamic characteristics is complicated by an order of magnitude.

The first attempts at a theoretical description of dissipative properties of materials are associated with the names of Voigt, Maxwell, and Kelvin. Further, new models were proposed, and known models were improved to describe dissipative processes occurring in various systems during dynamic processes. However, the results obtained did not always agree well with experimental data.

To eliminate this, other, more adequate models were used that take into account dissipative processes in the material, such as the model with hysteresis absorption or hereditary viscoelastic models of the Boltzmann-Volterra type, although their implementation is rather difficult and experimental materials are scarce [3–10].

Recently, when determining the natural frequencies and vibration modes of various high-rise structures, much attention has been paid to accounting for elastic properties of structure material only.

For instance:

– an experimental determination of dynamic characteristics of high-rise monolithic reinforced concrete buildings was considered in [11] and the results obtained were recommended for certification of buildings;

– the energy method for estimating the cylindrical shells vibration was given in [12], where the effect of uniform external pressure and symmetrical boundary conditions on eigenfrequency of homogeneous and multilayer isotropic cylindrical shells was studied;

– the oscillatory process of rigid composite cylindrical shells taking into account the bending behavior of stiffness ribs and their effect on eigenfrequencies of a shell, a change in its thickness and boundary conditions were studied in [13];

– in [14] the eigenfrequencies of a cylindrical shell were studied at different boundary conditions;

– change in dynamic characteristics of the structure in order to detect damage to reinforced concrete building structures was considered in [15];

– to effectively evaluate the eigenfrequency and attenuation coefficient, a reliable mathematical model was proposed in [16] based on the use of the probability distribution function of eigenfrequency.

– along with this, the stress-strain state, dynamic behavior, and wave phenomenon in various systems were studied in [17–26], taking into account the design features.

These are just some of the studies devoted to the determination of dynamic characteristics of various designs and systems.

The above review of well-known studies shows that the dynamic characteristics of spatial axisymmetric structures, such as ventilation and smokestacks of nuclear and heat power plants, are evaluated differently in different works, and each theory or method used has its advantages and disadvantages.

Therefore, the development of an adequate model, effective methods and algorithm for assessing the dynamic characteristics of high-rise axisymmetric structures, taking into account their design features and dissipative properties of their material, is an urgent task of the mechanics of a deformable rigid body.

The aim of this study is to develop an adequate mathematical model, methods and algorithm for solving the problem of spatial natural vibrations of viscoelastic axisymmetric structures using the theory of shells and studying the dynamic characteristics (i.e., frequency and decrement of vibrations) of real structures, as well as comparing the results with the results of field experiments.

2. Methods

Consider a high axisymmetric structure (Fig. 1), modeled as an axisymmetric viscoelastic shell with a rectilinear axis, with a variable slope and a variable wall thickness, the lower part of the structure *(z*=0) is on a rigid base, and the upper ($z=H$) is free. The spatial natural vibrations of the structure under consideration are investigated (Fig. 1).

To determine the dynamic characteristics of the structures (Fig. 1), it is necessary to study spatial natural vibrations, i.e. the structure motion in which all of its points oscillate according to the same harmonic law - real or complex one - with different amplitudes in the absence of external influences, i.e.

$$\vec{u}(\vec{x},t)=\vec{u}^{*}(\vec{x})e^{-i\omega t} \tag{1}$$

Here $\vec{u}^{*}(\vec{x})=\{u_z(\vec{x}),u_\theta(\vec{x}),u_r(\vec{x})\}$is the displacement vector of the structure point (Fig. 1) in the directions of the coordinate axes $\vec{x}=\{z,\theta,r\}$, respectively.

In the case of conservative systems ω, $\vec{u}^*(\vec{x})$ are the frequency and natural vibration of the structures. In the case of non-conservative systems ω, $\vec{u}^*(\vec{x})$ are the complex quantities, i.e. $\omega = \omega_R - i\omega_I$, the real part ω_R of the complex parameter ω in its physical essence is the frequency of free damped vibrations of the structure, and the imaginary part ω_I accurate to the sign is equal to the damping coefficient of vibrations and determines the dissipative properties of the structure as a whole.

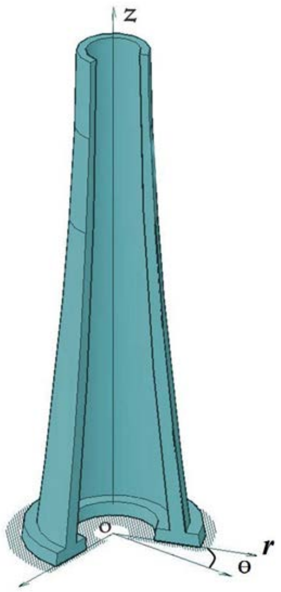

Figure 1. Model of axisymmetric structure.

To simulate the process of structure strain (Fig. 1) under its natural vibrations, the principle of virtual displacements is used according to which the sum of all active forces acting on the structure, including the inertia forces on virtual displacements, is zero, i.e.:

$$-\int_F \left(\tilde{M}_s\delta\gamma_s + \tilde{M}_\theta\delta\gamma_\theta + \tilde{M}_{s\theta}\delta\gamma_{s\theta} + \tilde{N}_s\delta\varepsilon_s + \tilde{N}_\theta\delta\varepsilon_\theta + \tilde{N}_{s\theta}\delta\varepsilon_{s\theta}\right)dF - \int_F \rho\left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right)dF = 0 \tag{2}$$

In this case, kinematic boundary conditions are used

$$z = 0:\ \vec{u} = 0;\ \frac{\partial w}{\partial z} = 0 \tag{3}$$

Here:

$\vec{u}(\vec{x},t) = \{u(\vec{x},t), v(\vec{x},t), w(\vec{x},t)\}$; $\vec{x} = \{z,\theta,r\}$ are the coordinates of the point in cylindrical coordinates;

u, v, w are displacements in the directions of axes (z,θ,r) of cylindrical coordinates, respectively; $\tilde{M}_s, \tilde{M}_\theta, \tilde{M}_{s\theta}, \tilde{N}_s, \tilde{N}_\theta, \tilde{N}_{s\theta}$ are bending, torsional and membrane forces; $\delta\gamma_s, \delta\gamma_\theta, \delta\gamma_{s\theta}, \delta\varepsilon_s, \delta\varepsilon_\theta, \delta\varepsilon_{s\theta}$ are isochronous variations in the curvature and components of the strain tensor; $\delta u, \delta v, \delta w$ - variations of displacements in longitudinal, circumferential and tangent directions; ρ is the density of the shell material.

The relationship between the components of the forces and strains at an arbitrary point of the structure (Fig. 1) is taken in the form:

$$\begin{gathered} \tilde{N}_s = \tilde{\lambda}(\varepsilon_s + \nu\varepsilon_\theta);\ \tilde{M}_s = \tilde{\mu}(\gamma_s + \nu\gamma_\theta) \\ \tilde{N}_\theta = \tilde{\lambda}(\varepsilon_\theta + \nu\varepsilon_s);\ \tilde{M}_\theta = \tilde{\mu}(\gamma_\theta + \nu\gamma_s)\ ; \\ \tilde{N}_{s\theta} = \tilde{K}\varepsilon_{s\theta};\ \tilde{M}_{s\theta} = \tilde{G}\gamma_{s\theta} \end{gathered} \tag{4}$$

where

$$\tilde{\lambda} = \frac{\tilde{E}h}{1-\nu^2};\ \tilde{\mu} = \frac{\tilde{E}h^3}{12(1-\nu^2)};\ \tilde{K} = \frac{\tilde{E}h}{2(1+\nu)};\ \tilde{G} = \frac{\tilde{E}h^3}{12(1+\nu^2)}$$

To describe the viscoelastic properties of the material, the Boltzmann-Volterra hereditary theory [5–8] is used according to which the long-term elastic modulus is expressed by the integral operator

$$\tilde{E}[\varphi] = E\left[\varphi(t) - \int_0^t \Gamma(t-\tau)\varphi(\tau)d\tau\right] \tag{5}$$

E is the instant modulus of elasticity of the material;

$\Gamma(t-\tau)$ is the kernel of relaxation.

If the function $\varphi(t)$ has the form

$$\varphi(t) = \psi(t)e^{-i\omega_R t} \tag{6}$$

where ψ is a slowly changing function of time, *i* is an imaginary unit, ω_R is a real constant.

Assuming that the integral terms in (5) are small compared to $\varphi(t)$, and using the freezing method [5, 6, 8], the integral relation (5) can be reduced to the complex one, i.e.:

$$\tilde{E}[\varphi] \approx E\ \left[1 - \Gamma^c(\omega_R) - i\Gamma^s(\omega_R)\right]\psi\},\tag{7}$$

where

$$\begin{aligned} \Gamma^c(\omega_R) &= \int_0^\infty \Gamma\ (\tau)\cos\omega_R\tau d\tau, \\ \Gamma^s(\omega_R) &= \int_0^\infty \Gamma\ (\tau)\sin\omega_R\tau d\tau, \end{aligned} \tag{8}$$

Γ^S, Γ^C are the sines and cosines of the image of Fourier kernel $\Gamma\ (\tau)$.

The relationship between the components of the strain tensor and the displacement vector is described by Cauchy relations

$$\begin{aligned}
\varepsilon_s &= \frac{\partial u}{\partial s} \\
\varepsilon_\theta &= \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{r}(w\cos\varphi + u\sin\varphi) \\
\varepsilon_{s\theta} &= \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial s} - \frac{1}{r}v\sin\varphi \\
\gamma_s &= -\frac{\partial^2 w}{\partial s^2} \\
\gamma_\theta &= -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta}\frac{\cos\varphi}{r^2} - \frac{\partial w}{\partial s}\frac{\sin\varphi}{r} \\
\gamma_{s\theta} &= 2\left(-\frac{1}{r}\frac{\partial^2 w}{\partial s\partial\theta} + \frac{\sin\varphi}{r^2}\frac{\partial w}{\partial \theta} + \frac{\cos\varphi}{r}\frac{\partial v}{\partial s} - \frac{\sin\varphi\cos\varphi}{r^2}v\right)
\end{aligned} \tag{9}$$

Here s is the coordinate measured along the neutral line of the shell; φ is the angle between the tangent to the generatrix and the axis z of the shell; r is the variable radius of the middle surface of the shell.

Substituting (1) into (2), (3), (4), (9), taking into account (7), reduces the problem under consideration to a complex variational eigenvalue problem of the form

$$-\int_F \left(\overline{M}_s\delta\gamma_s+\overline{M}_\theta\delta\gamma_\theta+\overline{M}_{s\theta}\delta\gamma_{s\theta}+\overline{N}_s\delta\varepsilon_s+\overline{N}_\theta\delta\varepsilon_\theta+\overline{N}_{s\theta}\delta\varepsilon_{s\theta}\right)dF+$$

$$+\rho\omega^2\int_F\left(u^*\delta u^*+v^*\delta v^*+w^*\delta w^*\right)dF=0 \qquad (10)$$

$$z=0:\ \vec{u}^*=0;\ \delta\left(\frac{dw^*}{ds}\right)=0$$

Now, the problem under consideration of finding the eigenfrequencies and natural modes of viscoelastic shell vibrations (Fig. 1) has been reduced to finding the constant ω^2 - ($\omega=\omega_R-i\omega_I$) and vector function $\vec{u}^*(\vec{x})$ - ($\vec{u}^*(\vec{x})=\vec{u}_R^*(\vec{x})-i\vec{u}_I^*(\vec{x})$), satisfying equation (10) for any kinematic virtual displacement $\delta\vec{u}^*$.

When solving the variational problem (10) on the spatial natural vibrations of a viscoelastic axisymmetric shell (Fig. 1), the solution along one coordinate is taken in the form of an exact trigonometric (in the circumferential direction at the angle θ) dependence

$$u^*(s,\theta)=\overline{u}(s)\cos n\theta$$

$$v^*(s,\theta)=\overline{v}(s)\sin n\theta \qquad (11)$$

$$w^*(s,\theta)=\overline{w}(s)\cos n\theta$$

$n=0,1,2,\ldots$ is number of harmonics, the finite element discretization is used along coordinate s with finite elements in the form of a truncated cone with 8 degrees of freedom.

The finite element method (FEM) procedure described in [27] allows us to reduce the variational problem (10) to a complex algebraic eigenvalue problem for a structure (Fig. 1):

$$\left(\left[\overline{K}\right]-\omega^2\left[M\right]\right)\{\overline{z}\}=0\,, \qquad (12)$$

where $\left[\overline{K}\right]$ is the complex stiffness matrix, the value of which depends on the sought for parameter ω_R; $[M]$ is the matrix of the structure mass; $\omega=\omega_R-i\omega_I$ is the complex eigenfrequency; $\{\overline{z}\}=\{z_R\}-i\{z_I\}$ is the complex eigenvector corresponding to the eigenfrequency ω of the structure.

Usually, the order of the equations to be solved (12) exceeds 1500. Therefore, the eigenvalues $\lambda=\omega^2$ of algebraic equation (12) are found using the Muller method [28], because there is no other, more efficient method for calculating complex eigenvalues, and the eigenvector $\{\overline{z}\}$ is determined by the Gauss method using specially developed algorithms and a calculation program for IBM. The entire calculation process outlined in this section is automated and runs on an IBM PC. An author's certificate of the Intellectual Property Agency under the Ministry of Justice of the Republic of Uzbekistan was obtained for the developed software.

3. Results and Discussion

3.1. Study of the method and algorithm convergence when solving model problems

In this section, consider the convergence and the solutions accuracy obtained on model problems of natural vibrations for cylindrical shells, taking into account the elastic and viscoelastic properties of the material. The obtained solutions are compared with the known exact solutions.

Task 1.

The axisymmetric own vibrations of an elastic cylindrical shell are considered here. The boundary conditions are: both ends of the shell are hinged, i.e.:

$$u\Big|_{\substack{z=0\\z=L}}=0;\ w\Big|_{\substack{z=0\\z=L}}=0;\ w''\Big|_{\substack{z=0\\z=L}}=0 \qquad (13)$$

In the calculation, the following initial data were used:

r_1/r_2=0.98; L/r_2=6.0.

The shell material is hypothetical, its elastic modulus is E, the density is ρ , and the Poisson's ratio is ν: E/ρ=1.0; ν=0.3.

where r_1,r_2 are the inner and outer radii of the cylinder; L is the shell length.

When solving the problem under consideration (at n=0 in expression (11)) we obtain axisymmetric eigenfrequencies of the shell. The task is to determine the axisymmetric frequencies of elastic cylindrical shells.

Table 1 shows a comparison of axisymmetric eigenfrequencies of an elastic cylindrical shell obtained by exact solution and using the finite element method.

An analysis of the comparison shows a satisfactory agreement between the numerical results and the exact solution.

Task 2.

The axisymmetric natural vibrations of a viscoelastic cylindrical shell with a hinged support at the ends are considered, i.e. the conditions are similar to (13). Geometrical and mechanical parameters of a shell, are similar to the ones in Task 1.

The Boltzmann–Volterra hereditary theory with the Rzhanitsyn–Koltunov relaxation kernel in the form of [6–8] was used to describe the viscoelastic properties of the shell material:

$$\Gamma(t) = Ae^{-\beta t}t^{\alpha - 1} \tag{14}$$

with parameters A=0.008; β=0.05; α=0.1.

The results obtained are presented in Table 1, which shows the complex eigenfrequencies of a viscoelastic cylindrical shell obtained by exact solution and using the finite element method.

Table 1. Natural frequencies of an elastic and viscoelastic cylindrical shell.

No. of eigen frequency	Eigenfrequencies			
	Elastic shell		Viscoelastic shell	
	Exact solution	Solutions obtained using the developed algorithm	Exact solution	Solutions obtained using the developed algorithm
ω_1	0.5152	0.5153	0.5132-i3.090*10^{-4}	0.5144-i3.096*10^{-4}
ω_2	0.8959	0.8962	0.8925-i5.223*10^{-4}	0.8944-i5.344*10^{-4}
ω_3	0.9732	0.9734	0.9695-i5.643*10^{-4}	0.9716-i5.653*10^{-4}
ω_4	0.9877	0.9880	0.9840-i5.724*10^{-4}	0.9859-i5.731*10^{-4}
ω_5	0.9930	0.9937	0.9896-i5.754*10^{-4}	0.9899-i5.769*10^{-4}

Analysis of the solutions of test problems (tasks 1, 2) allows us to draw the following conclusions:

The obtained eigenfrequencies and their comparison with the exact ones show satisfactory accuracy of the numerical results for elastic and viscoelastic shells.

Summarizing the results obtained, it can be stated that the studies of the convergence of numerical solutions, and their comparison with the exact ones, show the reliability and validity of the developed methods and compiled software for PC-IBM when solving problems of natural vibrations of elastic and viscoelastic shell-like structures.

3.2. Study of spatial natural vibrations of high stacks taking into account elastic properties of the material

In this section, consider the spatial natural vibrations (frequencies and modes) of high-rise smoke stacks of the Novo-Angren, Syrdarya, Azerbaijan and Ekibastuz TPPs and the ventilation pipe of the Armenian NPP according to the theory of shells in elastic statement using the developed methods and PC-IBM calculation programs.

In calculations, a high-rise smokestack is modeled by an elastic axisymmetric shell of variable thickness with separate variable slopes of both internal and external surfaces that describe the real geometry of the structures.

All geometric dimensions of the considered structures are taken from design documentation. Some of geometric dimensions of these structures are as follows (H is height, R is outer diameter and h is pipe wall thickness, z is pipe mark from the base of the structure):

Smokestack of the Novo-Angren TPP, H = 325.0 m; at the mark: z = 0.0 m: R = 19.0 m, h = 1.10 m; at the mark: z = 325.0 m: R = 8.35 m, h = 0.40 m.

Smokestack of Syrdarya TPP, H = 325.0 m; at the mark: z = 0.0 m: R = 21.0 m, h = 0.85 m; at the mark: z = 325.0 m: R = 6.00 m, h = 0.22 m.

Smokestack of the Azerbaijan TPP, H = 330.0 m; at the mark: z = 0.0 m: R = 19.0 m, h = 1.00 m; at the mark: z = 330.0 m: R = 7.52 m, h = 0.60 m.

Smokestack of Ekibastuz TPP, H = 420.0 m; at the mark: z = 0.0 m: R = 22.0 m, h = 1.20 m; at the mark: z = 420.0 m, R = 7.10 m, h = 0.30 m.

Ventilation pipe of the Armenian NPP, H = 150.0 m; at the mark: z = 0.0 m: R = 8.45 m, h = 0.90 m; at the mark: z = 150.0 m, R = 2.35 m, h = 0.16 m.

The parameters of the physico-mechanical characteristics of the material under consideration are taken as:

$$E = 2.9\times10^4 \text{ MPa};\ \nu = 0.17;\ \rho = 2.5 \text{ t/m}^3;\ \Gamma(t) = 0.0.$$

For all the aforementioned high-rise smokestacks, non-axisymmetric natural vibrations corresponding to different numbers *(n)* of harmonics were studied. At harmonic number *n* = 0, the spatial form splits into axisymmetric and torsional vibration modes.

For axisymmetric vibrations of a shell in one-dimensional problem, there is a one-dimensional equivalent – longitudinal vibrations of a beam, and for torsional vibrations of the shell a one-dimensional equivalent is torsional vibration of a beam. At n = 1, for non-axisymmetric vibrations of the shell, there also exists a one-dimensional equivalent – bending vibrations of a beam. At *(n* = 2,3, ...) non-axisymmetric vibrations of one-dimensional equivalents do not exist.

For all stacks listed above, at each harmonic (n = 0,1,2,3, ...), at least 5 eigenfrequencies were obtained and the corresponding vibration modes were constructed.

Table 2 shows the spatial (at n = 0,1,2,3, ...) eigenfrequencies for some high-rise stacks obtained using the developed methods and software.

Table 2. Frequency of spatial natural modes of vibration (rad/sec) of structures (high-rise stacks) obtained in elastic statement.

No. of harmonics	Novo-Angren TPP			Ekibastuz TPP			Syrdarya TPP		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
n=0 torsion.	20.10	34.92	55.70	17.76	29.05	45.36	24.42	39.92	60.43
n=0 axisym.	24.66	49.78	81.84	20.51	40.04	65.48	27.43	53.56	87.16
n=1	1.89	6.30	14.29	1.27	4.09	9.20	2.96	7.59	16.38
n=2	13.02	15.75	19.29	11.92	14.69	16.59	11.23	15.85	19.65
n=3	36.89	38.31	40.11	26.99	35.86	39.31	32.30	35.05	44.31
n=4	68.51	70.67	73.48	54.54	60.32	69.02	56.79	64.01	73.19
n=5	107.35	109.18	116.12	76.07	90.20	97.38	86.63	98.79	105.06

(tr – torsional, as – axisymmetric frequencies)

In the studies conducted at the Institute of Earth Physics of the Academy of Sciences of the Russian Federation [29] it was found that the predominate periods of soil vibrations during strong earthquakes are within the range of 0.1–0.5 sec.

When analyzing the values of obtained eigenfrequencies of the structures under consideration, there is a probability that they would coincide with the ground motion frequency during the earthquake; this can lead to a dangerous phenomenon – resonant vibrations of a pipe. This indicates that in assessing the seismic resistance of such structures, one cannot limit oneself by only a few bending eigenmodes of vibration (at n=1).

3.3. Study of spatial own vibrations of high-rise stacks taking into account viscoelastic properties of the material

In this section, spatial natural vibrations (frequencies, modes and decrements of vibrations) of the above structures in viscoelastic statement (i.e. considering viscoelastic properties of the structure material) are studied using the developed methods and PC-IBM computation. The structures under consideration are modeled by a viscoelastic axisymmetric shell with a variable slope and thickness, which makes it possible to take into account their real geometry.

To describe the viscoelastic properties of the material, the Boltzmann-Volterra hereditary theory with the Rzhanitsyn-Koltunov kernel is used (14).

The choice of viscoelastic models to describe the properties of structure material (concrete) is explained by the closeness of experimental and theoretical results obtained for the stress state of concrete [30]. The parameters of the relaxation kernel are found on the basis of the technique [8], the essence of which lays in the comparison of experimental creep curve and the theoretical curve. In this work, the theoretical strain values for various time instants are tabulated in detail and the curves are plotted for a wide range of kernel parameters A, β, α.

To determine the values of the kernel parameters for concrete, several experimental creep curves for concrete, presented in [30], were processed. By superimposing the obtained experimental curve on the assemblage (set) of theoretical curves [8] and shifting it to the abscissa and ordinate axes, we find the one theoretical curve that coincides with experimental one. The values corresponding to this theoretical curve are taken as the sought for values of the kernel parameters.

In the problem to be solved below, the parameters of the relaxation kernel (14) for concrete were used, obtained with the above method from experimental creep curves given in [30]:

$$A=0.0194;\ \beta=0.00000014;\ \alpha=0.075.$$

All geometric dimensions of the above structures are taken from design documentation. Some dimensions of these structures are given in section 3.2 of this paper.

Table 3 shows the complex eigenfrequencies of spatial vibrations obtained for the above listed structures using the developed methods and software taking into account viscoelastic properties of the structure material.

Table 3. The frequency of spatial natural vibrations (rad/sec) of structures (high-rise stacks) obtained in viscoelastic statement.

No. of harmonics	Novo-Angren TPP			Ekibastuz TPP			Syrdarya TPP		
	$\omega_1=\omega_{1R}-i\omega_{1I}$	$\omega_2=\omega_{2R}-i\omega_{2I}$	$\omega_3=\omega_{3R}-i\omega_{3I}$	$\omega_1=\omega_{1R}-i\omega_{1I}$	$\omega_2=\omega_{2R}-i\omega_{2I}$	$\omega_3=\omega_{3R}-i\omega_{3I}$	$\omega_1=\omega_{1R}-i\omega_{1I}$	$\omega_2=\omega_{2R}-i\omega_{2I}$	$\omega_3=\omega_{3R}-i\omega_{3I}$
n=0 torsion.	22.14--i0.32	44.46--i0.60	74.22--i0.95	18.38--i0.27	36.09--i0.49	59.27--i0.77	24.64--i0.35	48.40--i0.64	79.09--i1.01
n=0 axisym.	18.01--i0.26	31.44--i0.43	50.36--i0.67	15.89--i0.24	26.12--i0.37	40.94--i0.55	21.02--i0.31	35.99--i0.49	54.67--i0.72
n=1	1.66--i0.03	5.58--i0.09	12.76--i0.19	1.10--i0.02	3.61--i0.06	8.18--i0.13	2.19--i0.04	6.74--i0.11	14.65--i0.22
n=2	12.55--i0.19	14.04--i0.21	17.54--i0.26	10.62--i0.16	13.12--i0.19	14.84--i0.22	10.01--i0.15	14.17--i0.21	17.60--i0.26
n=3	33.22--i0.46	34.53--i0.47	36.16--i0.49	24.25--i0.34	32.29--i0.45	36.31--i0.49	29.07--i0.41	31.57--i0.44	39.98--i0.54
n=4	64.29--i0.84	64.31--i0.83	66.58--i0.86	49.38--i0.66	54.55--i0.72	62.48--i0.81	51.34--i0.68	57.94--i0.76	66.32--i0.86
n=5	98.43--i1.23	100.31--i1.25	105.50--i1.31	68.94--i0.89	81.86--i1.04	93.69--i1.18	79.85--i1.02	91.43--i1.15	97.88--i1.22

The real parts (ω_R) of eigenfrequencies ($\omega=\omega_R-i\omega_I$) given in Table 3 are the frequencies of structure natural vibrations and the imaginary parts (ω_I) carry information about the damping coefficients of structure vibrations.

Comparison of the results presented in Table 2 (elastic statement) and Table 3 (viscoelastic statement), shows that the real part of the complex frequencies is less than the corresponding values of the eigenfrequencies obtained in elastic statement, and the logarithmic decrement $\delta = -2\pi \frac{\omega_I}{\omega_R}$ slightly decreases with increasing frequency number. This means that an account for viscoelastic properties of the structure material leads to a weak dependence of the logarithmic decrement of vibrations on frequency.

Table 4 shows the comparison between the periods of bending vibrations obtained by one-dimensional theory (in elastic statement) and by the theory of shells (in elastic and viscoelastic statements) and the results of field experiments [17, 31].

Table 4. Periods of bending vibrations of various smokestacks.

High-rise smoke stacks	No. of period	Vibrations periods (sec)			
		Elastic statement		Viscoelastic statement	Field experiment
		One-dimensional theory	Theory of shells (at n=1)	Theory of shells (at n=1)	
Novo-Angren TPP	T_1	3.26	3.32	3.8	3.4
	T_2	0.91	0.99	1.12	1.0
	T_3	0.38	0.44	0.49	0.5
	T_4	0.21	0.25	0.28	0.3
Syrdarya TPP	T_1	2.5	2.12	2.88	2.8
	T_2	0.80	0.83	0.93	0.9
	T_3	0.33	0.38	0.42	0.4
	T_4	0.19	0.23	0.25	0.2
Armenian NPP	T_1	1.56	1.42	2.03	1.6
	T_2	0.46	0.46	0.52	0.5
	T_3	0.21	0.22	0.24	0.2
	T_4	0.11	0.12	0.14	-

An analysis of the above results shows that the values of bending vibrations periods obtained in field experiments [17, 31] and the ones found theoretically by the developed method are quite close.

Table 5 shows the values of logarithmic decrement of vibrations of the Novo-Angren TPP high smokestack obtained by field experiment [17] and the ones found theoretically for the three lower bending modes (at n=1), using the developed methods and software that take into account viscoelastic properties of the structure material.

Table 5. Logarithmic decrement of bending vibrations of the Novo-Angren TPP smokestack.

Decrement definition method	Logarithmic decrement		
	δ_1	δ_2	δ_3
Experimental	0.15	0.25	0.37
Theoretical	0.121	0.102	0.094

Significant differences in the values of experimentally and theoretically determined logarithmic decrement (Table 5), especially for the second and third modes, are apparently explained by a failure to consider dry friction and energy entrainment from the structure to infinity.

4. Conclusions

1. A mathematical model, methods and algorithm to study the spatial natural vibrations of axisymmetric structures, with account for dissipative properties of the material are developed based on the hereditary theory of viscoelasticity in the framework of the theory of shells.

2. The reliability of the developed methods and algorithms was verified by solving a number of test problems and comparing the results obtained by known exact solutions and the results of field experiments.

3. The spatial natural vibrations of a number of real axisymmetric structures (ventilation and smokestacks of nuclear power plants and thermal power plants) were studied taking into account the elastic and viscoelastic properties of the structure material of the structure.

4. It was revealed that not only the frequencies of bending modes of vibration are in the dangerous range of earthquakes frequencies, but also some other modes of spatial vibrations of structures, determined by the theory of shells.

5. It was found that the value of the logarithmic decrement of structure vibrations when accounting for viscoelastic properties of the structure material weakly depends on eigenfrequencies of vibrations.

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