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Assessment of Dynamic Characteristics of High-Rise Structures Taking into Account Dissipative Properties of the Material

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Abstract. The methods and algorithm to assess dynamic characteristics of high-rise structures are given in the paper taking into account the variability of slopes and the structure thickness in the framework of one-dimensional theory of viscoelasticity. The Boltzmann-Volterra hereditary theory was used to describe dissipative processes in the structure material. The reliability of results was verified by comparing the obtained results with the exact solution of a number of test problems. Natural vibrations of high-rise chimney stacks and ventilation pipes of thermoelectric and nuclear power plants have been investigated. It was revealed that the natural frequencies of the considered structures fall into the dangerous range of earthquake frequencies. The obtained frequencies of natural vibrations of real structures are compared with the results of field experiments.

1. Introduction

In the dynamics of research such dynamic characteristics as natural frequencies, modes and decrements of vibrations, occupy a special place, since they are a passport of the structure and allow us to assess the structure dynamic properties as a whole, without examining its behavior under various influences.

Determination of natural frequencies and vibration modes for elastic structures is an independent and rather difficult task. The determination of structure dynamic characteristics is complicated by an order of magnitude when dissipative properties of the material are taken into account.

High-rise structures include high-rise chimney stacks and ventilation pipes of thermoelectric (TPP) and nuclear power plants (NPP), cooling towers of TPP and NPP and protective shells of NPP. Due to their design features and geometric dimensions, they are unique structures.

Today, a large number of different high-rise structures are operated and built all over the world,including high-rise chimney stacks; the height of some of them reaches 150 m - 600 m [1-3].

If for the pipes of a height of 50 m the ratio of wall thickness δ to the radius R of its middle surface at the base is $\delta/R = 1/5\div1/7$, then for the pipes of a height of 250 - 300 m it is$\delta/R = 1/12\div1/15$, and for the pipes of a height of H = 420 m, the ratio δ/R is $\delta/R\approx1/23$. With increase in height H and radius R, the wall thickness of the pipe δ grows slowly [1-3]. Along with this, the pipe radius, thickness and slope of the cone change along its height, gradually moving from a conical section to a cylindrical one.

In the existing normative documents in many countries, an elastic cone-shaped console with a constant slope is used as a calculation model for such structures; it does not take into account such features as real geometry, construction features of structures and viscoelastic (dissipative) properties of their material; these features have a direct impact on the values of dynamic characteristics of structures [2,4].

Dissipative processes in the materials are taken into account by the Voigt, Maxwell, and Kelvin models, but they do not always agree with experimental data, therefore, to eliminate this, the models with hysteresis absorption or hereditary viscoelastic Boltzmann-Volterra models are used, although their implementation is rather difficult and available experimental data are scarce [5–9].

Recently, the researchers have paid great attention to the determination of natural frequencies and vibration modes of various high-rise structures, taking into account the elastic properties of the material.

In [10], the effect of uniform external pressure and symmetric boundary conditions on the natural frequencies of homogeneous and multi-layer isotropic cylindrical shells was studied.

The natural frequencies were analyzed in [11, 12], and the influence of various boundary conditions on the changes in shell thickness(cylindrical shells) and in natural frequencies of the shell were studied.

In [13], on the basis of a reliable mathematical model IC, it was proposed to efficiently calculate the natural frequency and damping coefficient using the stochastic distribution function of the natural frequency.

The stress-strain state, dynamic behavior and wave phenomenon in various systems were studied in [14–27], taking into account the features of various structures. These are just some of the publications devoted to determining the dynamic characteristics of various structures and systems.

So, the development of effective methods and algorithm for assessing dynamic characteristics of high-rise structures, taking into account their design features and dissipative properties of their material, is an urgent task.

2. Methods and algorithms for determining dynamic characteristics of high-rise structures.

Consider a high-rise structure (Figure 1.), modeled by a viscoelastic beam of variable cross section with a variable slope of the generatrix; the lower part of the structure (z=0) is on a rigid base, and the upper part (z=L) is free. In the structure sections, a longitudinal force is applied, which is the weight of the upper part of the structure. The slopes of various sections of the inner and outer surfaces of the structure are variable, $i_n^н$, i_n^e - the slopes of the outer and inner surfaces of the n-th section of the structure, respectively. Fig. 1 shows the natural vibrations of the structure under consideration.

The functional is used to describe the process of dynamic strain in a structure

$$L = \int_0^t (T - U - \Pi) dt , \tag{1}$$

and kinematic conditions

$$z = 0 : \; u = 0 ; \; w = 0 ; \; \left(\frac{\partial w}{\partial z} \right) = 0 \; , \tag{2}$$

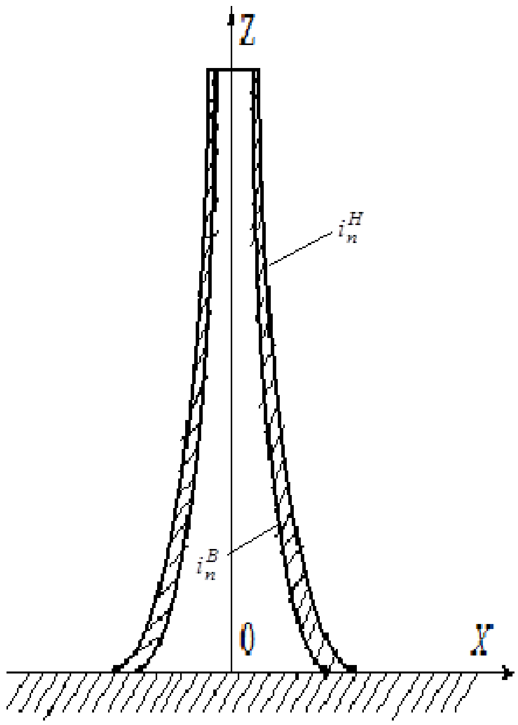

Figure 1. High-rise structure model

Here: T is the kinetic energy of the model, U is the potential energy; P is the potential of external forces:

$$U=\frac{1}{2}\int_0^L \widehat{E}F(z)\left(\frac{\partial u}{\partial z}\right)^2 dz+\frac{1}{2}\int_0^L \widehat{E}J(z)\left(\frac{\partial^2 w}{\partial z^2}\right)^2 dz, \tag{3a}$$

$$T=\frac{1}{2}\int_0^L \rho F(z)\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dz, \tag{3b}$$

$$\Pi=-\int_0^L \rho g(L-z)udz, \tag{3c}$$

Further, to describe the dissipative processes in the structure material under dynamic processes, the Boltzmann-Volterra hereditary theory [7,8] where the long-term elastic modulus is expressed by the integral operator

$$\widehat{E}[\varphi]=E\left[\varphi(t)-\int_0^t \Gamma(t-\tau)\varphi(\tau)d\tau\right], \tag{4}$$

E is theinstant modulus of elasticity of the material; $\Gamma(t-\tau)$ is the relaxation kernel.

Function $\varphi(t)$ has the form

$$\varphi(t)=\psi(t)e^{-i\omega_R t}, \tag{5}$$

Where ψ is theslowly varying time function, i is the imaginary unit, ω_R is a real constant. Assuming that the integral terms in (4) are small in comparison with $\varphi(t)$, and using the freezing method [28], we can reduce the integral relation (4) to the complex one, i.e.:

$$\widehat{E}[\varphi] \approx E\left[1-\Gamma^c(\omega_R)-i\,\Gamma^s(\omega_R)\right]\psi \quad (6)$$

where

$$\Gamma^c(\omega_R)=\int_0^\infty \Gamma(\tau)\cos\omega_R\tau d\tau, \quad (7a)$$

$$\Gamma^s(\omega_R)=\int_0^\infty \Gamma(\tau)\sin\omega_R\tau d\tau, \quad (7b)$$

Γ^S, Γ^C are the sinus and cosines of the Fourier image of the kernel $\Gamma(\tau)$.Here *u*, *w* are the longitudinal and transverse displacements in section *z*, *J(z)* is the moment of inertia, *F(z)* is the cross-sectional area, *L* is the height of the structure and ρ is the density of the material.

When considering a real structure, the numerical finite element method (FEM) is used, which consists in discretizing the model under study by finite elements [29], with the displacements of nodes satisfying the minimum condition of the total energy functional.

The displacements of any point inside the element, its strains and stresses are approximated based on the obtained displacements of the nodal points. Cross-sectional areas and moment of inertia are determined by the following formulas

$$F^n(z)=\pi\left[\left(R^H(z)\right)^2-\left(R^B(z)\right)^2\right], \quad (8a)$$

$$J^n(z)=\frac{\pi}{4}\left[\left(R^H(z)\right)^4-\left(R^B(z)\right)^4\right] \quad (8b)$$

where $R^H(z)=R_i^H-i^H z,\ \ R^B(z)=R_i^B-i^B z$ are the outer and inner radius of the circular section, respectively;

i^H, i^B are the outer and inner slope of the wall;

$\rho=\gamma F^n(z)$ is the linear mass of finite element;

γ is the specific mass of the material.

The displacements field inside the element is approximated linearly by u_i, u_j and cubically by w_i, w_j Angular displacements are determined by formulas

$$\beta_i=\frac{\partial w_i}{\partial z},\ \beta_j=\frac{\partial w_j}{\partial z}. \quad (8c)$$

Further use of the FEM procedure reduces the considered variation problem (1) and (2) to a complex system of algebraic equations for eigenvalues

$$\left(\left[\overline{K}\right]-\omega^2[M]\right)\{\overline{q}\}=0, \quad (9)$$

The solution of equation (9) allows us to determine the complex parameters ω, $\vec{u}^*(\vec{x})$: i.e. the complex natural frequency $\omega = \omega_R - i\omega_I$; the real part of ω_R of the complex parameter ω in its physical essence is the frequency of free damped vibrations of the structure, and the imaginary part ω_I is equal to the damping coefficient of oscillations and determines the dissipative properties of the structure as a whole, and $\{\bar{q}\} = \{q_R\} - i\{q_I\}$ is the complex eigenvector, corresponding to the natural frequency of the structure ω.

Here $[\bar{K}]$ is the complex stiffness matrix, its elements values depend on the sought for parameter ω_R; $[M]$ is the mass matrix of the structure.

The eigenfrequencies ω of algebraic equation (9) are determined using the Muller method [30], because there is no other, more efficient method to calculate complex eigenvalues, the eigenvector $\{\bar{q}\}$ is determined by the Gauss method.

3. The study of solution accuracy on test examples

The accuracy of the obtained solutions is investigated on model problems of natural vibrations for elastic and viscoelastic beams according to the proposed methods. The obtained solutions are compared to the known exact solutions.

Problem 1

The natural vibrations of an elastic beam of a circular cross section are considered. The inner and outer radii of the ring are r_1 and r_2 respectively. The lower end of the beam is fixed, the upper one is free. The task is to determine the natural frequencies of the bending, longitudinal and torsional vibrations of the beam.

Table 1 shows the bending and longitudinal natural frequencies of beam vibrations determined according to the proposed method for the following beam parameters (the material of the beam istaken as a hypothetical one):E/ρ=1.0; r_2/r_1=1.5; L/r_1=30 (r_1=10 sm)

Table 1. Bending and longitudinal natural frequencies of a beam

№ of natural frequencies	Natural frequencies			
	Bending natural frequencies		Longitudinal natural frequencies	
	Exact solution	Solution on the proposed methods	Exact solution	Solution on the proposed methods
ω_1	0.000352	0.000352	0.005236	0.005239
ω_2	0.002207	0.002206	0.015708	0.015708
ω_3	0.006180	0.006179	0.026180	0.026182
ω_4	0.012109	0.012108	0.036652	0.036660
ω_5	0.020018	0.020016	0.047124	0.047141

The frequencies of torsional natural vibrations for a beam of constant cross section are the same as of longitudinal ones, so these values are not given.

A comparison of results shows a rather high accuracy of the method and the reliability of the results obtained using the developed program for PC-IBM.

Problem 2

The longitudinal natural vibrations of a viscoelastic rod with rigidly fixed edges are considered.

To describe the viscoelastic properties of the material, the Boltzmann-Volterra hereditary theory with the Rzhanitsyn-Koltunov relaxation kernel [8] is used:

$$\Gamma(t) = Ae^{-\beta t}t^{\alpha-1}, \tag{10}$$

Table 2 gives the comparison of an exact solution and the one obtained by the proposed method for the complex eigenfrequencies $\omega = \omega_R - i\omega_I$ for longitudinal vibrations of a viscoelastic rod with rigidly fixed edges obtained at the following initial conditions
$L = 150.0;\ E = 10^5;\ \gamma = 1.0;\ A = 0.008;\ \beta = 0.05;\ \alpha = 0.1.$

Table 2. Complex natural frequencies of longitudinal vibrations of a viscoelastic rod

№ Nat. freq.	Exact solution	Solution obtained with the FEM
ω_1	6.413-i0.0336	6.4134-i0.0336
ω_2	12.855-i0.0628	12.855-i0.0628
ω_3	19.306-i0.0904	19.308-i0.0903
ω_4	25.763-i0.1170	25.771-i0.1170
ω_5	32.224-i0.1430	32.236-i0.1435

Comparison of the obtained values of ωwith the exact ones shows a satisfactory accuracy of the method and the reliability of the results obtained using the developed program for PC-IBM in solving the indicated class of problems. An analysis of the results obtained, namely, the proportionality of the real ω_R and imaginary ω_I parts of the complex frequencies ω of a viscoelastic rod, states that the logarithmic decrement of oscillations is independent of frequency.

4. The study of the natural vibrations of high-risestructures

The bending, longitudinal and torsional vibrations of high-rise chimney stacks of the Novo-Angren, Syrdarya, Azerbaijan and Ekibastuzhydro-electric power stations and the ventilation pipes of the Armenian NPP are studied using the beam theory with the proposed methods and PC-IBM computational programs.

The smokestack is modeled by an elastic beam with a variable slope and variable thickness of the cross section.

All geometric dimensions of the considered structures are taken from the design documentation. Some of the geometric dimensions of these structures are as follows:
Chimney stack of Novo-Angren TPP, L = 325.0 m; at the mark: z = 0.0 m: R = 19.0 m, h = 1.10 m; at the mark: z=325.0m: R=8.35m, h=0.40m.Chimney stack of the Syrdarya TPP, L = 325.0 m; at the mark: z = 0.0 m: R = 21.0 m, h = 0.85 m; at the mark: z=325.0 m: R=6.00 m,h=0.22m.Chimney stack of the Azerbaijan TPP, L=330.0 m; at the mark: z=0.0 m: R=19.0 m, h=1.00 m; at the mark: z=330.0 m: R=7.52 m, h=0.60 m. Chimney stack of Ekibastuz TPP, L = 420.0 m; at the mark: z = 0.0 m: R = 22.0 m, h = 1.20 m; at the mark: z = 420.0 m: R=7.10 m, h=0.30 m. Ventilation pipe of the Armenian NPP, L=150.0 m; at the mark: z=0.0 m: R=8.45 m, h=0.90 m; at the mark: z=150.0 m: R=2.35 m, h=0.16 m (L-height, R-outer diameter and h-pipe wall thickness, z – pipe marks from the base of the structure).

The parameters of the material physico-mechanical characteristics of the considered structures are:

$$E = 2.9 \times 10^4 MPa;\ \nu = 0.17;\ \gamma = 2.5t/m^3;\ \Gamma(t) = 0$$

In calculations, the potential of external forces P in (1) was not considered. The values of five lowest frequencies and the corresponding modes of natural vibrations were determined for these structures.

In studies conducted at the Institute of Earth Physics of the Academy of Sciences of the Russian Federation, it was found that the prevailing periods of soil vibrations during strong earthquakes are in the range of 0.1-0.5 sec. Therefore, in structure calculation for seismic effects, several lower frequencies and natural modes of structures located in the frequency spectrum of 2 - 10 Hz are used.

Table 3 shows the natural frequencies of bending vibrations for all above structures, obtained using the proposed methods.

Table 4 shows natural frequencies of bending, longitudinal and torsional vibrations ofthe Novo-Angren TPP smokestack.The most dangerous of them - in terms of seismic hazard- are revealed.

Table 3. Natural frequencies of bending vibrations of high-risestacks (rad/sec).

High-rise stacks	Natural frequencies	Obtained by the proposed methods
Novo-Angren TPP	ω_1	1.9246
	ω_2	6.8944
	ω_3	16.3258
	ω_4	30.6408
	ω_5	49.2861
SyrdaryaTPP	ω_1	2.5188
	ω_2	7.8423
	ω_3	18.7443
	ω_4	33.1236
	ω_5	51.1196
Azerbaijan TPP	ω_1	1.6735
	ω_2	6.2989
	ω_3	14.7466
	ω_4	27.5144
	ω_5	44.4424
Ekibastuz TPP-1	ω_1	1.1556
	ω_2	4.0199
	ω_3	9.4238
	ω_4	17.3573
	ω_5	27.9408
Armenian NPP	ω_1	4.0044
	ω_2	13.7226
	ω_3	30.3993
	ω_4	54.9257
	ω_5	87.5563

Table 4. Natural frequencies of bending, longitudinal and torsional vibrations of thesmokestack of the Novo-AngrenTPP (rad/sec)

№ of natural frequencies	Eigenfrequencies obtained by the proposed methods		
	Bending ones	Longitudinal ones	Torsional ones
ω_1	1.9246	28.9589	18.9309
ω_2	6.8944	47.4113	30.9937
ω_3	16.3258	83.4104	54.5271
ω_4	30.6408	119.9753	78.4303
ω_5	49.2861	138.8048	90.7904

If to write down these frequencies in increasing order, then the following frequencies of natural oscillations of the considered stacks fall into the range of prevailing earthquake frequencies (from 2 to 10 Hz) (Table 5).

Table 5. Vibration frequencies of the Novo-AngrenTPPsmokestacks in the range from 2 to 10 Hz

№№	Frequency(Hz)	Mode of oscillations
1	2.6	The third bending
2	3.0	The first torsional
3	4.6	The first longitudinal
4	4.8	The fourth bending
5	4.9	The second torsional
6	7.55	The second longitudinal
7	7.85	The fifth bending
8	8.68	The third torsional

Thus, there is a probability that these frequencies coincide with the frequency of the earthquake, which can lead to a dangerous - resonant - vibrations of the stack. This indicates the fact that when calculating a chimneystack for seismic effects, one cannot limit oneself to bending vibration modes only, especially to the first two eigenmodes, whose frequencies are not in the dangerous range. Table 6 compares the periods of bending vibrations of different smokestacks obtained by the proposed methods with the results of field experiments [14,27].

Table 6. Bending periods of high-rise smokestacks

Chimney stacks	№ of period	Periods of vibrations (sec)	
		Obtained by the proposed methods	Field experiment
Novo-Angren TPP	T_1	3.26	3.4
	T_2	0.91	1.0
	T_3	0.38	0.5
	T_4	0.21	0.3
SyrdaryaTPP	T_1	2.5	2.8
	T_2	0.80	0.9
	T_3	0.33	0.4
	T_4	0.19	0.2
Armenian NPP	T_1	1.56	1.6
	T_2	0.46	0.5
	T_3	0.21	0.2
	T_4	0.11	-

An analysis of the results shows that the values of the periods of bending vibrations of high-rise stacks obtained by the developed methods are quite close to the results of field experiments [14,27].

5. Conclusions

1. The methods and an algorithm are proposed for assessing the dynamic characteristics of high-rise structures taking into account the variability of the slope and structure thickness in the framework of the one-dimensional theory of viscoelasticity.
2. To describe the dissipative processes in the structure material the use of the Boltzmann-Voltaire hereditary theory of viscoelasticity is proposed.
3. The results reliability was verified by comparing the obtained results with the exact solutions of a number of test problems.
4. The natural vibrations of several high-rise chimney stacks and ventilation pipes of thermal and nuclear power plants have been investigated.

5. It was revealed that not only the bending, but the longitudinal and torsional natural frequencies of the considered structures fall into the dangerous range of earthquake frequencies.
6. A sufficiently good agreement between the obtained frequencies of natural vibrations of real structures and the results of field experiments proves the adequacy of the proposed methods for solving such problems.

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