Estimation of the stress-strain state of orthotropic plates on elastic foundation using the bimoment theory

M.K. Usarov[1], *F.A.* Usanov[1*], *D.M.* Usarov[1], *G.U.* Isaev[1], and *E.S.* Toshmatov[2]

[1]Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan
[2]National Research University - TashkentInstitute of Irrigation and Agricultural Mechanization Engineers, 39 KoriNiyoziy str., 100000, Tashkent, Uzbekistan

Abstract. The study is devoted to the development of the bimoment theory and the method for calculating thick plates in the framework of the three-dimensional theory of elasticity. The basic relations and equations of motion of the plate presented are constructed with respect to the forces, moments and bimoments arising from the nonlinearity of the law of distribution of displacements and stresses over the plate thickness. Bending and vibrations of isotropic and orthotropic plates are considered an example of calculations. The resulting solution showed the efficiency and accuracy of the proposed bimoment theory in assessing the stress-strain state of thick plates. **Keywords:** thick orthotropic plate, Hooke's law, three-dimensional theory, bimoment theory, infinite series, transverse load, exact solution.

1 Introduction

The calculation of plates and shells occupies a special place in the study of structural elements. The main provisions of the general methodology for constructing the classical and refined theory of plates and shells are built on the basis of a number of simplifying hypotheses. The problem of bending, stability and oscillation of orthotropic cantilevered plates has not been sufficiently studied due to the complexity of the main differential equation of the problem and boundary conditions. It requires the development of reliable numerical-analytical methods for its solution. We consider the plate material to be ideally elastic, and then there is an infinite number of critical loads that change the shape of the plate equilibrium. In [1], we study the natural oscillations of a rectangular plate, two adjacent edges of which are fixed, and the other two are free; it is an element of many building structures. The deflection function is chosen as a sum of two hyperbolic trigonometric series. Both series obey the main equation of free vibration. This eigenvalue problem is similar to the problem of determining the frequency spectrum of free vibrations of a plate [2-4]. The study in [4] considered stability problems for anisotropic plates and

[*] Corresponding author: usanovfurqat94@mail.ru

shells and the methods for solution applicable to cantilevered plates. The dynamic stability of round three-layer plates made of viscoelastic composite materials was studied in [5, 6].

In [7, 8], an orthotropic viscoelastic plate of arbitrarily varying thickness is presented. The plate was subjected to dynamic periodic loading. Within the framework of the Kirchhoff–Love hypothesis, a mathematical model was constructed in a geometrically nonlinear statement, taking into account the shear forces of inertia. The Bubnov–Galerkin method based on polynomial approximation of deflection and displacement was used. The problem was reduced to solving systems of nonlinear integro-differential equations. The solution to the system was obtained for an arbitrarily varying plate thickness. For a weakly singular Koltunov–Rzhanitsyn kernel with variable coefficients, the resulting system was solved numerically based on quadrature formulas. The computational algorithm was developed and implemented in the Delphi algorithmic language. The dynamic stability of the plate was studied depending on the geometric parameters of the plate, viscoelastic and non-homogeneous properties of the material. It was found that the results of the viscoelastic problem obtained using the exponential relaxation kernel practically coincide with the results of the elastic problem. When using the Koltunov–Rzhanitsyn kernel, the differences between the problems of elasticity and viscoelasticity are significant and amount to more than 40%.

References [9-12] are devoted to the development of a mathematical model and a method for solving the problem of contact interaction of various multilayer structures with a combined base. As a result of solving specific problems, a number of new mechanical effects associated with the manifestation of internal force factors in three-layer beam systems interacting with combined bases were revealed.

The studies in [13, 14] are devoted to solving problems of plate bending in the framework of the Reissner theory, in which the finite element method is used to calculate square fixed and hinged plates under a uniformly distributed load. In [15], analytical and numerical methods for calculating the edge elements of buildings in the form of axisymmetric thick plates for nuclear reactors were developed. The bending problem of a Reissner rectangular plate was solved by the iterative method [16].

Articles [17-19] are devoted to dynamic calculations of elements of the box-shaped structure of buildings for seismic resistance, taking into account the spatial work of box-shaped elements under the action of dynamic influences specified by the displacement of their lower part according to the sinusoidal law. The equations of motion for each of the plate and beam elements of the box structure of the building are given on the basis of the Kirchhoff-Love theory. Expressions are given for the forces, moments and stresses of plate elements that balance the movement of box elements, as well as boundary conditions and full contact conditions through displacements and force factors in the contact zones of plate and beam elements.

It should be noted that the theory of thick plates and methods for their calculation, taking into account forces, moments, and bimoments, were discussed in [20, 21]. The study in [22] is devoted to the dynamic calculation of the box-shaped structure of buildings for seismic resistance, taking into account the spatial work of box-shaped elements under the action of dynamic action. The article develops a mathematical model and a numerical-analytical method for solving the problem of dynamics by the finite difference method and expanding the solution by the modes of natural vibrations in the spatial formulation of box-shaped elements under kinematic action.

In the spatial case of deformation of the plate along its thickness, nonlinear laws of distribution of displacements, deformations and stresses hold. Therefore, it is necessary to take into account all components of the stress and strain tensor: σ_{ij}, ε_{ij}, $(i, j = 1,3)$. In this case, in contrast to the conventional case of setting the problem for describing the field of spatial deformation of the plate, an account for tensile and shear forces, bending and torque

 https://doi.org/10.1051/e3sconf/202340207020

moments is not enough; it is necessary to take into account bimoments. When constructing a general theory of plates within the framework of the three-dimensional theory of elasticity, researchers use various methods, for example, the method of hypotheses, the method of expansion of displacements in a series or the method of asymptotic solution, etc.

This article briefly presents the constitutive relations, the equations of motion, and the boundary conditions of the bimoment theory of plates developed in [22].

2 Formulation of the problem

Consider an orthotropic thick plate of constant thickness $H = 2h$ with dimensions a, b in plan. Let us introduce the notation: E_1, E_2, E_3– elastic moduli and G_{12}, G_{13}, G_{23}– shear moduli; $\nu_{12}, \nu_{13}, \nu_{23}$ - Poisson's ratios of the plate material.

We introduce the Cartesian coordinate system x_1, x_2 and z. oz -axis is directed vertically down. Let the distributed surface normal and tangential loads be applied to the lower and upper front surfaces of the plate $z = h$ and $z = -h$. Normal loads in the direction of the OZ -axis are denoted by $q_3^{(+)}$, $q_3^{(-)}$, shear loads in the direction ox_1, ox_2 are denoted by $q_k^{(+)}, q_k^{(-)}, (k = \overline{1,2})$.

The components of the displacement vector are determined by the functions of three spatial coordinates and time

$u_1 = u_1(x_1, x_2, z, t)$, $u_2 = u_2(x_1, x_2, z, t)$, $u_3 = u_3(x_1, x_2, z, t)$. The components of the strain tensor are determined by the Cauchy relations.

The technique for constructing the bimomental theory of plates is based on the Cauchy relations, the generalized Hooke law, three-dimensional equations of the theory of elasticity and boundary conditions on the front surfaces, as well as on the expansion of displacements in an infinite Maclaurin series.

It should be noted that the expressions for forces, moments and bimoments, as well as the equations for these force factors, are constructed within the framework of the method of integrating three-dimensional equations of motion of the theory of elasticity along one of the coordinates, directed along the small dimension (along the thickness) of the plate. The method of expansion of the displacement component in one of the coordinates is used only when approximating the boundary conditions of the problem, set on the front surfaces of the plate.

The bimoment theory of plates [22] is described by two independent problems, each of which is formulated on the basis of nine equations with the corresponding boundary conditions. The first problem describes the symmetric problem of longitudinal vibrations, taking into account the transverse compression, and the second describes the asymmetric bending problem, taking into account the transverse shear of the material of the thick plate.

In the article, we consider bending-shear vibrations of a thick plate on an elastic foundation. Let us consider the problem of bending vibrations of a thick plate within the framework of the bimoment theory of thick plates [22].

3 Bimoment theory of thick plates

The problem of bending vibrations of a thick plate in the framework of the bimoment theory is described by equations for moments, shear forces and bimoments, as well as three equations for generalized displacements of the front surfaces of the plate. Forces, moments and bimoments are determined with respect to nine unknown functions defined by the following relations:

 https://doi.org/10.1051/e3sconf/202340207020

$$\widetilde{W}=\frac{u_3^{(+)}+u_3^{(-)}}{2},\ \ \widetilde{r}=\frac{1}{2h}\int_{-h}^{h}u_3dz,\ \ \widetilde{\gamma}=\frac{1}{2h^3}\int_{-h}^{h}u_3z^2dz,$$
$$\widetilde{u}_k=\frac{u_k^{(+)}-u_k^{(-)}}{2},\ \ \widetilde{\psi}_k=\frac{1}{2h^2}\int_{-h}^{h}u_kzdz,\ \ \widetilde{\beta}_k=\frac{1}{2h^4}\int_{-h}^{h}u_kz^3dz,\quad (k=1,2). \tag{1}$$

The equations of motion in moments and forces have the form:

$$\frac{\partial M_{11}}{\partial x_1}+\frac{\partial M_{12}}{\partial x_2}-Q_{13}+H\widetilde{q}_1=\frac{H^2}{2}\rho\ddot{\widetilde{\psi}}_1,\quad \frac{\partial M_{21}}{\partial x_1}+\frac{\partial M_{22}}{\partial x_2}-Q_{23}+H\widetilde{q}_2=\frac{H^2}{2}\rho\ddot{\widetilde{\psi}}_2, \tag{2}$$

$$\frac{\partial Q_{13}}{\partial x_1}+\frac{\partial Q_{23}}{\partial x_2}+2\widetilde{q}_3=\rho H\ddot{\widetilde{r}}\,. \tag{3}$$

Bending and torque moments are defined as:

$$M_{11}=\frac{H^2}{2}\left(E_{11}H\frac{\partial\widetilde{\psi}_1}{\partial x_1}+E_{12}H\frac{\partial\widetilde{\psi}_2}{\partial x_2}-E_{13}\frac{2(\widetilde{r}-\widetilde{W})}{H}\right),$$
$$M_{22}=\frac{H^2}{2}\left(E_{12}H\frac{\partial\widetilde{\psi}_1}{\partial x_1}+E_{22}H\frac{\partial\widetilde{\psi}_2}{\partial x_2}-E_{23}\frac{2(\widetilde{r}-\widetilde{W})}{H}\right),\ M_{12}=M_{21}=G_{12}\frac{H^2}{2}\left(\frac{\partial\widetilde{\psi}_1}{\partial x_2}+\frac{\partial\widetilde{\psi}_2}{\partial x_1}\right), \tag{4}$$

where $E_{11},\ E_{12},...,E_{33}$ are the elastic constants determined in terms of the coefficients. The shear forces are defined as

$$Q_{13}=G_{13}(2\widetilde{u}_1+H\frac{\partial\widetilde{r}}{\partial x_1}),\ \ Q_{23}=G_{23}(2\widetilde{u}_2+H\frac{\partial\widetilde{r}}{\partial x_2})\,. \tag{5}$$

In equations (2), (3), the load terms are determined by the formulas:

$$\widetilde{q}_k=\frac{q_k^{(+)}+q_k^{(-)}}{2},\ (k=1,2),\quad \widetilde{q}_3=\frac{q_3^{(+)}-q_3^{(-)}}{2}\,.$$

The bimoments $P_{11},\ P_{22}\ P_{12}$, generated during bending and shearing of the plate, are determined by the following formulas:

$$P_{11}=\frac{H^2}{2}\left(E_{11}\frac{\partial\widetilde{\beta}_1}{\partial x_1}+E_{12}\frac{\partial\widetilde{\beta}_2}{\partial x_2}-E_{13}\frac{2(3\widetilde{\gamma}-\widetilde{W})}{H}\right),$$
$$P_{22}=\frac{H^2}{2}\left(E_{12}\frac{\partial\widetilde{\beta}_1}{\partial x_1}+E_{22}\frac{\partial\widetilde{\beta}_2}{\partial x_2}-E_{23}\frac{2(3\widetilde{\gamma}-\widetilde{W})}{H}\right),\ P_{12}=P_{21}=\frac{H^2}{2}G_{12}\left(\frac{\partial\widetilde{\beta}_1}{\partial x_2}+\frac{\partial\widetilde{\beta}_2}{\partial x_1}\right). \tag{6}$$

The intensities of the transverse shear and normal bimoments $\widetilde{p}_{13},\ \widetilde{p}_{23}$ and $\widetilde{p}_{33}$ are determined by the expressions

$$\widetilde{p}_{k3}=G_{k3}\left(\frac{2\widetilde{u}_k-4\widetilde{\psi}_k}{H}+\frac{\partial\widetilde{\gamma}}{\partial x_k}\right),\ (k=1,2),\ \widetilde{p}_{33}=E_{31}\frac{\partial\widetilde{\psi}_1}{\partial x_1}+E_{32}\frac{\partial\widetilde{\psi}_2}{\partial x_2}-E_{33}\frac{2(\widetilde{r}-\widetilde{W})}{H}. \tag{7}$$

The equations for the bimoments in bending and transverse shear are obtained in the form:

$$\frac{\partial P_{11}}{\partial x_1}+\frac{\partial P_{12}}{\partial x_2}-3\tilde{p}_{13}+H\tilde{q}_1=\frac{H^2}{2}\rho\ddot{\tilde{\beta}}_1,\quad \frac{\partial P_{21}}{\partial x_1}+\frac{\partial P_{22}}{\partial x_2}-3\tilde{p}_{23}+H\tilde{q}_2=\frac{H^2}{2}\rho\ddot{\tilde{\beta}}_2, \tag{8}$$

$$H\frac{\partial\tilde{p}_{13}}{\partial x_1}+H\frac{\partial\tilde{p}_{23}}{\partial x_2}-4\tilde{p}_{33}+2\tilde{q}_3=H\rho\ddot{\tilde{\gamma}}. \tag{9}$$

Equations (2), (3), (8) and (9) form a consistent system of six equations for nine unknown functions: $\tilde{\psi}_1,\ \tilde{\psi}_2,\ \tilde{\beta}_1,\ \tilde{\beta}_1,\ \tilde{u}_1,\ \tilde{u}_2,\ \tilde{r},\ \tilde{\gamma},\ \tilde{W}$.

The constructed six equations (2), (3), (8) and (9) contain nine unknown functions determined by formulas (1). As seen, three more equations are missing here. To construct these missing equations, we used the method of expansion of displacements in the Maclaurin series.

For the problem of bending vibrations, these equations can be written in the following form:

$$\frac{\partial\tilde{\sigma}_{11}}{\partial x_1}+\frac{\partial\tilde{\sigma}_{12}}{\partial x_2}+\frac{\tilde{\sigma}_{13}^*}{H}=\rho\ddot{\tilde{u}}_1,\quad \frac{\partial\tilde{\sigma}_{21}}{\partial x_1}+\frac{\partial\tilde{\sigma}_{22}}{\partial x_2}+\frac{\tilde{\sigma}_{23}^*}{H}=\rho\ddot{\tilde{u}}_2, \tag{10}$$

$$\frac{\partial\tilde{q}_1}{\partial x_1}+\frac{\partial\tilde{q}_2}{\partial x_2}+\frac{\tilde{\sigma}_{33}^*}{H}=\rho\ddot{\tilde{W}}. \tag{11}$$

Here $\tilde{\sigma}_{11},\tilde{\sigma}_{12},\tilde{\sigma}_{22}$ are determined from Hooke's law, taking into account the conditions on the front surfaces:

$$\tilde{\sigma}_{11}=E_{11}^*\frac{\partial\tilde{u}_1}{\partial x_1}+E_{12}^*\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{13}}{E_{33}}\tilde{q}_3,\quad \tilde{\sigma}_{22}=E_{12}^*\frac{\partial\tilde{u}_1}{\partial x_1}+E_{22}^*\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{23}}{E_{33}}\tilde{q}_3,\ , \tag{12}$$
$$\tilde{\sigma}_{12}=G_{12}\left(\frac{\partial\tilde{u}_1}{\partial x_2}+\frac{\partial\tilde{u}_2}{\partial x_1}\right)$$

here $E_{11}^*=E_{11}-\frac{E_{13}}{E_{33}}E_{31},\quad E_{22}^*=E_{22}-\frac{E_{23}}{E_{33}}E_{32},\quad E_{12}^*=E_{21}-\frac{E_{23}}{E_{33}}E_{31}.$

$$\frac{\tilde{\sigma}_{3k}^*}{H}=G_{k3}\frac{210\left(33\tilde{\beta}_k-9\tilde{\psi}_k-4\tilde{u}_k\right)}{H^2}+G_{k3}\frac{\partial}{\partial x_k}\left(\frac{\tilde{q}_3}{E_{33}}-\frac{E_{31}}{E_{33}}\frac{\partial\tilde{u}_1}{\partial x_1}-\frac{E_{31}}{E_{33}}\frac{\partial\tilde{u}_2}{\partial x_2}\right)+$$
$$+\frac{42}{H}\left(\tilde{q}_k-G_{k3}\frac{\partial\tilde{W}}{\partial x_k}\right),\ (k=1,2), \tag{13}$$
$$\frac{\tilde{\sigma}_{33}^*}{H}=E_{33}\frac{210\left(9\tilde{\gamma}-2\tilde{W}-\tilde{r}\right)}{H^2}+E_{31}\frac{\partial}{\partial x_1}\left(\frac{\tilde{q}_1}{G_{13}}-\frac{\partial\tilde{W}}{\partial x_1}\right)+E_{32}\frac{\partial}{\partial x_2}\left(\frac{\tilde{q}_2}{G_{23}}-\frac{\partial\tilde{W}}{\partial x_2}\right)+$$
$$+\frac{30}{H}\left(\tilde{q}_3-E_{31}\frac{\partial\tilde{u}_1}{\partial x_1}-E_{32}\frac{\partial\tilde{u}_2}{\partial x_2}\right)+$$

.

The system of equations (2), (3), (8) - (11) is a consistent system with respect to nine unknown functions $\tilde{\psi}_1,\ \tilde{\psi}_2,\ \tilde{u}_1,\ \tilde{u}_2,\ \tilde{\beta}_1,\ \tilde{\beta}_1,\ \tilde{r},\ \tilde{\gamma},\ \tilde{W}$.

https://doi.org/10.1051/e3sconf/202340207020

Note that when constructing the equations of motion (10) and (11), eight terms of the Maclaurin series are retained, and these equations are constructed up to the sixth-order accuracy with respect to the plate parameter $H/10a$.

Example. As an application of the bimoment theory of plates to applied problems, the dynamic problems of bending and vibrations of a plate on an elastic foundation is considered under the influence of an external dynamic load in the form of a Heaviside function applied, respectively, to the front surface $z=-h$:

$$q_3^{(-)}=\begin{cases}0, & \text{at } t\le 0;\\ -q_0, & \text{at } t>0,\end{cases}$$

where q_0 is the load parameter.

When solving the problem of oscillations of plates on an elastic foundation, we assume that the generalized displacements $\bar{u}_1,\ \bar{u}_1,\ \bar{W}$ are small compared to the generalized displacements $\tilde{u}_1,\ \tilde{u}_1,\ \tilde{W}$. This assumption ensures the independence of the second problem of bending and vibrations of plates of the bimoment theory from the first problem which describes tension-compression with allowance for the transverse compression of the plate, which facilitates the construction of a solution to the problem. The following expressions are constructed for the contact forces that appear between the plate and the elastic foundation:

$$\tilde{q}_1=\frac{k_1}{2}\tilde{u}_1+q_1^{(-)},\ \ \tilde{q}_2=\frac{k_2}{2}\tilde{u}_2+\frac{1}{2}q_2^{(-)},\ \ \tilde{q}_3=\frac{k_3}{2}\tilde{W}-\frac{1}{2}q_3^{(-)},$$

where k_1, k_2, k_3 are the coefficients of bed of elastic foundation.

It is assumed that the edge of the plate $x_2=0$ is rigidly fixed. The remaining edges of the plate are free from supports. On the fixed edge of the plate, the displacements are zero:

$$\tilde{\psi}_1=0,\quad \tilde{\beta}_1=0,\quad \tilde{\psi}_2=0,\quad \tilde{\beta}_2=0,\ \tilde{u}_1=0,\ \ \tilde{u}_2=0,\ \ \tilde{r}=0,\ \ \tilde{\gamma}=0,\ \ \tilde{W}=0, \tag{12}$$

And on the free edges of the plate $x_1=0,\ \ x_1=a,\ \ x_2=b$, the forces, moments and bimoments are zero.

$$M_{11}=0,\ M_{12}=0,\ P_{11}=0,\ P_{12}=0,\ \ Q_{13}=0,\ \tilde{p}_{13}=0,\ \ \tilde{\sigma}_{11}=0,\ \ \tilde{\sigma}_{12}=0,\ \ \tilde{\sigma}_{11}^{*}=0. \tag{13}$$

$$M_{22}=0,\ M_{12}=0,\ P_{22}=0,\ P_{12}=0,\ \ Q_{23}=0,\ \tilde{p}_{23}=0,\ \ \tilde{\sigma}_{22}=0,\ \ \tilde{\sigma}_{12}=0,\ \ \tilde{\sigma}_{22}^{*}=0. \tag{14}$$

Values $\tilde{\sigma}_{11},\tilde{\sigma}_{12},\tilde{\sigma}_{22}$ are determined from Hooke's law, taking into account the conditions on the front surfaces:

$$\tilde{\sigma}_{11}=E_{11}^{*}\frac{\partial\tilde{u}_1}{\partial x_1}+E_{12}^{*}\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{13}}{E_{33}}\tilde{q}_3,\quad \tilde{\sigma}_{22}=E_{12}^{*}\frac{\partial\tilde{u}_1}{\partial x_1}+E_{22}^{*}\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{23}}{E_{33}}\tilde{q}_3,\quad \tilde{\sigma}_{12}=G_{12}\left(\frac{\partial\tilde{u}_1}{\partial x_2}+\frac{\partial\tilde{u}_2}{\partial x_1}\right), \tag{15}$$

here $E_{11}^{*}=E_{11}-\frac{E_{13}}{E_{33}}E_{31},\quad E_{22}^{*}=E_{22}-\frac{E_{23}}{E_{33}}E_{32},\quad E_{12}^{*}=E_{21}-\frac{E_{23}}{E_{33}}E_{31}.$

The problem posed is described by the equations of motion of the bimoment theory of plates (2), (3), (8) - (11) and boundary conditions (12) - (14) and is solved by the finite difference method.

For the numerical solution to the problem posed, the method of finite differences was applied. To approximate the derivatives of displacements with respect to spatial coordinates, we use the formulas of central difference schemes. In this case, $\Delta x_1 = \frac{a}{N}$, $\Delta x_2 = \frac{b}{M}$ is the calculation step, N, M are the numbers of partitions, and Δt is the time step. The calculation steps in terms of spatial coordinates and time are chosen as follows:

$$\Delta x_1 = \frac{a}{N},\ \Delta x_2 = \frac{b}{M},\quad c\Delta t \le \min(\Delta x_1, \Delta x_2)\cdot$$

Dimensionless variables $x=x_1/a,\ y=x_2/b,\ \tau=c_t/H,$ where $c = \sqrt{E/\rho}$ are introduced.

The maximum stresses of the plate are determined by expressions (15).

4 Analysis of numerical results

Calculations are made for isotropic and orthotropic square plates on an elastic foundation. Dimensionless numerical results of calculation of displacements and stresses for a square orthotropic plate SVAM 15:1 with elastic characteristics are obtained.

$E_1 = 4{,}6 * E_0,\ E_2 = 1{,}6 * E_0,\ E_1 = 1{,}12 * E_0,\ G_{12} = 0{,}56 * E_0,$
$G_{13} = 0{,}43 * E_0,\ G_{23} = 0{,}33 * E_0,\quad \nu_{21} = 0{,}27,\ \nu_{31} = 0{,}07,\ \nu_{23} = 0.3,$ here $E_0 = 10^4\ MPa$.

The dimensionless values of the interaction coefficients are set in the following form:

$$k_1 = 0,\ k_2 = 0,\ \frac{k_3 H}{E_0} = 0.03\,.$$

Numerical results are determined for orthotropic plates on an elastic foundation, obtained by the bimoment theory and by the Timoshenko theory. Below are the numerical results of the dynamic calculation of a thick plate on an elastic base with one free $(y_1 = b)$ and the other fixed edges $(x_1 = 0,\ \ x_1 = a,\ \ y_1 = 0)$.

Let us consider the problems of vibrations of an isotropic and orthotropic plate on an elastic foundation with one free and other fixed edges. The boundary conditions of the problem on the fixed edges of the plate $x_1 = a$, $x_1 = a$ and $y_1 = 0$ have the form:

$$\tilde{\psi}_1 = 0,\ \tilde{\psi}_2 = 0,\ \tilde{\beta}_1 = 0,\ \tilde{\beta}_2 = 0,\ \tilde{r} = 0,\ \tilde{\gamma} = 0,\ \tilde{u}_1 = 0,\ \tilde{u}_2 = 0,\ \tilde{W} = 0\,,$$

The following conditions must be met at the free edge of the plate $y_1 = b$:

$$M_{11} = 0,\ M_{12} = 0,\ P_{11} = 0,\ P_{12} = 0,\ Q_{13} = 0,\ \tilde{p}_{13} = 0,\ \tilde{\sigma}_{11} = 0,\ \tilde{\sigma}_{12} = 0,\ \tilde{\sigma}^*_{11} = 0\,.$$

 https://doi.org/10.1051/e3sconf/202340207020

The stresses σ_{11}, σ_{22}, σ_{12} in the plate are calculated by formulas (15). The article presents the numerical results calculated by the authors according to the Timoshenko theory with shift coefficient $k^2 = 2/3$.

Tables 1 and 2 present the results of calculations for displacements and stresses for isotropic and orthotropic square plates on an elastic foundation, depending on their dimensions in plan.

Table 1 presents the results of calculations on normal displacements and stresses for isotropic square plates with dimensions in plan $a = b = 10H$ on an elastic foundation, obtained by the bimoment theory and the Timoshenko theory. The maximum value of the dimensionless stress σ_{11} of the plate is reached at point $x_1 = 0,\ y_1 = b$, according to the bimoment theory and the Timoshenko theory, they are $\sigma_{11} = -27.325 q_0$ and $\sigma_{11} = -22.715 q_0$, respectively.

The difference between them is more than 15%. The maximum value of the dimensionless stress of an isotropic plate σ_{22} is reached at point $x_1 = a/2,\ y_1 = 0$ and according to the bimoment theory and the Timoshenko theory is equal to $\sigma_{22} = -18.885 q_0$ and $\sigma_{22} = -14.964 q_0$, Here the difference is approximately 20%.

According to the bimoment theory, the maximum value of the normal displacement of a plate on an elastic foundation is $\tilde{r} = -104.488 q_0$, and according to the Timoshenko theory - $\tilde{r} = -100.197 q_0$. The difference is only 4%.

Table 1. Displacements and stresses of an isotropic plate with dimensions $a = b = 10H$ in plan on an elastic foundation according to the bimoment theory and the Timoshenko theory

Bimoment Theory				Timoshenko Theory			
$\frac{\sigma_{11}}{q_0}$	$\frac{\sigma_{22}}{q_0}$	$\frac{2M_{11}}{H^2 q_0}$	$\frac{\tilde{r}E_1}{Hq_0}$	$\frac{\sigma_{11}}{q_0}$	$\frac{\sigma_{22}}{q_0}$	$\frac{2M_{11}}{H^2 q_0}$	$\frac{\tilde{r}E_1}{Hq_0}$
−27.325	−18.885	−6.597	−104.488	−22.715	−14.964	−5.210	−100.197

At the same point, the bending moment M_{11} also reaches the maximum and its value is $M_{11} = -6.597 \frac{H^2}{2} q_0$- according to the bimoment theory and $M_{11} = -5.210 \frac{H^2}{2} q_0$- according to the Timoshenko theory. Here, the error is more than 22% according to Timoshenko's theory.

Table 2 shows the numerical results for square thick orthotropic plates on an elastic foundation for three dimensions $a = b = 3H$ $a = b = 5H$ and $a = b = 10H$ in plan according to the bimoment theory and the Timoshenko theory.

Table 2. Stresses of an orthotropic plate according to the bimoment theory and the Timoshenko theory depending on three dimensions of the plate in plan: $a = b = 3H;\ 5H;\ 10H$.

Plate dimensions	Bimoment theory		Timoshenko theory	
	σ_{11}	σ_{22}	σ_{11}	σ_{22}
$a = b = 3H$	$-19.669 q_0$	$-10.344 q_0$	$-9.252 q_0$	$-5.021 q_0$

$a = b = 5H$	$-28.885q_0$	$-16.885q_0$	$-19.131q_0$	$-10.674q_0$
$a = b = 10H$	$-62.696q_0$	$-21.081q_0$	$-31.261q_0$	$-17.582q_0$

The values of the normal stresses of an isotropic plate on an elastic foundation at its point $x_1 = 0,\ y_1 = b$ with dimensions $a = b = 3H$ and $a = b = 5H$ in plan are calculated. and according to the bimoment theory, they are $\sigma_{11} = -28.855q_0$ and $\sigma_{11} = -19.131q_0$, according to Timoshenko's theory - $\sigma_{11} = -19.6691q_0$ and $\sigma_{11} = -9.252q_0$. Here the error according to Timoshenko's theory is 65-100%.

The maximum stress value σ_{22} was determined at the point $x_1 = a/2,\ y_1 = b$ of orthotropic plates on an elastic foundation with the same dimensions; according to the bimoment theory, they are $\sigma_{22} = -16.880q_0$ and $\sigma_{22} = -10.674q_0$. According to Timoshenko's theory – $\sigma_{22} = -10.344q_0$ and $\sigma_{22} = -5.021q_0$. Here the error according to Timoshenko's theory is 60-100%.

For orthotropic plates of medium thickness $a = b = 10H$ on an elastic foundation, the numerical results obtained by the bimoment theory and the Timoshchenko theory (Table 2) differ by about 50%. At the point of the plate $x_1 = 0,\ y_1 = b$, stress σ_{11} reaches its maximum value. According to the bimoment theory, it is $\sigma_{11} = -62.696q_0$, and according to the Timoshenko theory, it is $\sigma_{11} = -31.261q_0$. Here the error of Timoshenko's theory is 100%. The maximum stress value σ_{22} was found at the point of orthotropic plates $x_1 = a/2,\ y_1 = b$ with the same dimensions: according to the bimoment theory, it is $\sigma_{22} = -21.081q_0$ and according to the Timoshenko theory, it is $\sigma_{22} = -17.582q_0$. Here, the error according to Timoshenko's theory is approximately 20%.

The calculations performed according to the Timoshenko theory and the bimoment theory showed that with an increase in the thickness of the plate on an elastic foundation, the difference in the numerical results obtained by these theories increases. In addition, it was found that the stress values for orthotropic plates are much higher than for isotropic plates.

In calculations, the number of partitions into steps of difference schemes in dimensionless coordinates is taken as follows $N = 30, M = 30$. The calculations were performed in the Delphi environment.

5 Conclusions

1. A bimoment theory and a method were developed for estimating the stress state of thick plates on an elastic foundation based on the equations of the three-dimensional theory of elasticity.

2. The basic relations and equations of motion of the plate were constructed with respect to the forces, moments and bimoments arising from the nonlinearity of the law of distribution of displacements and stresses over the plate thickness.

3. As an example of the calculation, the bending and vibrations of isotropic and orthotropic plates on an elastic foundation were considered.

4. The effectiveness and accuracy of the proposed bimoment theory in solving applied problems of vibrations of plates on an elastic foundation was shown.

5. The results obtained for plates on an elastic foundation showed that with an increase in the thickness of the plate, the difference in the numerical results obtained by the bimoment theory and the Timoshenko theory increases. The stress values for orthotropic plates are much higher than the values for isotropic plates.

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