IOP Conference Series: Materials Science and Engineering

Torsional vibrations of a cylindrical shell in a linear viscoelastic medium

IOP Conf. Series: Materials Science and Engineering **883** (2020) 012190 doi:10.1088/1757-899X/883/1/012190

Torsional vibrations of a cylindrical shell in a linear viscoelastic medium

I Safarov[1*], M Teshaev[3], E Toshmatov[1], Z Boltaev[3] and F Homidov[3]

[1]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan
[2]Tashkent Institute of Chemical Technology, Tashkent, Uzbekistan
[3]Bukhara Engineering and Technology Institute, Bukhara, Uzbekistan

t.elyor85@mail.ru

Abstract. In this paper, we consider the natural vibrations of inhomogeneous mechanical systems, i.e., cylindrical bodies located in a deformable viscoelastic medium. The theory and methods for studying the natural vibrations of a cylindrical shell in a viscoelastic medium are constructed. The viscoelastic properties of the medium are taken into account using the hereditary Boltzmann-Walter theory. For the statement of the problem, the general equation of the theory of viscoelasticity in the potentials of displacements in a cylindrical coordinate system is used. An algorithm has been developed to solve the tasks posed on a computer using the Bessel, Hankel, and Mueller and Gauss methods.
The considered problems were reduced to finding complex natural frequencies for the system of equations of motion of a cylindrical shell in an infinite viscoelastic medium using radiation conditions. It is shown that the problem has a discrete complex spectrum. The eigen frequencies of oscillations of a low-contrast heterogeneity are found. Revealed that the imaginary part of the eigen frequencies is comparable with the real one, which can lead to aperiodic movements of the systems considered.

1. Introduction
The study of inhomogeneities is of great interest in predicting an important tectonophysical phenomenon, i.e., the behavior of the hearth of an upcoming earthquake. Now it is widely accepted that the zone of upcoming seismic shocks is an area with elastic-density properties that change as a result of tectonic motions. This corresponds to the study of inhomogeneities with slightly changed relative to the external elastic medium velocities of longitudinal and transverse waves, as well as, possibly, the density.

Any inhomogeneity in the medium must have, like any elastic mechanical system, a certain spectrum of natural frequencies. Because of variations in inclusion and environment are interrelated, will be damped due to radiation of elastic waves and, therefore, the eigenfrequencies are complex [1]. This is, the attenuation in an ideal elastic medium is explained by the radiation of the energy of waves excited by their vibrations due to diverging elastic waves. The interest in studying the natural frequencies of the elastic inclusion – the medium system is also due to the following circumstance.
In this case, the state of an inhomogeneous body is described by a linear one-to-one relationship between stress and strain over the entire period of alternating stress. It follows that stress and strain are always in phase. The energy dissipation will occur if the stress and strain are not uniquely related during the oscillation period. When time derivatives appear in the equation linking these values, the

absence of such an unambiguous relationship occurs [2]. As you know, in this case, to calculate the wave field, the solution should be integrated in frequency along with the spectrum of the specified incident pulse. The resulting integral can, in General, be calculated by some direct numerical method. In this case, preference should be given to the method of integration using the theory of deductions in the form of expansion along the poles of the integrand function. Note that, the poles coincide with the roots of the equation of natural frequencies and, thus, to continue to have the opportunity to engage in problems of nonstationary diffraction of elastic waves, depending on the ratio apoplectic environmental parameters and inclusions, the study of the properties of the roots of the frequency equations [3].

In [4], the problem is considered in a stationary setting, when the incident wave is an infinite sinusoid in space and time. In this case, several difficulties arise since the eigenfunctions of the problem under study cannot be considered as a vector in a Hilbert space: they are not normalized due to the exponential growth of the distance. To eliminate it, it is proposed to take into account that fluctuations cannot exist for an infinitely long period of time, and, therefore, we come to the need for a restriction at a small initial time [5, 6]. In [7], the problem of "exponential catastrophe" is solved by developing special radiation conditions and boundary conditions.

This article discusses the vibrations of cylindrical shells in a deformable medium. The main attention will be paid to the study of low-contrast inhomogeneities. Besides, the behavior of complex eigenfrequencies depending on the geometric and physical-mechanical parameters of an inhomogeneous system will be investigated. The physical nature of the considered inhomogeneities is closely related to convective flows in The earth's interior, as well as to various areas of faults and fractures [8,9,10]. Such inclusions are very common and, therefore, have a significant impact on the scattering of seismic waves in various media [11, 12, 13].

Along with this, the dynamics of various systems and structures, taking into account the features and working conditions, were studied in [19, 20, 21, 22, 23].

These are just some of the works in which the assessment of the oscillatory processes of various systems and structures is considered taking into account the viscoelastic properties of materials.

2. Methods

2.1 Problem statement

The equations of motion of the Kirchhoff-Love shell, taking into account the reaction of the deformable (viscoelastic) medium under torsional vibrations, can be written as

$$G_0 h \frac{d^2 v}{dX^2} - p_0 h \frac{d^2 v}{dt^2} = q_c \qquad (1)$$

where G_0 is the modulus of elasticity of the shell material; *h* is the thickness of the shell, *v is the* torsional displacement of the point of the middle surface of the shell, $q_c -$ is external load from the environment[14].

The values of non-zero components of the stress tensor in the medium are determined using tangential offsets using the formulas

$$\sigma_{r0} = \widetilde{G}_c\left(\frac{du_0}{dr} - \frac{u_0}{r^2}\right); \sigma_{G0} = \widetilde{G}_c \frac{du_c}{dx}$$

where

$$\widetilde{G}_c f(t) = G_{0c}[f(t) - \int_0^t R_G(t-\tau) f(\tau) d\tau] \qquad (2)$$

f(x) is an arbitrary function of time; $R_G(t-\tau)$ is the core of the relaxation, $G_{oc} -$ is the instantaneous modulus of elasticity, $u_0 -$is the torsional motion of the medium.

Take the integral terms in (2) small. Next, using the freezing procedure, replace the ratios (2) with approximations of the form [15].

$$\overline{G}_c f(t) = G_{oc}[1 - \Gamma_G^C(\omega_R) - i\Gamma_G^S(\omega_R)]f(t) \tag{3}$$

here

$$\Gamma_G^C(\omega_R) = \int_0^\infty R_G(\tau)\cos\omega_R\tau d\tau \quad \Gamma_G^C(\omega_R) = \int_0^\infty R_G(\tau)\cos\omega_R\tau d\tau$$

are the cosine and sine Fourier images; ω_R –is real value. As a model of viscoelastic material, we take the relaxation core of Koltunov-Rzhanitsyn

$$R_G(t) = \frac{A_0 e^{-\beta t}}{t^{1-\alpha_1}}$$

Given that only shear waves are excited about the twisting load, we obtain the equation of motion of the medium[16]

$$\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r}\frac{\partial u_0}{\partial r} - \frac{u_0}{r^2} + \frac{\partial^2 u_0}{\partial x^2} = \frac{p_c}{G_{0c}[1 - \Gamma_G^C(W_R) - i\Gamma_G^C(W_R)]}\frac{\partial^2 u_0}{\partial t^2} \tag{4}$$

The boundary conditions of the problem for r=R have the form

$$u_0 = \vartheta + \frac{h_0}{2}, \sigma_{r\theta} = -q_c \tag{5}$$

2.2 Method of solution

Considering axisymmetric oscillations of the shell, we are will looking for the solution of equations (1) and (4) in the form:

$$\begin{pmatrix} u_0 \\ \vartheta \\ q_c \end{pmatrix} = \begin{pmatrix} U(r) \\ V \\ q_{co} \end{pmatrix} \cos m e^{i\omega t} \tag{6}$$

where π/m is wavelength along the generating line; $\omega = \omega_R + i\omega_i$ –is the complex circular frequency of natural oscillations.

Substituting the expression (6) in (1), we get the relationship between the amplitudes of the reaction of the elastic medium and the displacement of the shell

$$q_{co} = h_R^2[G_{oc}(1 - \Gamma_G^C(\omega_R) - i\Gamma_G^C(\omega_R))](h_R^2\Omega^2 - M_R^2)V^0 \tag{7}$$

where $h_R = \frac{h}{R}; M_R = mR; \Omega = \frac{p_0 R^2}{G_{oc}(1 - \Gamma_G^C(\omega_R) - i\Gamma_G^S(\omega_R))]h_R^2}\omega^2, V^0 = \frac{V}{R}$

Dependencies (7) allow the boundary condition for the environment for $r = R$ to be written as

$$\sigma_{r\theta}^0 = -G_{oc}(1 - \Gamma_G^C(\omega_R) - i\Gamma_G^S(\omega_R))]h_R^2(h_R^2\Omega^2 - M_R^2)U_0$$

Substituting the expression (6) in (4), we get

$$\frac{\partial^2 u_0}{\partial r_*^2} + \frac{1}{r_*}\frac{\partial u_0}{\partial r_*} - \frac{u_0}{r_*^2} - \frac{u_0}{r_*^2} - (M_R^2 - \delta^2)U_0 = 0 \tag{8}$$

where

$$r_1 = \frac{r}{R};\ U_0 = \frac{u}{h};\ \delta^2 = (\frac{h_R^2\gamma}{p_*})\Omega^2;\ \ \gamma = \frac{G_0}{G_{oc}(1 - \Gamma_G^C(\omega_R) - i\Gamma_G^S(\omega_R))]};\ p_* = \frac{p_0}{p_c}$$

Solving equations (8), taking in to account the damping condition of oscillation sat infinity, we have

$$U_0 = AK_1(\beta r_1), \delta_1 < M_R; U_0 = A\frac{1}{r}, \delta_1 = M_R; U_0 = AH_1^{(1)}(\overline{\beta} r_1), \delta_1 < M_R. \tag{9}$$

Here

$$\beta = \sqrt{M_R^2 - \delta^2};\ \overline{\beta} = \sqrt{\delta^2 - M_R^2}.\ \ \delta_1^2 = (\frac{h_R^2\gamma^0}{p_*})\Omega^2;\ Y^0 = \frac{G_0}{G_{0c}}$$

$K_1(x), H_1^{(1)}(x)$ are Mc Donald and Hankel functions of the 1st kind [17].

If we use Winkler bases, then $p_c = 0, (\delta = 0)$, and the solution of equation (8) takes the form

$$U_0 = AK_1(M_R r_1) \tag{10}$$

Substituting solutions (9) in (4) with the boundary condition (5), we obtain the following frequency equation

$$h_R\gamma(M_R^2 - h_R^2\Omega^2)\overline{\beta}\frac{H_0^{(1)}(\overline{\beta})}{H_1^{(1)}(\overline{\beta})} = 0; \delta > M_R \tag{11}$$

By $\delta < M_R$ in equation (11), insert an expression with a plus sign instead of the last term $\beta K_0(\beta)/K_1(\beta)$. If $\delta = M_R$ is executed, then satisfies the condition $p_c < \gamma^0 + \frac{2}{h_R M_R^2}$

This case is not of practical interest, since for most materials $p_c < \gamma^0$. In the case when the shell oscillation frequency is in a vacuum $(\gamma_0 \to \infty)$, *then* $\omega = \frac{M_R}{h_R}$. For a shell in an elastic medium, the complex eigenfrequencies of oscillations are determined by solving the transcendental equation (11) on a computer.

2.3 Shell oscillations are described by Timoshenko type equations

We describe the shell oscillations with Timoshenko-type equations. Then for an axisymmetric torsional motion, we have

$$\frac{d^2v}{dx^2} - \frac{k^2}{R}\alpha - \frac{p_0}{G_0}\frac{d^2v}{dt^2} = \frac{1}{Gh}q_c$$
$$\frac{d^2\alpha}{dx^2} - 12\frac{k^2}{h^2}\alpha - \frac{p_0}{G_0}\frac{d^2\alpha}{dt^2} = \frac{6}{Gh^2}q_c \tag{12}$$

Here α is the angle of rotation of the normal in the tangential direction; k_2 *is* the coefficient of Tymoshenko.

The boundary conditions for the medium at *r*= *R* have the form

$$u_0 = v + \frac{h}{2}\alpha; \sigma_{r0} = -q_c \tag{13}$$

Representing the solution of equations (12) as (6), we define the parameters q_{c0}, α_o through the move *V*.

$$q_{c0} = G_0 h_R^3\frac{\alpha_1\alpha_2}{6h_0^2 + h_R\alpha_1}V^*;\ \alpha_0 = 6h_R - \frac{\alpha_2}{6h_R^2 + h_R\alpha_1}V^*$$

$$a_1 = h_R^2\Omega^2 - M_R^2 - 12\frac{k^2}{h_R^2};\ a_2 = h_R^2\Omega^2 - M_R^2 \tag{14}$$

Given the expressions (14), we transform the boundary condition for the environment when $r_1=1$

$$\sigma_{r0}^{0}=-G_0 h_R^2 \frac{\alpha_2}{\alpha_3\alpha_1}U_0$$

where

$$\alpha_3=1+\frac{k^2}{h_R}\frac{6}{\alpha_1};\ \alpha_4=1+\frac{3\alpha_2}{\alpha_3\alpha_1} \quad (15)$$

Since the solution of the equation of motion of the medium does not depend on the accepted shell model, the further derivation of the equations is similar to the one discussed above for the Kirchhoff — Love shell. The choice of the shell model only affects the type of boundary conditions (13). Instead of the characteristic equation (11), we write an equation of the form

$$h_R\gamma\frac{a_2}{a_3a_4}+2-\bar{\beta}\frac{H_0^{(1)}(\bar{\beta})}{H_1^{(1)}(\bar{\beta})}=0;\quad \delta>M_R \quad (16)$$

For the case $p_c=0,(\delta=0)$ we obtain a characteristic equation for the Timoshenko shell in a non-inertial medium. By G_c=0 we find the frequency of torsional vibrations of the Timoshenko shell in a vacuum

$$\Omega_1=\frac{M_R}{h_R};\ \Omega_2=\frac{1}{h_R}\sqrt{M_R^2+12\frac{k^2}{h_R^2}} \quad (17)$$

The first frequency, as for the Kirchhoff - love shell, corresponds to the rotation of the section as a ring, the second is the form of oscillations caused by the rotation of the normal in the tangential direction.

Putting in equations (11), (16) $h_0=0,(h_R=0)$ we obtain a characteristic equation for the proper axisymmetric torsional oscillations of an elastic inertial array with a cylindrical cavity, given in the article [8].

2.4 Shell vibrations are described by the equations of elasticity theory

We are looking for an exact solution to the problem for the case when the shell motion is described by equation (2). Then the boundary conditions will look like [18], [19]:

$$\text{for } r=R: u_\theta^{(1)}=u_\theta;\ \sigma_{r0}^{(1)}=\sigma_{r0};$$

$$\text{for } r=R-h: \sigma_{r\theta}^{(1)}=0 \quad (18)$$

Here, the index 1 denotes the shell.

The General solution of the equations of motion of a cylindrical layer is written as

$$U_0^{(1)}=A_1K_1(\beta_1 r_1)+B_1I_1(\beta_1 r_1),\ h_R\Omega<M_R;$$

$$U_0^1=A_1\frac{1}{r_1}+B_1r_1, h_R\Omega=M_R \quad (19)$$

$$U_0^{(1)}=A_1K_1(\beta r_1)+B_1i_1(\beta_1 r_1), h_R\Omega>M_R$$

where $\beta_1=\sqrt{M_R^2-h_R^2\Omega^2}$; $\bar{\beta}_1=\sqrt{h_R^2\Omega^2-M_R^2}$ Substituting the expressions (19) in (18) and satisfying the radiation conditions at infinity, we obtain the characteristic equation det $||\alpha_{ij}||=0,\ i,j=1,2,$ 3.

The elements of the determinant (20) for

$$h_R\Omega<\sqrt{\frac{p_*}{\gamma_R}}M_R,\ h_R\Omega<\sqrt{\frac{p_*}{\gamma_R}}M_R<h_R\Omega<M_R,\ M_R<h_R\Omega \quad (20)$$

they have a different appearance. For example, for the third case

$$a_{11} = Y_1(\bar{\beta}_1)\ ; \qquad a_{12} = J_1(\bar{\beta}_1)\ ; \qquad a_{13} = -H_1^{(1)}(\bar{\beta}_1)\ ;$$

$$a_{21} = \bar{\beta}_1 Y_0(\bar{\beta}_1) - 2Y_1(\bar{\beta}_1)\ ; \quad a_{22} = \bar{\beta}_1 J_0(\bar{\beta}_1) - 2J_1(\bar{\beta}_1)\ ;$$

$$a_{23} = -(1/\gamma)[\bar{\beta}H_0^{(1)}\bar{\beta} - 2H_1^{(1)}(\bar{\beta})]\ a_{31} = \bar{\beta}_1 Y_0(\bar{\beta}_1\varepsilon) - (2/\varepsilon)Y_1(\bar{\beta}_1\varepsilon)$$

$$a_{32} = \bar{\beta}_1 J_0(\bar{\beta}_1\varepsilon) - (2/\varepsilon)J_1(\bar{\beta}_1\varepsilon)\ ; \qquad a_{31} = 0, \quad \varepsilon = 1 - h_R$$

Now we use asymptotic representations of the Bessel function of the first and second kind. Then we get the asymptotic formulas for the first eigenvalue frequency corresponding to long waves $R_G = 0$

$$\Omega^2 = \frac{p_*}{(\chi^2\gamma)}(a + \sqrt{a^2 + b^2}) \tag{21}$$

$$a = \frac{64h_R\gamma M_R^2 + 96 - 9h_R p_*}{128h_R p_*} \quad b = \frac{9(2 + h_R\gamma M_R^2)}{64h_R p_*}$$

If you use a Winkler base instead of an elastic array, then the frequency expression takes the form

$$\Omega^2 = \frac{(h_R\gamma + 1)M_R^2 + 2}{h_R^3\gamma} \tag{22}$$

Calculations have shown that for $(1/M_R) \geq 1.46$ the results, obtained using asymptotic formulas almost coincide with the exact solutions.

3. Results and Discussion

The frequency equation (11) is solved by the Muller method [18]. Complex Eigen frequencies depending on the wavelength for various parameters (stiffness) of the viscoelastic medium are shown in Figure 1 ($x = 0{,}01, A = 0{,}048; \beta = 0{,}05; \gamma = 0{,}1; \alpha = 0{,}1$) for curves:
$1 - \gamma = 120,\ \ p_c = 5;\ \ 2 - \gamma = 30,\ \ p_c = 5;\ \ 3 - \gamma = 5\ \ \ p_c = 2$. A continuous line shows the real parts of the shell's natural oscillation frequencies in an elastic medium (or in an inertial array), a dotted line –on the basis of a Winkler (or a non-inertial one), and a dashed line-in a vacuum.

Figure 1 shows that the real parts of the frequency approach the asymptotic with increasing medium stiffness. As seen in figure 2, with increasing medium stiffness, the error increases, especially in the region of short waves.

Figure 2 shows the dependence of the real part of the dimensionless complex frequency of torsional vibrations on the relative thickness of the shell. Numerical results are obtained $M_R = 3,\ \ \gamma_R = 30,\ \ \gamma_1 = 0{,}001,\ \ p_c = 5\ \ k^2 = 2/3$. For figure 2 curve 1 corresponds to the first mode of oscillation, 2 - to the second, and 3 - to the third. A continuous line marks the shell in an elastic medium, a dotted line marks it in a non - inertial medium, and a dashed line marks it in a vacuum. The results obtained when describing the shell motion by equations of elasticity theory correspond to the curves 1, 2 (–o-).

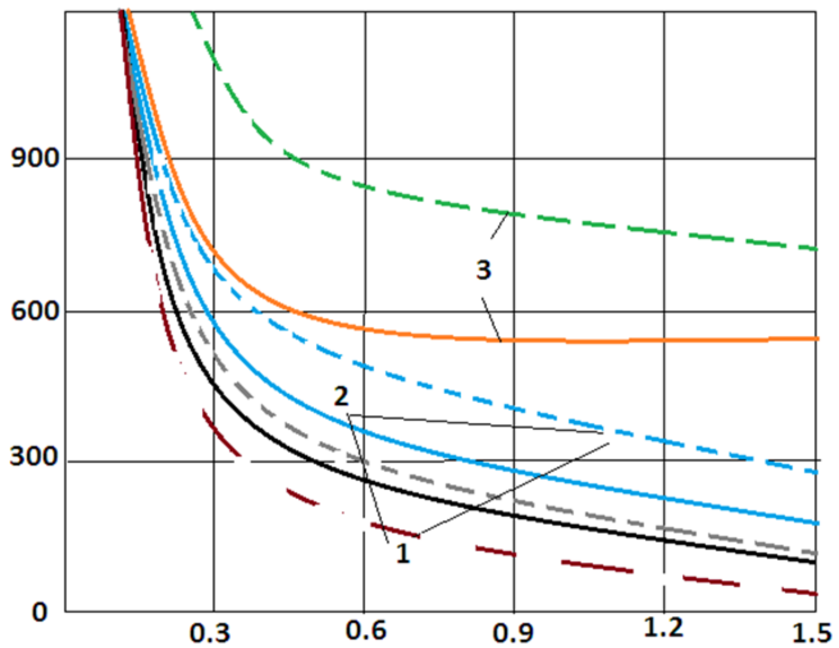

Figure 1. Changing ω_{R1} in function $\frac{1}{M_R}$

Numerical results show that for the first mode of motion, the results of calculations for the shell theory and the elasticity theory are almost identical, and the influence of the inertia of the medium on the complex oscillation frequencies is particularly significant for thin shells ($h_R < 0.025$). If the inertia of the medium is not taken into account, the value of the first frequency is overstated.

The real and imaginary parts of the third frequency are almost unaffected by the inertia of the medium. So, for a cylindrical shell with a thickness of $h_R < 0.064$ ($R_G = 0$) the obtained values of the real parts of complex frequencies practically coincide with the exact ones. If the shell motion is described by the equations of elasticity theory, a second mode (curve 2) appears, caused by the presence of a viscoelastic environment.

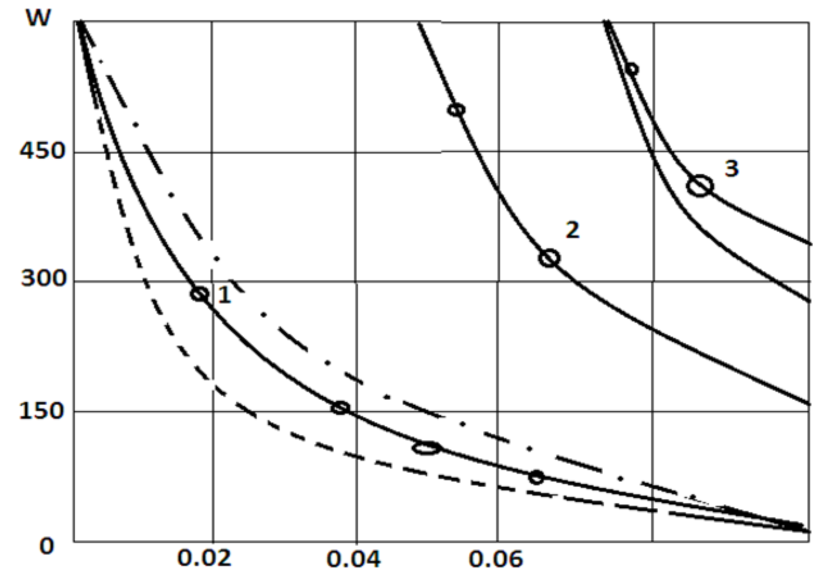

Figure 2. Changing ω_{R1} in function of the wavelength $\frac{R}{\lambda}$

Obtained the numerical results that differ from the known results of V. A. Dubrovsky [6, 7] by up to 10-15%.

4. Conclusions

1. The theory and methods for studying the natural vibrations of a cylindrical shell located in a viscoelastic medium are constructed.
2. An algorithm and computer calculation program have been developed to determine the complex eigenfrequencies and vibration modes of a cylindrical shell in a viscoelastic medium. using the functions of Bessel, Hankel, and the methods of Mueller and Gauss.
3. The natural vibrations of cylindrical shells in an infinite viscoelastic medium are studied for various parameters of both the shell and the medium.
4. The problem considered when using the radiation conditions for a reduced infinite region to a finite one has a nonzero solution in the class of infinitely differentiable functions. It is shown that the problem may have a discrete spectrum.
5. It was revealed that at some values of the viscoelastic density parameters of the system, low-frequency eigen-oscillations arise. In this case, the imaginary parts of the natural switching frequency can be commensurate with the real one. This can lead to some aperiodic movements, and the obtained results allow us to predict the scattering of viscoelastic (or seismic) waves in deformable media in the presence of an inclusion.

Reference

[1] Kabisov K 2010 Oscillations and wave processes (Moscow, Kom Book)
[2] Mirsaidov M M, Sultanov T Z, Abdikarimov R A, Ishmatov A N, Yuldoshev B S, Toshmatov E S and Jurayev D P 2018 Strength parameters of earth dams under various dynamic effects *Mag Civ Eng* doi.org/10.18720/MCE.77.9
[3] Grigorenko Y M 1996 Approaches to the numerical solution of linear and nonlinear problems in shell theory in classical and refined formulations *Int Appl Mech* doi.org/10.1007/BF02088409
[4] Grigorenko Ya M, Vasilenko AT and Bespalov E I 1975 et al Numerical solution of boundary value problems of the statics of orthotropic shells with variable parameters Kiev Nauk Dumka

[5] Kovaleva A, Manevitch L, Kosevich Yu. 2011 Fresnel integrals and irreversible energy transferin an oscillatory system with time-dependent parameters *Physical Review E* **83(1)** p 12
[6] Brinks R 2008 On the convergence of derivatives of B-splines to derivatives of the Gaussian function *Comp Appl Math* **27** pp 1-17
[7] Mirsaidov M M, Abdikarimov R A, Vatin N I, Zhgutov V M, Khodzhaev D A and Normuminov B A 2018 Nonlinear parametric oscillations of viscoelastic plate of variable thickness *Mag Civ Eng* doi.org/10.18720/MCE.82.11
[8] PanovkoYa G 2010 Fundamentals of the applied theory of oscillations and shock (Moscow, Librocom)
[9] Cherentsov D A 2014 Study of damped oscillations manometric tubular spring in a viscous medium *News of higher educational institutions* Oil and gas **5** pp 66-72
[10] Mirsaidov M, Mekhmonov Ya 1987 Nonaxisymmetric vibrations of axisymmetric structures with associated masses and hollows (protrusions) *Strengthof Materials* **19(3)** pp 424-430 DOI: 10.1007/BF01524147
[11] GrigorenkoYa M, Vlaikov G G, 2006 Numerical and analytical solution of problems of shell mechanics based on various models (Kiev Acad Period)
[12] Korotkov AV, Kulikov Yu A 2009 Free vibrations of multilayer curved pipes made of reinforced plastics *Mech Compos Mater Struct* **15** pp 203–220
[13] Koltunov M A, Maiboroda V P and Zubchaninov V G 1983 Strength calculations of products from polymeric materials (Moscow Mech Eng)
[14] Safarov I I, Koldashov N U and Boltaev Z I 2018 Oscillations and waves in a layered homogeneous Viscoelastic medium *Int J new Eng Res Technol* **6** pp 27–32
[15] Amabili M, Garziera R 2002 Vibrations of circular cylindrical shells with nonuniform constraints, elastic bed and added mass *Part II Shells containing or immersed in axial flow J Fluids Struct* doi.org/10.1006/jfls.2001.0402
[16] GrigorenkoYa M, Rozhok L S 2011 Analysis of Stress State of Hollow Layered Cylinders with the CrossSection of Complex- Shaped *Int J Appl Mech* **47** pp 48–57
[17] Grigorenko Y M, Rozhok L S 2010 Influence of curvature on the stress state of hollow cylinders with complex-shaped noncircular cross-section *Int Appl Mech* doi.org/10.1007/s10778-010-0362-9
[18] Teshaev M Kh, Safarov I I and Mirsaidov M 2019 OSCILLATIONS OF MULTILAYER VISCOELASTIC COMPOSITE TOROIDAL PIPES *J Serbian Soc Comput Mech* doi.org/10.24874/jsscm
[19] Koltunov M A, Mirsaidov M and Troyanovskii I E 1978 Transient vibrations of axissymmetric viscoelastic shells *Polym Mech* doi.org/10.1007/BF00857468
[20] Brinks R. 2008 On the convergence of derivatives of B-splines to derivatives of the Gaussian function *Comp Appl Math* **27** pp 1-17 doi.10.18720/MCE.85.7
[22] Sultanov K S, Loginov P V, Ismoilova S I and Salikhova Z R 2019 Quasistaticity of the process of dynamic strain of soils *Magazine of Civil Engineering* **1** pp 71–91 doi.org/10.1016/j.jappmathmech
[23] Bakhodirov A A, Ismailova S I and Sultanov K S 2015 Dynamic deformation of the contact layer when there is shear interaction between a body and the soil *Journal of Applied Mathematics and Mechanics* **79(6)** pp 587–595
[24] Wulfson II 2011Common-mode and spatial localization of oscillations of the working bodies of cyclic machines with a symmetrical dynamic structure Problems of mechanical engineering and machine reliability **1** pp 12–18