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Thermodynamic bases of mechanical working of metals by cutting

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Abstract. Thermo-mechanical phenomena that occur during the metals cutting in a cylindrical form are difficult in terms of systematic and interconnected mathematical description for a three-dimensional deformable body. In this connection, empirical formulas of the cutter interaction with the processed surface of metal products are used in calculations. In this paper, we propose the most general approach for modeling the process of metalworking by thermodynamically nonlinear strain. Using the non-classical theory of deformation, the three-dimensional problem of metalworking by cutting is reduced to a two-dimensional one. The proposed approach made it possible to compose resolving equations and boundary conditions for the problem under consideration, taking into account heat release and wear on contact sur faces with a cutting tool.

1. Introduction

Currently, in industrial production, the approaches to develop "smart technologies" with machine learning are becoming relevant; on the bases of these approaches lie the simulation modeling. In such studies, it is necessary to justify, systematize and ensure the interconnection of the theory of friction, wear and filings formation, the thermodynamics in the mechanical working of metals by cutting, ensuring the reliability of the cutting tool.

High-precision technologies for the production of industrial items are closely related to thermo-mechanical processing of metal parts surface. The machining of metal cylinders by cutting is accompanied by thermodynamic processes with significant heat release. The heat source in metal cutting is the work spent on the finite strain in the cut off a layer, and the work to overcome friction on the rear surfaces of the cutter. During the finite strain in cutting, the material points of the sample move relative to each other forming an additional source of heat build-up [1, 2, 3, 4, 5].

It is known that the process of thermo-mechanical cutting of rigid bodies, due to the complexity of the processes occurring there, is difficult to model mathematically [6, 7, 8, 9, 10]. Here, at the tip of the cutter, eleven relatively independent phenomena are focused that have not yet come to a holistic description. These are the filings formation processes, the mechanics and thermodynamics of metal cutting, the theory of friction and wear during metalworking, the theory of resistance, and reliability of cutting tools.

Here we propose a thermomechanical theory of cutting cylindrical metals based on the fundamental laws of thermodynamics and nonlinear mechanics of a deformable rigid body. In this process, along the cutting line (due to the occurrence of critical internal stresses at the tip of the cutting tool) metal filings tear off from the base. In this case, after the tear-off, the filings in the form of a metal strip make finite displacements [11, 12, 13, 14, 15, 16].

2. Methods

Mechanical processing of metals by cutting is accompanied by significant thermodynamic processes. The source of heat when cutting metals is the work spent on deformations in the cut layer and the layers adjacent to the treated surface and the cutting surface and on overcoming friction on the front and back surfaces of the cutter. During the deformation process, metal grains move relative to each other, this is a source of heat generation.

3. Results and discussion

The heat released during the cutting process is not concentrated in the places of its formation, but according to the laws of thermodynamics it spreads over the volume of the sample, from points with a higher temperature to points with a lower temperature. About 95% of the mechanical work of deformation and friction passes into heat when cutting metals.

Besides, when cutting metals, heat occurs due to the friction of the chip on the front surface of the cutter and the friction of the back surface of the cutter on the cutting surface. Therefore, when cutting metals, chips and contact surfaces are heated in the range of 500-1000°, while the generated heat is mainly absorbed by chips – 50-86%, by cutter – 10-40% and by workpiece – 3-9%, about 1% heat is radiated into the surrounding space [11].

The stress state under nonlinear deformation of a cylindrical body for a stationary mode of thermomechanical processing in the absence of volumetric forces, in the Euler coordinate system, is described by the equilibrium equation [7, 12, 13].

$$\begin{aligned} &\nabla_j P_{ij} + P_{iz},_z - P_{22}/x_1\delta_{i1} + P_{21}/x_1\delta_{i2} = 0, \\ &\nabla_j P_{zj} + P_{zz},_z = 0,\ \nabla_1 = \frac{\partial}{\partial x_1} + \frac{1}{x_1},\ \nabla_2 = \frac{\partial}{x_1\partial x_2}. \end{aligned} \tag{1}$$

when boundary conditions are met

$$\begin{aligned} &U_i\big|_{\Sigma_1} = U_i^{\Sigma}, \quad U_z\big|_{\Sigma_1} = U_z^{\Sigma}, \\ &\left[P_{ij}n_j + P_{rz}n_z\right]\Big|_{\Sigma_2} = S_i,\ \left[P_{zj}n_j + P_{zz}n_z\right]\Big|_{\Sigma_2} = S_z. \end{aligned} \tag{2}$$

Where $P_{ij},\ P_{zz}, P_{iz}, P_{zi},\ U_i, U_z$ are the components of the second Piola - Kirchhoff tensor and the displacement vector, -are the displacements $\Sigma=\Sigma_1+\Sigma_2$given on the part of the boundary, $U_i^0,\ V_i^0,\ U_z^0,\ V_z^0$ are the coordinate functions characterizing the initial state of the body, n_j, n_z are the external normal.

According to the theory of V.V. Novozhilov, nonlinear deformation of the elementary volume of the considered three-dimensional body is accompanied by finite rotations and displacements but with small relative changes in geometric dimensions.

In these conditions, the actual (after deformation) curvilinear triple-orthogonal coordinate system $O\xi_1\xi_2\xi_3$ practically remains unchanged, i.e. the condition $\xi_1 \approx x_1, \quad \xi_2 \approx x_2, \quad \xi_1 \approx z,$

Then the second Piola-Kirchhoff tensor P_{ij} through the symmetric Cauchy tensor σ_{kj} in the initial coordinate system is expressed as follows [7]

$$P_{ij} = \left(\delta_{ik} + U_{i,k}\right)\sigma_{kj},, \tag{3}$$

The defining relations between the symmetric Cauchy stress tensor σ_{ij} and the finite strain tensor for elastic isotropic materials in the thermodynamic form of recording the defining Duhamel-Neumann relations take the following form

$$\sigma_{ij} = \lambda\left(e - \beta\vartheta\right)\delta_{ij} + 2\mu e_{ij}, \ \vartheta = T - T_0 \tag{4}$$

Where

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}$$

Lame coefficients, E is the modulus of elasticity, ν is Poisson's ratio; T, T_0 are the temperature of the current and initial state, β is characterizing the influence of temperature stresses.

The Fourier heat equations for the steady-state process in cylindrical bodies will have the following form

$$c_\varepsilon\dot{\vartheta} = \lambda_\varepsilon\left(\frac{\partial}{\partial x_1} + \frac{\partial}{x_1\partial x_2} + \frac{\partial}{\partial z}\right)^2 \vartheta - T\beta\dot{e}_{kk}, \tag{5}$$

Where c_ε, λ_ε – coefficients of heat capacity and thermal conductivity with constant deformation.

Thus, we have the general task of the nonlinear thermodynamic theory of elasticity for thermal-mechanical processing of metal parts by cutting method solving which we

can study the stress-strain state of the objects under consideration, including metal strips and formed chips. Due to the difficulties encountered in solving nonlinear three-dimensional problems (1)-(5) in applied problems the mechanics of a deformable solid usually turn to two-dimensional problems.

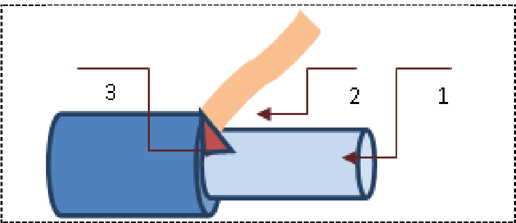

Figure 1. Schematic description of the process of thermomechanical processing of a metal cylinder by cutting: 1-cylindrical sample after thermomechanical processing; 2-formed metal chips; 3-cutter.

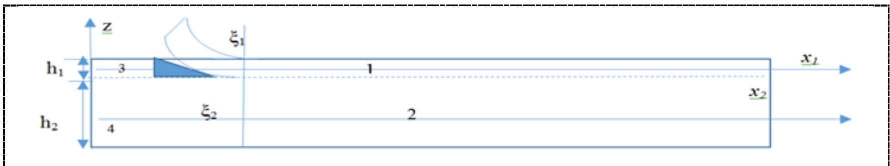

Figure 2. Kinematics of chip formation during machining. Zone 1, 3 the strip above the cutting tool before and after the impact of the cutting tool. Zone 2, 4 the corresponding area under the cutting tool

In this problem, the stress-strain state of a metal cylinder before the formation of chips is described in the framework of the linear thermodynamic theory of elasticity

$$\begin{cases} P_{ij} = \sigma_{ij}, \\ \varepsilon_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right), \end{cases}$$

After the impact of the cutter, at the time of chip formation, the metal strip undergoes final displacements, and the strip bending is accompanied by a predominance of deflection compared to the others, i.e. the condition $U_z >> U_i$is satisfied.

Therefore, in expressions (1)-(9), when the plates are bent by nonlinear terms along the U_i displacement vector axial displacements they are usually neglected. At that, the nonlinear components of the derivative $U_z$with respect to the normal coordinate z are also taken to be negligible.

Then, for the components of the second Piola – Kirchhoff tensor and the Lagrange – Green strain tensor, we have:

$$\begin{gathered} P_{ij} = \sigma_{ij}, P_{iz} = \sigma_{iz}, P_{zz} = \sigma_{zz},\ P_{zj} = \left(\delta_{zk} + \nabla_k U_z - \frac{U_z}{x_1}\delta_{k1}\right)\sigma_{kj}, \\ \varepsilon_{11} = U_{1,1} + \frac{1}{2}U_{z,1}U_{z,1},\quad \varepsilon_{22} = U_{2,2}/x_1 + U_1/x_1 + \frac{1}{2x_1^2}U_{z,2}U_{z,2},\quad \varepsilon_{zz} = U_{z,z}, \\ \varepsilon_{z1} = \frac{1}{2}\left(U_{z,1} + U_{1,z}\right),\quad \varepsilon_{z2} = \frac{1}{2}\left(U_{z,2}/x_1 + U_{2,z}\right), \\ \varepsilon_{21} = \frac{1}{2}\left(U_{2,1} + U_{1,2}/x_1 + U_{z,1}U_{z,2}/x_1\right), \varepsilon_{kk} = \nabla_k U_k + \frac{1}{2}U_{z,k}U_{z,k}. \end{gathered} \tag{6}$$

The design diagram of a three-dimensional deformable body is a metal cylindrical disk with a thickness $h=h_1 + h_2$ in the direction of Oz, with a radius-R on which the cutter acts (Fig. 2) Mentally carved from a cylindrical body. In this case, the strip-1 with a thickness of h_1 is above the cutting tool and the strip-2 with a thickness of h_2 below it. Cylindrical-coordinate systems $Ox_1x_2\xi_i$ $,i=1,2$ are located in the mid-plane of the bands and disk.

Here it should be noted that the thickness h_2 of a mentally cut disk from a metal cylinder, and even more so the thickness h_1 of the formed chip, is small compared to its radius-R. In the process of mechanical processing of metals by cutting, the formation of chips is accompanied by final rotations and movements, but with small relative changes in geometric dimensions, relative to the coordinate plane

These bands, until the moment of impact of the cutter $x > ct$, represent a single whole composition, therefore, along the line of separation of regions 1 and 2 ($\xi_2 = \frac{h_2}{2}$ or $\xi_1 = -\frac{h_1}{2}$) for homogeneous isotropic bodies, the continuity conditions are met:

$$\begin{gathered} U_z^{(1)} = U_z^{(2)},\quad U_i^{(1)} = U_i^{(2)},\quad \vartheta^{(1)} = \vartheta^{(2)}. \\ \left(U_z^{(1)}\right)_{,\xi} = \left(U_z^{(2)}\right)_{,\xi},\ \left(U_i^{(1)}\right)_{,\xi} = \left(U_i^{(2)}\right)_{,\xi},\ \left(\vartheta^{(1)}\right)_{,\xi} = \left(\vartheta^{(2)}\right)_{,\xi}. \end{gathered} \tag{7}$$

In the process of thermomechanical processing of metals by cutting at the tip of the cutter there is infinite tangential stress in the direction x_2, resulting in the chip separation from the base(area 3). After the impact of the cutter: $x_2 \le ct$ on the upper plane of the lower band $\xi_2 = \frac{h_2}{2}$ (area 4), tangential stresses occur due to the friction of the cutter when moving on the plane of the selected disk Ox_1x_2 Then the following boundary conditions take place:

$$\sigma_{2z}^{(4)} = kP_z H(x_2 - ct)H(ct + a - x_2), \quad \sigma_{1z}^{(4)} = 0, \quad U_z^{(4)} = 0,$$
$$\left(\vartheta^{(4)}\right)_{,z} = \gamma\left[\vartheta^{(2)} + \vartheta_{tr} H(x_2 - ct)H(ct + a - x_2)\right]. \quad (8)$$

Under these conditions, the following boundary conditions occur at the lower border $\xi_1 = -\frac{h_1}{2}$ of the upper band (area 3) when the cutter is tangentially impacted P_θ the following boundary conditions apply

$$\sigma_{2z}^{(3)} = P_\theta \delta(Rx_2 - ct), \quad \sigma_{1z}^{(3)} = 0,$$
$$U_z^{(3)} = ct\, tg\eta + f_T \vartheta^{(3)}, \quad \left(\vartheta^{(3)}\right)_{,\xi} = \gamma\vartheta^{(3)}. \quad (9)$$

Before and after the impact of the cutter, the following boundary conditions for the corresponding components of the stress tensor and the temperature field take place in the lower bounds of the lower cylindrical disk $\xi_2 = -\frac{h_2}{2}$ area 2,4), as well as for the upper plane $\xi_1 = \frac{h_1}{2}$ of the upper band (area 1,3) there are the following boundary conditions for the corresponding components of the stress tensor and the temperature field (they are written without an upper index):

$$\sigma_{2z} = 0, \quad \sigma_{1z} = 0, \quad \sigma_{zz} = 0, \quad (\vartheta)_{,\xi} = \gamma\vartheta. \quad (10)$$

In the above expressions, c is the rate of displacement of the cutter along the cylinder forming part. k is the coefficient of friction when the cutter interacts with the sample, at a given temperature-T_0. η is the angle of attack of the cutter, f_T is the coefficient reflecting the influence of temperature on the chip displacement.

The desired movement is represented as:

$$U_i^{(I)} = u_i^{(I)} + \xi\psi_i^{(I)} - \Phi_1^{(I)}(\xi)C_i^{(I)} - \frac{3h^2}{5}\Phi_2^{(I)}(\xi)D_i^{(I)},$$
$$U_z^{(I)} = w^{(I)} + \xi V^{(I)} - \Phi_1^{(I)}(\xi)\theta^{(I)}, \quad (11)$$
$$\vartheta^{(I)} = \vartheta_0^{(I)} + \xi\vartheta_1^{(I)} - \Phi_1^{(I)}(\xi)\vartheta_2^{(I)}.$$

Where

$$u_i^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_i^{(I)} d\xi, \quad \psi_i^{(I)} = \frac{12}{h_I^3}\int_{-h_I/2}^{h_I/2} U_i^{(I)}\xi d\xi, \quad w^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_z^{(I)} d\xi,$$
$$V^{(I)} = \frac{12}{h_I^3}\int_{-h_I/2}^{h_I/2} U_z^{(I)}\xi d\xi, \quad \vartheta_0^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} \vartheta^{(I)} d\xi, \quad \vartheta_1^{(I)} = \frac{12}{h_I^3}\int_{-h_I/2}^{h_I/2} \vartheta^{(I)}\xi d\xi,$$
$$\Phi_1^{(I)}(\xi) = \frac{h_I^2}{12}\left[1 - 12\left(\frac{\xi}{h_I}\right)^2\right], \quad \Phi_2^{(I)}(\xi) = \frac{1}{4}\left[1 - \frac{20}{3}\left(\frac{\xi}{h_I}\right)^2\right]\xi.$$

In (11), in contrast to traditional representations, 8 new integral functions for radial, meridional coordinates and time $-C_i, D_i, V, \theta, \vartheta_1, \vartheta_2$. This is the number of unknowns required to meet the eight boundary conditions set on the face planes of the cylindrical disks under consideration.

Taking into account (4), for elastic isotropic plates, it is possible to obtain analytical expressions for nonlinear components of the stress tensor via integral quantities.

Satisfying the boundary condition (7) will lead to the following relations

$$u_i^{(1)} - \frac{h_1}{2}\psi_i^{(1)} + \frac{h_1^2}{6}C_i^{(1)} - \frac{h_1^3}{20}D_i^{(1)} = u_i^{(2)} + \frac{h_2}{2}\psi_i^{(2)} + \frac{h_2^2}{6}C_i^{(2)} + \frac{h_2^3}{20}D_i^{(2)},$$
$$w^{(1)} - \frac{h_1}{2}V^{(1)} + \frac{h_1^2}{6}\theta^{(1)} = w^{(2)} + \frac{h_2}{2}V^{(2)} + \frac{h_2^2}{6}\theta^{(2)} \tag{12}$$
$$\vartheta_0^{(1)} - \frac{h_1}{2}\vartheta_1^{(1)} + \frac{h_1^2}{6}\vartheta_2^{(1)} = \vartheta_0^{(2)} + \frac{h_2}{2}\vartheta_1^{(2)} + \frac{h_2^2}{6}\vartheta_2^{(2)}$$

$$\psi_i^{(1)} - h_1 C_i^{(1)} + \frac{3h_1^2}{5}D_i^{(1)} = \psi_i^{(2)} + h_2 C_i^{(2)} + \frac{3h_2^2}{5}D_i^{(2)},$$
$$V^{(1)} - h_1\theta^{(1)} = V^{(2)} + h_2\theta^{(2)}, \tag{13}$$
$$\vartheta_1^{(1)} - h_1\vartheta_2^{(1)} = \vartheta_1^{(2)} + h_2\vartheta_2^{(2)}$$

By solving the last two equations (12) and (13) with respect to unknowns, $V^{(2)}, \theta^{(2)}, \vartheta_1^{(2)}, \vartheta_2^{(2)}$ one can write

$$V^{(2)} = 3\frac{w^{(1)} - w^{(2)}}{h_2} - \frac{1}{2}\left(1 + 3\frac{h_1}{h_2}\right)V^{(1)} + \frac{h_1}{2}\left(1 + \frac{h_1}{h_2}\right)R^{(1)},$$
$$\vartheta_1^{(2)} = 3\frac{\vartheta_0^{(1)} - \vartheta_0^{(2)}}{h_2} - \frac{1}{2}\left(1 + 3\frac{h_1}{h_2}\right)\vartheta_1^{(1)} + \frac{h_1}{2}\left(1 + \frac{h_1}{h_2}\right)\vartheta_2^{(1)}, \tag{14}$$
$$h_2\theta^{(2)} = -3\frac{w^{(1)} - w^{(2)}}{h_2} + \frac{3}{2}\left(1 + \frac{h_1}{h_2}\right)V^{(1)} - \frac{h_1}{2}\left(3 + \frac{h_1}{h_2}\right)\theta^{(1)},$$
$$h_2\vartheta_2^{(2)} = -3\frac{\vartheta_0^{(1)} - \vartheta_0^{(2)}}{h_2} + \frac{3}{2}\left(1 + \frac{h_1}{h_2}\right)\vartheta_1^{(1)} - \frac{h_1}{2}\left(3 + \frac{h_1}{h_2}\right)\vartheta_2^{(1)}.$$

Solving the first two equations (12) and (13) with respect to unknowns $h_2 C_i^{(2)}, h_2^2 D_i^{(2)}$, , we have

Solving the first two equations (12) and (13) with respect to the unknowns we will have

$$h_2 C_i^{(2)} = 2\left(1 + \frac{h_1}{h_2}\right)h_1 C_i^{(1)} + \frac{3}{5}\left(1 + \frac{h_1}{h_2}\right)h_1^2 D_i^{(1)} - 12\frac{u_i^{(1)} - u_i^{(2)}}{h_2} - 5\psi_i^{(2)} +$$
$$+3\left(1 + 2\frac{h_1}{h_2}\right)\psi_i^{(1)}, \tag{15}$$
$$h_2^2 D_i^{(2)} = \frac{10}{3}\left(1 + \frac{h_1}{h_2}\right)h_1 C_i^{(1)} + \left(2 + \frac{h_1}{h_2}\right)h_1^2 D_i^{(1)} +$$
$$+20\frac{u_i^{(1)} - u_i^{(2)}}{h_2} - \frac{20}{3}\psi_i^{(2)} + \frac{10}{3}\left(1 + 3\frac{h_1}{h_2}\right)\psi_i^{(1)}.$$

To meet the boundary conditions for tangential stresses in (10) by $\xi_2 = -\frac{h_2}{2}$ on the lower plane, as well as for the upper plane $\xi_1 = \frac{h_1}{2}$:

$$\psi_i^{(1)} + h_1 C_i^{(1)} + \frac{3h_1^2}{5}D_i^{(1)} + \left(w^{(1)} + \frac{h_1}{2}V^{(1)} + \frac{h_1^2}{6}\theta^{(1)}\right)_{,i} = 0, \tag{16}$$
$$\psi_i^{(2)} - h_2 C_i^{(2)} + \frac{3h_2^2}{5}D_i^{(2)} + \left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\left(w^{(2)} - \frac{h_2}{2}V^{(2)} + \frac{h_2^2}{6}R^{(2)}\right) = 0.$$

taking into account the expressions for the components of the stress tensor and the relations (14) and (15) and the resulting equations resolving relatively unknown ones $h_1C_r^{(1)},\ h_1C_\theta^{(1)},\ h_1^2D_r^{(1)},\ h_1^2D_\theta^{(1)}$, we will have:

$$h_1C_i^{(1)} = 24\frac{u_i^{(1)}-u_i^{(2)}}{h_2}+\psi_i^{(2)}-\psi_i^{(1)}-$$
$$-\left(3w^{(2)}-3w^{(1)}+\frac{1}{2}\left(h_2+h_1\right)V^{(1)}-\frac{1}{2}\left(h_2+h_1\right)h_1R^{(1)}\right)_{,i} \tag{17}$$
$$h_1^2D_i^{(1)} = -40\frac{u_i^{(1)}-u_i^{(2)}}{h_2}-\frac{10}{3}\psi_i^{(2)}-\frac{5}{3}\psi_i^{(1)}-$$
$$-\frac{5}{3}\left(3w^{(2)}-2w^{(1)}+\frac{1}{2}\left(h_2+2h_1\right)V^{(1)}-\frac{1}{6}\left(3h_2+2h_1\right)h_1R^{(1)}\right)_{,i}.$$

Boundary conditions for normal stresses and temperature fields from (10) lead to the following system of equations

$$V^{(1)}+R^{(1)}h_1+\frac{\nu}{1-\nu}\left(\varepsilon_{kk}^{(1)}-\beta\vartheta^{(1)}\right)=0,$$
$$\vartheta_1^{(1)}+h_1\vartheta_2^{(1)}=\gamma\left(\vartheta_0^{(1)}+\frac{h_1}{2}\vartheta_1^{(1)}+\frac{h_1^2}{6}\vartheta_2^{(1)}\right), \tag{18}$$
$$V^{(2)}-R^{(2)}h_2+\frac{\nu}{1-\nu}\left(\varepsilon_{kk}^{(2)}-\beta\vartheta^{(2)}\right)=0,$$
$$\vartheta_1^{(2)}-h_2\vartheta_2^{(2)}=\gamma\left(\vartheta_0^{(2)}-\frac{h_2}{2}\vartheta_1^{(2)}+\frac{h_2^2}{6}\vartheta_2^{(2)}\right).$$

In the heat transfer equations in (18) substituting the corresponding expressions from (14) and the resulting equations, resolving the relatively unknown $\vartheta_1^{(1)},\ \vartheta_2^{(1)}$:

$$h_1\vartheta_2^{(1)}=a_1\vartheta_0^{(1)}+a_2\vartheta_0^{(2)},\quad \vartheta_1^{(1)}=b_1\vartheta_0^{(1)}+b_2\vartheta_0^{(2)},$$
$$h_2\vartheta_2^{(2)}=c_1\vartheta_0^{(1)}+c_2\vartheta_0^{(2)},\quad \vartheta_1^{(2)}=d_1\vartheta_0^{(1)}+d_2\vartheta_0^{(2)},$$
$$\vartheta^{(1)}\left(\pm\frac{h_1}{2}\right)=\vartheta_0^{(1)}\pm\frac{h_1}{2}\vartheta_1^{(1)}+\frac{h_1^2}{6}\vartheta_2^{(1)}, \tag{19}$$
$$\vartheta^{(2)}\left(\pm\frac{h_2}{2}\right)=\vartheta_0^{(2)}\pm\frac{h_2}{2}\vartheta_1^{(2)}+\frac{h_2^2}{6}\vartheta_2^{(2)}$$
$$a_0=2+\frac{\gamma h_2}{2}+\frac{h_1}{h_2}+\frac{\gamma h_1}{3}+\frac{1}{3}\frac{6-\gamma h_1}{2-\gamma h_1}\left(2+\frac{3}{4}\gamma h_2+3\frac{h_1}{h_2}+\frac{5}{4}\gamma h_1\right),$$
$$a_1=\frac{1}{a_0}\left[\frac{2\gamma}{2-\gamma h_1}\left(2+\frac{3}{4}\gamma h_2+3\frac{h_1}{h_2}+\frac{5}{4}\gamma h_1\right)-2\frac{3+\gamma h_2}{h_2}\right],$$
$$a_2=\frac{1}{a_0}\left[\gamma+2\frac{3+\gamma h_2}{h_2}\right],\ b_1=\frac{2}{2-\gamma h_1}\left[\gamma-\left(1-\frac{\gamma h_1}{6}\right)a_1\right],$$
$$b_2=-\frac{2}{2-\gamma h_1}\left(1-\frac{\gamma h_1}{6}\right)a_1.$$

$$c_1 = -\frac{3}{h_2} + \frac{3}{2}\left(1+\frac{h_1}{h_2}\right)b_1 - \frac{1}{2}\left(3+\frac{h_1}{h_2}\right)a_1,\ c_2 =$$
$$= \frac{3}{h_2} + \frac{3}{2}\left(1+\frac{h_1}{h_2}\right)b_2 - \frac{1}{2}\left(3+\frac{h_1}{h_2}\right)a_2,$$
$$d_1 = \frac{3}{h_2} - \frac{1}{2}\left(1+3\frac{h_1}{h_2}\right)b_1 + \frac{1}{2}\left(1+\frac{h_1}{h_2}\right)a_1,$$
$$d_2 = \left[-\frac{3}{h_2} - \frac{1}{2}\left(1+3\frac{h_1}{h_2}\right)b_2 + \frac{1}{2}\left(1+\frac{h_1}{h_2}\right)a_2\right].$$

Expressions from (14), (15), (17) and (19) substituting in the corresponding equations (18) we get the following system of differential equations for determining $V^{(1)}, R^{(1)}$:

$$\begin{aligned}
&\nabla_k\nabla_k\left(a_{11}V^{(1)} + a_{12}h_1\theta^{(1)} + a_{17}W^{(1)} + a_{18}W^{(2)}\right) + \\
&+\nabla_i\left(a_{13}u_i^{(1)} + a_{14}u_i^{(2)} + a_{15}h_1\psi_i^{(1)} + a_{16}h_2\psi_i^{(2)}\right) - \\
&\beta\vartheta^{(1)}\left(\frac{h_1}{2}\right) + \frac{1-\nu}{\nu}\left(V^{(1)} + h_1\theta^{(1)}\right) = 0, \\
&\nabla_k\nabla_k\left(a_{21}V^{(1)} + a_{22}h_1\theta^{(1)} + a_{27}W^{(1)} + a_{28}W^{(2)}\right) + \\
&+\nabla_i\left(a_{23}u_i^{(1)} + a_{24}u_i^{(2)} + a_{25}h_1\psi_i^{(1)} + a_{26}h_2\psi_i^{(2)}\right) - \\
&\beta\vartheta^{(2)}\left(-\frac{h_2}{2}\right) + \frac{1-\nu}{\nu} + \\
&+\left(6\frac{w^{(1)} - w^{(2)}}{h_2} - \frac{2h_2 + 3h_1}{h_2}V^{(1)} + h_1\frac{2h_2 + h_1}{h_2}\theta^{(1)}\right) = 0,
\end{aligned} \tag{20}$$

$$a_{11} = \frac{1}{8}h_1\left(h_1 + h_2\right);\ a_{12} = \frac{1}{72}\left(6h_2 + 5h_1\right)h_1;\ a_{13} = 1 + 2\frac{h_1}{h_2};$$
$$a_{14} = -2\frac{h_1}{h_2};\ a_{15} = \frac{h_1}{3};$$
$$a_{16} = 0;\quad a_{17} = \frac{2h_1}{3};\quad a_{18} = -\frac{3h_1}{4};$$
$$a_{21} = \frac{1}{12}\left[\left(h_1 + h_2\right)^2 - \frac{1}{2}\left(4h_2 + 7h_1\right)(h_2 + 2h_1)\right];$$
$$a_{22} = -\frac{1}{12}\left[\left(h_1 + h_2\right)^2 - \frac{1}{6}\left(4h_2 + 7h_1\right)(h_2 + 2h_1)\right];$$
$$a_{23} = -11h_2 - 14h_1;\quad a_{24} = 11h_2 + 14h_1;\ a_{25} = \frac{2h_2 + h_1}{12};$$
$$a_{26} = -\frac{h_2 + 8h_1}{6};\ a_{27} = \frac{7h_2 + 10h_1}{6};\quad a_{28} = -\frac{6h_2 + 9h_1}{4};$$

The boundary conditions (8) and (9) are met with the following expressions for the desired values

$$
\begin{aligned}
&h_1\vartheta_2^{(3)} = a_3\vartheta_0^{(3)}, \quad \vartheta_1^{(3)} = b_3\vartheta_0^{(3)},\ h_1\vartheta_2^{(4)} = a_4\vartheta_0^{(4)},\\
&\vartheta_1^{(4)} = b_4\vartheta_0^{(4)}, b_3 = \frac{12\gamma}{12-\gamma^2 h_1^2}, \quad a_3 = \frac{6\gamma^2 h_1}{12-\gamma^2 h_1^2},\\
&b_4 = \frac{12\gamma}{12-\gamma^2 h_2^2}, \quad a_4 = \frac{6\gamma^2 h_2}{12-\gamma^2 h_2^2},\\
&\vartheta^{(3)}\left(\pm\frac{h_1}{2}\right) = \left[1 \pm \frac{6\gamma h_1}{12-\gamma^2 h_1^2} + \frac{\gamma^2 h_1^2}{12-\gamma^2 h_1^2}\right]\vartheta_0^{(3)},\\
&\vartheta^{(4)}\left(\pm\frac{h_2}{2}\right) = \left[1 \pm \frac{6\gamma h_2}{12-\gamma^2 h_2^2} + \frac{\gamma^2 h_2^2}{12-\gamma^2 h_2^2}\right]\vartheta_0^{(4)} +\\
&\left[1 \pm \frac{6+\gamma h_2}{12-\gamma^2 h_2^2} + \frac{6+\gamma h_2}{12-\gamma^2 h_2^2}\gamma h_2\right]\frac{\gamma h_2}{2}\\
&\vartheta_{tr} H(\theta - ct)H(ct + a - \theta).
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&h_1^2 D_i^{(3)} = \frac{5}{3}\left[\frac{P_\theta}{2}\delta(x_2 - ct)\delta_{i2} - \psi_i^{(3)} - \left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\left(W^{(3)} + \frac{1}{6}h_1^2\theta^{(3)}\right)\right],\\
&h_1 C_i^{(3)} = -\left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\frac{h_1}{2}V^{(3)} - \frac{P_\theta}{2}\delta(x_2 - ct)\delta_{i2}, \quad h_2^2\theta^{(4)} = -6\left(W^{(4)} + \frac{h_2}{2}V^{(4)}\right),\\
&h_2^2 D_i^{(4)} = \frac{5}{3}\left[\frac{P_\theta}{2}H(x_2 - ct)H(ct + a - x_2)\delta_{i2} - \psi_i^{(4)} - \left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\left(W^{(4)} - \frac{1}{6r}h_2^2\theta^{(4)}\right)\right],\\
&h_1 C_i^{(4)} = -\left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\frac{h_1}{2}V^{(4)} + \frac{P_\theta}{2}H(x_2 - ct)H(ct + a - x_2)\delta_{i2},\\
&V^{(3)} = \frac{1}{2(1-\nu)}\left[\nabla_i\left(u_i^{(3)} - \frac{7h_1}{6}\psi_i^{(3)}\right) - \frac{\alpha}{6h_1}\nabla_i\nabla_i\vartheta^{(3)}\left(-\frac{h_1}{2}\right)\right] + 3\frac{W^{(3)}}{h_1} +\\
&+3\frac{tc - x_2}{h_1}tg\eta + 6\frac{\alpha}{h_1}\vartheta^{(3)}\left(-\frac{h_1}{2}\right),\\
&h_1\theta^{(3)} = -\frac{3}{2(1-\nu)}\left[\nabla_i\left(u_i^{(3)} - \frac{7h_1}{6}\psi_i^{(3)}\right) - \frac{\alpha}{6h_1}\nabla_i\nabla_i\vartheta^{(3)}\left(-\frac{h_1}{2}\right)\right] +\\
&+3\frac{W^{(3)}}{h_1} - 3\frac{tc - x_2}{h_1}tg\eta - 3\frac{\alpha}{h_1}\vartheta^{(3)}\left(-\frac{h_1}{2}\right),\\
&\nabla_i\left(u_i^{(4)} - \frac{h_2}{3}\psi_i^{(4)}\right) - \frac{3h_2}{5}\nabla_i\nabla_i\vartheta^{(3)} - \beta\vartheta^{(4)}\left(-\frac{h_2}{2}\right) +\\
&+\frac{1-\nu}{\nu}\left(4V^{(4)} + 6\frac{W^{(4)}}{h_2}\right) = 0.
\end{aligned}
\tag{22}
$$

Thus, within the framework of the proposed non-classical theory of deformation of metal machining by cutting, the exact fulfillment of the boundary conditions that occur on the front planes of the processed metal disk and the formed chips after the impact of the cutter is achieved.

According to the non-classical theory of deformation when machining metals by cutting, ignoring the values of the high order of smallness, in order to move from the original three-dimensional problem to a two-dimensional one, we produce a standard procedure for integrating the thickness of the system of equilibrium equations [16];

$$\begin{aligned}
&\nabla_j N_{ij}^{(I)} - N_{22}^{(I)} / x_1\delta_{i1} + N_{21}^{(I)} / x_1\delta_{i2} + \sigma_{iz}^{(I)}\left(\frac{h_I}{2}\right) - \\
&-\sigma_{iz}^{(I)}\left(-\frac{h_I}{2}\right) = 0, \\
&\nabla_j M_{ij}^{(I)} - M_{22}^{(I)} / x_1\delta_{i1} + M_{21}^{(I)} / x_1\delta_{i2} - Q_i + \\
&+\frac{h_I}{2}\left[\sigma_{iz}^{(I)}\left(\frac{h_I}{2}\right) - \sigma_{iz}^{(I)}\left(-\frac{h_I}{2}\right)\right] = 0, \\
&\nabla_j\left[Q_j + \left(\nabla_k - \frac{\delta_{k1}}{x_1}\right)W\, N_{kj} + \left(\nabla_k - \frac{\delta_{k1}}{x_1}\right)V\, M_{kj}\right] + \\
&+\sigma_{zz}^{(I)}\left(\frac{h_I}{2}\right) - \sigma_{zz}^{(I)}\left(-\frac{h_I}{2}\right) = 0.
\end{aligned} \tag{23}$$

Where

$$N_{ij}^{(I)} = \int_{-h_I/2}^{h_I/2} \sigma_{ij}^{(I)} d\xi, \quad M_{ij}^{(I)} = \int_{-h_I/2}^{h_I/2} \sigma_{ij}^{(I)} \xi d\xi, \quad Q_i^{(I)} = \int_{-h_I/2}^{h_I/2} \sigma_{iz}^{(I)} d\xi,$$

$$\begin{aligned}
&\sigma_{iz}^{(I)}\left(\pm\frac{h_I}{2}\right) = \mu\left[\psi_i^{(I)} \pm h_I C_i^{(I)} + \frac{3h_I^2}{5} D_i^{(I)} + \left(\nabla_i - \frac{\delta_{i1}}{x_1}\right)\left(w^{(I)} \pm \frac{h_I}{2} V^{(I)} + \frac{h_I^2}{6}\theta^{(I)}\right)\right], \\
&\sigma_{zz}^{(I)}\left(\pm\frac{h_I}{2}\right) = \lambda\left[\varepsilon_{kk}^{(I)}\left(\pm\frac{h_I}{2}\right) - \beta\vartheta^{(I)}\left(\pm\frac{h_I}{2}\right) + \frac{1-\nu}{\nu}\left(V \pm R h_I\right)\right], \\
&\varepsilon_{kk}^{(I)}\left(\pm\frac{h_I}{2}\right) = \nabla_k\left(u_k \pm \frac{1}{2}\psi_k + \frac{h_I^2}{6} C_k \pm \frac{h_I^3}{20} D_k\right)
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\sigma_{iz}^{(I)}\left(\frac{h_I}{2}\right) - \sigma_{iz}^{(I)}\left(-\frac{h_I}{2}\right) = 2\mu\left[h_I C_i^{(I)} + \left(\nabla_i - \frac{\delta_{i1}}{x}\right)\frac{h_I}{2} V^{(I)}\right], \\
&\sigma_{iz}^{(I)}\left(\frac{h_I}{2}\right) + \sigma_{iz}^{(I)}\left(-\frac{h_I}{2}\right) = \\
&= 2\mu\left[\psi_i^{(I)} + \frac{3h_I^2}{5} D_i^{(I)} + \left(\nabla_i - \frac{\delta_{i1}}{x}\right)\left(w^{(I)} + \frac{h_I^2}{6}\theta^{(I)}\right)\right], \\
&\sigma_{zz}^{(I)}\left(\frac{h_I}{2}\right) - \sigma_{zz}^{(I)}\left(\frac{h_I}{2}\right) = \\
&= \lambda\left[\nabla_i\left(h_I\psi_i^{(I)} + \frac{h_I^3}{10} D_i^{(I)}\right) + 2\frac{1-\nu}{\nu}\theta h_I\right],
\end{aligned} \tag{25}$$

at the same time, $h_3 = h_1$, $h_4 = h_2$.

Similarly, by integrating the heat conduction equations, we will have a nonlinear equation for each band, in terms of integral quantities

$$c_\varepsilon \dot{\bar{\vartheta}}^{(I)} = \lambda_\varepsilon \Delta\bar{\vartheta}^{(I)} + h_I \vartheta_1^{(I)} - T\beta\dot{u}_{k,k}^{(I)} \tag{26}$$

In this case, the equation of equilibrium (23) of thermal conductivity (26) taking into account (14), (15), (17), (19), (20), (21) and (22) become closed with respect to the following unknown integral quantities $u_i^{(I)}, \psi_i^{(I)}, W^{(I)}, \bar{\vartheta}^{(I)}$.

In terms of integral quantities, the boundary conditions corresponding to various continuity and anchorage conditions are written as follows [18].

$$I.\quad \begin{cases} u_i^{(I-1)} = u_i^{(I)},\ \psi_i^{(I-1)} = \psi_i^{(I)},\ \ w^{(I-1)} = w^{(I)},\ \ V^{(I-1)} = V^{(I)},\ \ \vartheta^{(I-1)} = \vartheta^{(I)}, \\ N_{ij}^{(I-1)} n_j = N_{ij}^{(I)} n_j,\ M_{ij}^{(I-1)} n_j = M_{ij}^{(I)} n_j,\ \ Q_i^{(I-1)} n_i = Q_i^{(I)} n_i, \\ V^{(I-1)},_i n_i = V^{(I)},_i n_i,\ \vartheta^{(I-1)},_i n_i = \vartheta^{(I)},_i n_i. \end{cases}$$

$$I = 2,4 \ \ npu\, x_1 = r_1, \quad x_2 = ct, t > 0.$$

$$II.\ N_{ij}^{(I)} n_j \big|_\Sigma = 0,\ M_{ij}^{(I)} n_j \big|_\Sigma = 0,\ Q_i^{(I)} n_i \big|_\Sigma = 0,\ V^{(I)},_i n_i \big|_\Sigma = 0,\ \vartheta^{(I)},_i n_i \big|_\Sigma = \gamma \vartheta^{(I)},$$

$$I = \overline{1,4} \ \ npu\ x_1 = 0, R,\ x_2 = 0, 2\pi,$$

It should be noted that the first type of boundary conditions correspond to the continuity conditions, and the second type corresponds to the free edge taking into account the heat exchange process [17]–[24].

The initial conditions for the temperature field taking into account (11) can similarly be written in terms of integral quantities. The resulting problem can be solved by numerical methods [25,26].

4. Conclusions

1. A mathematical model describing the process of cutting cylindrical objects based on the thermodynamic nonlinear theory of elasticity has been developed;

2. Solving equations and boundary conditions for the description of thermomechanical processes in the processing of three-dimensional metal cylindrical bodies by cutting, taking into account heat generation and wear on the surfaces in contact with the cutting tool, are compiled.

3. Boundary conditions are formulated taking into account the interaction of the cutting tool and the surface of the object under consideration;

4. An approach is proposed for reducing three-dimensional problems of cutting cylindrical bodies to the corresponding two-dimensional problems.

5. The resulting heat transfer formulas take into account the possibility of additional heat from the impact of the cutting tool on the surface of the objects under consideration;

6. The developed model and approach of thermomechanical processing of metals allows us to solve the related problem of thermodynamics to determine the stress-strain state and distribution patterns of the temperature field and in the formed chips after cutting.

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