Dynamic analysis of displacement and stress in multi-story structures under transverse vibrations

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Abstract. The article provides a brief overview of scientific works on modeling the dynamic behavior of buildings and structures worldwide and presents the results of calculations of displacements, accelerations, and stresses obtained in the numerical study of forced vibrations of multi-story buildings subject to external harmonic impacts, corresponding to an earthquake with an intensity of 9 points in the soil conditions of the republic Uzbekistan within the framework of the plate model.

1 Introduction

At present, various models of the dynamic behavior of buildings and structures under external influences are proposed and calculation methods corresponding to them are developed. Each model is characterized by considering the most important factors from the point of view of a researcher of the dynamic behavior of buildings and structures, and the type of external influence. The most important type of external influence is seismic impact. As is known, the development of dynamic spatial models of buildings and structures, the deformation of which is spatial, is one of the most difficult but urgent problems in the mechanics of a deformable body and the dynamics of structures. Below is a brief overview of publications devoted to this topic.

Article [1] considers the issue of determining the natural vibration frequencies of modular buildings. The relationship between the frequency of intersection of the first one-story buildings and the frequency of a separate block was determined. The influence of the ratio of the rigidities of the horizontal and vertical elements of the building on the value of the first frequency of natural vibrations was shown.

The study in [2] discusses the calculation of a multi-story monolithic concrete building for an earthquake. The problem is solved in the time domain using the direct dynamic method. Direct integration of the equations of motion is conducted using an explicit scheme. This method allows for solving the problem in a nonlinear dynamic statement,

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considering geometric and physical nonlinearities.

Article [3] presents the results of a numerical study of the behavior of a multi-story reinforced concrete wall frame structure under special combination loads, considering the seismic impact corresponding to a destructive earthquake.

In [4-6], the organization and conduct of dynamic tests of a multi-story residential panel building in Krasnoyarsk are discussed. For dynamic testing, a hardware and software complex was developed that implements the standing wave method, which makes it possible to determine the dynamic characteristics of a building by recording micro-seismic vibrations of building structures. Based on the results of dynamic tests, the actual (resonant) eigenfrequencies and vibration modes of building structures were determined.

Article [5] describes the damage and destruction of building structures due to thermal effects caused by fire. The method used there is a cellular model of a thermally insulated plate, based on the localization of a heat source in a certain position above the plate and characterized by the temperature distribution in the cells.

Reference [7] considers the analytical calculation of brickwork for a barrel-shaped vault, the material structure of which has a pronounced variability of elastic constants. A mathematical solution to a fourth-order partial differential equation with two variables for an anisotropic orthotropic body in polar coordinates is presented to create mathematical models that describe the change in the elastic modulus of the arch material.

Article [8] presents a technology for producing building ceramics based on anorthite using semi-dry powder stressing based on sintering raw material mix consisting of low-melting clay and blast furnace sludge (BFS) in various proportions. The manufactured ceramic samples are sintered at a temperature of 1050°C. The properties of the raw material mix to increase the content of the anorthite phase in ceramic samples were studied.

In [9], the issue that a multi-story building is subject to equivalent static and dynamic analysis is considered. For the purpose of the study, the building was modeled in SAP2000 software. For dynamic analysis, the building was subject to ground motion to obtain its response.

Article [10] considers the contact interaction of deformable building structures or their parts. The subject of the study is the formulation of the contact interaction problem as a linear complementarity problem. An extension of the existing formulations of the problems of contact without friction and contact with a known friction boundary is proposed in the form of a problem of linear complementarity to the formulation of frictional contact. Ultimately, a heuristic formulation of the contact problem with friction was obtained in the form of a linear complementarity problem.

Publications [11-13] investigated the behavior and stress-strain state of structures and soils, considering the nonlinear strain of soil surrounding the structures and showed the existence of a near-contact layer of soil near the contact, which can behave as seismic protection for structures. To solve the problems, the numerical finite difference method was used and results were obtained for elasto-plastic interaction problems considering dynamic processes for the design of structures and soil.

In [14, 15], the influence of the moisture content of loess soil and soil foundations on sedimentary deformation was considered. Subsidence of loess soils occurs due to a decrease in the strength parameters of soil and transformation of the strain state of soil.

Articles [16-18] are devoted to dynamic calculations of elements of box-shaped buildings for seismic resistance, considering the spatial work of box-shaped elements. In this case, the dynamic effect is specified as harmonic oscillations of the base movements according to a sinusoidal law. The equations of motion are given for each of the plate and beam elements of the box-shaped building structure based on the Kirchhoff-Love theory.

Article [19] presents the numerical solution to the problem of transverse vibrations of a multi-story building within the framework of a solid slab model under seismic influence. A cantilever anisotropic plate is proposed as a dynamic model of the building. This model was developed within the framework of the three-dimensional dynamic theory of elasticity and takes into account not only structural forces and moments but also bi-moments.

Article [20] discusses the method developed for the dynamic calculation of a box-shaped structure consisting of interconnected longitudinal and transverse plate and beam elements. The problem is posed about the spatial vibrations of a box-shaped building structure under dynamic influence determined by the movement of its base according to a sinusoidal law.

2 Reduced elastic characteristics of a multi-story building

The reduced density of the building is determined by the following formula:

$$m_{\text{reduced}} = \rho_{\text{reduced}} V_1 = \rho_{\text{reduced}} V_0 \tag{1}$$

here V_1 – the volume of the slab forming one floor of the building. V_0 – the volume of one floor of the building. Considering the geometric parameters of the building in question, to calculate these volumes, we obtain the following formulas:

$$V_0 = ab_1H,\ V_1 = ab_1h_2 + (n-2)Hb_1h_2 + aHh_2, \tag{2}$$

where $a,\ H$ – building length and width; b_1 – height of one floor of a building; k – number of internal transverse walls of the building; h_1 - thickness of external load-bearing walls; h_2 – thickness of internal walls; $h_{\text{thickness}}$ – floor thickness.

In general, the given elastic characteristics and density of the building are determined by the following formulas:

$$\begin{gathered} E_1^{\text{reduced}} = \zeta_{11}E_0,\ E_2^{\text{reduced}} = \zeta_{22}E_0,\ E_3^{\text{reduced}} = \zeta_{33}E_0, \\ G_{12}^{\text{reduced}} = \zeta_{12}G_0,\ G_{13}^{\text{reduced}} = \zeta_{13}G_0, G_{23}^{\text{reduced}} = \zeta_{23}G_0,\quad \rho_{\text{reduced}} = \rho_0\varsigma_0. \end{gathered} \tag{3}$$

It should be noted that the coefficient values $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$ for each cell (room) of a discrete part of the building are determined as functions of two spatial variables, $E_0,\ G_0$ – moduli of elasticity and shear of the strongest load-bearing panel of a cell of a discrete part of a building.

Let us write formulas for determining coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$ reduced moduli of elasticity of a discrete part of the building:

$$\begin{gathered} \xi_{11} = \alpha\frac{S_{11}}{S_{01}},\quad \xi_{22} = \alpha\frac{S_{22}}{S_{02}},\quad \xi_{33} = \alpha\frac{S_{33}}{S_{03}},\ \xi_{12} = \alpha\frac{S_{12}}{S_{01}}, \\ \xi_{13} = \alpha\frac{h_{nep}}{b_1}\lambda^*,\quad \xi_{23} = \alpha\frac{h_2}{a_1},\quad \zeta_0 = \frac{V_1}{V_0}. \end{gathered} \tag{4}$$

where $S_{01},\ S_{02},\ S_{03}$ – cross-sectional area of the building in three coordinate planes of one floor of the building; S_{11}, S_{22}, S_{33} – total cross-sectional areas of slabs in coordinate planes forming one floor of the building; λ^* – coefficient characterizing the voids in the cross-section of the floor slab. Coefficient α is determined depending on the cellular structure of the building structure.

Depending on the size of the slabs, rooms, and the building itself, the above areas are determined using the following method:

$$S_{01} = E_0 b_1 H, \quad S_{02} = E_0 a H, \quad S_{03} = E_0 a b_1, \tag{5}$$

$$\begin{aligned} &S_{11} = b_1 h_2 E_b^{(2)} + H h_{thicknes} E_{thicknes}, \quad S_{12} = b_1 h_2 E_b^{(2)}, \\ &S_{22} = a h_2 E_b^{(2)} + (k-2) H h_2 E_b^{(2)}, \; S_{33} = a h_2 E_b^{(2)} + (k-2) b_1 h_2 E_b^{(2)}. \end{aligned} \tag{6}$$

Here $G_{thicknes}$ – building floor shear module; G_2 – internal walls shear module; $E_b^{(2)}$ – modulus of elasticity of internal walls; $E_{thicknes}$ – floor elastic modulus.

3 Statement of the problem

We formulate the problem of vibrations of a multi-story building within the framework of the bi-moment theory of plate structures proposed in [18-21]. The system of equations of motion of a multi-story building under transverse vibrations consists of three equations regarding bending and torque moments and shear forces, as well as three equations regarding three bi-moment equations, described by nine unknown kinematic functions [18-21]:

$$\begin{aligned} &\widetilde{u}_k = \frac{u_k^{(+)} - u_k^{(-)}}{2}, \quad \widetilde{\psi}_k = \frac{1}{2h^2}\int_{-h}^{h} u_k z dz, \quad \widetilde{\beta}_k = \frac{1}{2h^4}\int_{-h}^{h} u_k z^3 dz, \; (k=1,2), \\ &\widetilde{W} = \frac{u_3^{(+)} + u_3^{(-)}}{2}, \quad \widetilde{r} = \frac{1}{2h}\int_{-h}^{h} u_3 dz, \quad \widetilde{\gamma} = \frac{1}{2h^3}\int_{-h}^{h} u_3 z^2 dz. \end{aligned} \tag{7}$$

To describe the system of equations for multi-story buildings, expressions for forces, moments, and bi-moments are proposed in [18-21]. Expressions for determining shear forces have the form:

$$\begin{aligned} &Q_{13} = \int_{-h}^{h} \sigma_{13} dz = G_{13}\left(2\widetilde{u}_1 + H\frac{\partial \widetilde{r}}{\partial x_1}\right), \\ &Q_{23} = \int_{-h}^{h} \sigma_{23} dz = G_{23}\left(2\widetilde{u}_2 + H\frac{\partial \widetilde{r}}{\partial x_2}\right). \end{aligned} \tag{8}$$

Expressions for bending and torque moments M_{11}, M_{22}, M_{12} are written in the form:

$$
\begin{aligned}
&M_{11}=\int_{-h}^{h}\sigma_{11}zdz=\frac{H^2}{2}\left(E_{11}\frac{\partial\tilde{\psi}_1}{\partial x_1}+E_{12}\frac{\partial\tilde{\psi}_2}{\partial x_2}-E_{13}\frac{2(\tilde{r}-\tilde{W})}{H}\right),\\
&M_{22}=\int_{-h}^{h}\sigma_{22}zdz=\frac{H^2}{2}\left(E_{12}\frac{\partial\tilde{\psi}_1}{\partial x_1}+E_{22}\frac{\partial\tilde{\psi}_2}{\partial x_2}-E_{23}\frac{2(\tilde{r}-\tilde{W})}{H}\right),\\
&M_{12}=M_{21}=\int_{-h}^{h}\sigma_{12}zdz=G_{12}\frac{H^2}{2}\left(\frac{\partial\tilde{\psi}_1}{\partial x_2}+\frac{\partial\tilde{\psi}_2}{\partial x_1}\right).
\end{aligned}
\tag{9}
$$

Bending and torsional bi-moments P_{11}, P_{22}, P_{12} have expressions:

$$
\begin{aligned}
&P_{11}=\frac{1}{h^2}\int_{-h}^{h}\sigma_{11}z^3dz=\frac{H^2}{2}\left(E_{11}\frac{\partial\tilde{\beta}_1}{\partial x_1}+E_{12}\frac{\partial\tilde{\beta}_2}{\partial x_2}-E_{13}\frac{2(3\tilde{\gamma}-\tilde{W})}{H}\right),\\
&P_{12}=P_{21}=\frac{1}{h^2}\int_{-h}^{h}\sigma_{12}z^3dz=\frac{H^2}{2}G_{12}\left(\frac{\partial\tilde{\beta}_1}{\partial x_2}+\frac{\partial\tilde{\beta}_2}{\partial x_1}\right),\\
&P_{22}=\frac{1}{h^2}\int_{-h}^{h}\sigma_{22}z^3dz=\frac{H^2}{2}\left(E_{12}\frac{\partial\tilde{\beta}_1}{\partial x_1}+E_{22}\frac{\partial\tilde{\beta}_2}{\partial x_2}-E_{23}\frac{2(3\tilde{\gamma}-\tilde{W})}{H}\right).
\end{aligned}
\tag{10}
$$

Specific, transverse normal $\tilde{p}_{33}$ and shear bi-moments $\tilde{p}_{13}$, $\tilde{p}_{23}$ built using normal σ_{33} and shear stresses σ_{13}, σ_{23} as:

$$
\tilde{p}_{33}=\frac{1}{2h^2}\int_{-h}^{h}\sigma_{33}zdz=E_{31}\frac{\partial\tilde{\psi}_1}{\partial x_1}+E_{31}\frac{\partial\tilde{\psi}_1}{\partial x_1}-E_{33}\frac{2(\tilde{r}-\tilde{W})}{H}. \tag{11}
$$

$$
\tilde{p}_{k3}=\frac{1}{2h^3}\int_{-h}^{h}\sigma_{k3}z^2dz=G_{k3}\left(\frac{2\tilde{u}_k-4\tilde{\psi}_k}{H}+\frac{\partial\tilde{\gamma}}{\partial x_k}\right),\ (k=1,2). \tag{12}
$$

According to the bi-moment theory of plate structures [18-21], generalized external forces are introduced for antisymmetric problems:

$$
\tilde{q}_k=\frac{q_k^{(+)}+q_k^{(-)}}{2}\quad (k=1,2),\qquad \tilde{q}_3=\frac{q_3^{(+)}-q_3^{(-)}}{2}. \tag{13}
$$

The equations of motion for asymmetrical transverse vibration of a multi-story building within the framework of a plate model are also described using a system of six equations.

The first three equations for transverse vibrations of a building are constructed with respect to bending, torque, and shear forces:

$$
\begin{aligned}
&\frac{\partial M_{11}}{\partial x_1}+\frac{\partial M_{12}}{\partial x_2}-Q_{13}+H\tilde{q}_1=\frac{H^2}{2}\rho\ddot{\tilde{\psi}}_1,\\
&\frac{\partial M_{21}}{\partial x_1}+\frac{\partial M_{22}}{\partial x_2}-Q_{23}+H\tilde{q}_2=\frac{H^2}{2}\rho\ddot{\tilde{\psi}}_2,
\end{aligned}
\tag{14}
$$

$$
\frac{\partial Q_{13}}{\partial x_1}+\frac{\partial Q_{23}}{\partial x_2}+2\tilde{q}_3=\rho H\ddot{\tilde{r}}. \tag{15}
$$

The remaining three equations for transverse vibrations of a multi-story building based on a plate model of a multi-story building will be written relative to bi-moments. From these, two equations for transverse vibrations of a building are obtained with respect to bending and torsional bi-moments and have the form:

$$\begin{aligned}&\frac{\partial P_{11}}{\partial x_1}+\frac{\partial P_{12}}{\partial x_2}-3H\tilde{p}_{13}+H\tilde{q}_1=\frac{H^2}{2}\rho\ddot{\tilde{\beta}}_1,\\&\frac{\partial P_{21}}{\partial x_1}+\frac{\partial P_{22}}{\partial x_2}-3H\tilde{p}_{23}+H\tilde{q}_2=\frac{H^2}{2}\rho\ddot{\tilde{\beta}}_2.\end{aligned} \tag{16}$$

The third equation for the transverse vibrations of a multi-story building within the framework of a plate model of a multi-story building relative to the intensity of transverse shear and longitudinal bi-moments will be written in the form:

$$H\frac{\partial\tilde{p}_{13}}{\partial x_1}+H\frac{\partial\tilde{p}_{23}}{\partial x_2}-4\tilde{p}_{33}+2\tilde{q}_3=H\rho\ddot{\tilde{\gamma}}\cdot \tag{17}$$

It should be noted that to represent the transverse vibrations of a multi-story building within the framework of the bi-moment theory of plate structures, a joint system of six equations was obtained for nine unknown generalized displacement functions $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{\beta}_1$, $\tilde{\beta}_1$, $\tilde{r}$, $\tilde{\gamma}$, $\tilde{W}$: (12) - (17). Consequently, here too three more equations are missing.

Three missing kinematic equations of motion of a plate model under transverse vibrations of a multi-story building are constructed with respect to generalized displacement functions $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{W}$ in the following form:

$$\tilde{u}_k=\frac{1}{2}\left(21\tilde{\beta}_k-7\tilde{\psi}_k\right)-\frac{1}{30}H\frac{\partial\tilde{W}}{\partial x_k}+\frac{1}{30}\frac{H\tilde{q}_k}{G_{k3}}\quad(k=1,2), \tag{18}$$

$$\tilde{W}=\frac{1}{4}\left(21\tilde{\gamma}-3\tilde{r}\right)-\frac{1}{20}H\left(\frac{E_{31}}{E_{33}}\frac{\partial\tilde{u}_1}{\partial x_1}+\frac{E_{32}}{E_{33}}\frac{\partial\tilde{u}_2}{\partial x_2}\right)+\frac{H\tilde{q}_3}{20E_{33}}. \tag{19}$$

Bi-moments $\tilde{\sigma}_{11}$, $\tilde{\sigma}_{12}$, $\tilde{\sigma}_{22}$ are defined as:

$$\begin{aligned}&\tilde{\sigma}_{11}=\left(E_{11}-\frac{E_{13}}{E_{33}}E_{31}\right)\frac{\partial\tilde{u}_1}{\partial x_1}+\left(E_{12}-\frac{E_{13}}{E_{33}}E_{32}\right)\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{13}}{E_{33}}\tilde{q}_3,\\&\tilde{\sigma}_{22}=\left(E_{21}-\frac{E_{23}}{E_{33}}E_{31}\right)\frac{\partial\tilde{u}_1}{\partial x_1}+\left(E_{22}-\frac{E_{23}}{E_{33}}E_{32}\right)\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{E_{23}}{E_{33}}\tilde{q}_3,\\&\tilde{\sigma}_{12}=G_{12}\left(\frac{\partial\tilde{u}_1}{\partial x_2}+\frac{\partial\tilde{u}_2}{\partial x_1}\right).\end{aligned} \tag{20}$$

Based on Hooke's law and expressions (18), we obtain the following expressions for bi-moments $\tilde{\sigma}^*_{11}$ and $\tilde{\sigma}^*_{22}$:

$$\tilde{\sigma}_{11}^{*} = -E_{11}H\frac{\partial^2\tilde{W}}{\partial x_1^2} - E_{12}H\frac{\partial^2\tilde{W}}{\partial x_2^2} + E_{13}\frac{60\left(3\tilde{W}+4\tilde{r}-21\tilde{\gamma}\right)}{H} + E_{11}H\frac{\partial}{\partial x_1}\left(\frac{\tilde{q}_1}{G_{13}}\right) + E_{12}H\frac{\partial}{\partial x_2}\left(\frac{\tilde{q}_2}{G_{23}}\right),$$
$$\tilde{\sigma}_{22}^{*} = -E_{12}H\frac{\partial^2\tilde{W}}{\partial x_1^2} - E_{22}H\frac{\partial^2\tilde{W}}{\partial x_2^2} + E_{23}\frac{60\left(3\tilde{W}+4\tilde{r}-21\tilde{\gamma}\right)}{H} + E_{12}H\frac{\partial}{\partial x_1}\left(\frac{\tilde{q}_1}{G_{13}}\right) + E_{22}H\frac{\partial}{\partial x_2}\left(\frac{\tilde{q}_2}{G_{23}}\right). \quad (21)$$

At the base of the plate model of a multi-story building, the boundary conditions for flexural-shear vibrations have the form:

$$\tilde{\psi}_1 = 0,\ \tilde{\psi}_2 = 0,\ \tilde{\beta}_1 = 0,\ \tilde{\beta}_2 = 0,\ \tilde{u}_1 = 0,\ \tilde{u}_2 = 0,\ \ \tilde{r} = u_0(t),\ \tilde{\gamma} = \frac{1}{3}u_0(t),\ \tilde{W} = u_0(t). \quad (22)$$

where $u_0(t)$ – is the law of motion of the foundation of a building in the horizontal transverse direction.

On the free side faces of the plate model of a multi-story building we have conditions for forces, moments, and bi-moments and force factors equal to zero:

$$M_{11} = 0,\ M_{12} = 0,\ P_{11} = 0,\ P_{12} = 0,\ Q_{13} = 0,\ \tilde{p}_{13} = 0,\ \ \tilde{\sigma}_{11} = 0,\ \tilde{\sigma}_{12} = 0,\ \tilde{\sigma}_{11}^{*} = 0. \quad (23)$$

On the free upper face of the building, we have the following conditions:

$$M_{12} = 0,\ M_{22} = 0,\ P_{12} = 0,\ P_{22} = 0,\ Q_{23} = 0,\ \tilde{p}_{23} = 0,\ \ \tilde{\sigma}_{11} = 0,\ \tilde{\sigma}_{12} = 0,\ \tilde{\sigma}_{22}^{*} = 0. \quad (24)$$

Now we will define the boundary conditions for a unique solution to the system of equations of a multi-story building.

In the calculations, we assume zero initial conditions of the problem.

To solve the problem numerically, the finite difference method was chosen. To approximate the derivatives of displacements along spatial coordinates, we will use the formulas of central difference schemes.

We select the calculation steps based on spatial coordinates and time as follows:

$$\Delta x_1 = \frac{a}{N},\ \Delta x_2 = \frac{b}{M},\quad c\Delta t \le \min(\Delta x_1, \Delta x_2).$$

where $c = \sqrt{E/\rho}$.

4 Initial problem data

We consider that the generalized external forces applied on the front surfaces are zero $\tilde{q}_1 = 0,\ \tilde{q}_2 = 0,\ \tilde{q}_3 = 0.$

Law of motion of the foundation of a multi-story building $\ddot{u}_0(t)$ is given according to the harmonic law in the form:

$$\ddot{u}_0(t) = a_0\cos(\omega_0 t), \quad (25)$$

where $a_0 = k_c g$ - maximum acceleration and $\omega_0 = 2\pi\nu_0$ – circular frequency of the soil base, k_c and ν_0 earthquake magnitude coefficient and natural frequency of external impact, respectively.

From the differential dependence of the displacement u_0 on time, we can determine the displacement of the building base in the form:

$$u_0(t) = \frac{A_0}{2}(1 - \cos(\omega_0 t)). \tag{26}$$

Here A_0 – amplitude of movement of the base, determined by the formula:

$$A_0 = \frac{2k_c g}{\omega_0^2}. \tag{27}$$

Note that the seismicity coefficients for seven and eight magnitude earthquakes are, respectively, $k_c = 0.1;\ 0.2;\ 0.4$.

The geometric characteristics of the room panels and the external dimensions of the buildings must be specified as initial data.

We assume that the external walls consist of reinforced concrete with an elastic modulus $E = 20000\,\text{MPa}$, density $\rho = 2500\,\text{kg/m}^3$, Poisson's ratio $\nu = 0.3$.

We consider the internal walls to consist of expanded clay concrete with the following physical characteristics: modulus of elasticity $E = 7500\,\text{MPa}$, density $\rho = 1200$ кg/m^3, Poisson's ratio $\nu = 0.3$.

The results of calculations of forced vibrations of a building within the framework of a thick plate model are presented for the following sizes of building slabs:

$$h_1 = 0.40m,\ h_2 = 0.25m,\ h_{\text{thickness}} = 0.2m,\ a_1 = 5m,\ b_1 = 3m,$$

Table 1 shows the values of reduction coefficients and reduced elasticity and shear moduli of a multi-story building.

Table 1. Reduction coefficients and reduced elasticity and shear moduli of a multi-story building.

Thickness	Coefficients and elastic characteristics of the building											
H (m)	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{22}	ξ_{23}	ξ_{33}	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}
11	0.129	0.095		0.164			1545	1965		458.2	320.0	240.0
13	0.114	0.081		0.149			1369	1789		215.4	177.8	133.3
15	0.103	0.070		0.138			1240	1660		129.2	123.1	92.31
18	0.092	0.058		0.127			1100	1520		82.35	94.12	70.59
20	0.086	0.053	0.067	0.121	0.05	0.102	1030	1450	1220	60.00	76.19	57.14
22	0.081	0.048		0.116			972.7	1393		45.82	64.00	48.00
24	0.077	0.044		0.112			925.0	1345		36.21	55.17	41.38
26	0.074	0.040		0.109			884.6	1305		29.37	48.48	36.36
28	0.071	0.,038		0.106			850.0	1270		24.32	43.24	32.43

5 Solution method

The problem is solved using the explicit scheme of the finite difference method. To develop finite-difference expressions for approximating the derivatives of generalized displacements along spatial coordinates, we will use the formulas of central difference schemes.

We present the formulas for the method of finite-difference approximation of the first derivatives of generalized functions with respect to central points:

$$\frac{\partial f_{i,j}^{k}}{\partial x_1}=\frac{f_{i+1,j}^{k}-f_{i-1,j}^{k}}{2\Delta x_1},\quad \frac{\partial f_{i,j}^{k}}{\partial x_2}=\frac{f_{i,j+1}^{k}-f_{i,j-1}^{k}}{2\Delta x_2}. \tag{28}$$

Here $\Delta x_1=\frac{a}{N}$, $\Delta x_2=\frac{b}{M}$ – grid method calculation step, N, M – number of divisions per grid.

When approximating the derivatives of stresses, forces, moments, and bi-moments, central finite-difference half-step schemes are used, which have the second order of accuracy:

$$\frac{\partial F_{i,j}^{k}}{\partial x_1}=\frac{F_{i+\frac{1}{2},j}^{k}-F_{i-\frac{1}{2},j}^{k}}{\Delta x_1},\quad \frac{\partial F_{i,j}^{k}}{\partial x_2}=\frac{F_{i,j+\frac{1}{2}}^{k}-F_{i,j-\frac{1}{2}}^{k}}{\Delta x_2}\quad \left(i=1,N;\ j=1,M\right)\cdot \tag{29}$$

Here $\Delta x_1=\frac{a}{N}$, $\Delta x_2=\frac{b}{M}$.

When approximating the condition that the force factors on the side faces and edges of a multi-story building are zero, we use the following expressions:

$$F_{N+\frac{1}{2},j}^{k}+F_{N-\frac{1}{2},j}^{k}=0\ \left(j=1,M\right);\quad F_{i,M+\frac{1}{2}}^{k}+F_{i,M-\frac{1}{2}}^{k}=0\quad \left(i=1,N\right)\cdot \tag{30}$$

When using formulas (28) and (29), it is necessary to approximate the derivatives of the generalized displacement functions at the central point between two points x_i and x_{i+1} or y_j and y_{j+1}. In these cases, formulas are used, replacing accordingly i – with $i-\frac{1}{2}$ and j – with $j-\frac{1}{2}$.

$$\frac{\partial f_{i-\frac{1}{2},j}^{k}}{\partial x_1}=\frac{f_{i,j}^{k}-f_{i-1,j}^{k}}{\Delta x_1},\quad \frac{\partial f_{i,j-\frac{1}{2}}^{k}}{\partial x_2}=\frac{f_{i,j}^{k}-f_{i,j-1}^{k}}{\Delta x_2},\quad \left(i=1,N;\ j=1,M\right), \tag{31}$$

$$\frac{\partial f_{i-\frac{1}{2},j}^{k}}{\partial x_1}=\frac{\partial}{\partial x_1}\left(\frac{f_{i,j}^{k}+f_{i-1,j}^{k}}{2}\right),\quad \frac{\partial f_{i,j-\frac{1}{2}}^{k}}{\partial x_2}=\frac{\partial}{\partial x_2}\left(\frac{f_{i,j}^{k}+f_{i,j-1}^{k}}{2}\right),\quad \left(i=1,N;\ j=1,M\right). \tag{32}$$

We represent the second derivative of the function with respect to time within the framework of the finite difference method in the form:

$$\frac{\partial^2 f_{i,j}^{k}}{\partial t^2}=\frac{f_{i,j}^{k+1}-2f_{i,j}^{k}+f_{i,j}^{k-1}}{\Delta t^2}, \tag{33}$$

where Δt –the time step.

When implementing a numerical method for solving the problem, we select steps in spatial coordinates and time as follows:

$$\Delta x_1 = \frac{a}{N}, \ \Delta x_2 = \frac{b}{M}, \quad c\Delta t \le \min(\Delta x_1, \Delta x_2). \tag{34}$$

To construct a method for numerically solving the equation of transverse vibrations of a multi-story building, an algorithm and program for calculating a multi-story building for seismic resistance were compiled.

6 Analysis of numerical results

Let us present the results of stress calculations at the lowest points of load-bearing walls of multi-story buildings.

Calculations were performed for multi-story buildings under seven, eight and nine magnitude earthquakes, which are specified through the corresponding seismicity coefficients k_s. In the calculations, the building walls, floors, and ceilings are considered to be made of reinforced concrete.

6.1 Calculation results for a 20-story building

Calculations were performed at the following values: frequency of external influence $\nu_0 = 3.8\ Hz$, the period of the main oscillation tone $T_0 = 1/\nu_0 = 0.263$ sec. Amplitude of external influence A_0 was determined depending on the intensity of the earthquake.

The height and width of a twenty-story building are assumed to be a=30m, b=72m, and H=18m, respectively. The value of the natural frequency of a twenty-story building is p_1=0.905 Hz, and the period of the fundamental tone of natural oscillations is T_1=1/p_1=1.11 sec.

Let us present the numerical results of stresses obtained during transverse vibrations of a 20-story building during seven, eight and nine magnitude earthquakes.

Figures 1 and 2 show graphs characterizing changes in the maximum normal stress σ_{11}, σ_{12}, σ_{22} in the middle of the first floor of a twenty-story building in time t during nine magnitude earthquakes $k_c = 0{,}4$ and $\nu_0 = 3{,}8$.

σ_{11},
MPa

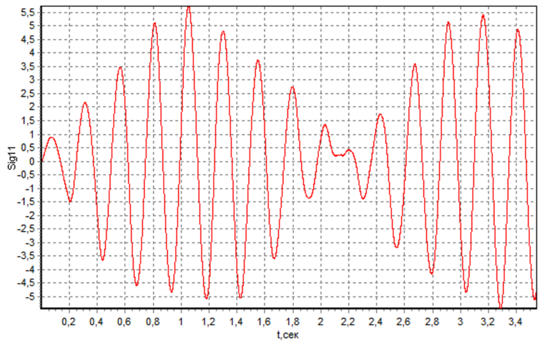

Fig. 1. Graph of changes in normal σ_{11} stress over time in the middle of the first floor of a twenty-story building.

As seen in the Figure 1, in the middle of the first floor of the building module, the maximum value of the normal stress turned out to be $\sigma_{11} = 5.71 MPa$.

Figure 2 shows a graph characterizing changes in the maximum normal stress σ_{22} in the middle of the first floor of a twenty-story building in time t.

σ_{22},
MPa

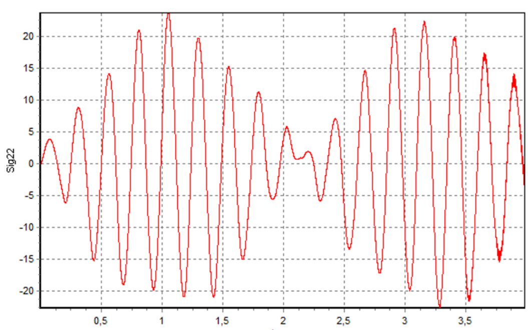

Fig. 2. Graph of changes in normal stress σ_{22} in time in the middle of the first floor of a twenty-story building.

As seen in Figure 2, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be σ_{22}=25.1 MPa.

Table 2 shows the minimum and maximum stress values obtained during transverse vibrations of a 20-story building.

Table 2. Values of the first natural frequency, maximum and minimum stresses of a 20-story large-panel building during earthquakes of magnitude 7-8-9.

№	H,m	k_s	v_0, Hz	p_1, Hz	σ_{11}, MPa		σ_{22}, MPa		$\ddot{\vec{r}}, m/\sec^2$,	
					min	max	min	max	min	max
1	18	0.1	3.8	0.905	-1.35	1.40	-5.30	5.5	-7.95	8.55
2		0.2			-2.5	3.02	-11.2	11.9	-16.0	16.7
3		0.4			-5.51	5.71	-24.9	25.1	-30.6	31.8

Note that the minimum and maximum values of shear and normal stresses were found during forced transverse vibrations in the middle and quarter of the length *a=30m* of the multi-story high-rise building in question.

It was established that the module maximum values of normal stresses σ_{11} and σ_{22} in the middle of the lower part of a twenty-story building during seven-, eight- and nine-magnitude earthquakes are:

$$\sigma_{11} = 1.40\,\text{M}Pa, \quad \sigma_{11} = 3.02\,\text{M}Pa, \quad \sigma_{11} = 5.71\,\text{M}Pa,$$
$$\sigma_{22} = 5.5\,\text{M}Pa, \quad \sigma_{22} = 11.9\,\text{M}Pa, \quad \sigma_{22} = 25.1\,\text{M}Pa.$$

When obtaining numerical results for a twenty-story building, the number of divisions on the grid along spatial coordinates is $N = 30, \;\; M = 72$.

6.2 Calculation results for a 24-story building

Calculations were performed at the following values: frequency of external influence $\nu_0 = 3.9$ Hz, the period of the main oscillation tone $T_0 = 1/\nu_0 = 0.256$ sec. Amplitude of external influence A_0 is determined depending on the intensity of the earthquake. During calculations, natural frequency values for a 24-story building p_1=2.2 Hz and the period of the fundamental tone of oscillations $T_1=1/p_1$=1.18 sec. were obtained.

Let us present the numerical results of stresses obtained during transverse vibrations of a 24-story building during seven, eight and nine magnitude earthquakes. The height and width of the building are *a=30m*, *b=75m* and *H=20m*, respectively.

Figure 3 shows a graph characterizing changes in the maximum normal stress σ_{11} in the middle of the first floor of a twenty-four-story building in time t.

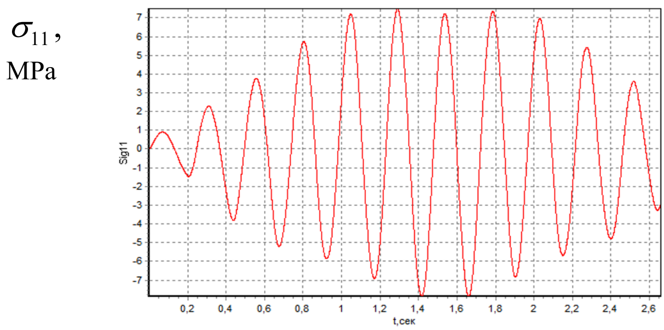

Fig. 3. Graph of changes in normal stress σ_{11} in time in the middle of the first floor of a twenty-four-story building.

As seen in Figure 3, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{11} = 7.6\ MPa$.

Figure 4 shows a graph characterizing changes in the maximum normal stress σ_{22} in the middle of the first floor of a twenty-four-story building in time t.

σ_{22},
MPa

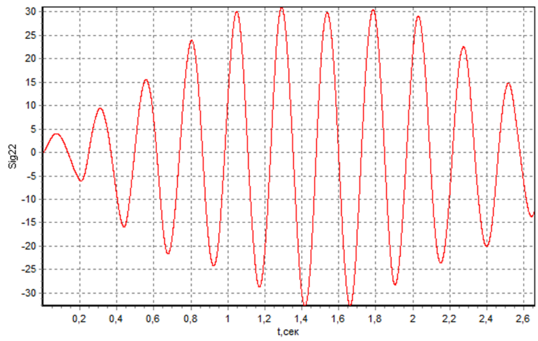

Fig. 4. Graph of changes in normal stress σ_{22} in time in the middle of the first floor of a twenty-four-story building.

As seen in Figure 4, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{22} = 32.2$.

Table 3 shows the minimum and maximum stress values obtained during transverse vibrations of a twenty-four-story building. The height and width of the building are assumed to be *a*=30*m, b=75m,* and H=20*m,* respectively. Note that the minimum and maximum values of normal stresses were found during forced transverse vibrations in the middle and quarter of the length *a=30m* of the multi-story high-rise building in question.

Calculations have shown that under external dynamic influences with a large amplitude, quite dangerous stresses appear in different upper levels of external walls during magnitude seven, eight, and nine earthquakes.

Table 3. Values of the first natural frequency, maximum and minimum stresses of a 24-story large-panel building during earthquakes of magnitude 7-8-9.

№	*H, m*	k_s	ν_0, Hz	p_1, Hz	σ_{11}, MPa		σ_{22}, MPa		$\ddot{\tilde{r}}, m/\sec^2$,	
					min	max	min	max	min	max
1	20	0.1	3.9	2.2	-1.83	1.81	-8.02	7.82	-9.99	10.3
2		0.2			-3.8	3.7	-16.2	14.1	-19.9	20.5
3		0.4			-7.5	7.6	-32.2	31.0	-40.2	40.1

It was established that the module maximum values of normal stresses σ_{11} and σ_{22} in the middle of the lower part of a twenty-four-story building during seven-, eight- and nine-magnitude earthquakes are:

$$\sigma_{11} = 1.83\,\mathrm{M}Pa, \quad \sigma_{11} = 3.8\,\mathrm{M}Pa, \quad \sigma_{11} = 7.6\,\mathrm{M}Pa,$$
$$\sigma_{22} = 8.02\,\mathrm{M}Pa, \quad \sigma_{22} = 16.6\,\mathrm{M}Pa, \quad \sigma_{22} = 32.2\,\mathrm{M}Pa.$$

6.3 Calculation results for a 28-story building

Calculations were performed for the following values: frequency of external influence $\nu_0 = 3.5$ Hz, the period of the main oscillation tone $T_0 = 1/\nu_0 = 0{,}29$ sec. In the calculations, natural frequency values for a 28-story building p_1=0.9 Hz and the period of the fundamental tone of oscillations T_1=1/p_1=1.1 sec. were obtained. The height and width of the building are assumed to be *a*=30*m, b*=84*m* and *H=22*, respectively. Let us present the

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numerical results of stresses obtained during transverse vibrations of a 28-story building, under seven, eight, and nine magnitude earthquakes.

Figure 5 shows a graph characterizing changes in the maximum normal stress σ_{11} in the middle of the first floor of a twenty-eight-story building in time t.

σ_{11}, MPa

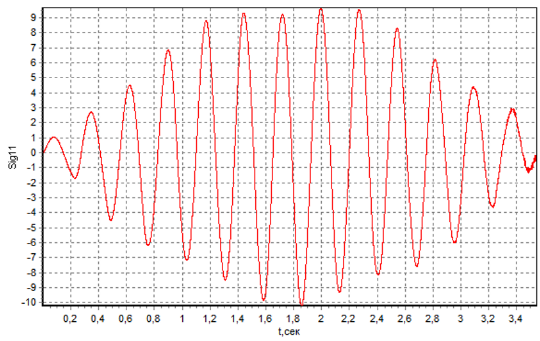

Fig. 5. Graph of changes in normal stress σ_{11} in time in the middle of the first floor of a twenty-eight-story building.

As seen in Figure 5, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{11} = 10.1\,MPa$.

Figure 6 shows a graph characterizing changes in the maximum normal stress σ_{22} in the middle of the first floor of a twenty-eight-story building in time t.

σ_{22}, MPa

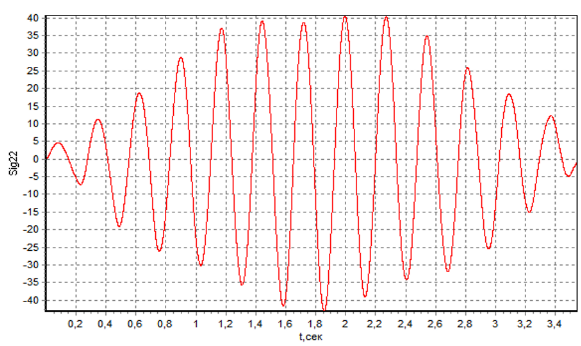

Fig. 6. Graph of changes in normal stress σ_{22} in time in the middle of the first floor of a twenty-eight-story building.

As seen in Figure 6, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{22} = 41.1MPa$.

Table 4 shows the minimum and maximum stress values obtained during transverse vibrations of a 28-story building.

Table 4. Values of maximum and minimum stresses of a 28-story large-panel building during an earthquake of 7-8-9 magnitude.

№	*H,m*	k_s	ν_0, *Hz*	p_1, *Hz*	σ_{11}, *MPa*		σ_{22}, *MPa*		$\ddot{r}$, $m/\sec^2$	
					min	max	min	max	min	max
1	22	0.1	3.5	0.9	-3.48	3.31	-10.9	10.6	-11.9	12.0
2		0.2			-4.84	4.51	-20.2	19.8	-22.5	22.7
3		0.4			-10.1	10	-41.1	40.0	-44.1	43.2

The height and width of the building are *b=84m, a=30m* and *H=22*, respectively. Calculations have shown that under external dynamic influences with a large amplitude, quite dangerous stresses appear at different upper levels of external walls during magnitude seven, eight and nine earthquakes.

It was established that the module maximum values of normal stresses σ_{11} and σ_{22} in the middle of the lower part of a 28-story building during seven, eight and nine magnitude earthquakes are:

$$\sigma_{11} = 3.48\,\mathrm{M}Pa, \quad \sigma_{11} = 4.51\,\mathrm{M}Pa, \quad \sigma_{11} = 10.1\,\mathrm{M}Pa,$$
$$\sigma_{22} = 10.9\,\mathrm{M}Pa, \quad \sigma_{22} = 20.2\,\mathrm{M}Pa, \quad \sigma_{22} = 41.1\,\mathrm{M}Pa.$$

When obtaining numerical results for a twenty-story building, the number of divisions on the grid along spatial coordinates is $N = 30,\ \ M = 72$.

Note that based on the application of a continuum model of multi-story buildings using the finite difference method, dynamic calculation methods, algorithms, and programs for multi-story buildings under seismic impacts were developed.

6.4 Calculation results for a 32-story building

Calculations were performed for the following values: frequency of external influence $\nu_0 = 2{,}98$ Hz, the period of the main oscillation tone $T_0 = 1/\nu_0 = 0.34\sec$. The amplitude of the external influence A_0 is determined depending on the magnitude of the earthquake.

The height and width of the building are assumed to be *b=110m* and *a=30m, H=25m* respectively. Let us present numerical stress results obtained from transverse vibrations of a thirty-two-story building during seven, eight and nine-point earthquakes.

The values of the natural frequency of a five-story building $p_0 = 3.5$ Hz, and the period of the fundamental vibration tone $T_0 = 1/p_0 = 0.29$ sec, are calculated.

Figures 7 and 8 show graphs characterizing changes in the maximum normal stresses σ_{11}, σ_{12}, σ_{22} in the middle of the first floor of a thirty-two-story building in time t during seven, eight and nine-point earthquakes.

σ_{11},
MPa

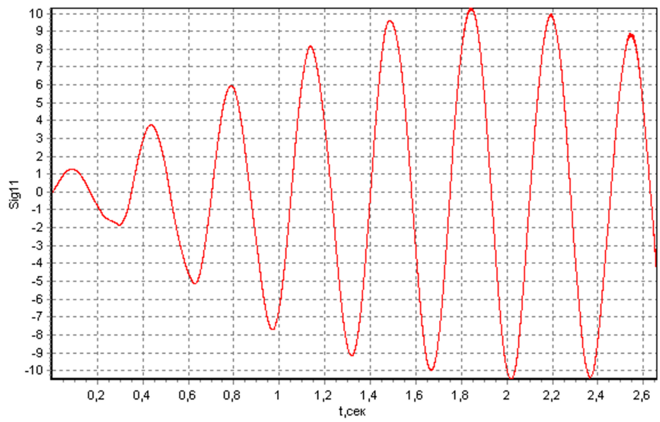

Fig. 7. Graph of changes in normal stress σ_{11} in time in the middle of the first floor of a thirty-two story building.

As seen in Figure 7, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{11} = 10.3MPa.$

σ_{22},
MPa

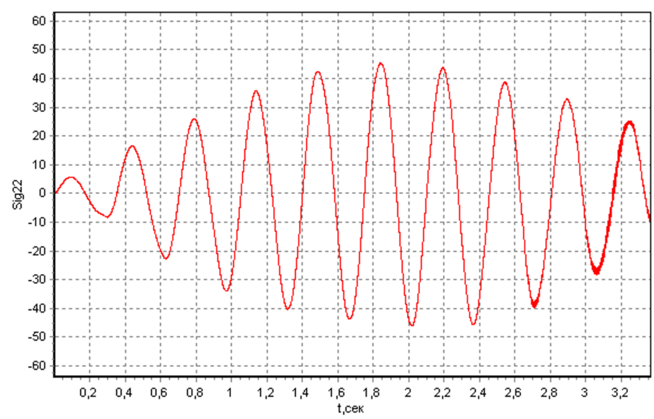

Fig. 8. Graphs characterizing changes in the maximum normal stress σ_{22} in the middle of the first floors of a thirty-two story building in time t.

As seen in Figure 8, in the middle of the first floor of the module building, the maximum value of the normal stress turned out to be $\sigma_{22} = 43.2MPa.$

Table 5 shows the minimum and maximum stress values obtained from transverse vibrations of a 32-story building. The height and width of the building are assumed to be *b=110m* and *a=30m*, respectively. It should be noted that the minimum and maximum values of shear and normal stresses were found under forced transverse vibrations in the middle and quarter of the length a = 30 m of the multi-story high-rise building under consideration.

Table 5. The values of the first natural frequency, maximum and minimum stresses of a 32-story large-panel building during an earthquake of 7-8-9 points.

№	H, m	k_s	ν_0 Hz	p_1 Hz	σ_{11} MPa,		σ_{22} MPa,		$\ddot{r}$, $m/\sec^2$,	
					min	ma x	min	max	min	max
1	25	0.1	2.98	3.05	-2.5	2.6	-11.8	11.9	-8.25	8.01
2		0.2			-5.07	5.02	-23.5	23.9	-16.1	16.2
3		0.4			-10.3	10.2	-43.1	43.2	-35.0	35.1

It is established that the module has the maximum values of normal stresses σ_{11} and σ_{22} in the middle of the lower part of a thirty-two-story building during seven-, eight- and nine-point earthquakes:

$$\sigma_{11} = 2.6 \text{ M}Pa, \quad \sigma_{11} = 5.07 \text{ M}Pa, \quad \sigma_{11} = 10.3\,\text{M}Pa,$$
$$\sigma_{22} = 11.9\,\text{M}Pa, \quad \sigma_{22} = 23.9\,\text{M}Pa, \quad \sigma_{22} = 43.2\,\text{M}Pa.$$

When obtaining numerical results for a five-story building, the number of divisions on the grid according to spatial coordinates is assumed $N = 30,\ M = 110$.

Thus, for various options of geometric dimensions, numerical values of accelerations and stresses during transverse vibrations of multi-story buildings with a number of floors ranging from twenty to thirty-two were obtained.

Based on the analysis of the numerical results presented in Tables 1-4, obtained from a continuum plate model of a multi-story building under seismic influence in the form of base acceleration, it was established that the values of displacements and accelerations of the building floors increase from 1.2 to 8 on different floors compared to their values in the base.

7 Conclusions

Based on the application of the resonance method, the first three values of the natural frequency of a multi-story building were determined.

For various options of geometric dimensions, numerical results of calculations of displacements, accelerations, and stresses during transverse and longitudinal vibrations of multi-story buildings were obtained.

Based on the analysis of numerical results, it was established that the plate model is suitable for describing the dynamic behavior and calculating the stress-strain state of multi-story buildings under seismic impacts.

The developed methods for the dynamic calculation of buildings for seismic resistance made it possible to determine displacements, stresses, and accelerations, and dangerous zones of multi-story buildings under seismic influences.

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