To the solution of the problem of longitudinal vibrations of multi-storey buildings on the basis of the plate model

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Abstract. The article is devoted to the development of a continual spatial plate model of a multi-story building, developed in the framework of the bimoment theory of thick plates. A technique for dynamic spatial calculation for the seismic resistance of buildings under longitudinal seismic impacts is proposed. Formulas are given for determining the reduced moduli of elasticity. Numerical results of eigenfrequencies and displacements are obtained.

1 Introduction

Among the numerous objects of study in the mechanics of a deformable rigid body, a special place is occupied by multi-story buildings and structures. The development of dynamic models of buildings and structures, and the spatial nature of their deformation are complex topical problems of mechanics. To date, a universal model of a multi-story building has not been yet developed. This is due to the complex structure, diversity, and multiplicity of elements of the building. There are many articles and monographs devoted to the development of the theory of seismic resistance of a building and methods for calculating buildings and structures for seismic effects, taking into account various important factors. Multi-story buildings erected in seismic areas must meet the requirements of seismic resistance.

One of the important tasks of the modern theory of seismic resistance of structures is the development of calculation models of buildings that adequately describe their vibrations during earthquakes.

Buildings are complex objects of study in structural mechanics. To date, there are no universal methods for the dynamic calculation of their stress-strain state (SSS), due to the large number of elements and the complex structure of high-rise buildings. The theory of seismic resistance of buildings and structures is developing as one of the topical areas in structural mechanics. There are numerous studies devoted to the development of the theory

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of seismic stability. Various methods were developed for calculating buildings and structures for seismic effects, taking into account such important factors as seismic load, soil conditions of the area and design features of building structures. Note that the analysis of the consequences of many strong earthquakes showed the shortcomings of the existing methods for calculating buildings and structures for seismic resistance.

The study in [1] is devoted to the static consideration of higher vibration modes in the problems of the dynamics of building structures under an external harmonic load. Using a computing complex, the displacements of nodes and internal forces in the elements of the structures under consideration were determined. The influence of displacements and fractures of the axes of wall panels during their installation on the operation of large-panel structures were considered in [2]. The analysis of design schemes was performed considering various types of installation errors. Forces in structural elements exceeding the allowable ones were determined to account for the error in the installation of parts.

The most reliable and accurate methods for determining the parameters of reinforcement were shown. In [3], [4], the dynamic characteristics and vibrations of various axisymmetric and plane structures are considered, taking into account different geometries, spatial factors, and inelastic properties of materials.

In [5], an analytical calculation of the brickwork of a barrel-shaped vault was considered; the structure of its material has a pronounced variability of elastic constants. In regulatory documents, brickwork is considered a complex two-component building material with elastic-plastic properties. However, there are no clear recommendations that consider the variability of the elastic properties of brickwork. A mathematical solution to a fourth-order partial differential equation with two variables for an anisotropic orthotropic body in polar coordinates is given to create mathematical models that describe the change in the modulus of elasticity of the vault material. Based on the solution of the anisotropy problem for a curvilinear orthotropic body, the authors obtained correlations between the elastic constants in the main anisotropy directions.

A technology for the production of anorthite-based building ceramics using semi-dry powder pressing based on the sintering of raw mixes consisting of low-melting clay and blast-furnace sludge (BFS) in various proportions is presented in [6]. The fabricated ceramic samples are sintered at a temperature of 1050°C. The properties of the raw mix to increase the content of the anorthite phase in ceramic samples were studied. Studies of the physical and mechanical properties of ceramic samples show that the addition of BFS to the mix composition provides the compressive strength of the obtained samples up to 48.8 MPa, which is 25% higher than that of the control sample. The higher compressive strength is due to the formation of an anorthite phase, which is proven by X-ray studies. According to the differential thermal analysis of the obtained samples, the exo-effect occurs during sintering at a temperature of 1050°C, which is typical for the formation of an anorthite phase.

Reference [7] considers the foundations of buildings and structures laid of weakly viscoelastic soils and the features of the theoretical justification of their deformations. The need for this study is due to the discrepancy between the theory of seepage compaction and field and laboratory experiments. Within the framework of the proposed model, designs are constructed for solving problems of loading the soil surface with typical loads that describe the stress-strain state of each phase of a two-phase medium (soil skeleton + pore water), taking into account the residual pore pressure. The deviation of the calculated residual pore pressure from the experimental data is no more than 5% (in laboratory experiments), 7% (in field experiments). The calculation method presented in the article makes it possible to predict the deformation of the foundations of structures on weak water-saturated soils.

Articles [8,9] are devoted to the dynamic problems of the deformed state of earth dams under seismic impacts. A method was developed for solving wave problems for determining the stress-strain state of earthworks, in particular earth dams. Using the finite difference method, calculation formulas and an algorithm for solving problems were developed.

In [10], the behavior and stress-strain state of structures and soils were studied, taking into account the nonlinear deformation of soil surrounding the structure. The existence of a near-contact soil layer which can play the role of seismic protection for structures, was shown.

There are numerous studies devoted to the development of the theory of seismic stability. Various methods were developed for calculating buildings and structures for seismic effects, taking into account such important factors as seismic load, soil conditions of the area and design features of building structures. These studies include the publications of the authors of this article [12-14].

Articles [15-18] are devoted to the dynamic calculation of the box-shaped structure of buildings for seismic resistance, taking into account the spatial work of box-shaped elements under the action of dynamic impact. A mathematical model and a numerical-analytical method for solving the problem of dynamics by the method of finite differences and expanding the solution by the modes of natural vibrations in the spatial statement of elements of box-shaped structures under kinematic action were developed. The forced oscillations of box structures under harmonic influences applied to the base of the structure were studied. The areas where the highest values of shear forces and bending moments occur under harmonic influences were determined.

Article [19] is devoted to solving the problem of transverse vibrations of a multi-story building in the framework of a spatial model of multi-story buildings under seismic action using an explicit scheme of the finite difference method. As a dynamic model of a multi-story building, a continuum model in the form of an orthotropic plate is proposed, the theory of which is developed in the framework of the three-dimensional theory of elasticity and takes into account not only traditional forces and moments, but also bimoments [20].

This paper proposes a spatial continuum lamellar dynamic model, methods and programs for calculating multi-story buildings for seismic resistance, which make it possible to determine dangerous sections and butt joints of its elements at different intensities of seismic loads. Recommendations for the application of the bimoment theory of flexural and longitudinal vibrations of plate models of buildings were developed.

A multi-story building is modeled as a continuous thick plate. Seismic vibrations of a building are modeled by the movement of a thick anisotropic cantilevered plate, the deformation of which is described on the basis of the bimoment theory of thick plates. This model is considered the most suitable model for a multi-story building to conduct a dynamic spatial analysis for seismic resistance of buildings under seismic impacts.

Definitions of the reduced density and modulus of elasticity of the plate model are given in [19]; the reduced density of the building is determined by the following formula:

$$m_{\text{пр}} = \rho_{\text{пл}} V_1 = \rho_{\text{пр}} V_0 \,. \tag{1}$$

Here V_1 is the volume of the plate forming one floor of the building. V_0 is the volume of one floor of the building.

Taking into account the geometric parameters of the building under consideration, we obtain the following formulas to calculate these volumes:

$$V_0 = ab_1H,\ V_1 = ab_1h_2 + (n-2)Hb_1h_2 + aHh_2, \tag{2}$$

where $a,\ H$ are the length and width of the building; b_1 is the height of one floor of the building; k is the number of internal transverse walls of the building; h_1 is the thickness of external load-bearing walls; h_2 is the thickness of internal walls; h_{nep} is the floor thickness.

In the general case, the reduced elastic characteristics and building density are determined by the following formulas:

$$E_1^{\text{пр}} = \zeta_{11} E_0, \quad E_2^{\text{пр}} = \zeta_{22} E_0, \quad E_3^{\text{пр}} = \zeta_{33} E_0,$$
$$G_{12}^{\text{пр}} = \zeta_{12} G_0, \quad G_{13}^{\text{пр}} = \zeta_{13} G_0, G_{23}^{\text{пр}} = \zeta_{23} G_0, \quad \rho_{\text{пр}} = \rho_0 \varsigma_0. \tag{3}$$

It should be noted that the values of coefficients $\xi_{11}, \xi_{22}, \xi_{33}, \xi_{12}, \xi_{13}, \xi_{23}, \zeta_0$ for each cell (room) of the discrete part of the building are determined as functions of two spatial variables, E_0, G_0 are the moduli of elasticity and shear of the strongest load-bearing panel of the cell of the discrete part of the building.

Let us write the formulas for determining coefficients $\xi_{11}, \xi_{22}, \xi_{33}, \xi_{12}, \xi_{13}, \xi_{23}, \zeta_0$ of the reduced moduli of elasticity of the discrete part of the building:

$$\xi_{11} = \alpha \frac{S_{11}}{S_{01}}, \quad \xi_{22} = \alpha \frac{S_{22}}{S_{02}}, \quad \xi_{33} = \alpha \frac{S_{33}}{S_{03}}, \xi_{12} = \alpha \frac{S_{12}}{S_{01}},$$
$$\xi_{13} = \alpha \frac{h_{nep}}{b_1} \lambda^*, \quad \xi_{23} = \alpha \frac{h_2}{a_1}, \quad \zeta_0 = \frac{V_1}{V_0}. \tag{4}$$

Where S_{01}, S_{02}, S_{03} are the cross-sectional areas of the building in three coordinate planes of one floor of the building; S_{11}, S_{22}, S_{33} are the total cross-sectional areas of the plates in the coordinate planes that form one floor of the building; λ^* is the coefficient characterizing the voids in the cross-section of the floor plate. The coefficient α is determined depending on the cellular structure of the building structure.

Depending on the dimensions of the plates, rooms and the building itself, the above areas are determined using the methodology presented in [19] in the following form:

$$S_{01} = E_0 b_1 H, \quad S_{02} = E_0 a H, \quad S_{03} = E_0 a b_1, \tag{5}$$

$$S_{11} = b_1 h_2 E_b^{(2)} + H h_{nep} E_{nep}, \quad S_{12} = b_1 h_2 E_b^{(2)},$$
$$S_{22} = a h_2 E_b^{(2)} + (k-2) H h_2 E_b^{(2)}, \; S_{33} = a h_2 E_b^{(2)} + (k-2) b_1 h_2 E_b^{(2)}. \tag{6}$$

Here G_{nep} is the shear modulus of the building floor; G_2 is the shear module of internal walls; $E_b^{(2)}$ is the modulus of elasticity of internal walls; E_{nep} is the modulus of elasticity of the floor plate.

When determining the reduced moduli of elasticity and shear of the external walls, taking into account window openings, we apply the technique given in [21] in the form of approximate formulas:

$$E_1^{прив} = E_1\left(1-\frac{\eta}{\eta_0}\right),\ E_2^{прив} = E_2\left(1-\frac{\eta}{\eta_0}\right),\ G_{12}^{прив} = G_{12}\left(1-\frac{\eta}{\eta_0}\right),\ G_{13}^{прив} = G_{13}\left(1-\frac{\eta}{\eta_0}\right). \quad (7)$$

where $E_1,\ E_1,\ G_{12},\ G_{13}$ are the moduli of elasticity and shear of the external walls, η, η_0 are constant coefficients.

The resulting formulas are the values of the reduced moduli of elasticity of the considered plate model of the building.

The values of coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$ for each cell (room) of the building are determined as functions of two spatial variables, $E_0,\ G_0$ are the moduli of elasticity and shear of the strongest load-bearing panel of the building.

Formulas (1) - (7) determine the reduced moduli of elasticity of the discrete part of the plate model of the building. According to these formulas, the reduced moduli of elasticity are 8–30 times less than the elastic modulus of the panels, and the reduced density of the plate model of the discrete part of the building is 7–20 times less than the density of the panel material. Such a discrepancy between the modules is explained by the presence of a large number of voids in the cellular structure of the building.

2 Formulation of the problem

Longitudinal oscillations of a multi-story building within the framework of a plate model of a multi-story building are considered in the Cartesian coordinate system $x_1,\ x_2$ and z. The origin of coordinates is located in the lower left corner of the middle surface of the continual plate model of a multi-story building. Let us direct the OX_1 and OX_2 axes along the length and height, and the OZ-axis - along the thickness (width of the building) of the plate model.

The problem of longitudinal vibrations of a multi-story bimoment theory of plate structures consists of two equations for longitudinal and shear forces and four additionally constructed bimoment equations for nine unknown kinematic functions:

$$\begin{gathered}\bar{u}_k = \frac{u_k^{(+)} + u_k^{(-)}}{2}, \quad \overline{\psi}_k = \frac{1}{2h}\int_{-h}^{h} u_k dz, \quad \bar{\beta}_k = \frac{1}{2h^3}\int_{-h}^{h} u_k z^2 dz, \quad (k=1,2), \\ \overline{W} = \frac{u_3^{(+)} - u_3^{(-)}}{2}, \quad \bar{r} = \frac{1}{2h^2}\int_{-h}^{h} u_3 z dz, \quad \bar{\gamma} = \frac{1}{2h^4}\int_{-h}^{h} u_3 z^3 dz.\end{gathered} \quad (8)$$

Forces $N_{11},\ N_{12},\ N_{22}$ from stresses $\sigma_{11}, \sigma_{12}, \sigma_{22}$ are defined by the following expressions:

$$\begin{gathered}N_{11} = E_{11}H\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{12}H\frac{\partial\overline{\psi}_2}{\partial x_2} + 2E_{13}\overline{W}, \\ N_{22} = E_{12}H\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{22}H\frac{\partial\overline{\psi}_2}{\partial x_2} + 2E_{23}\overline{W},\ N_{12} = N_{21} = G_{12}\left(H\frac{\partial\overline{\psi}_2}{\partial x_1} + H\frac{\partial\overline{\psi}_2}{\partial x_1}\right).\end{gathered} \quad (9)$$

The bimoments generated under longitudinal vibrations of the plate model of the building T_{11}, T_{22}, T_{12} from stresses $\sigma_{11}, \sigma_{12}, \sigma_{22}$ are defined as:

$$T_{11} = H\left(E_{11}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{12}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{13}\frac{2\overline{W} - 4\overline{r}}{H} \right),$$
$$T_{12} = T_{21} = HG_{12}\left(\frac{\partial\overline{\beta}_2}{\partial x_1} + \frac{\partial\overline{\beta}_1}{\partial x_2} \right), \quad T_{22} = H\left(E_{12}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{22}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{23}\frac{2\overline{W} - 4\overline{r}}{H} \right). \tag{10}$$

The intensities of the transverse bimoments generated under longitudinal vibrations of the plate model of the building $\overline{p}_{13}, \overline{p}_{23}$ and $\overline{\tau}_{13}, \overline{\tau}_{23}$ from shear stresses σ_{13}, σ_{23} are constructed in the following form:

$$\overline{p}_{k3} = G_{k3}\left(\frac{\partial\overline{r}}{\partial x_k} + \frac{2(\overline{u}_k - \overline{\psi}_k)}{H} \right), \quad \overline{\tau}_{k3} = G_{k3}\left(\frac{\partial\overline{\gamma}}{\partial x_k} + \frac{2(\overline{u}_k - 3\overline{\beta}_k)}{H} \right), \quad (k = 1,2). \tag{11}$$

For the intensity of bimoments $\overline{p}_{33}$ and $\overline{\tau}_{33}$ from normal stress σ_{33}, we have the following equations:

$$\overline{p}_{33} = E_{31}\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{32}\frac{\partial\overline{\psi}_2}{\partial x_2} + E_{33}\frac{2\overline{W}}{H}, \quad \overline{\tau}_{33} = E_{31}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{32}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{33}\frac{2\overline{W} - 4\overline{r}}{H}. \tag{12}$$

The system of equations of motion of the plate model of the building relative to the longitudinal and shear forces generated under longitudinal oscillations of the plate model of the building are built in the following form:

$$\frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{12}}{\partial x_2} = \rho H\ddot{\overline{\psi}}_1, \quad \frac{\partial N_{21}}{\partial x_1} + \frac{\partial N_{22}}{\partial x_2} = \rho H\ddot{\overline{\psi}}_2 \tag{13}$$

It should be noted that the system of two equations (15) contains three unknown functions $\overline{\psi}_1$, $\overline{\psi}_2$, $\overline{W}$.

To determine the longitudinal and shear bimoments generated under longitudinal vibrations of the plate model of the building, two equations of motion are constructed:

$$\frac{\partial T_{11}}{\partial x_1} + \frac{\partial T_{12}}{\partial x_2} - 4\overline{p}_{13} = \rho H\ddot{\overline{\beta}}_1, \quad \frac{\partial T_{12}}{\partial x_1} + \frac{\partial T_{22}}{\partial x_2} - 4\overline{p}_{23} = \rho H\ddot{\overline{\beta}}_2. \tag{14}$$

Two more equations of plate motion with respect to the intensity of transverse bimoments, which are absent in the conventional theory of plates, are constructed in the following form:

$$\frac{\partial\overline{p}_{13}}{\partial x_1} + \frac{\partial\overline{p}_{23}}{\partial x_2} - \frac{2\overline{p}_{33}}{H} = \rho\ddot{\overline{r}}, \quad \frac{\partial\overline{\tau}_{13}}{\partial x_1} + \frac{\partial\overline{\tau}_{23}}{\partial x_2} - \frac{6\overline{\tau}_{33}}{H} = \rho\ddot{\overline{\gamma}} \tag{15}$$

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Three missing kinematic equations of motion of the plate model with longitudinal vibrations of a multi-story building, which determine the generalized displacements $\bar{u}_1,\ \bar{u}_2,\ \overline{W}$ are rewritten in the following form:

$$\bar{u}_k = \frac{1}{4}\left(21\bar{\beta}_k - 3\bar{\psi}_k\right) - \frac{1}{20}H\frac{\partial \overline{W}}{\partial x_k},\quad (k=1,2), \tag{16}$$

$$\overline{W} = \frac{1}{2}\left(21\bar{\gamma} - 7\bar{r}\right) - \frac{1}{30}H\left(\frac{E_{31}}{E_{33}}\frac{\partial \bar{u}_1}{\partial x_1} + \frac{E_{32}}{E_{33}}\frac{\partial \bar{u}_2}{\partial x_2}\right). \tag{17}$$

Let us write down the boundary conditions for the considered problem of vibrations of multi-story buildings. When describing the boundary conditions for the equations of longitudinal vibrations of buildings (16) - (17), we introduce the intensities of bimoments $\bar{\sigma}_{11},\ \bar{\sigma}_{22},\ \bar{\sigma}_{12},\ \bar{\sigma}^*_{11},\ \bar{\sigma}^*_{22}$, determined by the following formulas:

$$\begin{aligned}
\bar{\sigma}_{11} &= \left(E_{11} - \frac{E_{13}}{E_{33}}E_{31}\right)\frac{\partial \bar{u}_1}{\partial x_1} + \left(E_{12} - \frac{E_{13}}{E_{33}}E_{32}\right)\frac{\partial \bar{u}_2}{\partial x_2},\\
\bar{\sigma}_{22} &= \left(E_{21} - \frac{E_{23}}{E_{33}}E_{31}\right)\frac{\partial \bar{u}_1}{\partial x_1} + \left(E_{22} - \frac{E_{23}}{E_{33}}E_{32}\right)\frac{\partial \bar{u}_2}{\partial x_2},\\
\bar{\sigma}_{12} &= G_{12}\left(\frac{\partial \bar{u}_1}{\partial x_2} + \frac{\partial \bar{u}_2}{\partial x_1}\right).
\end{aligned} \tag{18}$$

Based on Hooke's law and expressions (18), we obtain the following expressions for bimoments
$\bar{\sigma}^*_{11},\ \bar{\sigma}^*_{22}$:

$$\begin{aligned}
\bar{\sigma}^*_{11} &= -E_{11}H\frac{\partial^2 \overline{W}}{\partial x_1^2} - E_{12}H\frac{\partial^2 \overline{W}}{\partial x_2^2} + E_{13}\frac{60\left(7\overline{W} + 42\bar{r} - 105\bar{\gamma}\right)}{H},\\
\bar{\sigma}^*_{22} &= -E_{12}H\frac{\partial^2 \overline{W}}{\partial x_1^2} - E_{22}H\frac{\partial^2 \overline{W}}{\partial x_2^2} + E_{23}\frac{60\left(7\overline{W} + 42\bar{r} - 105\bar{\gamma}\right)}{H}.
\end{aligned} \tag{19}$$

At the base of the plate model of a multi-story building, the boundary conditions for flexural-shear vibrations have the following form:

$$\bar{\psi}_1 = u_0(t),\ \bar{\psi}_2 = 0,\ \bar{\beta}_1 = \frac{1}{3}u_0(t),\ \bar{\beta}_2 = 0,\ \bar{u}_1 = \frac{1}{3},\ \ \bar{u}_2 = 0,\ \ \bar{r} = 0,\ \ \bar{\gamma} = 0,\ \ \overline{W} = 0. \tag{20}$$

where $u_0(t)$ is the law of motion of the base.

On the free side faces of the building, we have the conditions of equality to zero of forces, moments and bimoments and force factors:

$$\begin{aligned}
&N_{11} = 0,\ \ N_{12} = 0,\ \ T_{11} = 0,\ \ T_{12} = 0,\ \ \bar{p}_{13} = 0,\ \ \bar{\tau}_{13} = 0,\\
&\bar{\sigma}_{11} = 0;\ \ \bar{\sigma}_{12} = 0\ \ \bar{\sigma}^*_{13} = 0.
\end{aligned} \tag{21}$$

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On the free upper face of the building we have the following conditions:

$$N_{12}=0,\quad N_{22}=0,\quad T_{12}=0,\quad T_{22}=0,\quad \bar{p}_{23}=0,\quad \bar{\tau}_{23}=0,$$
$$\bar{\sigma}_{12}=0;\quad \bar{\sigma}_{22}=0\quad \bar{\sigma}^{*}_{23}=0. \tag{22}$$

It is assumed that the seismic ground motion occurs in the direction of the OZ-axis (width or thickness of the building).

Based on the consideration, the external seismic impact is given as the acceleration of the base $\ddot{u}_0(t)$ in the following form:

$$\ddot{u}_0(t)=a_0\cos(\omega_0 t)\,, \tag{23}$$

where $a_0=k_c g$ and $\omega_0=2\pi\nu_0$ are, respectively, the maximum acceleration and the circular frequency of the soil foundation, k_c and ν_0 are the magnitude factor of the earthquake and the natural frequency of the external impact.

From the expression of acceleration, the displacements of the base of the building are determined in the form:

$$u_0(t)=\frac{A_0}{2}(1-\cos(\omega_0 t)). \tag{24}$$

Here, A_0 is the amplitude of displacement of the base.

The amplitude of external impact A_0 depends on the magnitude of the earthquake, determined from condition $A_0\omega_0^2=2k_c g$, where k_c, g are the seismicity coefficient and the free fall acceleration.

3 Solution method

For the numerical solution to the problem posed, the method of finite differences was applied. To approximate the derivatives of displacements with respect to spatial coordinates, we use the formulas of central difference schemes. In this case, $\Delta x_1=\frac{a}{N},\ \Delta x_2=\frac{b}{M}$ is the calculation step, $N,\ M$ are the numbers of partitions, and Δt is the time step. The calculation steps in terms of spatial coordinates and time are chosen as follows:

$$\Delta x_1=\frac{a}{N},\ \Delta x_2=\frac{b}{M},\quad c\Delta t\le \min(\Delta x_1,\Delta x_2)\cdot$$

Dimensionless variables x=x1/a, y=x2/b, τ=ct /H, where $c=\sqrt{E/\rho}$ are introduced.

4 Analysis of numerical results

Calculations were made for nine- and twelve-story buildings at seven-, eight- and nine-point earthquakes, which are specified through the corresponding seismicity coefficients kc.

At that, internal and external walls, ceilings and floors are considered to consist of reinforced concrete.

For a magnitude 7 earthquake, $k_c = 0.1; \ \nu_0 = 2.7 Hz$. Then the amplitude of the external influence is $A_0 = \frac{2k_c g}{\omega_0^2} = \frac{2 \cdot 0.1 \cdot 9.8}{16.95^2} = 0.68\, cm$.

For an eight magnitude earthquake, $k_c = 0.2; \ \nu_0 = 2.4 Hz$. Then the amplitude of the external influence is $A_0 = \frac{2k_c g}{\omega_0^2} = \frac{2 \cdot 0.2 \cdot 9.8}{15.07^2} = 1.74\, cm$.

For a nine magnitude earthquake, $k_c = 0.4; \ \nu_0 = 2\, Hz$. Then the amplitude of the external influence is $A_0 = \frac{2k_c g}{\omega_0^2} = \frac{2 \cdot 0.4 \cdot 9.8}{12.56^2} = 4{,}97.cm$.

The mechanical and geometric characteristics of the room panel materials and the external dimensions of the buildings are given as input data.

To obtain numerical results, the following initial data were used for the structures of the considered plate model of a multi-story building.

We consider that the external walls are made of reinforced concrete with elastic modulus $E = 20000\,\text{MPa}$, density $\rho = 2500\,\text{kg/m}^3$, Poisson's ratio $\nu = 0.3$. Internal walls are made of expanded clay concrete with the following physical characteristics: modulus of elasticity $E = 7500\,\text{MPa}$, density $\rho = 1200\, kg/m^3$, Poisson's ratio $\nu = 0.3 MPa$.

Let us introduce the following designations for the plate model of a multi-story building: b_1 – the height of one floor of the building; k - the number of internal transverse walls of the building; h_1 - the thickness of the external load-bearing walls; h_2 - the thickness of the internal walls; h_{nep} - the floor thickness.

The results of calculations of forced vibrations of a building within the framework of a thick plate model are given with the following dimensions of building plates:

$$h_1 = 0.40\, m, \ h_2 = 0.25\, m, \ h_{\text{пер}} = 0.2\, m, \ a_1 = 5\, m, \ b_1 = 3\, m,$$

The height and length of a multi-story building are $b = nb_1$ *and* $a = 30\, m$, respectively, and the width of the building H varies.

Then coefficients ξ_0, ξ_{11}, ξ_{22}, ξ_{33}, ξ_{12}, ξ_{13}, ξ_{23} calculated according to the corresponding formulas are:

$$\xi_{11} = 0.11, \ \xi_{22} = 0.137, \xi_{33} = 0.09, \ \xi_{12} = 0.077, \ \xi_{13} = 0.067, \ \xi_{23} = 0.04, \ \xi_0 = 0.142$$

The reduced elastic characteristics of the building are determined by the following formulas:

$$E_1^{\text{пр}} = \xi_{11} E_0, \ E_2^{\text{пр}} = \xi_{22} E_0, \ E_3^{\text{пр}} = \xi_{33} E_0, \ G_{12}^{\text{пр}} = \xi_{12} G_0, \ G_{13}^{\text{пр}} = \xi_{13} G_0, \ G_{23}^{\text{пр}} = \xi_{23} G_0 .$$

Modulus of elasticity and density of concrete are $E = 20000\ MPa,\ \ \rho = 2500\ \ kg/m^3$ .Then, according to the formulas, the reduced characteristics of the building are:

$$E_1^{пр} = 1323.08\,MPa,\ \ E_2^{пр} = 1643.08\,MPa, E_3^{пр} = 1120.00\ MPa,$$
$$G_{12}^{пр} = 369{,}23\ MPa,\ \ G_{13}^{пр} = 320.00\ MPa,\ \ G_{23}^{пр} = 192{,}00 MPa,\ \ \rho_{np} = 351.02\ \ kg/m^3.$$

The length of a nine-story, twelve-story and sixteen-story buildings is assumed to be the same, equal to a=30m.

Let us present the numerical results of stresses obtained under transverse vibrations of a 9-story building during a 9-magnitude earthquake.

Calculations were made for the following values: frequency of external action $\nu_0 = 2.8$ Hz, the period of the fundamental tone of vibrations $T_0 = 1/\nu_0 = 0.36$ c.

5 Calculation results for a 9-story building.

Calculations were made for the following values: frequency of external action $\nu_0 = 2.8$ Hz, the period of the fundamental tone of vibrations $T_0 = 1/\nu_0 = 0.36$ s. The amplitude of external impact A_0 is determined for a nine-story building during an eight-point earthquake $(k_c = 0.2)$ as equal to $A_0 = 0.0174m$.

Figure 1 shows graphs of changes in the values of horizontal longitudinal displacement $\overline{u}_1$ at the highest point of a nine-story building in time τ.

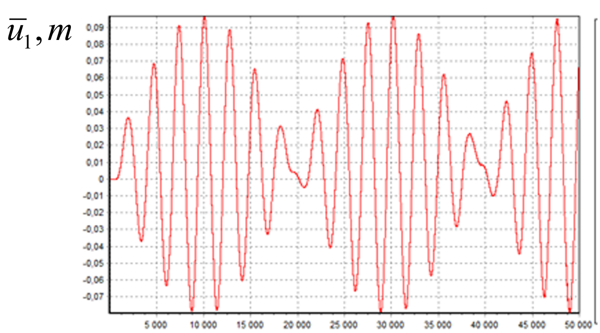

Fig. 1. Graphs of changes in longitudinal displacement $\overline{u}_1$ in time in the middle of the top level of a nine-story building

As it was found, the dynamic behavior of the nine-story building is close to the state of beating, since the values of its natural frequency are close to the frequency of the external influence.

In the graph (Figure 1) it is seen that in the middle of a nine-story building, the maximum displacement is $\overline{u}_1 = 9cm$.

Using the resonance method, the natural frequency values of a nine-story building were calculated depending on two values of the building width $H = 11$m и $H = 13$m, the

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frequencies are p1=8.917 Hz and p1=8.608 Hz, and the periods of fundamental tone of vibrations are T1=1/p1=0.112 s and T1=1/p1=0.116 s.

6 Calculation results for a 12-story building

Calculations were made for the following values: frequency of external impact $\nu_0 = 2.8$ Hz, and the period of the fundamental tone of vibrations $T_0 = 1/\nu_0 = 0.36$ s.

The amplitude of the external impact is $A_0 = 0.0174m$, which corresponds to an eight-point earthquake $(k_c = 0.2)$.

Figure 2 shows graphs of changes in the values of longitudinal horizontal displacements u1 at the edges of a twelve-story building in time t.

From the graph (Figure 2) it can be seen that in the middle of a twelve-story building, the maximum longitudinal displacement is. $\bar{u}_1 = 12cm$

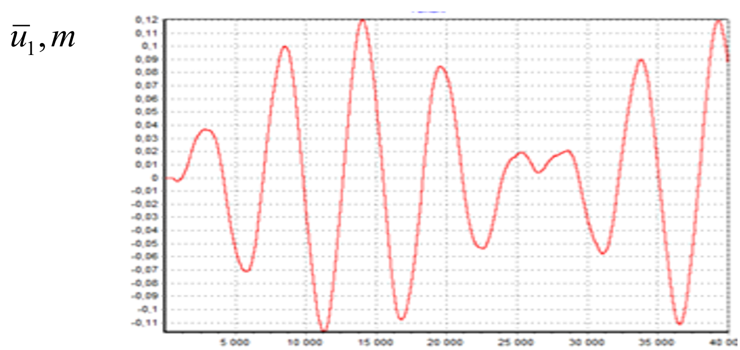

Fig. 2. Graph of changes in displacement u1 in time in the middle of a twelve-story building

7 Own vibrations of multi-story buildings

Calculations were made for nine-, twelve- and sixteen-story buildings for seven-, eight- and nine-point earthquakes, set by the corresponding seismicity coefficients kc. Internal and external walls, ceilings and floors are made of reinforced concrete.

Let us present the results of calculations of natural frequency, periods of oscillations and displacement of points for nine-, twelve-story buildings, near the resonant mode. The dynamic characteristics of a multi-story building under longitudinal vibrations are determined by the resonance method. The values of the external impact frequency, starting from some small value, gradually increase, and for each frequency value, displacements and stresses are calculated. With an increase in the value of the frequency of external influence, a state of beating is observed in the oscillatory process of the building. As the value of the dimensionless frequency of external impact ω_0 approaches dimensionless natural frequency p_1, the values of displacements, forces, and moments increase sharply, which indicates a gradual transition to the resonant mode.

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Table 1 shows the values of natural frequencies p_1 and the periods of natural vibrations T_1 of longitudinal oscillations of nine-, twelve-story buildings, depending on the value of the width of the building H.

The value of the natural frequency of the longitudinal vibrations of a nine-story building at $H = 11\text{m}$ is p0=3.4 Hz. The period of the fundamental tone of natural oscillations is T0=1/p0=0.294 s (Table 1).

The values of the natural frequency of the longitudinal vibrations of a twelve-story building are calculated depending on two values of the width of the building H=11 m and H=13m, which are p1=4.863 Hz and p1=5.178 Hz, and the periods of the fundamental tone of vibration are T1=1/p1=0.205 s and T1=1/p1=0.193 s (Table 1).

The value of the natural frequency of the longitudinal vibrations of a sixteen-story building, depending on two values of the width of the building H=11 m and H=13m, is p1=3.086 Hz, and the period of the fundamental tone of vibrations is T1=1/p1=0.323 s (Table 1).

Table 1. Natural frequencies ν_1 and p_1 and periods of natural oscillations T_{10} of nine-, twelve-, sixteen-story buildings, depending on the width of the building H

№	Number of floors	H, m	ρ_1, Hz	T_1, sec
1	9	11	8.917	0.112
2	12	11	4.863	0.205
		13	5.178	0.193
3	16	13	3.086	0.323

In conclusion, we note that the plate model of the building adequately reflects the modes of vibrations of the building under seismic effects. In calculations, the number of partitions into steps of difference schemes in dimensionless coordinates is taken as follows: for a nine-story building $N = 30,\ M = 30$, for a twelve-story building $N = 30,\ M = 42$, a sixteen-story large-panel building $N = 30,\ M = 52$. The stability of the computation in dimensionless time is ensured by an explicit scheme with step $\Delta\tau = 0.01$.

8 Conclusion

1. A spatial continuum plate model of longitudinal oscillations of multi-story buildings, developed in the framework of the bimoment theory of thick plates, was proposed. Formulas were given for determining the elastic characteristics of a plate model of multi-story buildings, taking into account design features.
2. A technique, algorithm and program for the numerical calculation of the displacements of a multi-story building within the framework of a plate model using an explicit scheme of the finite difference method were developed.
3. The numerical results of the maximum values of displacements at the upper level of nine- and twelve-story buildings were obtained, for which the first frequencies and periods of natural oscillations were determined. The laws of changes of displacements in time were determined in the form of graphs in the state of beating and in resonant mode.

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