PAPER • OPEN ACCESS

Vibrations of multilayer composite viscoelastic curved pipe under internal pressure

To cite this article: Ismoil Safarov *et al* 2021 *IOP Conf. Ser.: Mater. Sci. Eng.* **1030** 012073

View the article online for updates and enhancements.

Vibrations of multilayer composite viscoelastic curved pipe under internal pressure

Ismoil Safarov[2], Muhsin Teshaev[3,4] and J A Yarashev[1]

[1]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39, K.Niyazi str., Tashkent city

[2]Tashkent Chemical-Technological Institute, 32, Navoi str. Tashkent sity, Uzbekistan

[3]BukharaBranch of Institute of Mathematics AS RUz., 11, M.Iqbol street, Bukhara city, Republic of Uzbekistan

[4]Bukhara branch of the Tashkent Institute of Irrigation and Mechanization Engineering in Agricultural, 32, Gazli highway, Bukhara city, Republic of Uzbekistan,

E-mail: safarov54@mail.ru, muhsin_5@mail.ru

Abstract. The paper presents a mathematical statement and the methods to solve the problem of natural vibrations of thin-walled multilayer composite viscoelastic curved pipes under internal pressure. A brief overview of well-known publications devoted to this problem is given. The stress state and flexibility of the pipes are investigated for two types of boundary conditions, taking into account internal pressure and viscoelastic properties of the material and geometrical and physical parameters of the pipe. The paper provides the results of numerical analysis of the spectra of the lower complex eigenfrequencies of the pipe depending on internal pressure, geometrical, structural parameters, rheological properties and boundary conditions. Ten lower eigenfrequencies were found and the corresponding vibration modes constructed. The paper presents a comparison of solution results with known solutions and experimental results.

1. Introduction

The problem of vibration is a relevant one in the oil-and-gas and aircraft industries. Pulsating flows and the intense vibrations initiated by them occur in power plants too [1]. An analysis of the pipelines behavior in nuclear power plants showed [2, 3], that the frequencies of vibrations vary from 0.5 to 1000 Hz, and the vibration amplitudes are 3-5 mm. The phenomenon of pulsations in the coolant circulation circuit [4] and equipment vibrations are the most acute. Cases of the loss-of-piping integrity are known. The fracture pattern indicates the fatigue nature of breaking. The origin and development of cracks is associated with elastic vibrations and hydraulic shock. Thus, the problem of pipelines is gaining important economic importance. When the frequencies of the excitation spectrum coincide with the eigenfrequencies of the pipeline, resonance develops in the system. One way to reduce the amplitudes of vibrations is to detune the dynamic system from resonances at the design stage [5, 6]. The eigenfrequencies of the pipeline should not coincide with the fundamental frequencies of the excitation spectrum. For this, a thorough calculation of the

hydrodynamic loading parameters and analysis of the dynamic stability of a pipeline using classical mechanics methods are needed [7, 8]. At present, regulatory documents [9–11] and reference books [12, 13] contain methods for calculating the static and dynamic parameters of pipelines made of homogeneous and isotropic materials. In these cases, as a rule, simplified rod models were used. To calculate the dynamic characteristics of curved pipes, the linear theory of rod vibrations[14, 15] and the formulas for circular arches[16] [17] were used. The rod forms of vibration are taken into account. Obviously, this approach is justified for thick-walled structures only. Generally, for complex structures, a plane or spatial design scheme of the theory of elasticity were used [27-29], the solution to the problem of natural vibrations for such structures becomes greatly complicated. As is well-known, the development and creation of composite structures and methods to assess their dynamic characteristics is a more complex problem. At that, the subject of design is the material itself, or rather its structure. The new material is designed taking into account technological capabilities for a given structure design and a given load. Only in this unity can the potentialities embodied in the composite be realized. However, the method for calculating the dynamic parameters of multilayer pipes, considering the layered fiber structure and anisotropy of the material, is currently practically absent. The effect of inhomogeneity in the material structure and the initial ovality and non-uniform thickness of the cross section on the stress state and dynamic properties of composite pipe, has not been adequately studied.

In this regard, the task of studying the dynamic properties of multilayer pipes and pipelines made of composite materials is of great practical importance. The need to solve an important scientific and technical problem related to the development of methods for calculating and designing multilayer composite pipes and pipelines in order to improve their dynamic properties and increase reliability determines the relevance of this study.

2. Statement of the problem

Consider a viscoelastic composite pipe whose centerline is an arc of a circle of radius R and length L. The pipe has a cross-section with a nominal average radius r and wall thickness h. Let us restrict ourselves to thin-walled long pipes of small curvature: $h/r \leq 1/20, L/r \geq 4, r/R \leq 1/5$. The pipe is considered as an element of a pipeline that conveys fluid. Consider the internal flow as a homogeneous one, the fluid - as a single-phase, ideal and incompressible one. Rigid weightless flanges that rest on fixed hinged supports close the end sections of the pipe.

The boundary conditions at $s = 0$ and $s = L$ at points with coordinates $(0 \leq s \leq L)$ $\phi_0 = 90^0$ and $\phi_0 = 270^0$ are zero, i.e. the displacement components are $u = \vartheta = w = 0$. To describe the viscoelastic properties of the body material, we accept the linear hereditary Boltzmann–Volterra theory; physical relations for the m-th viscoelastic element of the system are defined as [18]

$$\sigma_{mk}^n(t) = \tilde{\lambda}_n \Theta^n(t)\delta_{mk} + 2\tilde{\mu}_n \varepsilon_{mk}^n(t) , \qquad (1)$$

where $\tilde{\lambda}_n, \tilde{\mu}_n$ are the Volterra integral operators. Poisson's ν_n ratio in the proposed statement of the problem is assumed constant. This means that for a structurally homogeneous viscoelastic system, the modes of natural vibrations are equal to the eigenvectors of the corresponding elastic problem [19, 20]. Expressing integral operators from the well-known formulas in terms of $\tilde{E}_n, \tilde{\nu}_n$, and considering that $\tilde{\nu}_n = \nu_n = const$, instead of (1) we get

$$\sigma_{mk}^n(t) = \frac{\tilde{E}_n}{1+\nu_n}\left[\frac{\nu_n}{1-2\nu_n}\Theta^n(t)\delta_{mk} + \varepsilon_{mk}^n(t)\right] , \qquad (2)$$

where $\tilde{E}_n$ is the Volterra operator of the following form

$$\tilde{E}_n\phi(t) = E_{0n}\left[\phi(t) - \int_0^t R_{En}(t-\tau)\phi(\tau)d\tau\right]. \qquad (3)$$

Here E_{0n} is the instantaneous modulus of elasticity, and R_{En} is the relaxation kernel.

Given (1), the time function in equality (3) is $\varphi(t) = \exp(-i\omega t)$ with slowly varying amplitude. Assuming the smallness of integral term $\int_0^\infty R(\tau)d\tau$, and using the freezing method [21], we replace relation (3) with an approximate one:

$$\tilde{E}_n \phi(t) \cong E_{0j}[1 - \Gamma_n^c(\omega_R) - i\Gamma_n^s(\omega_R)]\phi(t) \equiv \bar{E}_n\phi(t), \tag{4}$$

where $\Gamma_E{}^C(\omega_R) = \int_0^\infty R_E(\tau)\cos\omega_R\,\tau d\tau$, $\Gamma_E{}^S(\omega_R) = \int_0^\infty R_E(\tau)\sin\omega_R\,\tau d\tau$ are the cosine and sine of the Fourier images of the relaxation kernel of the material, respectively, ω_R is the real value. A three parametric relaxation kernel [30] $R_E(t) = Ae^{-\beta_1 t}/t^{1-\alpha_1}$ is taken as a sample of a viscoelastic material. Present the following forms of motion under given boundary conditions [27,28]:

$$w(s,\phi,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\cos n\,\phi\,\sin\frac{m\pi s}{L},$$

$$\vartheta(s,\phi,t) = -\sum_{m=1}^{\infty}\sin\frac{m\pi s}{L}\sum_{n=1}^{\infty}\frac{1}{n}w_{mn}\sin$$

$$u(s,\phi,t) = \frac{\pi r}{L}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m}{n^2}w_{mn}\cos n\,\phi\,\cos\frac{m\pi s}{L}. \tag{5}$$

Here u, ϑ and w are the displacement components of the points of the shell middle surface in the axial, circumferential, and radial directions. The wavenumbers m and n characterize the vibration mode: m is the number of half-waves in the axial direction, n is the number of waves in the circumferential direction.

The following equation determines the kinetic energy of the pipe motion:

$$K = \frac{1}{2}\rho_T r h_m \int_0^L \int_0^{2\pi}(\dot{u}^2 + \dot{\vartheta}^2 + \dot{w}^2)h(\phi)dsd\phi = \frac{1}{8}(m_T)\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{n^2+1}{n^2} + \frac{m^2\pi^2 r^2}{n^4 L^2}\right)\dot{w}_{mn}^2 \tag{6}$$

The viscoelastic potential, constructed based on the semi-momentless theory of multilayer thin shells and approximations (5), has the form:

$$\Pi = \frac{1}{2}r\int_0^L\int_0^{2\pi}(B_{1m}\varepsilon_1^2 + D_{2m}k_2^2)dsd\phi =$$

$$= \frac{\pi B_{1m}}{4}rL\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[-\left(\frac{m\pi}{L}\right)^2\frac{r}{n^2}w_{mn} + \frac{n-2}{n-1}\frac{w_{mn-1}}{D} + \frac{n+2}{n+1}\frac{w_{mn+1}}{D}\right]^2 +$$

$$+ \frac{\pi D_{2m}L}{4r^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(n^2-1)^2 w_{mn}^2 \quad. \tag{7}$$

The axial deformation of the wall middle surface has the following form:

$$\varepsilon_1 = \frac{1}{R}(w\cos\phi - \vartheta\sin\phi) =$$

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[-\left(\frac{m\pi}{L}\right)^2\frac{r}{n^2}w_{mn} + \frac{n-2}{n-1}\frac{w_{mn-1}}{D} + \frac{n+2}{n+1}\frac{w_{mn+1}}{D}\right]\cos n\,\phi\,\sin\frac{m\pi s}{L}. \tag{8}$$

The change in wall curvature in the circumferential direction is

$$k_2 = \frac{1}{r^2}\left(\frac{\partial^2 w}{\partial\phi^2} - \frac{\partial\vartheta}{\partial\phi}\right) = -\frac{1}{r^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[(n^2-1)w_{mn}]\cos n\,\phi\,\sin\frac{m\pi s}{L}. \tag{9}$$

The potential of external forces is

$$W = \frac{1}{2}p_m\Delta V = \frac{1}{4}\pi p_m L\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[(n^2-1)w_{mn}^2\right] \quad. \tag{10}$$

Here ΔV is the change in internal cavity volume due to the pipe wall bending. Under pipe vibrations, the pressure p_m resists the wall bending, D = 2R.

Using the Lagrange equations [22]:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial\dot{w}_{mn}}\right) + \frac{\partial\Delta}{\partial\dot{w}_{mn}} - \frac{\partial(K-\Pi-W)}{\partial w_{mn}} = Q_{mn},$$

and dependences (2) - (10), we obtain a coupled system of homogeneous differential equations with complex coefficients of the form

$$[A]\{\ddot{w}\} + D_{1m}(1 - \Gamma_n^\circ(\omega_R))[C]\{w\} = 0 \quad, \tag{11}$$

where $\Gamma_n^\circ(\omega_R) = \Gamma_n{}^C(\omega_R) + i\Gamma_n{}^S(\omega_R)$, $D_{1m} = \frac{h_m}{1-\nu_{12}\nu_{21}}E_{02}$, E_{02} is the instantaneous elastic modulus, $[A]$ is the diagonal matrix, the elements of which are determined by recurrence formulas: $a_{nn} = \frac{n^2+1}{n^2} + \frac{m^2\pi^2 r^2}{n^4 L^2}$, $[C]$ is the square matrix, the elements of which are determined by the following recurrence formulas:

$$c_{nn} = \frac{n^2+1}{n^2} + \frac{2\beta_m{}^2}{n^4} + \frac{1}{6}\varsigma\lambda^2(n^2-1)(n^2-1+3p_m'),$$

$$c_{nn+1} = -2\beta_m \frac{n^2+n+1}{n^2(n+1)^2},$$

$$\text{c}_{11} = 2\beta_m^2 \quad, c_{nn+2} = -2\beta_m \frac{n^2+2n-3}{2n(n+2)}. \tag{12}$$

Here $\beta_m = (\frac{m\pi}{L})^2 rR$ and $p_m' = \frac{p_m r^3}{3D_{2m}}$ are the dimensionless parameters, $D_{2m} = \frac{h_m^3}{1-\nu_{12}\nu_{21}}E_{02}$.

Now the problem of natural vibrations of a viscoelastic curved pipe made of composite materials can be formulated as follows: it is required to find a nontrivial solution, i.e. a nonlinear complex parameter ω^2 and a function w satisfying the homogeneous system of differential equation (11) with complex coefficients.

3. Solution method

To solve equation (11), note that the analysis of the structure of the matrix [C] shows that it corresponds to generalized coordinates related to each other.

The interaction of the generalized coordinates is due to elastic constraints, the intensity of which is characterized by off-diagonal elements of the matrix [C] and depends on the pipe length $L = \theta_0 R$, where θ_0 is the angle at center. The shorter the pipe, the larger the number of half-waves m in section L; and the greater the curvature parameter r/R, the stronger their interaction. As the radius of curvature R increases, the interaction of the generalized coordinates w_{mn} becomes weaker. At $1/R \Rightarrow 0$, a passage to the limit of a thin-walled cylindrical shell occurs. In this case, a system of independent equations is obtained from equations (11):

$$\ddot{w}_n + \Omega_n^2 w_n = 0 \quad (n=2, 3, 4....),$$

$$\Omega_n^2 = \omega_n^2(1 - \Gamma_n^\circ(\omega_R)), \quad \omega_n^2 = \frac{2\pi D_{2m} n^2(n^2-1)(n^2-1+3\tilde{p}_m)}{m_T r^3(n^2+1)}, \tag{13}$$

where Ω_n^2 - is the complex circular eigenfrequency.

At $w_n(t) = 0$, $(n=2, 3, 4....)$ we get a system of equations of bending vibrations of a hinged-supported straight rod:

$$\ddot{w}_m + \Omega_m^2 w_m = 0 \quad (m=1,2, 3,.),$$

where

$$\Omega_m^2 = (\frac{m\pi}{L})^2 \sqrt{\frac{E_{01}(1-\Gamma_n^\circ(\omega_R))}{m_T(1-\nu_{12}\nu_{21})}}.$$

At $\nu_{12} = 0$ or $\nu_{21} = 0$ $(\Gamma_n^\circ(\omega_R) = 0)$, the frequency Ω_m expression coincides with the exact solution [23]. Varying (7) and (10) with respect to the generalized coordinates w_{m2}, from the minimum condition for the total potential energy of the system we find:

$$w_{m2} = \frac{12rR\lambda^2 m_0}{10+12\varsigma_0(1-\Gamma_n^\circ(\omega_R))(1+p_m')+\beta_m^2} w_{m1}, \tag{14}$$

where $m_0 = \frac{m^2\pi^2}{L^2}, \lambda = \frac{r^2}{Rh}, \varsigma_0 = \frac{E_{02}}{E_{01}}$.

The coefficient of a composite curved pipe flexibility increase is determined by the formula [24]

$$k = \frac{m_0'}{m_0} = \frac{10+12\varsigma_0(1-\Gamma_n^\circ(\omega_R))(1+p_m')\lambda^{-2}+\beta_m^2}{1+12\varsigma_0(1-\Gamma_n^\circ(\omega_R))(1+p_m')\lambda^{-2}+\beta_m^2}. \tag{15}$$

Here m_0' and m_0 are the changes in the pipe axis curvature with / without considering the Karman effect, respectively.

4. Solution results and discussion

4.1 Flexibility assessment of viscoelastic composite curved pipes.

Consider the flexibility of viscoelastic composite pipe samples depending on boundary conditions and geometrical parameters. Table 1 presents geometrical characteristics of samples.

Table 1. Geometrical characteristics of samples.

Sample number	R, mm	r, mm	h, mm	φ_0	r/R	h/r	λ
1	835	80	4,2	180°	1/10	1/20	0,50
2	1250	80	4,2	180°	1/15	1/20	0,70

Structure samples considered were made of Kevlar 49/PR-286 organic plastic with physical characteristics $E_\alpha = 64,10 GPa, E_\beta = 5,38 GPa,\ G_{\alpha\beta} = 2,07 GPa, \nu_{\alpha\beta} = 0,35$ and the relaxation kernel parameters $A = 0,048;\ \ \beta_1 = 0,05;\ \ \alpha_1 = 0,1$. The number of monolayers is six. Effective elastic constants with a wall, as a multilayer orthotropic body, depending on reinforcement, are given in [25]. Table 2 presents the calculated values of the flexibility characteristics of a composite pipe with free edges; the values were calculated according to the formula (15). The results obtained showed that the greater the initial curvature of the pipe, the stronger the Karman effect.

Table 2. Design values of flexibility characteristics of the pipe with free edges

Sample number	Coefficients of flexibility increase k depending on reinforcement angles $\pm \varphi_m$				
	50°	60°	70°	80°	90°
1	2,5	1,67	1,35	1,26	3,25
2	1,74	1,31	1,16	1,12	2,6

4.2 Analysis of complex eigenfrequencies of vibrations

Consider the spectra of lower complex frequencies and the corresponding eigenmodes of hinged-supported multilayer viscoelastic curved pipes with parameters: r = 80 mm, h / r = 1/40, and relaxation kernel parameters: $A = 0,048;\ \ \beta_1 = 0,05;\ \ \alpha_1 = 0,10$, at reinforcement angle of $\phi_m = \pm 80^0$, depending on initial curvature r/R = 1/40, 1/20, 1/15. Pipes have the same length $L = 2.5$m, but different bending angles: θ_0= 45 °, 90 °, 135 °, 180 °. The material is organic plastic Kevlar 49/PR-286. The number of monolayers is six. The solution of homogeneous differential equations (11) is sought in the form

$$\{w\} = \{w_{mn}\}e^{-i\omega t} \qquad , \qquad (16)$$

where w_{mn} are the amplitudes of the generalized displacements, $\omega = \omega_R + i\omega_I$ is the complex frequency.

 Substituting (16) into (11), we obtain the following homogeneous algebraic equations

$$(-\omega^2[A] + D_{1m}(1 - \Gamma_n^\circ(\omega_R))[C])\{w_{mn}\} = 0.$$

The frequency equation of the eigenvalue problem is written as:

$$\left[-\omega^2[A] + D_{1m}(1 - \Gamma_n^\circ(\omega_R))[C]\right] = 0 \quad . \tag{17}$$

The roots of characteristic equation (17) are sought by the Mueller method [26].

Table 3. Change in real part of the complex eigenfrequency depending on initial curvature r / R

r/R	Real parts of complex eigenfrequencies $\omega_{Rm\,1}$, Hz			
	m=1	m=2	m=3	m=4
1/10	-	62.41	153.5	515.1
1/15	-	83.69	170.2	486.6
1/20	-	94.54	190.9	455.2
1/40	-	104.6	216.0	433.1
Straight section of a pipe	27.6	110.3	248.3	441.4

Table 4. Change in real part of the complex eigenfrequency depending on angle θ_0

θ_o	Real parts of complex eigenfrequencies, ω_{Rm2}, Hz			
	m=1	m=2	m=3	m=4
45°	247.6	251.6	281.7	269.6
90°	275.1	281.9	321.7	257.6
135°	315.6	324.1	366.4	242.8
180°	364.6	373.7	415.0	228.1
Straight section of a pipe	237.8	243.3	258.6	289.8

Tables 3 and 4 show the change in real part of complex eigenfrequencies of multi-layered viscoelastic curved pipes depending on initial curvature r/R and angle θ_0, respectively. As seen from Table 4, at a decrease in bending angle θ_0 the real parts of the lower complex frequency (ω_{Rm1}), corresponding to n = 1 and m = 2,3,4 modes, increase, while the real parts of the higher frequency ω_{Rm2} and ω_{Rm3}, corresponding to n = 2,3 and m = 1,2,3,4 modes, on the contrary, decrease. At the limit, they approach the eigenfrequencies of the vibrations of a straight section of composite elastic pipe.

5. Conclusions

1.   The paper provided a mathematical problem and a method to assess the natural vibrations of thin-walled composite viscoelastic curved pipes using the theoretical preconditions of the approximate energy method.

2.   A homogeneous differential equation was derived to estimate the natural vibrations of a cylindrical shell and a hinged-supported straight rod, taking into account the layered-fibrous structure and the anisotropy of the material.

3.   The lower complex eigenfrequencies and vibration modes of hinged-supported multilayer composite curved pipes were obtained.

4.   Based on the analysis, it was stated that at a decrease in initial curvature of the pipe, the lower frequencies increase, and the higher ones, on the contrary, decrease, and, at the limit, they approach the eigenfrequencies of a straight section of a pipe.

References

[1]   Voloshin, A..: Strength analysis of pipelines of ship power plants. Leningrad: Shipbuilding (1967).

[2]   Timoshenko, S.P, Young,D.H and Weaver, W.: Fluctuations in Engineering. Moscow: Engineering (1985).

[3]   Bozorov M B, Safarov I I, Shokin, Y.: Numerical simulation of oscillations of dissipatively homogeneous and inhomogeneous mechanical systems, (1996).

[4]   Biderman, V..: Theory of mechanical vibrations. Moscow: Higher School (1980).

[5]   Smooth, A .G and Khachaturian, S.A.: Vibrations in pipelines and methods for their elimination. Moscow: Mashgiz (1959).

[6]   Ibrahimovich, S.I., Khudoyberdiyevich, T.M., Sharipovich, A.M., Ixtiyorovich, B.Z.: Distribution Free Waves in Viscoelastic Wedge with an Arbitrary Angle Tops. Appl. Math. (2017). https://doi.org/10.4236/am.2017.85058.

[7]   Sarbaev, B..: Calculation of the power shell of a composite pressure cylinder. Moscow: Publishing House of MSTU, N.E.Bauman (2001).

[8]   Svetlitsky, V.A.: Mechanics of pipelines and hoses: Tasks of interaction of rods with fluid flow or air. Moscow: Mechanical Engineering (1982).

[9]   Standards for calculating the strength of equipment and pipelines of nuclear power plants (PNAE G-7-002-96). Moscow: Energoatomizdat (1997).

[10]   Standards for calculating the strength of pipelines of heating networks: RD10-400-01. Moscow, STC Industrial Safety. (2001).

[11]   Guidelines for the calculation of the strength and vibration of technological steel pipelines RTM38.001-94. Moscow: Russia (1995).

[12]   Ainbinder, A..: Calculation of trunk and industrial pipelines for strength and stability:a reference guide. Moscow: Nedra (1991).

[13]   Kamerstein, A.G , Rozhdestvensky, V.V and Ruchimsky, M..: Strength calculation of pipelines: A reference book. Moscow: Gostekhizdat (1963).

[14]   Stasenko, I.V and Rakhmanova, I.I.: Calculation of thin-walled pipes of large curvature Strength calculations. Moscow Mech. Eng. 27, 57–69 (1986).

[15]   Vasiliev, V.. .: Mechanics of structures made of composite materials. Moscow: Engineering (1988).

[16]   Kovrevsky, A.P.: Experimental and theoretical study of free oscillations of pipes containing a flowing fluid. Moscow Izv. High. Sch. Energy. 4, 89–94 (1964).

[17]   Adamo, F.: Assessment of the Uncertainty in Human Exposure to Vibration. IEEE Sens. J. 14, 474–481 (2014). https://doi.org/10.1109/JSEN.2013.2284257.

[18]   Korotkov, A.V and Kulikov, Y.A.: Vibration resistance of multilayer curved pipes of reinforced plastics. Mech. Compos. Mater. Struct. 16, 178–194 (2010).

[19] Korotkov, A.V and Kulikov, Y.A.: Free vibrations of multilayer curved pipes made of reinforced plastics. Mech. Compos. Mater. Struct. 15, 203–220 (2009).

[20] Ibrahimovich, S.I., Hudoyberdievich, T.M., Maqsud, M.: Natural Oscillations of Viscoelastic Lamellar Mechanical Systems with Point Communications. Appl. Math. 05, 3018–3025 (2014). https://doi.org/10.4236/am.2014.519289.

[21] Filatov, A.N.: Asymptotic methods and theory of differential and integro-differential equations. Tashkent: Fan (1974).

[22] Golovin, P.D.: Application of the quasi-sample interval method time for separate measurement of parameters of parametric sensors. Casp. J. Manag. High Technol. 149–154 (2013).

[23] Koltunov, M.A., Mirsaidov, M., Troyanovskii, I.E.: Transient vibrations of axisymmetric viscoelastic shells. Polym. Mech. 14, 233–238 (1978). https://doi.org/10.1007/BF00857468.

[24] Kulikov, Y..: Vibration resistance of a thin-walled curved pipeline by the action of pulsating internal pressure. Probl. Mech. Eng. Mach. Reliab. 3, 23–30 (1993).

[25] Kovalchuk, P.S , Kruk, L.A.: On nonlinear energy exchange between the intrinsic forms of circular cylindrical shells filled with liquid during their free vibrations. Prikl. Mech. 36, 115–122 (2000).

[26] Muller, D.E.: A Method for Solving Algebraic Equations Using an Automatic Computer. Math. Tables Other Aids to Comput. (1956). https://doi.org/10.2307/2001916.

[27] Mirsaidov, M.M., Khudainazarov, S.O.: Spatial natural vibrations of viscoelastic axisymmetric structures. Mag. Civ. Eng. 96, 118–128 (2020). https://doi.org/DOI: 10.18720/MCE.96.10.

[28] Mirsaidov, M., Mekhmonov, Y.: Nonaxisymmetric vibrations of axisymmetric structures with associated masses and hollows (protrusions). Strength Mater. 19, 424–430 (1987). https://doi.org/DOI: 10.1007/BF01524147.

[29] Mirsaidov, M.M., Toshmatov, E.S.: Spatial stress state and dynamic characteristics of earth dams. Mag. Civ. Eng. (2019). https://doi.org/10.18720/MCE.89.1.

[30] Koltunov, M.A., Kravchuk, A.S., Maiboroda, V.P.: Applied Mechanics of Deformable Solids. Moscow.:Higher school (1983).