TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI MILLIY TADQIQOTLAR UNIVERSITETI

S.T.Vafoyev, O.S.Vafoyeva, R.S.Vafoyev, O‘.S.Vafoyev

MATEMATIKA

BUXORO-2023

UO‘K:51(077)
V-29

Ushbu o‘quv qo‘llanma, TIQXMMI MTU Buxoro tabiiy resurslarni boshqarish instituti 2023 yil - yanvarda bo’lib o’tgan ilmiy Kengashning -sonli qarorlari bilan chop etishga tavsiya etilgan.

O‘quv qo‘llanmada matematika fanining barcha bo‘lim va boblarini qamrab olgan mavzularning nazariy asoslari, ulardagi ayrim misol va masalalarni yechish uslubiyatlari ko‘rsatilgan, shuningdek, abiturientlar o‘z bilimlarini sinab ko‘rishlari uchun test savollari, hamda berilgan variantlar bo‘yicha test savollarining javoblari ilovada berilgan.O‘quv qo‘llanma matematika fanidan tayyorlovchi kurslarda dars beradigan o‘qituvchilar hamda, elementar matematika fanini mustaqil o‘rganuvchilar uchun mo‘ljallangan. O‘quv qo‘llanmadan umumiy o‘rta maktablarida matematika fanini chuqurlashti-rilib o‘rganiladigan sinflar, akademik litseylarining o‘quvchilari va o‘qituvchilari ham foydalanishlari mumkin.

В учебном пособии теоретические основы тем, охватывающих все разделы и главы математики для поступающих в семейное образовательное учреждение, в них показаны некоторые примеры и способы решения задач, а также могут проверить свои знания. их результаты, а также ответы на вопросы теста по заданным вариантам приведены в приложении. Учебное пособие предназначено для учителей, преподающих курсы математики семейных образовательных учреждений, а также для самостоятельных изучающих начальную математику. Учебник может быть использован учащимися и учителями классов высшей математики общеобразовательных школ, академических лицеев. Учебник может быть использован учащимися и учителями классов высшей математики общеобразовательных школ, академических лицеев.

In the textbook, the theoretical foundations of topics covering all sections and chapters of mathematics for applicants to a family educational institution, they show some examples and ways to solve problems, and applicants can also test their knowledge. their results, as well as answers to the test questions for the given options, are given in the appendix. The textbook is intended for teachers teaching mathematics courses for applicants of family educational institutions, as well as for independent students of elementary mathematics. The textbook can be used by students and teachers of higher mathematics classes of secondary schools, academic lyceums. The textbook is intended for teachers teaching mathematics courses for applicants of family educational institutions, as well as for independent students of elementary mathematics. The textbook can be used by students and teachers of higher mathematics classes of secondary schools, academic lyceums.

O’quv qo’llanma O’z.R.FA ning akademigi t.f.d., professor M.M.Mirsaidov tahriri ostida tayyorlandi

T a q r i z ch i l a r:

Qutlimurotov A.R. Chirchiq davlat pedagogika universitetining fizika-matematika fanlari nomzodi, dotsent.

Yuldashov A.A. TIQXMMI MTU Buxoro tabiiy resurslarni boshqarish institutining “Kompyuter grafikasi va raqamli texnologiyalar” kafedrasi katta o’qituvchisi.

KIRISH

Ma’lumki, oily o‘quv yurtlarning deyarli barchasida abiturientlar matematika fanidan test imtihonlari topshiradilar. Matematika fanining qoida va teoremalarini puxta bilish, tanlangan soha bo‘yicha yetuk mutaxasiss bo‘lib chiqishni kafolatlaydi.

Ushbu o‘quv qo‘llanmada, o‘rta maktab va akademik litseylarida o‘qitiladigan matematika fanidagi barcha bo‘lim va boblarining mavzularni qamrab olgan bo‘lib, unda oily o‘quv yurtlariga kiruvchilar uchun matematika fanining nazariy asoslari, ularda uchraydigan misol va masalalarni yechish uslubiyati keltirilgan.

Matematik masalalarni yechishda, puxta nazariy bilim va ko‘nikmalarga ega bo‘lish talab etiladi. Masalani yechishda uning mazmunini (yechish uchun berilgan qiymatlar asosida aniqlanishi kerak bo‘lgan qiymatni topish uchun zarur bo‘lgan qoida, teorema va formulalarni asosli qo‘llagan holda) yaxshilab o‘rganib chiqish, lozim bo‘lsa chizmalarini chizish talab qilinadi.

O‘quv qo‘llanmada, elementar matematika fanining barcha bo‘limlari (arifmetika, algebra, geometriya, trigonometriya, analitik geometriya, matematik analiz asoslari) bo‘yicha nazariy bilimlar, ularga doir misol va masalalar, hamda 1996-2019 yillarda tushgan test savollarida uchraydigan masalalar (jami 300 ta misol va masalalar o‘ldiylikdan murakkablikka qarab) yechilish uslubiyati bilan berilgan.

Abiturientlar o‘z bilimlarini sinab ko‘rishlari uchun, o‘quv qo‘llanmada test savollaridan 14 ta variantda berilgan (test savollarining javoblari ilovada keltirilgan). Testda berilgan misol va masalalarni yechishda, abiturientlar, elementar matematikaning qoida, teorema va formulalarni yaxshi bilishlari bilan birga, ushbu o‘quv qo‘llanmada yechib ko‘rsatilgan misol va masalalarni yaxshi o‘zlashtirgan bo‘lishlari kerak.

O‘quv qo‘llanma, bosh muallifning 45 yillik abiturientlarni matematika fanidan test-ga tayyorlashda to‘plagan tajribalari asosida tuzildi.

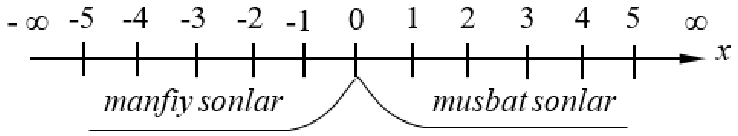
O‘quv qo‘llanma yuzasidan bildirgan fikr va takliflarni mualliflar mamnuniyat bilan qabul qiladi va oldindan o‘z minnatdorchiligini bildiradi.

Mualliflar

1. ARIFMETIKA

1.1. Ratsional sonlar, sonlar o‘qi, musbat va manfiy sonlar.

$(-\infty;\infty)$ oraliqdagi barcha (musbat va manfiy butun, kasr va iratsional) sonlarga ratsional sonlar deb ataladi.



1.1-rasm.

Sonlar o‘qi 1.1-rasmda ko‘rsatilgan.

1.2. Natural sonlar. Toq va juft sonlar. Natural sonlar deb, 0 dan boshqa barcha musbat butun yoki sanoqda ishlatiladigan sonlarga aytiladi.

Natural sonlar ketma-ketligi 1,2,3,4,5, ...

Toq sonlar deb, 2 ga bo`linmaydigan natural sonlarga aytiladi.

Toq sonlar 1,3,5,7,9, ...

Juft sonlar deb, 2 ga bo`linadigan natural sonlarga aytiladi.

Juft sonlar 2,4,6,8,10,12 ...

1.3. Tub (o‘ldiy) sonlar. Tub sonlar deb, faqat 1 ga va o`ziga bo`linadigan natural sonlarga aytiladi. 1 soni tub son emas, eng kichik tub soni 2.

Tub sonlar ketma-ketligi 2,3,5,7,11,13,17,19, ...

M: 20 dan 99 gacha bo‘lgan natural sonlar ichida nechta tub son bor.

23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. 17 ta.

**O`zaro tub sonlar deb,** umumiy bo`luvchisi bo`lmagan natural songa aytiladi.

M: $(2;3), (5;6), (5;18;29)$ ...

1.4. Murakkab sonlar. Murakkab son deb, 1 ga, o`ziga va boshqa sonlarga bo`li-nadigan musbat butun sonlarga aytiladi. ***M***: 4,6,8,9,10,12,14,15,16 ...

1.5. Berilgan sonlarning eng katta umumiy bo‘luvchisi (EKUB) ni topish.

Berilgan sonlarning EKUB ni topish uchun ularni asosi tub sondan, darajasi turlicha bo‘lgan ko‘paytuvchlarga ajratiladi va ularda qatnashgan eng kichik ko‘rsatkich daraja-laridan bittadan olib, ko‘paytiriladi. **1. *M***: 48 va 72 natural sonlarning EKUB ni toping. Bunda $48 = 6\cdot 8 = 2\cdot 3\cdot 2^3 = 2^4\cdot 3$ va $72 = 8\cdot 9 = 2^3\cdot 3^2$. Demak bu sonlarning EKUBi $2^3\cdot 3 = 24$ bo‘lar ekan. **2. *M***: 256, 344 va 120 natural sonlarning EKUB ni toping. Bunda $120 = 2\cdot 2\cdot 3\cdot 2\cdot 5 = 2^3\cdot 3\cdot 5$, $256 = 2^8$ va $344 = 2^3\cdot 43$ bo‘ladi. Bularni ichida eng kichik darajalisi $2^3 = 8$. Demak; 256, 344 va 120 natural sonlarnng EKUBi 8 ekan.

1.6. Berilgan sonlarning eng kichik umumiy karralisi (EKUK) ni topish.

Berilgan sonlarning EKUK ni topish uchun ularni asosi tub sondan, darajasi turlicha bo‘lgan ko‘paytuvchlarga ajratiladi va hosil bo‘lgan bir xil kopaytuvchilarni eng kqtta ko‘rsatkich darajalaridan bittadan olib, ko‘paytiriladi. **1. *M***: 36 va 54 natural sonlarning EKUK ni toping. Bunda $36 = 6\cdot 6 = 2^2\cdot 3^2$ va $54 = 3^3\cdot 2$. Demak bu sonlarning EKUKi $3^3\cdot 2^2 = 27\cdot 4 = 108$ bo‘lar ekan. **2. *M***: 54, 63 va 72 natural sonlarning EKUB ni toping. Bunda $54 = 3^3\cdot 2$, $63 = 3^2\cdot 7$ va $72 = 2^3\cdot 3^2$ bo‘ladi. Demak bu sonlarning EKUKi $2^3\cdot 3^3\cdot 7 = 1512$ bo‘lar ekan.

Berilgan n va m natural sonlarning EKUBi va EKUKi ning ko‘paytmasi shu sonlar ko‘paytmasiga teng bo‘ladi. $n\cdot m = \text{EKUB}\cdot\text{EKUK}$ (1.1)

Masalan: 28 va 21 sonlarning EKUB $(28;21)$i $= 7$,

EKUK $(28;21)$i $= 3\cdot 4\cdot 7 = 84$ ga teng. Demak, $7\cdot 84 = 588$.

1.7. Natural sonning natural bo`luvchi sonlarini aniqlash. Agar A natural soni berilgan bo'lsa, uni asosi tub sonlardan iborat bo'lgan daraja ko'rinishdagi ko'paytma sonlarga ajratamiz: $A = a^n \cdot b^m \cdot c^k$. Berilgan sonning natural bo'luvchilar soni quyida-gicha aniqlanadi:

$$NBS = (n+1)(m+1)(k+1) \qquad (1.2)$$

1. $\boldsymbol{M}$: 192 sonning natural bo'luvchi sonlarini aniqlang. $192 = 3 \cdot 64 = 3 \cdot 2^6$ bunda $NBS = (1+1)(6+1) = 2 \cdot 7 = 14$.

2. $\boldsymbol{M}$: 81000 sonning natural bo'luvchi sonlarini aniqlang. $81000 = 3^4 \cdot 2^3 \cdot 5^3$ bunda $NBS = (4+1)(3+1)(3+1) = 5 \cdot 4 \cdot 4 = 80$.

3. $\boldsymbol{M}$: 11 dan katta bo'lmagan natural sonlar ko'paytmasining natural bo'luvchi sonlarini aniqlang. 11 dan katta bo'lmagan natural sonlarning ko'paytmasini quyidagi ko'rinishda yozamiz: $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11$.

Buni $2 \cdot 3 \cdot 2^2 \cdot 5 \cdot 2 \cdot 3 \cdot 7 \cdot 2^3 \cdot 3^2 \cdot 2 \cdot 5 \cdot 11$ ko'rinishda yozib,

uni $2^8 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11$ ko'rinishga keltiramiz.

Shunda $NBS = (8+1)(4+1)(2+1)(1+1)(1+1) = 9 \cdot 5 \cdot 3 \cdot 2 \cdot 2 = 540$.

**1.8. Natural sonning natural bo`luvchi sonlarini yig'indisini aniqlash.** Natural sonning natural bo`luvchi sonlarini yig'indisini aniqlash uchun berilgan sonni asosi tub sonlardan iborat bo'lgan daraja ko'rinishdagi ko'paytma sonlarga ajratamiz:

Berilgan sonning natural bo`luvchi sonlarini yig'indisi quyidagi formula orqali aniqlanadi:

$$У_{NBS} = \frac{a^{n+1}-1}{a-1} \cdot \frac{b^{m+1}-1}{b-1} \cdot \frac{c^{k+1}-1}{c-1} \qquad (1.3)$$

$\boldsymbol{M}$: 540 sonning natural bo'luvchi sonlarining yig'indisini aniqlang.

$$540 = 2^2 \cdot 3^3 \cdot 5. \; У_{NBS} = \frac{2^{2+1}-1}{2-1} \cdot \frac{3^{3+1}-1}{3-1} \cdot \frac{5^{1+1}-1}{5-1} = 420.$$

1.9. Irratsional sonlar. Irratsional sonlar deb, cheksiz, davriy bo'lmagan o'nli kasr ko'rinishdagi sonlarga aytiladi. $\boldsymbol{M}$: $\sqrt{\mathbf{2}}, \sqrt{\mathbf{7}}$, p $= 3{,}14$ ..., $e = 2{,}71$ ... va hokozo.

1.10. Kasr sonlar. Kasr sonlar deb, surat va maxrajga ega bo'lgan sonlarga aytiladi.

$$\frac{a}{b} = \frac{a-surat}{b-maxraj} \quad \boldsymbol{M}: \frac{2}{3}; \frac{1}{5}; \frac{5}{3}; \frac{17}{25}; \frac{27}{19} \text{ va hokozo.}$$

1.11. O'ldiy rasrlar. O'ldiy kasr deb, surati maxrajidan kichik bo'lgan kasrga aytiladi. $\boldsymbol{M}$: $\frac{1}{3}; \frac{2}{5}; \frac{5}{9}; \frac{17}{25}; \frac{27}{57}$ va hokozo.

1.12. Noto'g'ri kasrlar. Noto'g'ri kasr deb, surati maxrajidan katta bo'lgan kasrga aytiladi. $\boldsymbol{M}$: $\frac{8}{3}; \frac{17}{5}; \frac{13}{9}; \frac{27}{25}; \frac{87}{53}$ va hokozo.

1.13. Aralash kasrlar. Aralash kasr deb, butun va kasr sonlardan tashkil topgan songa aytiladi. $\boldsymbol{M}$: $1\frac{3}{5}$; $2\frac{3}{5}$; $3\frac{3}{7}$; $4\frac{4}{13}$ va hokazo.

Aralash kasirdan noto'g'i kasrga quyidagi formula orqali o'tiladi:

$$a\frac{c}{b} = a + \frac{c}{b} = \frac{ab+c}{b} \quad \boldsymbol{M}: 3\frac{3}{5} = \frac{3 \cdot 5+3}{5} = \frac{18}{5}.$$

1.14. O‘nli kasrlar. O`nli kasr deb, kasr qismi o‘nliklar bilan ajratilgan kasrga aytiladi. $\boldsymbol{M}$: 2,3; 3,14; 12,678 va hokazo.

1.15. O‘nli davriy kasrlar. O‘nli davriy kasr deb, kasr qismi davriy ravishda qaytariladigan kasrga aytiladi.

$\boldsymbol{M}$: $2{,}333\ldots = 2,(3); 1{,}131313\ldots = 1,(13);\ 3{,}4165165\ldots = 3{,}4(165).$

O‘nli davriy kasrni aralash kasrga aylantirishga misollar: Bunda davriy qiamdagi o‘nliklardan bitta kam olinadi (buni sababi, qo‘llanmaning 2.3 qismidagi 3-misolda tushuntirilgan). Masalan: $0,(3) = \frac{3}{9};\ 2,(14) = 2\frac{14}{99}$.

$\mathbf{1}.\boldsymbol{M}$: $2,(3) = 2\frac{3}{9}$; $\mathbf{2}.\boldsymbol{M}$: $3,(13) = 3\frac{13}{99}$; $\mathbf{3}.\boldsymbol{M}$: $5{,}4(13) = 5\frac{413-4}{990} = 5\frac{409}{990}$

1.16. Sonlarni qo‘shish va ayirish. Bir xil ishorali sonlar qo‘shilganda natija umumiy ishorali bo‘ladi. $\mathbf{1}.\boldsymbol{M}$: $3+5+8=16$: $\mathbf{2}.\boldsymbol{M}$: $-2-4-7=-13$.

Ikkita turli ishorali sonlar qo‘shilganda ularning kattasidan kichigi ayirilib kattasini ishorasi qo‘yiladi. $\mathbf{1}.\boldsymbol{M}$: $-8+5=-3$; $\mathbf{2}.\boldsymbol{M}$: $13-7=6$..

1.17. Sonlarni ko‘paytirish va bo‘lish. Ikkita bir xil ishorali sonlar ko‘paytirilganda yoki bo`linganda natija musbat ishorali bo‘ladi.

$\boldsymbol{M}$: $-\cdot- = +,\ \frac{-}{-} = +,\ +\cdot+ = +,\ \frac{+}{+} = +$

yoki $-3\cdot(-5) = 15,\ \frac{-12}{-4} = 3,\ 4\cdot 7 = 28,\ \frac{27}{3} = 9.$

Ikkita turli ishorali sonlar ko‘paytiril-ganda yoki bo`linganda natija manfiy ishorali bo‘ladi. $\boldsymbol{M}$: $-\cdot+ = -,\ \frac{+}{-} = -,\ +\cdot- = -,\ \frac{-}{+} = -$

yoki $-4\cdot 6 = 24,\ \frac{32}{-4} = -8,\ 7\cdot(-3) = -21,\ \frac{-42}{6} = -7.$

1.18. Bir nechta sonlar ko‘paytmasining yig‘indisi va ayirmasida qatnashgan bir xil sonlarni qavusdan chiqarish, qavuslarni ochish. Bir nechta sonlar ko‘paytmasining yig‘indisi va ayirmasida qatnashgan bir xil sonlarni qavusdan chiqarish:

$\boldsymbol{M}$: $ab + ca - ad = a(b + c - d)$ yoki

$$14\cdot 7 - 8\cdot 14 + 14\cdot 3 = 14(7 - 8 + 3) = 28.$$

Qavuslarni ochish. $\boldsymbol{M}$: $(n+m)(a-b) = an - bn + am - bm.$

1.19. Qarama-qarshi va teskari sonlar, o‘zaro teskari sonlarning ko‘paytmasi.

<u>Qarama-qarshi sonlar deb</u>, berilgan sonning teskari ishorali qiymatiga aytiladi.

$\boldsymbol{M}$: 5 ga q – q → –5 , $-\frac{1}{4}$ ga q – q → $\frac{1}{4}$, va hokazo.

O‘zaro qarama-qarshi sonlarning yig‘indisi hamma vaqt nolga teng bo‘ladi.

$\mathbf{1}.\boldsymbol{M}$: $-4+4=0$; $\mathbf{2}.\boldsymbol{M}$: $\frac{3}{4} - \frac{3}{4} = 0.$

<u>Berilgan sonning teskarisi deb</u>, shu soning teskari qiymatiga aytiladi. Bunda ishora o‘zgarmaydi. $\boldsymbol{M}$: $-\frac{1}{4}$ tes – 4, 5 tes $\frac{1}{5}$ va hokazo. O‘zaro teskari sonlarning ko‘paytmasi hamma vaqt birga teng bo‘ladi.

$\boldsymbol{M}$: $-5\cdot\left(-\frac{1}{5}\right) = 1,\ \frac{2}{3}\cdot\frac{3}{2} = 1$ va hokazo.

1.20. Kasrlarni qo‘shish va ayrish. Kasrlarni qo‘shish va ayrish uchun, agar ular qisqarsa qisqartirilib, umumiy maxrajga keltiriladi yoki karralisi aniqlanadi.

Umumiy maxraj deb, berilgan sonlarning eng kichik bo‘linuvchisiga aytiladi.

$\boldsymbol{M}$: $\frac{a}{b}+\frac{c}{d}-\frac{k}{f}=\frac{adf+cbf-bdk}{bdf}$ yoki $\frac{2}{3}+\frac{7}{6}-\frac{5}{18}=\frac{2\cdot6+3\cdot7-5}{18}=\frac{28}{18}=\frac{14}{9}=1\frac{5}{9}$

.O‘nli kasrlarni qo‘shish va ayrishga doir misollar:

1) $13,176+7,2673=20,4433$. 2) $11,21-9,968=1,242$.

1.21. Kasrlarni ko‘paytirish va bo‘lish. Kasrni kasrga ko‘paytirish uchun, agar ular o‘zaro qisqarsa qisqartirilib, surat bilan surat ko`payib suratga, maxraj bilan maxraj ko`payib maxrajga yoziladi. $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$ **1.** $\boldsymbol{M}$: $\frac{8}{9}\cdot\frac{81}{64}=\frac{9}{8}=1\frac{1}{8}$. **2.** $\boldsymbol{M}$: $\frac{1}{3}\cdot\frac{7}{49}=\frac{1}{21}$

Butun son kasr son yoki kasr songa butun son ko‘paytirilsa, butun son kasrning suratiga ko‘paytirilib, kasrning maxraji o‘z holicha qoldiriladi:

$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$ yoki $\frac{b}{c}\cdot a=\frac{a\cdot b}{c}$

$\boldsymbol{M}$: $3\cdot\frac{2}{5}=\frac{3\cdot2}{5}=\frac{6}{5}=1\frac{1}{5}=1,2$.

Kasrni kasrga bo`lishning ikkita usuli mavjud. 1) Birinchi kasrga ikkinchi kasrning teskari qiymati ko`paytiriladi $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$ $\boldsymbol{M}$: $\frac{5}{7}:\frac{4}{3}=\frac{5}{7}\cdot\frac{3}{4}=\frac{15}{28}$

Butun son kasr songa bo‘linsa, butun songa berilgan kasrning teskari qiymati ko‘pay-tiriladi. $a:\frac{b}{c}=a\cdot\frac{c}{b}=\frac{a\cdot c}{b}$ $\boldsymbol{M}$: $8:\frac{2}{3}=8\cdot\frac{3}{2}=12$.

Kasr son butun songa bo‘linsa, kasr songa butun sonning teskari qiymati ko‘pay-tiriladi. $\frac{a}{b}:c=\frac{a}{b}\cdot\frac{1}{c}=\frac{a}{b\cdot c}$ $\boldsymbol{M}$: $\frac{3}{4}:5=\frac{3}{4}\cdot\frac{1}{5}=\frac{3}{20}$.

2) Berilgan kasr ustma-ust qilib yoziladi va kasrning chekki qismlari ko‘paytirilib, suratga, o‘rta qismlari ko‘paytirilib, maxrajga yoziladi.

$\frac{a}{b}:\frac{c}{d}=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$ $\boldsymbol{M}$: $\frac{5}{6}:\frac{4}{7}=\frac{\frac{5}{6}}{\frac{4}{7}}=\frac{5\cdot7}{6\cdot4}=\frac{35}{24}=1\frac{11}{24}$.

O‘nli kasrlarni ko‘paytirsh va bo‘lishga doir misollar:

1) $7,28\cdot3,107=22,61896\approx22,62$. 2) $17,560:12,829=1,36877$.

3) $13,17\cdot0,192=2,52864\approx2,53$. 4) $12,75:0,162=78,70$.

5) $121,13\cdot1,78=215,6$. 6) $0,672:0,021=32$.

Arifmetikada, oldin ko‘paytirish yoki bo‘lish undan so‘ng, qo‘shish va ayrish amallari bajariladi. Qavs ichida berilgan ifodalar birinchi navbatda bajariladi.

1.22. Masshtab haqida tushuncha. Masshtab deb, Jismning kattalshtirilgan yoki kichiklashtirilgan shakliga aytiladi. Undan asosan detallar chizmasini chizishda va xari-talarni tuzishda foydalaniladi. Jismning chizmadagi o‘lchami (ℓ_{ch}- chizmadagi uzunligi) ni uning haqiqiy o‘lchami (ℓ_h- haqiqiy uzunligi) ga bo‘lgan nisbati masshtab deb yuritiladi.

$$M=\frac{\ell_{ch}}{\ell_h} \qquad (1.4)$$

Agar chizmadagi o‘lcham haqiqiy o‘lcham bilan teng bo‘lsa: M 1:1.

Agar chizmadagi o‘lcham haqiqiy o‘lchamdan ikki marta kichik bo‘lsa: M 1:2.

Agar chizmadagi o‘lcham haqiqiy o‘lchamdan besh marta kichik bo‘lsa:

M 1:5 va hokazo.

1. $\boldsymbol{M}$: Ikki shahar orasidagi masofa 400 *km*. Shu masofa M 1:2500000 masshtabli xaritada necha *sm* bo‘ladi.

<u>***Yechish:***</u> $\frac{1}{2500000}=\frac{x}{400}$ bundan $x=\frac{400}{2500000}=\frac{4}{25000}\ km=\frac{4}{25000}\cdot 10^5=8\ sm.$

2. $\boldsymbol{M}$: M 1:3000000 masshtabli xaritadagi 12 *sm* li masofa necha *km* bo‘ladi.

<u>***Yechish:***</u> $\frac{1}{3000000}=\frac{12}{x}$ bundan $x=12\cdot 3000000=36\cdot 10^6\ sm,$ yoki

$$x=36\cdot 10^6\cdot 10^{-5}=360\ km.$$

3. $\boldsymbol{M}$: M 1:2 masshtabda chizilgan detalning chizmadagi uzunligi 120 *mm* bo‘lsa, bu detall M 1:5 masshtabli chizmada chizisa uning chizmadagi uzunligi qancha *mm* bo‘ladi.

$\frac{2}{5}=\frac{x}{120}$ bundan $x=\frac{2\cdot 120}{5}=\frac{240}{5}=48\ mm.$

Shunday detallar borki uning o‘lchami juda kichik bo‘ladi, bunday detallaning o‘l-chamlari chizmada kattalashtirilib chiziladi. Bularda masshtab M 2:1; M 5:1; M 10:1.

1.23. Natural sonlarni ko‘paytirishdan hosil bo‘lgan nollar sonini aniqlash.

Natural sonlarni ko‘paytirishdan hosil bo‘lgan nollar sonini aniqlash uchun quyidagi formuladan foydalaniladi:

$$\left|\frac{n}{5}\right|+\left|\frac{n}{25}\right|+\left|\frac{n}{125}\right|+\cdots \qquad (1.5)$$

Bu yerda $\left|\frac{n}{5}\right|$ − kasrning butun qismi olinadi.

$\boldsymbol{M}$: 1 dan 37 gacha natural sonlarni ko‘paytirishdan hosil bo‘lgan nollar sonni toping.

$$\left|\frac{37}{5}\right|+\left|\frac{37}{25}\right|+\left|\frac{37}{125}\right|=7+1+0=8.$$

1.24. Yuqori darajali soning oxirgi raqamini topish.

Masalan $2^n=\cdots x$

$2^1=2_1$
$2^2=4_2$
$2^3=8_3$
$\underline{2^4=16_0}$
$2^5=32$
$2^6=64$
$2^7=128$
$\underline{2^8=256}$

Bundan ko‘rinadiki, oxirgi raqam turlicha (to‘rt xil ko‘rinishda) bo‘lib, to‘rtinchi darajadan keyin oxirgi raqamlar takrorlanar ekan. Shuni inobat-ga olib, darajadagi *n*, 4 ga bo‘linadi, agar bir qoldiq qolsa oxirgi raqam 2 ga, ikki qoldiq qolsa oxirgi raqam 4 ga, uch qoldiq qolsa oxirgi raqam 8 ga, qoldiq qolmasa 6 ga teng bo‘ladi (qoldiqlar pastki indeksda berilgan). Xuddi shu usulda 3, 4, 5, 6, 7, 8, 9 larning yuqori darajalarini oxirgi raqamlari aniqlanadi. Beshning har qanday darajasining oxirgi raqami 5, oltining har qanday darajasining oxirgi raqami 6 ga teng bo‘ladi.

$3^1=3_1$	$4^1=4_1$	$7^1=7_1$	$8=8_1$	$9^1=9_1$
$3^2=9_2$	$4^2=\ldots 6_2$	$7^2=\ldots 9_2$	$8^2=\ldots 4_2$	$9^2=\ldots 1_2$
$3^3=\ldots 7_3$	$4^3=\ldots 4_3$	$7^3=\ldots 3_3$	$8^3=\ldots 2_3$	$9^3=\ldots 9_3$
$3^4=\ldots 1_0$	$4^4=\ldots 6_0$	$7^4=\ldots 1_0$	$8^4=\ldots 6_0$	$9^4=\ldots 1_0$

1. $\boldsymbol{M}$: 2^{1999}ni oxirgi raqamini toping. 1999 ni 4 ga bo‘lganda 3 qoldiq qoladi, demak berilgan sonning oxirgi raqami 8 ekan. **2.** $\boldsymbol{M}$: $7^{171}-3^{286}$ ayirmaning oxirgi raqamini toping. 7 ning darajasidagi 171 ni 4 ga bo‘lganda 3 qoldiq qoladi, demak berilgan sonning oxirgi raqami 3 ekan. 3 ning darajasidagi 286 ni 4 ga bo‘lganda 2 qoldiq qoladi, demak berilgan sonning oxirgi raqami 9 ekan.

Shunda $7^{171} - 3^{286} = \cdots 3 - \cdots 9 = \cdots 4$.

1.25. Bir no’malumli tenglama. Bir nomalumli birinchi darajali tenglama quyidagi ko‘rinishga ega: $ax + b = 0$, bu yerda x - no’malum son; a - no’malum son oldidagi koeffitsient; b - ozo‘l had yoki aniq son.

Tenglamani yechishda ikkita maqsad qo‘yiladi:

1) Amaldagi no’malumni topish.

2) Nazariy, ya’ni chiziqlarni kesishish nuqtasini topish.

Bir nomalumli tenglamalarni yechish uchun quyidagi ishlar amalga oshiriladi:

1. Agar berilgan tenglama standart shaklida bo‘lmasa uni standart shakliga keltiriladi. Ya’ni aralash va o‘nli kasrlar o‘ldiy yoki noto‘g‘ri ko‘rinishga keltirilib (agar qovis-lar bo‘lsa, ular ochiladi), umumiy maxraj beriladi, tenglikda umumiy maxraj tashlab yuboriladi.

2. Yuqoridagi amal bajarilgandan so‘ng, nomalm sonlar o‘z koeffitsientlari bilan birgalikda tenglikning bir tomoniga, aniq sonlar esa tenglikning ikkinchi tomoniga o‘tkazilib, jamlanadi. Ifodalar tenlikning bir tomonidan ikkinchi tomoniga o‘tganda o‘z isho-ralarini o‘zgartiradilar.

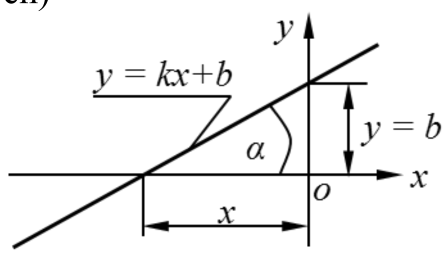
3. Nomalum x ni topish uchun aniq son nomalum son oldidagi koeffitsientga bo‘linadi.

$ax + b = 0$ tenglama bilan berilgan tenglamaning ildizi (to‘g‘ri chiziqning OX o‘qi bilan kesishish nuqtasi), yechimi yoki javobi quyidagicha aniqlanadi $x = -\frac{b}{a}$.

To‘g‘ri chiziq tenglamasi. To‘g‘ri chiziqning (t/ch) tenglamasi (funksiyasi) $y = kx + b$ ko‘rinishda bo‘ladi.

Bu yerda y - funksiya; x – argument; b - to‘g‘ri chiziqni OY o‘qi bilan kesishish nuqtasi; k - to‘g‘ri chi-ziqning burchak koeffitsienti bo‘lib, uni OX o‘qi bilan tashkil qilgan burchagini ko‘rsatadi.

$k = \frac{y}{x} = \frac{b}{x} = tg б$.



1.2-rasm.

To‘go‘ri chiziqning grafigi 1.2-rasmda ko‘rsatilgan.

Bir no’malumli tenglamalarga misollar.

1. ***M***: 5 kg sabzi 1500 so‘m tursa bir kg sabzi qancha turadi. $5x = 1500$ bundan $x == \frac{1500}{5} = 300$ so‘m.

2. ***M***: $7x - 2,(3)x + 2\frac{1}{6} + 2{,}5 = 3(x - 3) + 4$.

Tenglamadagi no’malum x ni toping.

1.Tenglamani standart shaklga keltiramiz:

$7x - \frac{7}{3}x + \frac{13}{6} + \frac{5}{2} = 3x - 9 + 4$. Umumiy maxrajdan so‘ng tenglama quyidagi ko‘-rinishga ega bo‘ladi: $42x - 14x + 13 + 15 = 18x - 30$.

2. No’malum sonlarni chap tomonga, aniq sonlarni o‘ng tomonga o‘tkazib, jamlaymiz:

$42x - 14x - 18x = -30 - 13 - 15$ yoki $10x = -58$

3. No'malum son x ni topamiz: $x = \frac{-58}{10} = \frac{-29}{5} = -0{,}58.$

3. ***M***: Otasi 26 yoshda bolasi 4 yoshda. Nech yildan keyin otasi bolasidan uch marta katta bo'ladi.

<u>Yechish:</u> *Masala shartidan kelib chiqib, quyidagicha tenglama tuzamiz:*

$26 + x = 3(4 + x)$ *bundan* $x = 7.$

4. ***M***: Otasi o'g'lidan 30 yosh katta, bobosi esa otasidan 30 yosh katta. To'rt yil oldin ota, bola va boboning yoshlarining yig'indisi 114 yosh bo'lgan bo'lsa, hozir boboning yoshi nechada.

<u>Yechish:</u> *Masala shartidan kelib chiqib,bolaning yoshini x deb olib, quyidagicha tenlama tuzamiz:* $x + x + 30 + x + 30 + 30 - 3 \cdot 4 = 114$ *bundan* $x = 12$, *boboning hozirgi yoshi* $x + 60 = 12 + 60 = 72.$

1.26. O'rta arifmetik va o'rta geometrik qiymatlar.

<u>Berilgan sonlarning o'rta arifmetigi deb</u>, berilgan sonlarning yig'indisini shu sonlar soniga bo'lgan nisbatiga aytiladi.

$$O'ar = \frac{a_1 + a_2 + \cdots + a_n}{n} \qquad (1.6)$$

1. ***M***: 10, 15 va 17 sonlarning o'rta arifmetigini toping. $O'ar = \frac{10+15+17}{3} = 14.$

2. ***M***: $1{,}2; 2{,}2; x$ va $-1{,}3$ sonlarning o'rta arifmetigi 1,7 bo'lsa, no'malum x ni toping.

$\frac{1{,}2+2{,}2+x-1{,}3}{4} = 1{,}7.$ Bundan $x = 6{,}8 - 2{,}1 = 4{,}7.$

<u>Berilgan sonlarning o'rta geometrigi deb</u>, berilgan sonlarning ko'paytmasidan shu sonlar soni darajasidagi olingan ildiziga aytiladi.

$$O'ge = \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \ldots \cdot a_n} \qquad (1.7)$$

M: 4 va 9 sonlarining o'rta geometrigini toping. $O'ge = \sqrt{4 \cdot 9} = 6.$

a va b sonlarning o'rta arifmetigi va o'rta geometrigi orasida quyidagi munosabat bor:

$$\frac{a+b}{2} \geq \sqrt{ab} \text{ yoki } a + b \geq 2\sqrt{ab}. \qquad (1.8)$$

1.27. Protsent (foiz) haqida tushuncha.

Sonning yuzdan bir ulushiga birlik deb ataladi. $a \cdot \frac{1}{100} \cdot$

Birlikning 100% ga ko'paytmasiga shu sonning protsenti deb ataladi.

$a \cdot \frac{1}{100} \cdot 100\%.$

A sonining a dan b qismi (bo'lagi yoki ulushi) ni B desak ($a > b$), unda $B = A \cdot \frac{b}{a}$ bo'ladi. Misol: 90 ning oltidan besh qismini toping $B = 90 \cdot \frac{5}{6} = 75.$

Foyda-tavarning sotish B_s va olish B_o (tannarxi) baholari farqini sotish bahosiga nisbatining 100% ga ko'paytmasidir.

$$F = \frac{B_s - B_o}{B_s} \cdot 100\%. \qquad (1.9)$$

Agar bank mejozning unga qo‘ygan s so‘m puli uchun $p\%$ foyda bersa, n yildan so‘ng mejoz necha S_n so‘mga ega bo‘lishini quyidagi formula orqali aniqlash mumkin:

$$S_n = s \cdot \left(1 + \frac{p}{100}\right)^n \qquad (1.10)$$

Protsentga doir masalalar.

1. No’malum sonning 12% i 48 ga teng bo‘lsa, shu no’malum sonni toping.

Yechish: *12%, 0,12 birlikka teng bo‘ladi, shuni inobatga olib, quyidagi tenglamani tuza-miz:* $0{,}12x = 48.$ *Bundan* $x = \frac{48}{0{,}12} = 400.$

2. 17 soni 51 sonidan necha foiz kam.

Yechish: *Quyidagicha tenglama tuzamiz:* $17 = 51(1 - x)$ *yoki* $1 = 3(1 - x)$ *bun-dan* $x = \frac{2}{3}$ *birlik, yoki* $x = \frac{2}{3} \cdot 100\% = 66\frac{2}{3}\%.$

3. 57 soni 19 sonidan necha foiz ko‘p.

Yechish: *Quyidagicha tenglama tuzamiz:* $57 = 19(1 + x)$ *yoki* $3 = 1 \cdot (1 + x)$ *bun-dan* $x = 2$ *birlik, yoki* $x = 2 \cdot 100\% = 200\%.$

4. Ishchilarning maoshi 15% ga oshirildi, ma’lm vaqitdan so‘ng 15% ga kamaytirildi. Maosh qanchaga o‘zgargan.

Yechish: *Ishchilarning maoshini bir birlik deb qabul qilamiz: Shunda maosh 1,15 birlikka oshadi. Maoshning 15% ga yoki 0,15 birlikka kamayishi, uning oshirilganini 85% yoki 0,85 birligiga teng bo‘ladi. Demak, maosh datlabkisidan* $1{,}15 - 0{,}85 = 0{,}30$ *birlik-ka, yoki 30% ga kamaygan.*

5. Ishchilarning maoshlari ketma-ket oldin 15% ga keyinroq 10% ga oshirildi. Hammasi bo‘lib maoshlar necha foizga oshirilgan.

Yechish: *Ishchilarning maoshini bir birlik deb qabul qilamiz: Shunda maoshlar dastlab 1,15 birlikka, so‘ng yana 1,1 birlikka oshirilgan.*

Demak, maoshlar datlabkisidan $1{,}15 \cdot 1{,}1 - 1 = 0{,}265$ *birlikka, yoki 26,5% ga oshgan.*

6. Go‘sht qaynatilganda o‘z massasini 20% ni yo‘qotish ma’lum bo‘lsa, 12 *kg* qaynatilgan go‘sht olish uchun qozonga necha *kg* toza qaynatilmagan go‘sht solish kerak.

Yechish: *Go‘sht qaynatilganda o‘z massasini 20% ni yoki 0,2 birligini yo‘qotsa, uning 80% i yoki 0,8 birligi qoladi, shuni inobatga olib quyidagi tenglikni tuzamiz:*

$$0{,}8x = 12 \text{ bun-dan } x = \frac{12}{0{,}8} = 15\ kg.$$

7. 45 *kg* qum – sementli aralashmaning tarkibida 25% sement bor, aralashma tar-kibida sementning miqdori 15% bo‘lishi uchun unga necha *kg* qum qo‘shish kerak.

Yechish: *Qo‘shiladigan qumning miqdorini x deb olib, quyidagi tenglamani tuzamiz:* $45 \cdot 25\% = 15\%(45 + x)$ *bundan* $x = 75 - 45 = 30\ kg.$

8. Massasi 36 *kg* bo‘lgan mis va rux qotishmaning tarkibida 45% mis bor. Qotish-maning tarkibida misning miqdori 60% bo‘lishi uchun unda yana necha *kg* mis qo‘shish kerak.

Yechish: *Foizlarni birlikka aylantirib, qo‘shiladigan misning miqdorini x deb bel-gilab, quyidagi tenglamani tuzamiz:*

$$36 \cdot 0{,}45 + x = 0{,}6 \cdot (36 + x) \text{ yoki } 0{,}4x = 36 \cdot (0{,}6 - 0{,}45) \text{ bundan } x = 13{,}5\ kg.$$

9. Qotishma mis va qo‘rg‘oshindan tashkil topgan bo‘lib, uning tarkibida 60% mis bor. Agar mis qo‘rg‘oshindan $3\frac{2}{5}\,kg$ ga ko‘p bo‘lsa, qotishma tarkibidagi misni miqdorini (kg) aniqlang.

Yechish: *Foizni birlikka aylantirib, qo‘shiladigan miqdorini x deb belgilaymiz. Masala shartiga asosan qotishmadagi misning miqdori* $0{,}6x$ *ga, qo‘rg‘oshinning miqdori* $0{,}4x$*, mis qo‘rg‘oshidan* $\frac{17}{5}$ *kg ga ko‘p, ya’ni* $0{,}6x = 0{,}4x + \frac{17}{5}$ *yoki*

$0{,}4x = 0{,}6x - \frac{17}{5}$ *ekanligini inobatga olib, quyidagi tenglamani tuzamiz:*

$x = 0{,}6x + 0{,}6x - \frac{17}{5}$ *bundan* $x = 17\ kg.$

Qotishmadagi misning miqdori $0{,}6x = 0{,}6 \cdot 17 = 10{,}2\ kg.$

10. Ishchi ish normasini birinchi kuni uchdan bir qismini, ikkinchi kuni birinchi kunda bajargan ishining to‘rtdan bir qismicha ko‘p ish bajargan bo‘lsa, u shu ikki kunda qancha ish bajargan.

Yechish: *Ish normasini bir birlik deb olib, u birinchi kunda ish normasini* $1/3$ *qismini, ikkinchi kuni esa ish normasining* $\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{4}$ *qismini bajargan, bularni qo‘shib, quyidagi munosabatni hosil qilamiz:*

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{4} = \frac{3}{4}$ *. Ikki kunda ish normasini 75% i bajargan.*

11. Yo‘lovchi butun yo‘lning 45% ni o‘tgach, yo‘l oxirigacha 110 *km* qolgani ma’lum bo‘lsa, butun yo‘lni uzunligini toping.

Yechish: *Butun yo‘l uzunligini x deb olib, quyidagi tenglamani tuzamiz:*

$x = 0{,}45x + 110$ *bundan* $x = 200\ km.$

12.Yog‘lig‘ligi 2% bo‘lgan 90 litr sutga, yog‘lig‘ligi 5% bo‘lgan necha litr sut qo‘shilsa, yog‘lig‘ligi 2,6% bo‘lgan sut hosil bo‘ladi.

Yechish: *Qo‘shiladigan sutning miqdorini x deb olib, quyidagi tenglamani tuzamiz:*

$90 \cdot 2 + 5x = 2{,}6(90 + x)$ *yoki* $2{,}4x = 90 \cdot (2{,}6 - 2) = 54$ *bundan* $x = 22{,}5\ l.$

13. Maxsulotning narxi 25% ga oshirildi, maxsulot yaxshi sotilmaganligi uchun uni narxi 15% ga kamaytirildi. Maxsulotning keyngi narxi dastlabki narxidan qanchaga farq qiladi.

Yechish: *Maxsulotning dastlabki narxini bir birlik deb olsak, unda u* $1{,}25$ *birlikka ortadi, shu nazdan 15% yoki* $0{,}15$ *birlik kamaytirilsa, oshirilgan narxdan* $0{,}15$ *birlikka kamayadi, ya’ni uning* 85% yoki $0{,}85$ *birligini tashkil qiladi. Demak* $1{,}25 \cdot 0{,}85 = 1{,}0625$ *bo‘ladi. Maxsulotning keyngi narxi dastlabki narxidan* $1{,}0625 - 1{,}00 = 0{,}0625$ *birlikka yoki* $6{,}25\%$ *ga ko‘p bo‘ladi.*

Foizning fizik masalalarga tadbig‘i.

Birlik vaqt (t) ichida bosib o‘tilgan yo‘l (S) ga tezlik (ϑ) deb ataladi: $\vartheta = S/t$

Harakatdagi transportlarni uchrashish vaqti: $t = \frac{S}{\vartheta_1 \pm \vartheta_2}$ bu yerda ϑ_1 va ϑ_2 – harakatdagi transportlarning tezliklari.

Agar harakat qarama-qarshi yo‘nalishda bo‘lsa, $(+)$ belgisi olinadi, agar harakat bir yo‘nalishda bo‘lsa, $(-)$ belgisi olinadi.

Birlik vaqt (t) ichida bajarilgan ish (A) ga ish unumdorligi (U) deb ataladi:

$$U = A/t$$

Fizik masalalardan namunalar:

1. Agar yo‘lni bosib o‘tishga sarflangan vaqt 20% ga tejalsa, tezlik necha foizga oshadi.

Yechish: *Bosib o‘tilgan yo‘l S va vaqtini bir birlik, tezlikni x deb olib, quyidagi teng-lamani tuzamiz:* $x = \frac{1}{1-0{,}2} = \frac{1}{0{,}8} = 1{,}25$ *birlikka yoki 25% ga oshadi.*

2. Ikki shahar orasidagi masofa $S = 440\ km$ ga teng. Birinchi shahardan ikkinchi shaharga qarab yuk avtomobili $\vartheta_1 = 50\ km/soat$ tezlik bilan yo‘lga chiqdi, bir vaqtning o‘zida ikkinchi shahardan birinchi shaharga qarab yengil avtomobil ham yo‘lga chiqdi. Agar yengil avtomobilning tezligi yuk avtomobilnikidan 20% ga ko‘p bo‘lsa, ular necha soatdan so‘ng uchrashadilar.

Yechish: *Harakatlar qarama-qarshi bo‘lgani uchun*$(+)$*belgisini olib, harakatdagi transportlarni uchrashish vaqti formulasidan foydalanamiz:*

$$t = \frac{S}{\vartheta_1 \pm \vartheta_2} = \frac{440}{50+50\cdot 1{,}2} = 4\ soat.$$

3. Ish unumdorligini 15% ga oshirish uchun uni bajarishga sarflanadigan vaqtni necha foizga tejash kerak.

Yechish: *Bajariladigan ishni bir birlik, ishni bajarishga sarflanadigan vaqtni x deb olamiz va ish unumdorligi formulasidan foydalanamiz:*

$$1{,}15 = \frac{1}{1-x}\ \ bundan\ x = 13\%.$$

1.28. To‘g‘ri va teskari proporsiyalar.

To‘g‘ri proporsiya deb, son o‘sishi bilan uning qiymatini mos ravishda o‘sib bori-shiga aytiladi. Masalan 1 kg sabzi 200 so‘m tursa, 2 kg mi 400 so‘m, 3 kg mi 600 so‘m.

Demak $\begin{matrix} a-kg \to b-so‘m. \\ c-kg \to d-so‘m. \end{matrix}$ bundan proporsiya $a \cdot d = b \cdot c$ yoki $a{:}b = c{:}d$. Proporsiyada chekki hadlarining ko‘paytmasi, o‘rta hadlarining ko‘paytmasiga teng bo‘ladi.

A sonini $a{:}b{:}c$ kabi nisbatga ajratish. Buning uchun proporsionallik koeffitsientini k ni topamiz: $k = \frac{A}{a+b+c}$. Shunda $A = k\cdot a + k\cdot b + k\cdot c$. Misol: 144 sonini $2{:}3{:}4$ kabi nisbatga ajrating. $k = \frac{144}{2+3+4} = 16$.

Demak $144 = 16\cdot 2 + 16\cdot 3 + 16\cdot 4 = 32 + 48 + 64$.

To‘g‘ri proporsiyaga doir masala:

1) Agar $21,(3)$ metr gazlama 128000 so‘m tursa, shu gazlamaning 3 metri qancha turadi.

Yechish: *O‘nli davriy kasrni noto‘g‘ri kasirga aylantirib, proporsiya tuzamiz:*

$$\begin{matrix} \frac{64}{3} m \to 128000\ so‘m \\ 3\ m \to x\ so‘m \end{matrix} \qquad \frac{64}{3} x = 3\cdot 128\cdot 1000\ bundan\ x = 18000\ so‘m.$$

2) 800 *kg* mevaning tarkibida 80% suv bor. Ma’lum vaqtdan so‘ng mevaning massasi 500 *kg* bo‘lgan bo‘lsa, uni tarkibidagi suvning miqdorini % da aniqlang.

Yechish: *800 kg mevaning tarkibidagi shiraning miqdori* $800\cdot 0{,}2 = 160\ kg$ *bo‘lib, u o‘zgarmaydi.*

500 *kg meaning tarkibidagi suvning miqdori* $500 - 160 = 340\ kg$ *bo'ladi.*

Proporsiya tuzamiz: $\begin{matrix} 500 \to 100\% \\ 340 \to x\% \end{matrix}$ *bundan* $x = 68\%$.

3) 1 *tonna* mevaning tarkibidagi suvning miqdori 82%. Ma'lum vaqtdan so'ng unda-gi suvning miqdori $76\frac{12}{13}$ % ga teng bo'ldi, unda meaning massasi necha *kg* bo'lib qoladi.

Yechish: *1 tonna meva 1000 kg gat eng va undagi shiraning miqdori*

1000·0,18 = 180 kg ga tebg. $76\frac{12}{13}$ % *ni birlik miqdorga o'tkazsak* $\frac{76 \cdot 13 + 12}{13 \cdot 100} = \frac{10}{13}$ *bo'ladi.*

Qurigan meva tarkibidagi suvni miqdorini x deb olamiz va quyidagi praporsiyani tuzamiz: $\begin{matrix} 180 + x \to 1 \\ x \to \frac{10}{13} \end{matrix}$ *bundan* $x = 600\ kg$.

Unda qurigan meaning massasi $180 + x = 780\ kg$.

Teskari praporsiya deb, son o'sishi bilan uning qiymatini mos ravishda kamayib borishiga (va aksincha) aytiladi. Masalan bir ishni 12 kishi 18 kunda bajarsa, 24 ta kishi shu ishni 9 kunda bajaradi.

Demak $\begin{matrix} a - ishchi \to b - kun. \\ c - ishchi \to d - kun. \end{matrix}$ bundan proporsiya $a \cdot b = c \cdot d$.

Teskari proporsiyaga doir masala: 1) Agar 20 ta ishchi bir ishni 12 kunda bajar-sa, 16 ta ishchi shu ishni necha kunda bajaradi.

Yechish: *Proporsiya tuzamiz:* $\begin{matrix} 20 \to 12 \\ 16 \to x \end{matrix}$ $16 \cdot x = 20 \cdot 12$ *bundan* $x = 15\ kunda$.

2) Meshdagi suvni otasi 4 kunda, bolasi 12 kunda ichib tugatadi. Agar meshdagi suvni ota bola birgalikda ishsa, meshdagi suv necha kunda tugaydi

Yechish: *Otaning ichgan suv miqdorini n, bolaning ichgan suv miqdorini m, birga-likda ichilgan suvning miqdorini k deb belgilasak, unda quyidagi tenglikni hosil qilish mumkin:* $\frac{1}{n} + \frac{1}{m} = \frac{1}{k}$ *bundan* $k = \frac{nm}{n+m} = \frac{4 \cdot 12}{4+12} = 3\ kunda$.

3) Suv havzasini bir nasos 5 soatda to'ldiradi. Shu nasos bilan birgalikda ishlaydi-gan ikkinchi nasos ishlatilganda havzani 4 soatda to'ldiradi. Havzani ikkinchi nasosning yolg'iz o'zi necha soatda to'ldiradi.

Yechish: *Havzani birinchi nasos 5 soatda to'ldirsa, ikkinchi nasos x soatda to'ldi-radi deb hisoblab, oldingi masaladagi formuladan foydalanamiz:*

$4 = \frac{5x}{5+x}$ *bundan* $x = 20$ *soat. Demak ikkinchi nasosning yolg'iz o'zi havzani 20 soatda to'ldirar ekan.*

To'g'ri va teskari proporsiyalar aralashgan masalalarni yechish.

To'rtta sonining yig'indisi *A* ga teng.

Ulardan dastlabki a_1, a_2, a_2 uchtasi k_1, k_2, k_2 sonlariga to'g'ri, $a_3$va a_4 sonlari esa $k_4$va k_5 sonlariga teskari proporsiya bo'lsa, birinchi sonni toping.

Yechish: *Demak* $a_1 + a_2 + a_3 + a_4 = A$. *Shartga asosan*

$$\left.\begin{array}{l} a_1 \to k_1 \\ a_2 \to k_2 \\ a_3 \to k_3 \end{array}\right\} \textit{larga to‘g‘ri prororsiya bo‘lgani uchun} \quad \begin{array}{l} a_2 = \frac{a_1 \cdot k_2}{k_1} \\ a_3 = \frac{a_1 \cdot k_3}{k_1} \end{array}$$

$$\left.\begin{array}{l} a_3 \leftrightarrow k_4 \\ a_4 \leftrightarrow k_5 \end{array}\right\} \textit{larga teskari prororsiya bo‘lgani uchun} \quad a_4 = \frac{a_3 \cdot k_4}{k_5} = \frac{a_1 \cdot k_3 \cdot k_4}{k_1 \cdot k_5}.$$ *Shunda*

$$a_1 + \frac{a_1 \cdot k_2}{k_1} + \frac{a_1 \cdot k_3}{k_1} + \frac{a_1 \cdot k_3 \cdot k_4}{k_5} = A.$$ *Yoki* $$\frac{a_1}{k_1}\left(k_1 + k_2 + k_3 + \frac{k_3 \cdot k_4}{k_5}\right) = A.$$ *Bundan*

$$a_1 = \frac{A \cdot k_1}{k_1 + k_2 + k_3 + \frac{k_3 \cdot k_4}{k_5}}.$$ *Unversal ko‘rinishi* $$a_i = \frac{A \cdot k_i}{k_1 + k_2 + k_3 + \frac{k_j \cdot k_4}{k_5}}$$

Misol: To‘rtta sonining yig‘indisi 161 ga teng. Ulardan dastlabki uchtasi 4, 5 va 8 sonlariga to‘g‘ri, uchinchi va to‘rtinchi sonlari esa 6 va 8 sonlariga teskari proporsiya bo‘lsa, ikkinchi sonni toping.

<u>Yechish:</u> *Kelirib chiqarilgan universal formuladan foydalanib, izlanayotgan ikkinchi sonni topamiz:* $a_2 = \frac{161 \cdot 5}{4+5+8+\frac{8 \cdot 6}{8}} = 35.$

2. ALGEBRA

2.1. Daraja haqida tushuncha.

1) har qanday sonning nolinchi darajasi birga teng. $a^o = 1$. $\boldsymbol{M}$: $7^o = 1;\ 13^o = 1$
2) har qanday sonning birnichi darajasi shu sonning o‘ziga teng. $a^1 = a$.
3) $a^2 = a \cdot a$; $a^3 = a \cdot a \cdot a$. $\boldsymbol{M}$: $2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$.
4) $a^n = a \cdot a \cdot a \cdot a$ ....
5) $a^{-n} = \frac{1}{a^n}$; $\boldsymbol{M}$: $3^{-4} = \frac{1}{3^4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$.
6) daraja darajaga oshirilganda daraja ko‘rsatkichlari ko‘paytiriladi. $(a^n)^m = a^{n \cdot m}$
$\boldsymbol{M}$: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$.
7) $a^n \cdot a^m = a^{n+m}$. $\boldsymbol{M}$: $\boldsymbol{3^{x+3}} = 3^x \cdot 3^3 = 27 \cdot 3^x$.
8) $\frac{a^n}{a^m} = a^{n-m}$. $\boldsymbol{M}$: $\frac{6^{15}}{6^{13}} = 6^{15-13} = 6^2 = 36$.
9) $a^n \cdot b^n = (a \cdot b)^n$. $\boldsymbol{M}$: $(2 \cdot x \cdot y)^3 = 2^3 \cdot x^3 \cdot y^3 = 8 \cdot x^3 \cdot y^3$.
10) $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$. $\boldsymbol{M}$: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$.
11) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\boldsymbol{M}$: $\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$.

13) Agar asoslari bir xil, daraja ko‘rsatkichlari har xil bo‘lgan ifodalar teng bo‘lsa, ularning darajalari teng bo‘ladi. $a^n = a^m$ bo‘lsa, $n = m$ bo‘ladi.

Misol uchun $3^x = 27 = 3^3$ *bundan* $x = 3$ *bo‘ladi.*

13) $(-a)^n$ da agar *n* juft bo‘lsa, a^n bo‘lai, agar *n* toq bo‘lsa, $-a^n$ bo‘ladi.

2.2. Qisqa ko‘paytirish formulalari.

1) ikki son yig‘indisining kvadrati $(a + b)^2 = a^2 + 2ab + b^2$.

2) ikki son ayirmasining kvadrati $(a-b)^2 = a^2 - 2ab + b^2$.

3) ikki son kvadratlari ayirmasini ko‘paytuchiga ajratish

$a^2 - b^2 = (a-b)(a+b)$.

4) ikki son ayirmasini ko‘paytuchiga ajratish $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$.

5) ikki son kublarining yig‘indisini ko‘paytuchiga ajratish

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2).$$

6) ikki son kublarining ayirmasini ko‘paytuchiga ajratish

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2).$$

7) ikki son yig‘indisining kubi $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

8) ikki son ayirmasining kubi $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

9) $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$.

10) $a^6 - b^6 = (a^3)^2 - (b^3)^2 = (a^3 - b^3)(a^3 + b^3) =$

$= (a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2)$.

11) $a^6 - b^6 = (a^2)^3 - (b^2)^3 = (a^2 - b^2)(a^4 + a^2b^2 + b^4) =$

$= (a-b)(a+b)(a^4 + a^2b^2 + b^4)$.

12)) $a^6 + b^6 = (a^2)^3 + (b^3)^2 = (a^2 + b^2)(a^4 - a^2b^2 + b^4)$.

2.3. Ayniyatlar. Ifodalarning chap tomoni o‘ng tomoniga teng bo‘lsa, u ayniyat bo‘ladi. Ayniyatda chap va o‘ng tomondagi no’malum sonlar oldidagi koeffitsientlar ham-da ozod hadlar teng bo‘ladi.

Ayniyatga doir misollar:

1) $a - b = -(b - a)$ *ni ayniyat ekanligini isbotlang. O‘ng tomondagi qavsni ochib, quyidagini hosil qilamiz:* $a - b = -b + a = a - b$, *demak, ayniyat ekan.*

2) $(x-a)(x+b) = x^2 - (a-b)x - ab$ *ni ayniyat ekanligini isbotlang. Teng-likning chap va o‘ng tomonidagi qavslarni ochib taqqoslaymiz:*

$x^2 - ax + bx - ab = x^2 - ax + bx - ab$, *demak, ayniyat ekan.*

3) $\frac{5}{(x+3)(x-5)} = \frac{a}{x-5} + \frac{b}{x+3}$ *ifoda ayniyat bo‘lsa, ayniyatdagi a va b koeffitsi-yenlarni aniqlang:* $\frac{5}{(x+3)(x-5)} = \frac{a(x+3)+b(x-5)}{(x+3)(x-5)}$ *yoki*

$5 = ax + 3a + bx - 5b$; *bundan* $(a+b)x + 3a - 5b = 5$; *o‘ng tomonda no’ma-lum bo‘lmagani uchun uning koeffitsiuenti 0 ga teng, chap tomondagi no’malim son oldidagi koeffitsientni nolga tenglaymiz:*

$a + b = 0$ *bundan* $b = -a$. *Ayniyatning chap tomonidagi aniq sonni, uning o‘ng tomonidagi aniq songa tenglaymiz:*

$3a - 5b = 5;\ 3a + 5a = 5;\ 8a = 5.\ a = \frac{5}{8};\ b = -\frac{5}{8}.$

4) $(бx + 2y)(3x + вy) = гx^2 + 2\frac{3}{4}xy + 3y^2$ayniyatdagi koeffitsient б ni toping.

Tenglikning chap tomonidagi qavslarni ochib, ixchamlab, no’malum sonlar oldida-gi koeffitsientlarni aniqlaymiz;$3бx^2 + (бв + 6)xy + 2вy^2 = гx^2 + \frac{11}{4}xy + 3y^2$.

Endi no’malum sonlar oldidagi koeffitsientlarni tenglashtiramiz:

$2в = 3$ *bundan* $в = \frac{3}{2}$ $бв + 6 = \frac{11}{4}$ *bundan* $б = -\frac{13}{6}$

2.4. Ildiz haqida tushuncha. Maxrajni irratsionallikdan qutqarish.

1) Agar ifodaning darjasi kasr ko‘rinishda bo‘lsa, kasrning maxraji ildiz ko‘rsatkich qilib, surati esa daraja ko‘rsatkich qilib yoziladi.

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$. Agar ildiz ko‘rsatkichi 2 bo‘lsa, unga kvadrat ildiz deb atalib, u ildiz ko‘rsatkichda yozilmaydi. Masalan $\sqrt{a}$.

M: $\sqrt[0,2]{3}$ ni hisoblang. $3^{\frac{1}{0,2}} = 3^5 = 243$.

2) $\sqrt[n]{a^n} = a$. **1.** ***M***: $\sqrt{16} = \sqrt{4^2} = 4$. **2.** ***M***: $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$.

3) $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

M: $\sqrt[3]{8 \cdot 27 \cdot 64} = \sqrt[3]{8} \cdot \sqrt[3]{27} \cdot \sqrt[3]{64} = \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{4^3} = 2 \cdot 3 \cdot 4 = 24$.

4) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ ***M***: $\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{2^4}}{\sqrt[4]{3^4}} = \frac{2}{3}$.

5) $\sqrt[n]{\sqrt[m]{\sqrt[k]{a}}} = \sqrt[nmk]{a}$. ***M***: $\sqrt[3]{\sqrt{\sqrt[3]{81}}} = \sqrt[3\cdot 2\cdot 3]{3^4} = \sqrt[18]{3^4} = \sqrt[9]{9}$.

6) $\sqrt[n]{a} \cdot \sqrt[m]{b} = \sqrt[nm]{a^m} \cdot \sqrt[nm]{b^n} = \sqrt[nm]{a^m \cdot b^n}$.

M: $\sqrt[3]{a} \cdot \sqrt{b} = \sqrt[6]{a^2} \cdot \sqrt[6]{b^3} = \sqrt[6]{a^2 \cdot b^3}$.

7) $\sqrt[n]{a \cdot \sqrt[n]{a \cdot \sqrt[n]{a \cdot \sqrt[n]{a \ldots}}}} = \sqrt[n-1]{a}$.

M: $\sqrt[4]{8 \cdot \sqrt[4]{8 \cdot \sqrt[4]{8 \cdot \sqrt[4]{8 \ldots}}}} = \sqrt[4-1]{8} = \sqrt[3]{2^3} = 2$.

8) $\sqrt[n]{a : \sqrt[n]{a : \sqrt[n]{a : \sqrt[n]{a \ldots}}}} = \sqrt[n+1]{a}$.

M: $\sqrt[3]{81 : \sqrt[3]{81 : \sqrt[3]{81 : \sqrt[3]{81 \ldots}}}} = \sqrt[3+1]{81} = \sqrt[4]{3^4} = 3$.

9) $\sqrt{a + \sqrt{a + \sqrt{a + \cdots}}} = \frac{\sqrt{4a+1}+1}{2}$.

M: $\sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}} = \frac{\sqrt{4\cdot 6+1}+1}{2} = 3$.

10) $\sqrt{a - \sqrt{a - \sqrt{a - \cdots}}} = \frac{\sqrt{4a+1}-1}{2}$.

M: $\sqrt{2 - \sqrt{2 - \sqrt{2 - \cdots}}} = \frac{\sqrt{4\cdot 2+1}-1}{2} = 1$.

11) $\sqrt{a \pm b\sqrt{c}} = \frac{b}{2} \pm \sqrt{c}$

$1M$: $\sqrt{28 - 10\sqrt{3}} = \frac{10}{2} - \sqrt{3} = 5 - \sqrt{3}$.

$2M$: $\sqrt{7 + 4\sqrt{3}} = \frac{4}{2} + \sqrt{3} = 2 + \sqrt{3}$.

Maxrajni irrotsionallikdan qutqarish. Maxrajni irrotsionallikdan qutqarish uchun maxrajdagi irrotsional son berilgan kasrning surat va maxrajiga ko‘paytiriladi.

Misollar. 1) $\frac{2}{\sqrt{3}}=\frac{2\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\frac{2\cdot\sqrt{3}}{3}$ 2) $\frac{\sqrt{5}}{\sqrt{3}}=\frac{\sqrt{5}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\frac{\sqrt{5\cdot3}}{3}=\frac{\sqrt{15}}{3}$

Agar maxrajda irratsional sonlarning yig‘indisi yoki ayirmasi qatnashsa, maxrajdagi yig‘indi yoki ayirmaning yig‘indisiga berilgan kasrning surat va maxrajiga ko‘paytiriladi.

Misollar. 1) $\frac{2}{\sqrt{5}-1}=\frac{2\cdot(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{2\cdot(\sqrt{5}+1)}{5-1}=\frac{(\sqrt{5}+1)}{2}$

2) $\frac{\sqrt{7}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{7}\cdot(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=\frac{\sqrt{7}\cdot(\sqrt{3}-\sqrt{2})}{3-2}=\sqrt{7}\left(\sqrt{3}-\sqrt{2}\right).$

Irratsional sonlarni hisoblashga doir misollar.

1) $\sqrt{3+\sqrt{5}}\cdot\sqrt{3-\sqrt{5}}$ ni hisoblang.

Yechish: *Ildiz haqidagi formulaning (2) dan fo‘lalanib, berilgan ifodani quyidagicha yozamiz:* $\sqrt{3+\sqrt{5}}\cdot\sqrt{3-\sqrt{5}}=\sqrt{(3+\sqrt{5})\cdot(3-\sqrt{5})}=\sqrt{9-5}=\sqrt{4}=2.$

2) $\sqrt{7+2\sqrt{6}}+\sqrt{7-2\sqrt{6}}$ ni hisoblang.

Yechish: *Berilgan ifodani a deb belgilab, uni ikkala tomonini kvadratga oshiramiz:*

$$a^2=\left(\sqrt{7+2\sqrt{6}}+\sqrt{7-2\sqrt{6}}\right)^2=$$

$$=\left(\sqrt{7+2\sqrt{6}}\right)^2+2\sqrt{7+2\sqrt{6}}\cdot\sqrt{7-2\sqrt{6}}+\left(\sqrt{7-2\sqrt{6}}\right)^2$$

$a^2=14+2\cdot5=24.$ *Bundan* $a=2\sqrt{6}.$

3) $\sqrt{11-2\sqrt{10}}-\sqrt{11+2\sqrt{10}}$ ni hisoblang.

Yechish: *Berilgan ifodani a deb belgilab, uni ikkala tomonini kvadratga oshiramiz:*

$$a^2=\left(\sqrt{11-2\sqrt{10}}-\sqrt{11+2\sqrt{10}}\right)^2=$$

$$=\left(\sqrt{11-2\sqrt{10}}\right)^2-2\sqrt{11-2\sqrt{10}}\cdot\sqrt{11+2\sqrt{6}}+\left(\sqrt{11-2\sqrt{10}}\right)^2$$

$a^2=22-2\cdot9=4.$ *Bundan* $a=\pm2.$ *Birinchi irratsional son, ikkinchi irratsional sondan kichik bo‘lgani uchun, berilgan ifodaning qiymati* -2 *ga teng bo‘ladi.*

2.5. Ikki nomalumli tenglama. Ikkita no’malum qatnashgan tenglamalarga ikki no’malumli tenglamalar deb ataladi. Ularni yechishda ikkita maqsad qo‘yiladi.

1) Amaliy, ya’ni amaldagi ikkita no’malumni topish.

2) Nazariy, ya’ni chiziqlarni kesishish nuqtasini aniqlash.

Ikkita no’malumli tenglamalar birgalikda sistema qilib yechiladi.

Agar ikkita to‘g‘ri chiziqlar $y_1=k_1\cdot x_1+b_1$ va $y_2=k_2\cdot x_2+b_2$ ko‘rinishdagi tenglama bilan berilgan bo‘lsa, ularni quyidagi sistema shakliga keltirib yechamiz:

$\begin{cases} y_1 = k_1 \cdot x_1 + b_1 \\ y_2 = k_2 \cdot x_2 + b_2 \end{cases}$ bu yerda k_1 va k_2 − to‘g‘ri chiziqlarning burchak koeffitsientlari; b_1 va b_2 − to‘g‘ri chiziqlarni OY o‘qi bilan kesishish nuqtalari.

Sistemani yechishning quyidagi usullari mavjud:

1. Qo‘shish yoki ayrish.
2. O‘rniga qo‘ysh.
3. Belgilab olish.
4. Yangi o‘zgartirish kiritish.

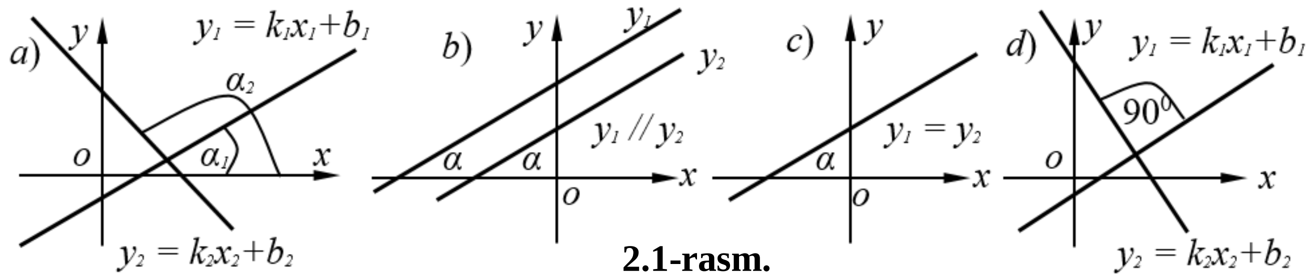
Sistema yechimlarining xususiy hollari (2.1-rasm):

1. Agar $k_1 \neq k_2$ bo‘lsa, sistema yechimga ega bo‘ladi, yani to‘g‘ri chiziqlar albatta kesishadi (2.1a-rasm).

2. Agar $k_1 = k_2$ bo‘lib, $b_1 \neq b_2$ bo‘lmasa, sistema yechimga ega bo‘lmaydi, ya’ni to‘g‘ri chiziqlar parallel $y_1 // y_2$ bo‘lib, kesishadi (2.1b-rasm).

3. Agar $k_1 = k_2$ va $b_1 = b_2$ bo‘lsa, sistema cheksiz ko‘p yechimga ega bo‘ldi, bunda chiziqlar ustma-ust tushib qoladi (2.1c-rasm).

4. Agar $k_1 = -\frac{1}{k_2}$ bo‘lsa, chiziqlar kesishish nuqtasida bir biri bilan 90^0 li burchak hosil qiladi, ya’ni $y_1 \perp y_2$ (2.1d-rasm).



2.1-rasm.

Ikki no’malumli tenglama va tenglamalar sistemasini yechishga doir misollar.

1. $\begin{cases} 2x + 3y = 10 \\ 3x - y = 4 \end{cases}$ tenglamalar sitemasini yeching.

Yechish: *Ikkinchi tenglamani 3 ga ko‘paytirib, birinchi tenglamaga qo‘shamiz, shunda* $11x + 0 = 22$ *bundan* $x = 2$. *Ikkinchi tenglamadan y ni topib, x ninig bu qiymatini qo‘yamiz* $y = 3x - 4 = 3 \cdot 2 - 4 = 2$. *Demak* $y = 2$.

2. Firmadagi tovuq va qo‘ylarning soni 90 ta, ularning oyoqlarining soni 210 ta bo‘lsa, firmadagi tovuq va qo‘ylarning sonini aniqlang.

Yechish: *Firmadagi tovuqlar sonini x va uning oyoqlari 2 bo‘lishini, qo‘ylar sonini y va uning oyoqlari soni 4 ta ekanligini bilgan holda quyidagi tenglamalar sistemasini tuzamiz:* $\begin{cases} x + y = 90 \\ 2x + 4y = 210 \end{cases}$ *birinchi tenglamani 2 ga ko‘paytirib,uni ikkinchi tenglamadan ayiramiz, shunda* $0 + 2y = 30$ *bundan* $y = 15$. *Birinchi tenglamadan x ni topib, y ninig bu qiymatini qo‘yamiz* $x = 90 - y = 90 - 15 = 75$.

Demak firmada 15 ta tovuq, 75 ta qo‘y bor ekan.

3. $\begin{cases} x - 2ay = 7 \\ 2x + y = 4 \end{cases}$ a ning qanday qiymatida tenglamalar sitemasini yechimga ega bo‘lmaydi.

Yechish: *Sistema yechimga ega bo‘lmasligi uchun* $k_1 = k_2$ *bo‘lib,* $b_1 \neq b_2$ *bo‘lmasligi kerak.Burchak koeffitsiyentlarini tenglashtirish uchun birinchi tenglamani ikkala tomonini 2 ga ko‘paytiramiz va y ni oldidagi koeffitsientlarni tenglashtirib , a ni aniqlaymiz:* *Shunda* $-4a = 1$ *bundan* $a = -\frac{1}{4}$.

4. $\begin{cases} ax + y = 6 \\ 2x - 3y = b \end{cases}$ a va b larning qanday qiymatlarida tenglamalar sitemasini cheksiz ko‘p yechimga ega bo‘ladi.

Yechish: *Sistema cheksiz ko‘p yechimga ega bo‘lishi uchun* $k_1 = k_2$ *bo‘lib,* $b_1 = b_2$ *bo‘lishi kerak. Burchak koeffitsiyentlarini tenglashtirish uchun birinchi tenglamani ikkala tomonini 2 ga, ikkinchi tenglamaning ikkala tomonini a ga ko‘paytiramiz; shunda sistema quyidagi ko‘rinishga ega bo‘ladi:*

$\begin{cases} 2ax + 2y = 12 \\ 2ax - 3ay = ab \end{cases}$ *y oldidagi koeffitsientlarni va ozo‘l hadlarni tenglashtirib, a ni topamiz:* $-3a = 2$ *bundan* $a = -\frac{2}{3}$ *,ozo‘l hadlarni tenglashtirib undan b ni qiymatlarini aniqlaymiz* $ab = 12$*:* $-\frac{2}{3}b = 12$ *bundan* $b = -18$.

2.6. Arifmetik progressiya. Arifmetik progressiya deb, biror o‘zgarmas d (progressiyani ayirmasi) sonni qo‘shishdan yoki ayrishdan hosil bo‘lgan sonlar ketma-ketligiga aytiladi.

Arifmetik progressiyani ayirmasi

$$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a_n - a_{n-1}$$

M: $-5 - 3 - 1 + 1 + 3 + 5 + 7 + \cdots$ $d = 2$. $3 + 5 + 8 + 11 + 14 + \cdots$ $d = 3$. $9 + 5 + 1 - 3 - 7 - 11 - 15 \ldots$ $d = -4$.

Arifmetik progressiyaning umumiy ifodasi $a_1 + a_2 + a_3 + a_4 + \cdots + a_n$.

Agar progressiyaning birinchi hadi a_1 berilgan bo‘lsa, uning ikkinchi hadi, uchinchi, to‘rtinchi va hokozo *n*- hadlari quyidagicha aniqlanadi:

$a_2 = a_1 + d$; $a_3 = a_2 + d = a_1 + d + d = a_1 + 2d$;

$a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d$; $a_5 = a_4 + d = a_1 + 3d + d = a_1 + 4d$;

demak arifmetik progressiyaning istalgan hadini quyidagicha aniqlash mumkin:

$$a_n = a_1 + (n-1) \cdot d. \quad (2.1)$$

Bundan

$$d = \frac{a_n - a_1}{n-1}. \quad (2.2)$$

bu yerda $a_n -$ progressiyaning oxirgi hadi; $a_1 -$ progressiyaning birinchi hadi; $n -$ progressiya hadlarining soni; $d -$ progressiyaning ayirmasi.

Progressiyaning *n* ta hadining yig‘indisi quyidagi formulalar orqali aniqlanadi:

$$S_n = \frac{a_1 + a_n}{2} \cdot n. \quad (2.3)$$

yoki

$$S_n = \frac{2a_1 + (n-1) \cdot d}{2} \cdot n. \quad (2.4)$$

$$a_n = S_n - S_{n-1}. \quad (2.5)$$

$$a_{n+1} = S_{n+1} - S_n. \quad (2.6)$$

$$a_{n+1} - a_n = a_n - a_{n-1} = d. \quad (2.7)$$

Agar $S_n = n^2$ bo‘lsa, $a_n = n^2 - (n-1)^2$ bo‘ladi (2.8)

Arifmetik progressiyaga doir misollar.

1) 2+4+6+… arifmetik progressiyaning 21-hadini toping.

Yechish: *Berilgan misol va uning shartidan quyidagilarni aniqlaymiz; progressiyaning birinchi hadi* $a_1 = 2$; *ayirmasi* $d = 2$; *hadlarining soni* $n = 21$. *Arifmetik progressiyaning istalgan hadini topish (2.1) formulasidan*

$a_{21} = a_1 + (n-1)\cdot d = 2 + (21-1)\cdot 2 = 42.$

2) Arifmetik progressiyaning o‘nbirinchi hadi 41 ga, ayirmasi 3 ga teng bo‘lsa, uni birinchi hadini toping.

Yechish: *Masala shartiga asosan* $a_{11} = 41,\ d = 3,\ n = 11$. *Arifmetik progressiyaning istalgan hadini topish (1) formulasi* $a_{11} = a_1 + (n-1)\cdot d$ *dan*

$a_1 = a_{11} - (n-1)\cdot d = 41 - (11-1)\cdot 3 = 11.$

3) 1 dan 81 gacha bo‘lgan natural sonlar yig‘indisini toping.

Yechish: *Masala shartiga asosan* $a_1 = 1, a_n = 81, d = 1, n = 81$. *Arifmetik progressiya hadlarining yig‘indisi formulasi (2.3) dan*

$$S_{81} = \frac{a_1 + a_{81}}{2}\cdot 81 = \frac{1+81}{2}\cdot 81 = 3321.$$

4) Agar arifmetik progressiya $a_n + a_{n+1} = 3(1+n)$ bo‘lsa, S_n ni toping.

Yechish: $n = 1$ *da berilgan tenglik* $a_1 + a_2 = 3(1+1) = 6$.

Yoki $a_1 + a_1 + d = 6$. *Bundan* $2a_1 = 6 - d$.

$a_n + a_{n+1} = a_1 + nd - d + a_1 + nd = 3 + 3n.$

$2a_1 + 2nd - d = 3 + 3n;\ \ 6 - d + 2nd - d = 3 + 3n;\ \ 2d(n-1) = 3(n-1).$

Bundan $d = \frac{3}{2};\ \ 2a_1 = 6 - \frac{3}{2} = \frac{9}{2}.$

Shunda $S_n = \frac{2a_1 + (n-1)d}{2}\cdot n = \frac{\frac{9}{2} + (n-1)\frac{3}{2}}{2}\cdot n = \frac{3n(n+2)}{2}.$

2.7. Geometrik progressiya. Geometrik progressiya deb, biror o‘zgarmas q (progressiyani maxraji) songa ko‘paytirshdan yoki bo‘lishdan hosil bo‘lgan sonlar ketma-ketligiga aytiladi. Geometrik progressiyaning maxraji $q \neq 0;\ q \neq 1$.

$$q = \frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = \frac{b_n}{b_{n-1}} \qquad (2.9)$$

M: $1 + 2 + 4 + 8 + \cdots;\ \ q = 2.\ \ 3 + 9 + 27 + 81 + \cdots;\ \ q = 3.$

$16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots;\ \ q = 4.$

Geometrik progressiyaning umumiy ifodasi $b_1 + b_2 + b_3 + b_4 + \cdots + b_n$.

Agar progressiyaning birinchi hadi b_1 berilgan bo‘lsa, uning ikkinchi hadi, uchinchi, to‘rtinchi va hokozo *n*- hadlari quyidagicha aniqlanadi:

$b_2 = b_1\cdot q;\ \ b_3 = b_2\cdot q = b_1\cdot q\cdot = b_1\cdot q^2;\ b_4 = b_3\cdot q = b_1\cdot q^2\cdot q = b_1\cdot q^3;$ $b_5 = b_4\cdot q = b_1\cdot q^3\cdot q = b_1\cdot q^4;\ b_6 = b_5\cdot q = b_1\cdot q^4\cdot q = b_1\cdot q^5;$ demak geometik pro-gressiyaning istalgan hadini quyidagicha aniqlash mumkin:

$$b_n = b_1\cdot q^{n-1}\ . \qquad (2.10)$$

bu yerda b_n −progressiyaning oxirgi hadi; b_1 −progressiyaning birinchi hadi; n − progressiya hadlarining soni; q − progressiyaning ayirmasi.

Progressiyaning *n* ta hadining yig‘indisi quyidagi formulalar orqali aniqlanadi:

o‘suvchi $$S_n = \frac{b_1\cdot(q^n - 1)}{q-1}. \qquad (2.11)$$

kamayuvchi $$S_n = \frac{b_1\cdot(1-q^n)}{1-q}. \quad (2.12)$$

$$b_n = S_n - S_{n-1}. \quad (2.13)$$

$$b_{n+1} = S_{n+1} - S_n. \quad (2.14)$$

$$q = \frac{b_{n+1}}{b_n}. \quad (2.15)$$

$$b_1^2 + b_2^2 + b_3^2 + \cdots + b_n^2 = \frac{b_1^2\left((q^n)^2-1\right)}{q^2-1} \quad (2.16)$$

$$\frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \cdots + \frac{1}{b_n} = \frac{\frac{1}{b_1}(1-q^n)}{1-q} \quad (2.17)$$

Geometrik progressiyaga doir misollar.

1) $1+3+9+27+\cdots$ geometrik progressiyaning 6-hadini toping.

Yechish: *Berilgan misol va uning shartidan quyidagilarni aniqlaymiz; progressiyaning birinchi hadi $b_1 = 1$; maxraji $q = 3$; hadlarining soni $n = 6$.*

Geometrik progressiyaning istalgan hadini topish (2.10) formulasidan

$b_6 = 1\cdot 3^{6-1} = 1\cdot 3^5 = 243$.

2) Geometik progressiyaning beshinchi hadi 32 ga, birinchi hadi 2 ga teng bo‘lsa, uni maxrajini toping.

Yechish: *Masala shartiga asosan $b_5 = 32,\ b_1 = 2,\ n = 5$.*

Geometrik progressiyaning istalgan hadini topish (2.10) formulasidan

$32 = 2\cdot q^{5-1} = 2\cdot q^4$. *Bundan* $q = 2$.

3) Geometik progressiyaning beshinchi hadi 4 ga, maxraji 2 ga teng. Shu progressiyaning oltita hadini yig‘indisini toping.

Yechish: *Masala shartiga asosan $b_1 = 5,\ q = 2 > 0,\ n = 6$. O‘suvchi geometrik progressiya hadlarining yig‘indisini topish (2.12) formulasidan* $S_6 = \frac{5\cdot\left(2^6-1\right)}{2-1} = 315$.

4) Geometrik progressiyada $c_n = ak^{n-6}$ va $c_2\cdot c_{10} = 25$ bo‘lsa, koeffitsient a ni toping.

Yechish: *Geometrik progressiyaning hadlari c ga, maxraji esa k ga o‘zgartirilganligini hisobga olib, $c_n = c_1\cdot k^{n-1} = a\cdot k^{n-6}$ va $c_2\cdot c_{10} = c_1k\cdot c_1k^9 = 25$ larni hosil qilamiz. Ikkinchi tenglikdan $c_1k^5 = 5$, bundan $c_1 = \frac{5}{k^5}$.*

Demak $\frac{5}{k^5}\cdot k^{n-1} = a\cdot k^{n-6}$, *bundan* $a = 5$.

2.8. Cheksiz kamayuvchi geometrik progressiya. Cheksiz kamayuvchi geometrik progressiya deb, maxraji o‘ldiy kasrdan tashkil topgan progressiyaga aytiladi. Cheksiz kamayuvchi geometrik progressiya hadlarining yig‘indisi quyidagi formulalar orqali aniqlanadi: $S_n = \frac{b_1}{1-q}$ (2.15); $S_n = \frac{b_1^2}{1-q^2}$. (2.18)

Cheksiz kamayyuvchi geometrik progressiyaga doir misollar.

1) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots;\ q = \frac{1}{2};\ \ S_n = \frac{1/2}{1-1/2} = 1.$

2) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots; q=\frac{1}{3};\ S_n=\frac{1/3}{1-1/3}=0{,}5.$

3) $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots; q=\frac{1}{4};\ S_n=\frac{1/4}{1-1/4}=0{,}333\ldots=0,(3).$

$0{,}3333\ldots=0{,}3+0{,}03+0{,}003+0{,}0003+\cdots=\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{1000}+\cdots$

Bundan ko‘rinadiki, bu ketma-ketlik maxraji $q=\frac{1}{10}$ ga teng bo‘lgan cheksiz kamayuvchi geometrik progressiyadir. Uning *n* ta hadini yig‘indisi (2.15) formula orqali aniqlanadi:

$$S_n=\frac{3/10}{1-1/10}=\frac{3/10}{9/10}=\frac{3}{9}.$$

Ayrim ketma-ket kelgan sonlarni yig‘indisini topish formulalari:

1) $2+5+13+\cdots+(2^{n-1}+3^{n-1})=2^n-1+\frac{3^n-1}{2}$

2) $1\cdot2+2\cdot3+\cdots+n\cdot(n+1)=\frac{n(n+1)(n+2)}{3}$

3) $1\cdot4+2\cdot7+3\cdot10+\cdots+n\cdot(n+3)=n(n+1)^2$

4) $3\cdot9+5\cdot27+7\cdot81+\cdots+(2n+1)\cdot3^{n+1}=n\cdot3^{n+2}$

5) $1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$

6) $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(4n^2-1)}{3}$

7) $1^3+2^3+3^3+\cdots+n^3=\frac{n^2(n+1)^2}{4}=\left(\frac{n\cdot(n+1)}{2}\right)^2$

8) $1^3+3^3+5^3+\cdots+(2n-1)^3=n^2(2n^2-1)$

9) $1^2-2^2+3^2-4^2+\cdots+(-1)^{n-1}\cdot n^2=(-1)^{n-1}\cdot\frac{n(n+1)}{2}$

2.9. Tengsizlik va qo‘shaloq tengsizliklar. Tengsizlik, bu - maxsus belgi bo‘lib, biron bir ifoda yoki qiymatni katta yoki kichik ekanligini ko‘rsatadi. Bunda agar qiymat belgi ichida bo‘lsa, u katta, belgi uchida bo‘lsa, kichik hisoblanadi. ***Misol*:** $7>5$ yoki $11<17$. Agar tengsizlikni ikkala tomoni biror manfiy songa ko‘paytirilsa yoki bo‘linsa, qiymatlar qarama-qarshi ishoraga, tengsizlik belgisi ham o‘zgaradi.

***Misol*:** $9>5$ ni ikkala tomoni -1 ga ko‘paytirsak $-9<-5$ bo‘ladi.

Qo‘shaloq tengsizliklar, agar bir qiymat boshqa bir qiymatdan katta, yana bir boshqa qiymatdan kichik bo‘lsa, u qo‘shaloq tengsizlikka olinadi.

Misol uchun $7<9<11$.

Qo‘shaloq tengsizlikka doir *misol*. $3\le x-5<7$ tengsizlikni yeching. Buni quyidagi sistema ko‘rinishiga keltiramiz: $\begin{cases}x-5\ge3\\x-5<7\end{cases}\Rightarrow\begin{cases}x\ge8\\x<12\end{cases};$

$8\le x<12$ yoki [8; 12).

2.10. To‘la kvadrat tenglama. To‘la kvadrat tenglamaning umumiy ko‘rinishi quyidagicha bo‘ladi. $ax^2+bx+c=0$; kvadrat tenglama bu parabola tenglamasi bo‘lib, u quyidagi ko‘rinishga ega $y=ax^2+bx+c$.

Bu yerda a, b, c - koeffitsientlar bo‘lib, a- parabola shoxining yo‘nalishini; agar $a > 0$ bo‘lsa, shox yuqoriga, $a < 0$ bo‘lsa, shox pastga yo‘nalgan bo‘ladi. b-parabola shoxining tik yoki yotiqligi, hamda uchining ko‘chishini ko‘rsatadi. c- ozo‘l had bo‘lib, para-bolani OY o‘qi bilan kesishish nuqtasini ko‘rsatadi.

To‘la kvadrat tenglama $ax^2 + bx + c = 0$ ning ildizi, ya’ni OX o‘qi bilan kesishish nuqtalari quyidagi formula orqali aniqlanadi: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Bunda a va c koeffit-sientlarning qaram-qarshi ishorasi olinadi.

Buni isboti quyidagicha; $ax^2 + bx + c = 0$ tenglikni ikkala tomonini a ga bo‘lib, quyidagi tenglikni hosil qilamiz: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. Buni quyidagicha qilib yozamiz: $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$; $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$; Buni ikkala tomonidan kvadrat ildiz olsak, $\left(x + \frac{b}{2a}\right)_{1,2} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

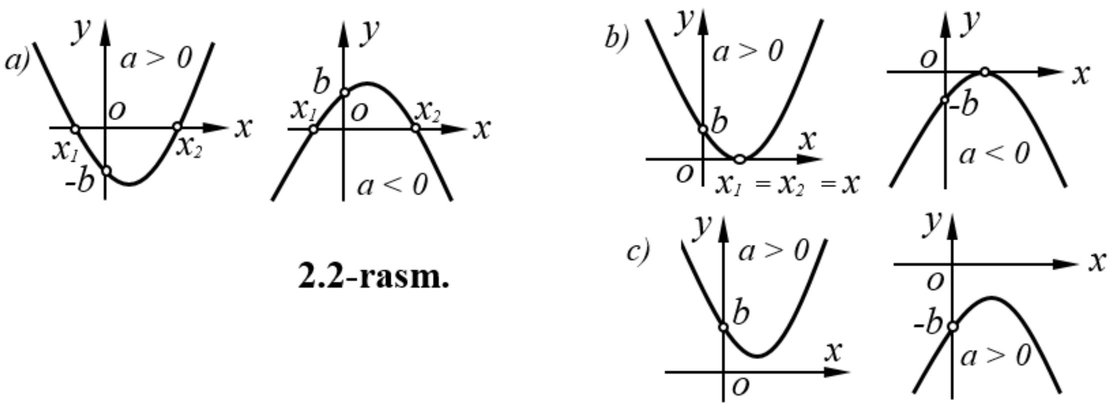
Bundan $x_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Bu yerda kvadrat ildiz ostidagi ifodaga diskirminant (kvadrat tenglamani ildizini bor yoki yo‘qligini aniqlovchi ko‘rsatkich) deb ataladi. $D = b^2 - 4ac$.

Buni inobatga olib, tenglama ildizlarini quyidagi formula yordamida aniqlash mumkin:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

Diskirminantning xususiy hollari va uning grafiklari 2.2-rasmda ko‘rsatilgan.



2.2-rasm.

1) Agar $D > 0$ bo‘lsa, tenglama haqiqiy ikkita ildizga ega, ya’ni parabola OX o‘qini albatta ikki nuqtasidan kesib o‘tadi (2.2a-rasm).

2) Agar $D = 0$ bo‘lsa, tenglama bitta ildizga ega, ya’ni $x_1 = x_2 = x$ bo‘lib, para-bola OX o‘qining ostida yoki ustida joylashgan bo‘ladi (2.2b-rasm).

3) Agar $D < 0$ bo‘lsa, tenglama ildizga ega emas, ya’ni parabola OX o‘qi bilan kesishmaydi (2.2c-rasm).

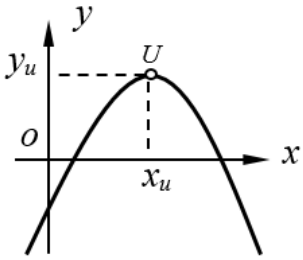
Parabola uchining koo‘linatalari. Agar parabola uchini U deb olsak, uning koor-dinatasi $U(x_u; y_u)$ bo‘ladi (2.3-rasn).

Agar (+; + →I), (−;+ → II), (−;− → III), (+;− → IV) bo‘ladi.

Diskriminantni $D = 0$ deb olib, parabolaning x_u uchini topamiz: $x_u = -\frac{b}{2a}$ buni parabola tenglamasidagi x ni o‘rniga qo‘yib, parabolaning y_u uchini topamiz:

$$y_u = a\cdot\frac{b^2}{4a^2} - \frac{b^2}{2a} + c = \frac{b^2}{4a} - \frac{b^2}{2a} + c = c - \frac{b^2}{4a}.$$

Demak $y_u = c - \frac{b^2}{4a}$

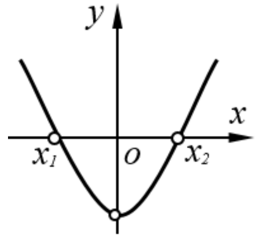


2.3-rasm.

2.11. Chala kvadrat tenglamalar. To‘la kvadrat tenglamadan $ax^2 + bx + c = 0$:

1) Agar $a = 0$ bo‘lsa, $bx + c = 0$ bo‘ladi va u bitta ildizga ega bo‘ladi, $x = -\frac{c}{b}$

2) Agar $b = 0$ bo‘lib, $c < 0$ bo‘lsa, $ax^2 - c = 0$, bo‘ladi, parabolaning uchi *OY* (ordinata) o‘qida joylashgan bo‘lib, ildizlari teng va qarama-qarshi ishorali bo‘ladi (2.4-rasm): $x_{1,2} = \pm\sqrt{\frac{c}{a}}$.

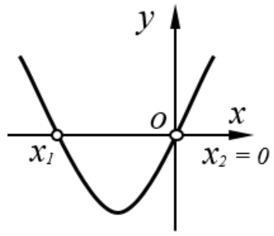


2.4-rasm.

Agar $c > 0$ bo‘lsa, tenglama yechimga ega emas.

3) Agar $c = 0$ bo‘lsa, $ax^2 + bx = 0$ bo‘ladi va parabolaning bitta ildizi 0 ga teng bo‘lib, parabola albatta koordinata boshidan o‘tadi (2.5-rasm). Berilgan tenglikni $x\cdot(ax + b) = 0$ ko‘rinishda yozib, ko‘paytmani har birini nolga tenglab, uning ildizlarini aniqlaymiz:

$$x_1 = -\frac{b}{a}\,;\ x_2 = 0.$$



2.5-rasm.

2.12. Keltirilgan kvadrat tenglama. Viyet teoremasi.

Keltirilgan kvadrat tenglama. To‘la kvadrat tenglamani $ax^2 + bx + c = 0$ ikkala tomonini a ga bo‘lib, quyidagi tenglikni hosil qilamiz:

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0.$ $\frac{b}{a} = p,$ $\frac{c}{a} = q$ kabi o‘zgartirishdan so‘ng, tenglama

$x^2 + px + q = 0$ ko‘rinishga ega bo‘ladi, bu esa keltirilgan kvadrat tenglamaning umumiy ko‘rinishi bo‘ladi. Uning ildizlari, ya’ni *OX* o‘qi bilan kesishish nuqtalari quyidagi formula orqali aniqlanadi: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$.

Buni isboti quyidagicha; $x^2 + px + q = 0$ tenglikdan quyidagi tenglikni hosil qilamiz: $\left(x + \frac{p}{2}\right)^2 - \frac{p^2}{4} + q = 0$; Buni ikkala tomonidan kvadrat ildiz olamiz:

$\left(x + \frac{p}{2}\right)_{1,2} = \pm\sqrt{\frac{p^2}{4} - q}$. Bundan $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$.

Viyet teoremasi. Keltirilgan kvadrat tenglama ildizlarining yig‘indisi $x_1 + x_2 = -p$ ga, ko‘paytmasi $x_1\cdot x_2 = q$ ga teng.

Isboti: $x_1 + x_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q} - \frac{p}{2} + \sqrt{\frac{p^2}{4} - q} = -\frac{2p}{2} = -p.$

$$x_1\cdot x_2 = \left(-\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}\right)\cdot\left(-\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}\right) = \left(-\frac{p}{2}\right)^2 - \left(\sqrt{\frac{p^2}{4} - q}\right)^2 = q.$$

Viyet teoremasiga doir misollar:

1) Ildizlari $x_1 = 3 - \sqrt{2}$ va $x_2 = 3 + \sqrt{2}$ ga teng bo‘lgan tenglama tuzing.

Yechish: *Keltirilgan kvadrat tenglama $x^2 + px + q = 0$ dagi p va q koeffitsient-larni aniqlaymiz:* $p = -(x_1 + x_2) = -\left(3 - \sqrt{2} + 3 + \sqrt{2}\right) = -6.$

$q = x_1 \cdot x_2 = \left(3 - \sqrt{2}\right) \cdot \left(3 + \sqrt{2}\right) = 9 - 2 = 7.$

Bu qiymatlarni o‘rniga qo‘yib, quyidagi tenglamani hosil qilamiz: $x^2 - 6x + 7 = 0.$

2) Agar x_1 va x_2 $x^2 + 6x - a = 0$ tenglamani ildizlari va $\frac{1}{x_1} + \frac{1}{x_2} = \frac{2}{3}$ bo‘lsa, no’malum koeffitsient a ni toping.

Yechish: $\frac{1}{x_1} + \frac{1}{x_2} = \frac{2}{3}$ *ni* $\frac{x_1+x_2}{x_1 \cdot x_2} = \frac{2}{3}$ *ko‘rinishda yozamiz, shunda* $\frac{-6}{-a} = \frac{2}{3}$ *bo‘ladi, bundan* $a = 9$.

3) Agar x_1 va x_2 $x^2 + ax - 6 = 0$ tenglamani ildizlari va $x_1^2 + x_2^2 = 13$ bo‘lsa, no’malum koeffitsient a ni toping.

Yechish: *Berilgan tenglamada* $x_1 \cdot x_2 = -6$ *ga teng.* $a = -(x_1 + x_2)$ *bu tenglikni ikkala tomonini kvadratga oshiramiz:*

$a^2 = x_1^2 + 2x_1x_2 + x_2^2 = 13 + 2(-6) = 1$,*bundan* $a = \pm 1$.

4) $(x^2 - 3x + 3)(x^2 - x + 3) = 3x^2$ tenglamaning ildizlari yig‘indisini toping.

Yechish: *Berilgan tenglamani quyidagi ko‘rinishda yozib, uni kvadrat tenglama shakliga keltiramiz:* $(x^2 - x + 3 - 2x)(x^2 - x + 3) = 3x^2$

$(x^2 - x + 3)^2 - 2x \cdot (x^2 - x + 3) - 3x^2 = 0.$ $(x^2 - x + 3)_{1,2} = x \pm \sqrt{x^2 + 3x^2} = x \pm 2x.$

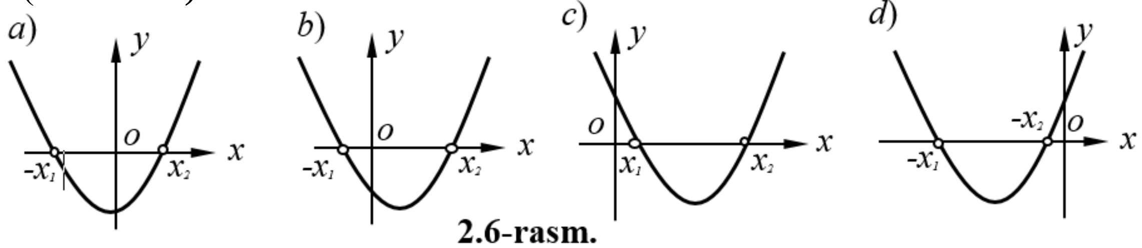
$(x^2 - x + 3)_1 = x - 2x = -x;$ $x^2 + 3 = 0;$ $x^2 \neq -3.$ $(x^2 - x + 3)_2 =$

$= x + 2x = 3x;$

$x^2 - 4x + 3 = 0.$ *Bundan viyet teoremasiga asosan* $x_1 + x_2 = -p = -(-4) = 4.$

Keltirilgan kvadrat tenglama ildizlarining ishoralari:

- Agar $p > 0,$ $q < 0$ bo‘lsa, tenglama ildizlari turli ishorali bo‘lib, $|-x_1| > x_2$ bo‘ladi (2.6*a*-rasm).
- Agar $p < 0,$ $q < 0$ bo‘lsa, tenglama ildizlari turli ishorali bo‘lib, $|-x_1| < x_2$ bo‘ladi (2.6*b*-rasm).



2.6-rasm.

- Agar $p < 0,$ $q > 0$ bo‘lsa, tenglama ildizlari musbat ishorali bo‘ladi, $x_1 > 0, x_2 > 0$ (2.6*c*-rasm).
- Agar $p > 0,$ $q > 0$ bo‘lsa, tenglama ildizlari manfiy ishorali bo‘ladi, $x_1 < 0, x_2 < 0$ (2.6*d*-rasm).

2.13. Kvadrat uch hadni ko’paytuvchilarga ajratish. Kvadrat uch hadni ko’pay-tuvchilarga ajratish uchun uni shartli ravishda nolga tenglab, ildizlari aniqlanadi va bu ildizlarning qiymatlari teskari ishora bilan bir biriga ko‘paytiriladi.

Kvadrat uch hadni ko'paytuvchilarga ajratishga doir misollar:

$$1)\ x^2+4x-5=(x+5)(x-1).\quad x_{1,2}=-2\pm\sqrt{4+5}=-2\pm 3;$$
$$x_1=-5;\ x_2=1.$$

$$2)\ 2x^2-3x-2=(x-2)(2x+1).\quad x_{1,2}=\frac{3\pm\sqrt{9+4\cdot 2\cdot 2}}{2\cdot 2}=\frac{3\pm 5}{4};$$
$$x_1=2;\ 2x_2=-1.$$

Ko'p hadni ko'p hadga bo'lish. Agar ko'p had koeffitsientlarining yig'indisi 0 ga teng bo'lsa, tenglama ildizlaridan biri 1 ga teng bo'ladi. *Masalan*: $x^3+79x-80=0$.

Tenglama koeffitsientlarining yig'indisi $1+79-80=0$. Demak tenglamaning ildizlaridan biri $x_1=1$ ga teng bo'lib, berilgan ko'p had $x-1$ ga qoldiqsiz bo'linadi.

$$\begin{array}{l|l} x^3+79x-80 & x-1 \\ \underline{x^3-x^2} & \overline{x^2+x+80} \\ \quad x^2+79x-80 & \\ \quad \underline{x^2-x} & \\ \qquad 80x-80 & \\ \qquad \underline{80x-80} & \\ \qquad\quad 0 & \end{array}$$

$$x^3+79x-80=(x-1)(x^2+x+800)$$

$$\begin{array}{l|l} x^4+2x^3-x^2+3x-8 & x^2-x-2 \\ \underline{x^4-x^3-2x^2} & \overline{x^2+3x+4} \\ \quad 3x^3+x^2+3x-8 & \\ \quad \underline{3x^3-3x^2-6x} & \\ \qquad 4x^2+9x-8 & \\ \qquad \underline{4x^2-4x-8} & \\ \qquad\quad 13x & \end{array}$$

$$x^4+2x^3-x^2+3x-8=(x^2-x-2)(x^2+3x+4)+13x$$

Kubik tenglamalar.

Kubik tenglamaning umumiy ko'rinishi $ax^3+bx^2+cx+d=0$.

Agar $a+b+c+d=0$ bo'lsa, tenglama ildizlaridan biri 1 ga teng bo'ladi. Berilgan kubik tenglama $x-1$ ga qoldiqsiz bo'linadi.

Keltirilgan kubik tenglamaning umumiy ko'rinishi $x^3+px^2+qx+r=0$. Bunda tenglamaning ildizlari uning koeffitsiyentlari bilan quyidagicha bog'langan bo'ladi:

$x_1+x_2+x_3=-p;\quad x_1x_2+x_2x_3+x_1x_3=q;\quad x_1\cdot x_2\cdot x_3=-r.$

2.14. Kvadrat tengsizliklar, ularni intervallar metodi bilan yechish.

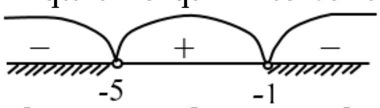
Kvadrat tengsizliklarni yechish uchun quyidagi ishlarni amalga oshirish kerak:

1. Kvadrat tengsizlikni shartli ravishda nolga tenglab, ildizlarini, ya'ni *OX* o'qi bilan kesishish nuqtalarini aniqlanadi.
2. Aniqlangan nuqtalar orqali interval (oraliq) lar tuziladi.
3. Intervallardagi ishoralar aniqlanadi.
4. Tengsizlik shartiga asosan javob aniqlanadi.

Agar kvadrat tengsizlik kasr sondan tashkil topgan bo'lsa, uni maxrajini nolga aylantiruvch, hamda noldan farq qiluvchi qiymatlar tengsizlik yechimga kiritilmaydi.

Kvadrat tengsizliklarni yechishga doir misollar:

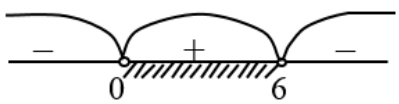
1) $x^2+6x+5\le 0$ tengsizlikni yeching. Uning ildizlarini aniqlaymiz: $x_{1,2}=-3\pm\sqrt{9-5}=-3\pm 2;\ x_1=-5;\ x_2=-1.$ Bu niqtalar orqali intervallar ajratib, ulardagi ishoralarni aniqlaymiz:



Tengsizlikni yechimi $(-\infty;-5]U[-1;\infty)$

2)) $6x-x^2>0$ tengsizlikni yeching va uning quyidagi qiymatlarini aniqlang.

Uning ildizlarini aniqlaymiz: $x(6-x)>0$ bundan $x_1=0;\ x_2=6$. Bu niqtalar orqali intervallar ajratib, ulardagi ishoralarni aniqlaymiz:

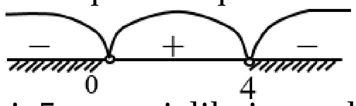


Tengsizlikni yechimi $(0;6)$.

- Nechta butun yechimi bor. 5 ta (1,2,3,4,5)
- Nechta tub yechimi bor. 3 ta (2,3,5)
- Eng katta va eng kichik butun yechimlarining ayirmasini toping. $5 - 1 = 4$.
- Butun yechimlarini o‘rta arifmetigini toping. $O‘ar = \frac{1+2+3+4+5}{5} = 3$.

3) $\frac{3x}{4-x} \leq 0$ tengsizlikni yeching va uning eng kichik musbat butun yechimi bilan eng katta manfiy butun yechimi ayirmasini toping.

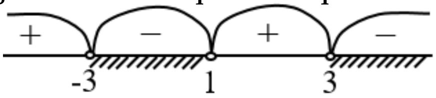
Uning ildizlarini aniqlaymiz: $x_1 = 0;\ x_2 = 4$. Bu nuqtalar orqali intervallar ajratib, ulardagi ishoralarni aniqlaymiz:



Tengsizlikni yechimi $(-\infty; 0]U(4; \infty)$.

Tengsizlikni eng kichik musbat butun yechimi 5, tengsizlikni eng katta manfiy butun yechimi -1 bo‘lgani uchun $5 - (-1) = 6$.

4) $\frac{1-x}{x^2-9} \leq 0$ tengsizlikni yeching. Uning ildizlarini aniqlaymiz: $x_1 = -3;\ x_2 = 1$ va $x_3 = 3$. Bu nuqtalar orqali intervallar ajratib, ulardagi ishoralarni aniqlaymiz:



Tengsizlikni yechimi $(-3; 1]U(3; \infty)$.

2.15. Bi kvadrat tenglamalar. $ax^4 + bx^2 + c = 0$ ko‘rinishdagi, ya’ni x^2 ga nisbatan kvadrat tenglamalarga bi kvadrat tenglamalar deb yuritiladi. Uning to‘rtta ildizi, ya’ni egri chiziq *OX* o‘qini to‘rt nuqtasidan kesib o‘tadi.

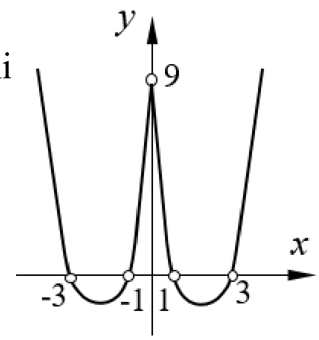
Uning ildizlari $x_{1,2,3,4} = \pm\sqrt{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$ ya’ni kvadrat tenglama ildizidan yana bir marta ildiz olinadi.

Misol: $x^4 - 10x^2 + 9 = 0$ tenglama ildizlarini ko‘paytmasini toping. Bu tenglamani x^2 ga nisbatan yechamiz:

$(x^2)_{1,2} = 5 \pm \sqrt{25 - 9} = 5 \pm 4;$

Demak $x_1^2 = 1$ bundan $x_{1,2} = \pm 1$ va $x_2^2 = 9$ bundan $x_{3,4} = \pm 3$. Shunda $x_1 = -3,\ x_2 = -1,\ x_3 = 1,\ x_4 = 3$.

Shartga asosan $x_1 \cdot x_2 \cdot x_3 \cdot x_4 = -3 \cdot (-1) \cdot 1 \cdot 3 = 9$.

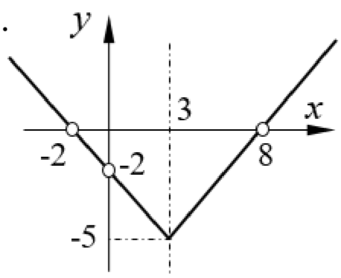


2.16. Modulli tenglamalar va tengsizliklar.

O‘lcham yoki qiymatning absalyut qiymatiga uning mo‘luli deb ataladi va u $|a|$ ko‘rinishda ifodalanadi. Bunda $a \in (-\infty; \infty)$ bo‘lib, $|a| \geq 0$ bo‘ladi.

Misol uchun $a = -5$ bo‘lsa, uning mo‘luli $|-5| = 5$ bo‘ladi.

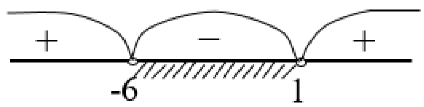
<u>Mo‘lulli tenglamalar</u> deb, mo‘lul ostida no’malum son qatnashgan tenglamalarga aytiladi. Misol: $|x - 3| = 5$ mo‘lulli tenglamani yeching. Buni $x - 3 = \pm 5$; yoki $x = 3 \pm 5$, bundan $x_1 = -2;\ x_2 = 8$; Bundan ko‘rinadiki, mo‘lulli tenglamalarni yechish uchun, mo‘lul ostidagi ifoda, mo‘luldan chiqqan ifodaning qarama-qarshisiga tenglashtirilar ekan.



<u>Mo‘lulli tengsizliklar.</u> Mo‘lulli tengsizliklar ham xuddi kvadrat tengsizliklarniki kabi yechiladi. Misol uchun $|2x + 5| < 7$ tengsizlikni yeching.

Bu tengsizlikni noldan farqli qilib, quyidagi $|2x+5|-7<0$ ko‘rinishda yozamiz.

Tengsizlikni mo‘luldan chiqarib, $2x+5<\pm 7$ yoki $2x<-5\pm 7$ ko‘rinishda yozish mumkin, bundan $x_1=-6;\ x_2=1$.



Bu nuqtalar orqali intervallar ajratib, ulardagi isho-ralarni aniqlaymiz: Tengsizlikni yechimi $(-6;1)$.

2.17. Irratsional tenglamalar va tengsizliklar.

Ildiz ostida no’malum son qatnashgan tenglamalarga, irratsional tenglamalar deb ataladi.

Misol uchun $\sqrt[n]{f(x)}=0$. Agar tenglama ildiz ko‘rsatkichi juft ($n=2;\ n=4;$ $n=6;$) bo‘lgan ildiz ostida bo‘lsa, $f(x)\geq 0$ shartga amal qilish kerak.

Irratsional tenglamalarga doir misollar:

1) $\sqrt{3x-2}=4$ irratsional tenglamani yeching. Tenglikni ikkala tomonini kvadrat-ga oshirsak $3x-2=16$ bo‘ladi, bundan $x=6$.

2) $3\sqrt{x-2}=\sqrt{2x+3}$ irratsional tenglamani yeching. Tenglikni ikkala tomonini kvadratga oshirsak $9x-18=2x+3$ bo‘ladi, bundan $x=3$.

3) $\sqrt[3]{3-x}=3$ irratsional tenglamani yeching. Tenglikni ikkala tomonini kubga oshirsak $3-x=27$ bo‘ladi, bundan $x=-24$.

4) $x-\sqrt{x+1}=1$ tenglamani yeching. Bu tenglamani quyidagi $\sqrt{x+1}=x-1$ ko‘rinishda yozamiz. Bunda tenglamaning ildizi $x-1\geq 0$ yoki $x\geq 1$ bo‘lishi kerak.

Tenglikni ikkala tomonini kvadratga oshirsak $x+1=(x-1)^2$ bo‘ladi, buni soddlashtirib, $x^2-3x=0$. Bundan $x_1\neq 0,\ x_2=3$.

5) $\sqrt{x}+2\sqrt[4]{x}-15=0$ tenglamani yeching.

Bu tenglamani quyidagi $\sqrt[4]{x^2}+2\sqrt[4]{x}-15=0$ ko‘rinishda yozamiz. Tenglamaning ildizi $x\geq 0$ bo‘lishi kerak. Bu tenglamani $\sqrt[4]{x}$ ga nisbatan kvadrat tenglama qilib yechamiz:

$\left(\sqrt[4]{x}\right)_{1,2}=-1\pm\sqrt{1+15}=-1\pm 4$ bo‘ladi. Unda $\left(\sqrt[4]{x}\right)_1\neq -5,\ \left(\sqrt[4]{x}\right)_2=3$ bo‘ladi, bundan $x=81$.

6) $\sqrt{\frac{3-x}{x-1}}+3\sqrt{\frac{x-1}{3-x}}=4$ tenglamani yeching. Tenglamaga quyidagicha o‘zgarti-rish kiritamiz: $t=\sqrt{\frac{3-x}{x-1}}$. Shunda $t+\frac{3}{t}-4=0$ yoki $t^2-4t+3=0$. Bu tenglamani yechib, $t_{1,2}=2\pm 1$ bundan $t_1=1;\ t_2=3$. t ning bu qiymatlarini belgilab olingan joyga qo‘yib, tenglamaning quyidagi yechimlarini hosil qilamiz: $\sqrt{\frac{3-x}{x-1}}=1$. Buni ikkala tomonini kvadratga oshirib, $3-x=x-1$ ni hosil qilamiz.

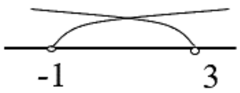
Bundan $x=2$. $\sqrt{\frac{3-x}{x-1}}=3$. Buni ikkala tomonini kvadratga oshirib,

$3-x=9x-9$ ni hosil qilamiz. Bundan $x=2{,}2$.

Irratsional tengsizliklarga doir misollar:

1) $\sqrt{x+1} < 2$ tengsizlikni yeching. Bunda $x+1 \geq 0$ yoki $x \geq -1$ bo‘lishi kerak.

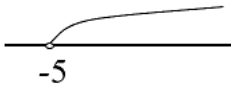
Tengsizlikni ikkala tomonini kvadratga oshirsak $x+1 < 4$ bo‘ladi, bundan $x < 3$.



Shunda tengsizlikni yechimi $-1 \leq x < 3$ yoki [-1;3)

2) $\sqrt{x+5} > -2$ tengsizlikni yeching. Bunda $x+5 \geq 0$ yoki $x \geq -5$ bo‘lishi kerak.

Shunda tengsizlikni yechimi $x \geq -5$ yoki [-5;∞).



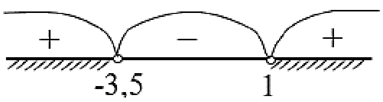
3) $(x-1)\sqrt{x} < 0$ tengsizlikni yeching. Bunda $x \geq 0$ hamda $x < 1$ bo‘lishi kerak.

Shartga asosan $x \neq 0$. Shunda tengsizlikni yechimi $(0; 1)$.

4) $\frac{\sqrt{2x^2+5x-7}}{x+6} > 0$ tengsizlikni yeching. Bunda birinchidan $2x^2+5x-7 \geq 0$, ikkinchidan $x+6 > 0$ bo‘lishi kerak. Birinchi tengsizlikni yechimi quyidagicha aniqlanadi. Uning absissa o‘qi bilan kesishish nuqtalarini aniqlaymiz:

$$x_{1,2} = \frac{-5 \pm \sqrt{25+4\cdot 2\cdot 7}}{2\cdot 2} = \frac{-5 \pm 9}{4}; \;\; x_1 = -3{,}5; \; x_2 = 1.$$

Bu nuqtalar orqali intervallar ajratib, ulardagi ishoralarni aniqlaymiz:



Birinchi tengsizlikni yechimi $(-\infty; -3{,}5]U[1; \infty)$

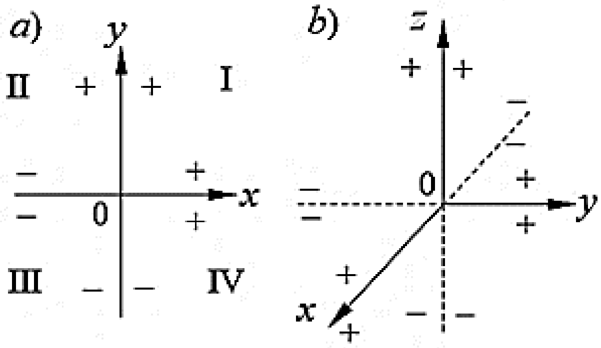
Ikkinchi tengsizlikni yechimi quyidagicha aniqlanadi: $x > -6$.

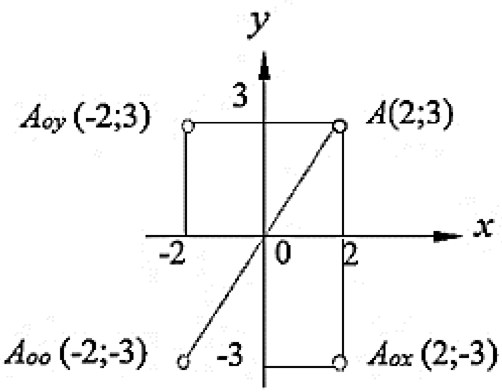
Bu tengsizliklarni yechimini birlashtirib, quyidagi umumiy yechimni aniqlaymiz: $(-6; -3{,}5]U[1; \infty)$.

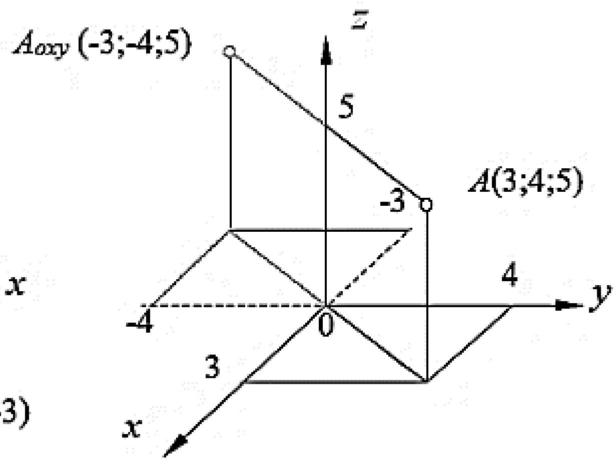
2.18. Koordinatalar sistemasi. Nuqtaning koordinatalari. Simmetirik nuqtalar.

Koordinatalar sistemasi. Matematikada koordinatalar sistemasining ikki turidan foydalaniladi.

Ulardan biri ikki o‘lchamli (tekislikda, Dekard koordinatalar sistemasi *a*-rasm). Ikkinchisi, uch o‘lchamli yoki qutub koordinatalar sistemasi deb yuritiladi (*b*-rasm).







Nuqtaning koordinatalari. Agar nuqta Dekard koordinatalar sistemasida bo‘lsa, x va y o‘qlaridan shu nuqtalarni topib, o‘qlarga paralellar o‘tkazamiz, paralellarning kesishishgan joyida berilgan nuqta joylashgan bo‘ladi. Agar nuqta qutub koordinatalar sistemasida bo‘lsa, x va y o‘qlaridan shu nuqtalarni topib, o‘qlarga paralellar o‘tkazamiz, paralellarning kesishishgan nuqtasidan z o‘qi bo‘ylab uning qiymati qo‘yiladi.

Simmetirik nuqtalar. Bu berilgan nuqtalarni, o‘qlardan tuzilgan tekislikka nisbatan akslantirishdan hosil bo‘lgan nuqtadir. Agar berilgan nuqtani koordinata boshiga nisbatan akslantirilsa, uning barcha qiymatlari qarama-qarshi ishorasiga o‘zgaradi.

2.19. Funksiya, uning xossalari (grafigi, aniqlanish sohasi, qiymatlar (o‘zgarish) sohasi, juft va toqliligi, teskari funksiya).

Funksiya deb, o‘zgaruvchi erkli omil x argumentning qabul qilishi mumkin bo‘lgan sonlar to‘plamiga aytiladi. Funksiya, bu erksiz o‘zgaruvchi omil bo‘lib, u y bilan belgilanadi.

Funksiyaning umumiy ko‘rinishi $y = f(x)$. Misol uchun fizikada S yo‘lni funksiya, vaqt o‘zgarishi t ni argument deb qabul qilingan.

Funksiyaning xossalari: Funksiyaning xossalari 17 tadan ortiq. Shulardan ayrimlarini ko‘rsatib o‘tamiz:

1. Funksiyaning grafigi. To‘g‘ri chiziq va parabolaning grafiklari ushbu qo‘llanmaning 8, 16, 20 betlarida berilgan, shuningdek, ayrim funksiyalarning grafiklari qo‘llanmaning ilova qismida ham berilgan.

2. Funksiyaning aniqlanish sohasi. Argument x ning qabul qilishi mumkin bo‘lgan sonlar to‘plamidir va u $D(f)$ deb belgilangan. O‘latda deyarli barcha funksiyalarning aniqlanish sohasi $(-\infty;\infty)$ oraliqda bo‘ladi. Ayrim funksiyalar bundan mustasno. Agar funksiya kasr ko‘rinishda berilgan bo‘lsa, uni maxrajini nolga aylantiruvchi qiymat aniqlanish sohasiga kiritilmaydi. Misol uchun $y = f(x) = \frac{3}{x-2}$ funksiyaning aniqlanish sohasi, $x \neq 2$ ekanligini hi-sobga olgan holda $D(f) = (-\infty; 2)\text{U}(2;\infty)$ bo‘ladi.

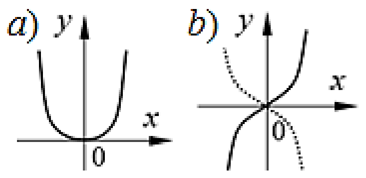
3. Funksiyaning qiymatlar (o‘zgarish) sohasi. Funksiyaning o‘zi, ya’ni y ni qabul qilishi mumkin bo‘lgan sonlar to‘plamidir va u $E(f)$ deb belgilangan. O‘latda deyarli barcha fungsiyalarning qiymatlar sohasi $(-\infty;\infty)$ oraliqda bo‘ladi. Ayrim funksiyalar bundan mustasno. Misol uchun $y = ax^2 + bx + c$ parabola ko‘rinishidagi funksiyalarda, parabola shoxining yuqori yoki pastga yo‘nalishiga qarab, parabola uchining ordinatasiga bog‘liq bo‘ladi.

Misol uchun (20 betdagi rasmlarga qarang)

$$(-\infty; -y_u);\quad (-\infty; y_u);\quad (-y_u;\infty);\quad (y_u;\infty).$$

4. Funksiyaning juft yoki toqligi. Agar funksiya o‘zining aniqlanish sohasidagi biron bir manfiy qiymatida musbat $f(-x) = f(x)$ bo‘lsa, yoki funksiyaning grafigi simmetrik bo‘lib, OY o‘qi bo‘yicha qatlaganda ustma-ust tushsa bunday funksiyalar juft funksiyalar deb ataladi.

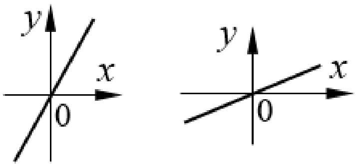
Misol uchun: $y = x^2$ funksiya o‘z aniqlanish sohasining manfiy qiymati $x = -1$ da funksiya $y = 1$ musbat bo‘lishini ko‘rish mumkin. Yoki yning grafigi (rasmga qarang) simmetrik bo‘lib, OY o‘qi bo‘yicha qatlaganda ustma-ust tushadi (a-rasim).



Agar funksiya o'zining aniqlanish sohasidagi biron bir manfiy qiymatida manfiy $f(-x) = -f(x)$ bo'lsa, yoki funksiyaning grafigi simmetrik bo'lib, *OY* o'qi bo'yicha qatlaganda ustma-ust tushmasa bunday funksiyalar toq funksiyalar deb ataladi (b-rasim).

Misol uchun: $y = x^3$ funksiya o'z aniqlanish sohasining manfiy qiymati $x = -1$ da funksiya $y = -1$ manfiy bo'lishini ko'rish mumkin.

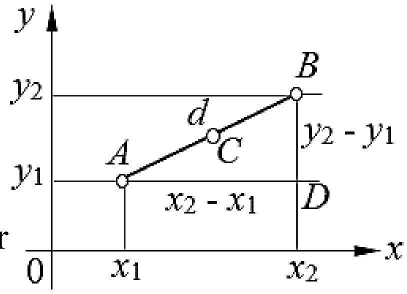
Yoki uning grafigi (rasmga qarang) simmetrik bo'lib, *OY* o'qi bo'yicha qatlaganda ustma-ust tushmaydi.



5. Teskari funksiya. Berilgan funksiyaga teskari bo'lgan funksiyani topish uchun, uni x ga nisbatan yechib, so'ngra x va y larning o'rni almashtiriladi. Masalan $y = 3x$ bundan $x == \frac{1}{3}y$ agar *x* va *y* larning o'rni almashtirsak $y = \frac{1}{3}x$.

2.20. Ikki nuqta orasidagi masofa va undan o'tuvchi to'g'ri chiziq tenglamasi.

Agar tekslikda ikki nuqta va ularning koordinatalari berilgan bo'lsa, ular orasidagi masofani (rasmga qarang) Pifagor teoremasiga asosan topish mumkin.



2.7-rasm.

To'g'ri burchakli uchburchak *ABD* da $AB = d$ gipotenuza (*A* va *B* nuqtalar orasidagi masofa, 2.7-rasm), $AD = x_2 - x_1$ va $BD = y_2 - y_1$lar katetlardir. Bu masofa Pifagor teoremasiga asosan

$AB = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y)^2}$ bo'ladi.

Agar nuqta uch o'lchamli sistemada bo'lsa, unda ular orasidagi masofa

$AB = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Chizmadagi *C* nuqta *AB* kesmaning o'rtasi bo'lsa, uning koordinatalarini $C(x_{o'r}; y_{o'r})$ bo'ladi. $x_{o'r} = \frac{x_1+x_2}{2}$, bundan $x_2 = 2x_{o'r} - x_1$; $y_{o'r} = \frac{y_1+y_2}{2}$, bundan $y_2 = 2y_{o'r} - y_1$; $z_{o'r} = \frac{z_1+z}{2}$, bundan $z_2 = 2z_{o'r} - z_1$.

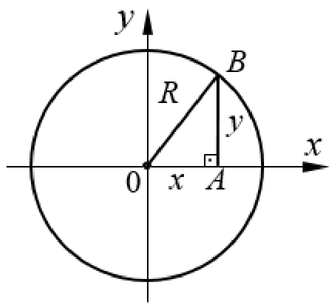
AB nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} = \frac{z-z_1}{z_2-z_1}$ ko'rinishda bo'ladi.

Misol: $A(-2;3)$ va $B(4;-5)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

$\frac{y-3}{-5-3} = \frac{x-(-2)}{4-(-2)}$. Buni hisoblab, $y = -\frac{4}{3}x + \frac{1}{3}$ ni hosil qilamiz.

2.21. Aylana tenglamasi. Markazi koordinata boshida bo'lgan aylananing tenglamasi quyidagi ko'rinishda bo'ladi $x^2 + y^2 = R^2$ (2.8-rasm). Bu yerda R aylananing radiusi. Buni, chizmadagi to'g'ri burchakli uchburchak *ABO* dan Pifagor teoremasiga asosan isbotlash mumkin.

Agar aylana markazi $M(a;b)$ koordinata boshidan a va b masofaga ko'chirilgan bo'lsa, uning tenglamasi quyidagi ko'rinishda bo'ladi $(x-a)^2 + (y-b)^2 = R^2$ (2.9-rasm).



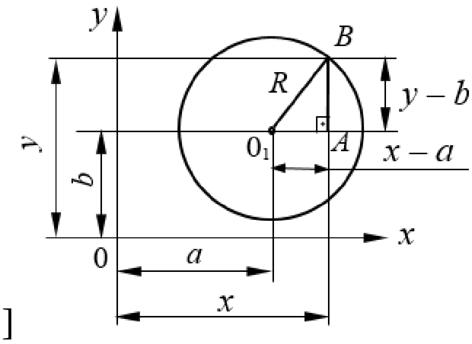
2.8-rasm.

Buni, chizmadagi to‘g‘ri burchakli uchburchak ABO_1 dan Pifagor teoremasiga asosan isbotlash mumkin. Aylana markazining $M(a;b)$ choraklarda joylashishi: $M(a;b)$ I - chorak; $M(-a;b)$ II - chorak; $M(-a;-b)$ III - chorak; $M(a;-b)$ IV - chorak.

Misol: $x^2+y^2+6x-8y+21=0$ tenglama bilan berilgan aylanani standart shaklga keltiring.

Berilgan tenglamani quyidagi ko‘rinishda yozamiz: $x^2+6x+9+y^2-8y+16-4=0$ bundan, aylananing standart ko‘rinishini quyidagicha yozish mumkin:

$$(x+3)^2+(y-4)^2=2^2.$$



2.9-rasm.

Bundan quyidagilarni aniqlash mumkin:

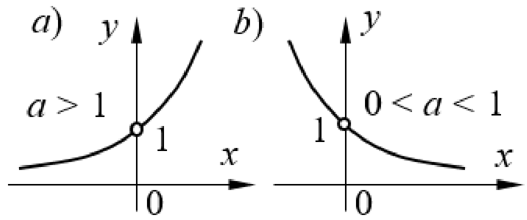
- Aylananing rariusi $R=2$.
- Aylana markazining koo‘linatalari $M(-3;4)$.]
- Aylana markazi II chorakda joylashgan.

2.22. Ko‘rsatkichli funksiya, tenglama va tengsizliklar.

Ko‘rsatkichli funksiya deb, darajasida no’malum qatnashgan funksiyalarga aytiladi.

Funksiyaning umumiy ko‘rinishi $y=a^x$. Bu yerda a asos bo‘lib, $a\neq 0$; $a\neq 1$. Daraja haqidagi formulalarning barchasi ko‘rsatkichli funksiya uchun ham o‘rinli bo‘ladi.

Agar $a>1$ bo‘lsa, funksiya o‘suvchi (2.10a-rasm), $0<a<1$ bo‘lsa, funksiya kamayuvchi (2.10b-rasm) bo‘ladi (grafikka qarang).



2.10-rasm.

$y=a^x$ tenglikda $x=0$ bo‘lsa, $y=1$ bo‘ladi. Demak funksiyaning grafigi OY o‘qining bir birligidan kesib o‘tadi (chizmaga qarang).

Ko‘rsatkichli funksiyaning **aniqlanish sohasi,** barcha haqiyqiy sonlar to‘plamidan iborat. $E(f)\rightarrow(-\infty;\infty)$.

Ko‘rsatkichli funksiyaning **qiymatlar (o‘zgarish) sohasi,** barcha musbat haqiyqiy sonlar to‘plamidan iborat. $D(f)\rightarrow(0;\infty)$.

Ko‘rsatkichli funksiya **juft ham toq ham** emas. Agar funksiya $y=a^x+a^{-x}$ ko‘rinishda berilgan bo‘lsa, u juft funksiya bo‘ladi.

Ko‘rsatkichli funksiyaga **teskari** funksiya, logarifmik funksiyadir.

$y=a^x\rightarrow y=log_a x$.

Ko‘rsatkichli tenglama va tengsizliklarga doir misollar.

Quyidagi ko‘rsatkichli tenglamalarni yeching:

1) $7^{x-2}=1$, yechimi $7^{x-2}=7^0$ bundan $x-2=0\rightarrow x=2$.
2) $9^{3-x}=\frac{1}{27}$, yechimi $3^{6-2x}=3^{-3}$ bundan $6-2x=-3\rightarrow x=4{,}5$.
3) $3^{5x-2}=9^{2x}$, yechimi $3^{5x-2}=3^{4x}$ bundan $5x-2=4x\rightarrow x=2$.
4) $2\cdot 9^x-3^{x+1}-9=0$, buni quyidagicha yozish mumkin

$$2\cdot 3^{2x}-3\cdot 3^x-9=0.$$

Bu esa 3^x ga nisbatan kvadrat tenglamadir buni yechamiz:

$(3^x)_{1,2} = \frac{3\pm\sqrt{9+4\cdot2\cdot9}}{2\cdot2} = \frac{3\pm9}{4}$ bundan $(3^x)_1 \neq -1{,}5;\ (3^x)_2 = 3; x = 1.$

Quyidagi ko‘rsatkichli tengsizliklarni yeching:

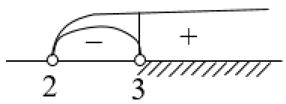
1) $5^{x-3} < \frac{1}{25}$ yechimi $5^{x-3} < 5^{-2}$ bundan $x - 3 < -2 \to x < 1$ yoki $(-\infty; 1)$.

2) $0{,}8^{x^2+3x} \geq 1$ yechimi $0{,}8^{x^2+3x} \geq 0{,}8^0$ yoki $x^2 + 3x \leq 0$ bundan $-3 \leq x \leq 0$ yoki $[-3; 0]$.

3) $\left(\frac{1}{8}\right)^{7-3x} \leq 64$ yechimi $8^{-7+3x} \leq 8^2$ bundan $-7 + 3x \leq 2 \to x \leq 3$ yoki $(-\infty; 3)$.

4) $5^{\sqrt{x-2}} > 5^{1-\sqrt{x-2}} + 4$ yechimi, bunda $x - 2 \geq 0 \to x \geq 2$, tengsizlikni quyida-gi ko‘rinishda yozamiz: $5^{\sqrt{x-2}} > \frac{5}{5^{\sqrt{x-2}}} + 4$ bundan

$\left(5^{\sqrt{x-2}}\right)^2 - 4 \cdot 5^{\sqrt{x-2}} - 5 > 0$ bu esa $5^{\sqrt{x-2}}$ ga nisbatan kvadrat tenglama uni yechamiz: $\left(5^{\sqrt{x-2}}\right)_{1,2} = 2 \pm \sqrt{4+5} = 2 \pm 3$; $\left(5^{\sqrt{x-2}}\right)_1 \neq -1$, $\left(5^{\sqrt{x-2}}\right)_2 = 5$ bundan $\sqrt{x-2} = 1$, yoki $x = 3$.



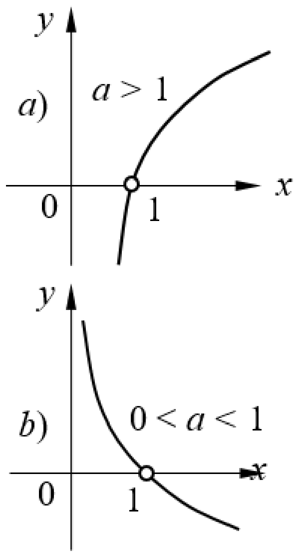
Quyidagicha intervalni tuzib, undgi ishoralarni aniqlaymiz: yechim $[3; \infty)$.

2.23. Logarifmik funksiya, tenglama va tengsizliklar.

$y = log_a x$ ko‘rinishdagi funksiyalarga logarifmik funksiyalar deb ataladi. Bu yerda x - logarifmning argumenti, a - logarifmning asosi bo‘lib, u har doim musbat, hamda $a \neq 0$; $a \neq 1$ bo‘lishi kerak. Agar $a > 1$ bo‘lsa, funksiya o‘suvchi, agar $0 < a < 1$ bo‘lsa, funksiya kamayuvchi bo‘ladi (grafikka qarang).

$y = log_a x$ tenglikda $x = 1$ bo‘lsa, $y = 0$ bo‘ladi. Demak funksiyaning grafigi OX o‘qining bir birligidan kesib o‘tadi (chizmaga qarang).

Logarifmar o‘nli, ya’ni asosi $a = 10$, bunda asos yozilmaydi, masalan lgx; natural, uning asosi $e = 2{,}71$ bo‘lib, u ham yozilmaydi, masalan lnx.



2.11-rasm.

Bu funksiyalar bir biridan grafiklari bilan farq qiladi.

Logarifmik funksiyaning **aniqlanish sohasi,** barcha musbat haqiyqiy sonlar to‘plamidan iborat. $D(f) \to (0; \infty)$.

Logarifmik funksiyaning **qiymatlar (o‘zgarish) sohasi,** barcha haqiyqiy sonlar to‘plamidan iborat. $E(f) \to (-\infty; \infty)$.

Logarifmik funksiya **juft ham toq ham** emas.

Logarifmik funksiyaga **teskari** funksiya, ko‘rsatkichli funksiyadir.

$y = log_a x \to y = a^x$

Logarifmik formulalar.

1) Logarifm har qanday asosga ko‘ra 1 nolga teng; $log_a 1 = 0$.

M: $log_3 1 = 0,\ log_{0,5} 1 = 0,\ lg1 = 0,\ ln1 = 0.$

2) Logarifmning asosi va argument teng bo‘lsa u 1 ga teng bo‘ladi.

$M: log_2 2 = 1,\ lg10 = 1,\ lne = 1.$

3) $log_a x^n = nlog_a x.$ $M: log_3 x^4 = 4log_3 x.$

4) $a^{log_b c} = c^{log_b a}.$ $M:\ 7^{log_3 5} = 5^{log_3 7}.$

5) $a^{log_a b} = b.$ $M:\ 3^{log_3 5} = 5.$

6) $log_a x = b \to x = a^b.$ $M: log_3 x = 2 \to x = 3^2 = 9.$

7) Ko‘paytmani yig‘indiga keltirish $log_a(x \cdot y) = log_a x + log_a y.$

$M: log_3(9x) = log_3 9 + log_3 x = log_3 3^2 + log_3 x = 2log_3 3 + log_3 x = 2 + log_3 x.$

8) Bo‘linmani ayirmaga keltirish $log_a \frac{x}{y} = log_a x - log_a y.$

$M:\ log_2 \frac{x}{8} = log_2 x - log_2 8 = log_2 x - log_2 2^3 = log_2 x - 3log_2 2 = log_2 x - 3.$

9) $log_a x = log_a y \to x = y.$

10) Bir asosdan boshqa asosga o‘tish formulasi $log_a b = \frac{log_c b}{log_c a}$.

$M: log_8 x = \frac{log_2 x}{log_2 8} = \frac{log_2 x}{log_2 2^3} = \frac{log_2 x}{3log_2 2} = \frac{log_2 x}{3} = \frac{1}{3}log_2 x.$

Logarifmik tenglama va tengsizliklarga doir misollar.

Quyidagi logarifmetik tenglamalarni yeching:

1) $log_3(2x-1) = 2$, yechimi (6) formuladan foydalanib, $2x - 1 = 3^2$ ekanligi aniqlanadi va bundan $2x = 10 \to x = 5.$

2) $log_5(x-10) = 2 + log_5 2$, yechimi bu tenglamani quyidagi ko‘rinishda yozamiz:

$log_5(x-10) = 2log_5 5 + log_5 2,\ \ log_5(x-10) = log_5 5^2 + log_5 2 =$
$= log_5 25 + log_5 2 == log_5 50$, (9) formuladan foydalanib, $x - 10 = 50 \to x = 60.$

3) $lg(3-x) - lg(x+2) = 2lg2,$ bu tenglamaning aniqlanish sohasi

$(-2 < x < 3)$ oraliqda bo‘ladi, tenglamani chap tomonini (8) va o‘ng tomonini (3) formuladan foydalanib, quyidagi ko‘rinishda yozamiz: $lg\frac{3-x}{x+2} = lg2^2 = lg4$, (9) for-mulaga asoslanib, bu tenglikni quyidagicha yozamiz:

$\frac{3-x}{x+2} = 4$ yoki $3 - x = 4x + 8 \to x = -1.$

4) $log_5\sqrt{2x+3} = log_{25}7$, bu tenglamaning aniqlanish sohasi $(x > 1{,}5)$.

Tenglikning chap tomonini (10) formuladan foydalanib, 5 asosga o‘tamiz:

$log_{25}7 = \frac{log_5 7}{log_5 25} = \frac{log_5 7}{log_5 5^2} = \frac{1}{2}log_5 7 = log_5\sqrt{7}.$ (9) formulaga asoslanib, bu tenglikni quyidagicha yozamiz: $\sqrt{2x+3} = \sqrt{7}$ yoki $2x + 3 = 7 \to x = 2.$

5) $log_3^2(27x) = log_3 x^6,$ bu tenglamani quyidagicha yozamiz;

$(log_3 27 + log_3 x)^2 = 6log_3 x \to (3 + log_3 x)^2 - 6log_3 x = 0$ yoki,
$9 + 6log_3 x + log_3^2 x - 6log_3 x = 0.$ Bundan $log_3^2 x \neq -9$, demak temglama yechimga ega emas.

6) $log_2\left(16 \cdot 4^{3(1-x)+1}\right) + 1 = 0.$ $log_2\left(2^4 \cdot 2^{2(3(1-x)+1)}\right) + 1 = 0,$
$log_2(2^{4+6-6x+2}) + 1 = 0,\ log_2(2^{12-6x}) + 1 = 0,\ 2^{12-6x} = 2^{-1}$ bundan $x = 2\frac{1}{6}$

7) $4 \cdot 2^{lgx^2} + 11 \cdot 2^{lgx} = 3.$ $\quad 4 \cdot \ (2^{lgx})^2 + 11 \cdot 2^{lgx} - 3 = 0$, bundan $(2^{lgx})_{1,2} = \frac{-11 \pm \sqrt{121+4\cdot4\cdot3}}{8} = \frac{-11\pm13}{8}$ $\ (2^{lgx})_1 \neq -3, \quad (2^{lgx})_2 = \frac{1}{4} = 2^{-1}$ $lgx = -1$ bundan $x = 0{,}01$.

8) $\left(\sqrt{3}\right)^{2log_{0,01}(x^2+1)} = \frac{1}{3\cdot 3^{log_{0,01}(x^2+1)}}$ tenglamaning eng katta ildizini toping.

Tenglamani quyidagi ko‘rinishga keltiramiz; $3^{log_{0,01}(x^2+1)} = 3^{-(log_{0,01}(x^2+1)+1)}$ yoki, $2log_{0,01}(x^2+1) + 1 = 0$, bo‘ladi, $x^2 + 1 = 0{,}01^{-\frac{1}{2}}$ bundan $x^2 = 10 - 1 = 9$, $x = \pm 3$, tenglamaning eng katta ildizini 3 ga teng bo‘ladi.

9) $log_2(x-1) + log_{(x-1)}\frac{1}{4} = 1$ tenglama ildizlarining yig‘indisini toping. $x \neq 1$

$log_2(x-1) + \frac{log_2 2^{-2}}{log_2(x-1)} - 1 = 0, \ \ log_2^2(x-1) - log_2(x-1) - 2 = 0.$

$\left(log_2(x-1)\right)_{1,2} = \frac{1}{2} \pm \frac{3}{2}$; $\ \left(log_2(x-1)\right)_1 = -1 \to x_1 = \frac{3}{2}$, $\left(log_2(x-1)\right)_2 = 2 \to$ $x_2 = 5$ demak. $x_1 + x_2 = \frac{3}{2} + 5 = 6\frac{1}{2}$.

<u>Quyidagi logarifmik tengsizliklarni yeching:</u>

1) $log_3(x-2) \leq 2$, aniqlanish sohasi $x - 2 > 0 \to x > 2$. $(x-2) \leq 3^2$ yoki $x \leq 11$.

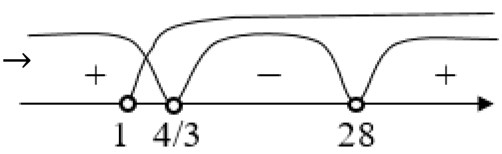
Tengsizlikni yechimi $2 < x \leq 11$.

2) $log_{\frac{1}{4}}(x+1) > -\frac{3}{2}$, aniqlanish sohasi $x + 1 > 0 \to x > -1$. $(x+1) < \left(\frac{1}{4}\right)^{-\frac{3}{2}}$ yoki $\quad x < 7$. Tengsizlikni yechimi $-1 < x < 7$.

3) $5^{log_5(2x-1)} < 7$, aniqlanish sohasi $2x - 1 > 0 \to x > 0{,}5$. (5) formuladan foydalanib, $5^{log_5(2x-1)} = 2x - 1$ ekanligini aniqlaymiz. Shunda berilgan tengsizlik quyidagi ko‘rinishga ega bo‘ladi: $2x - 1 < 7$ yoki $x < 4$.

Tengsizlikni yechimi $0{,}5 < x < 4$.

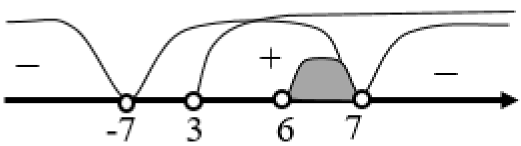
4) $log_3^2(x-1) - 2log_3(x-1) > 3. \ \ log_3^2(x-1) - 2log_3(x-1) - 3 > 0.$ Bu tengsizlikni shartli ravishda nolga tenglab, uni *ox* o‘qi bilan kesishish nuqtalarini topiib, ulardan intervallar tuzamiz: $x - 1 > 0$ yoki $x > 1$.



$\left(log_3(x-1)\right)_{1,2} = 1 \pm 2$; $\ \left(log_3(x-1)\right)_1 = -1 \to$ $x_1 = \frac{4}{3}$, $\left(log_3(x-1)\right)_2 = 3 \to x_2 = 28.$ Uechim $\left(1; \frac{4}{3}\right) \cup (28; \infty)$.

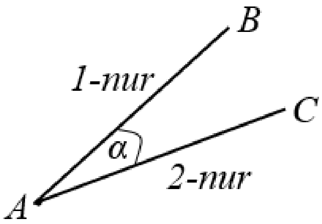
5) $(x-3)^{log_{(x-3)}(49-x^2)} \leq 13.$ $x - 3 > 0 \ \to x > 3, \ 49 - x^2 > 0, 49 - x^2 \leq 13$, yoki $x^2 - 36 \geq 0$.

Bu tengsizlikni shartli ravishda nolga tenglab, uni *ox* o‘qi bilan kesishish nuqtalarini topiib, ulardan intervallar tuzamiz:



Chizmadan tengsizlikning yechimi $[6; 7)$

3. GEOMETRIYA



3.1-rasm.

3.1. Burchak, uning turlari va o‘lchov birliklari. *A* nuqtadan chiqqan *AB* va *AC* nurlar orasida burchak б hosil bo‘ladi, uni o‘latda $\angle BAC$ deb yoziladi. Demak burchakni yozishda bur-chak uchidagi harf o‘rtaga olib yozilar ekan (3.1-rasm).

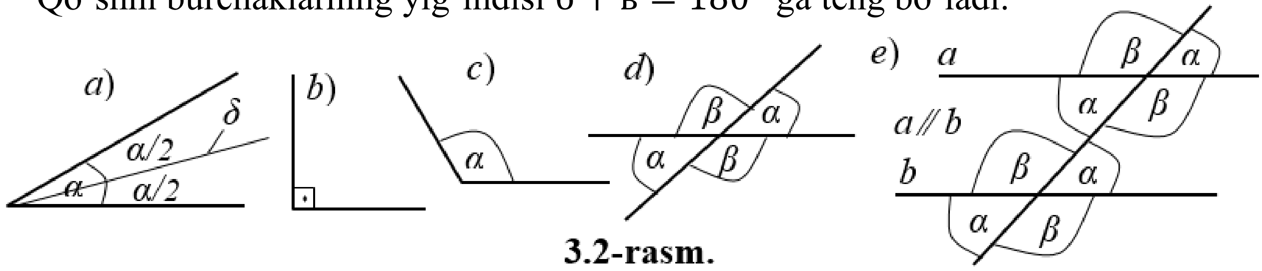
Burchakning turlari. 1) O‘tkir burchak $0 < б < 90^0$ (3.2*a*-rasm).

2) Burchakni teng ikkiga bo‘luvchi kesmaga ***bissektrisa*** (3.2*a*-rasm) deb ataladi $д = б/2$.

3) To‘g‘ri burchak $б = 90^0$ (3.2*b*-rasm). 4) O‘tmas burchak $90^0 < б < 180^0$ (3.2*c*-rasm).

5) Qo‘shni va qarama-qarshi burchaklar (3.2*d*-rasm).

Qo‘shni burchaklarning yig‘indisi $б + в = 180^0$ ga teng bo‘ladi.



3.2-rasm.

Agar ikkita *a* va *b* parallil to‘g‘ri chiziqlarni uchinchi to‘g‘ri chiziq kesib o‘tsa, ular-ning barcha qarama-qarshi borchaklari teng bo‘ladi (3.2*e*-rasm).

Burchakning o‘lchov birligi. Burchaklar gradus yoki radianda o‘lchanadi.

Ikki nurdan biri to‘liq bir marta aylansa 360^0 yoki 2p radian masofa bosadi. Demak $360^0 = 2p$ yoki $180^0 = p$. Bundan $1_{rad} = \frac{180^0}{p} = \frac{180^0}{3{,}14} = 57^0 3' 25''$.

Radiandan gradusga, gradusdan radianga o‘tish. Berilgan $б^0$dagi burchakdan radianga o‘tish uchun uni $\frac{p}{180^0}$ ga ko‘paytirish kerak. Misol uchun 120^0 li burchakni radianga aylantiring. $120^0 \cdot \frac{p}{180^0} = \frac{2p}{3}$ radian. Berilgan $б_{rad}$ dagi burchakdan gradusga o‘tish uchun uni $\frac{180^0}{p}$ ga ko‘paytirish kerak. Misol uchun $\frac{5p}{3}$ radianli burchakni gradus-ga aylantiring. $\frac{5p}{3} \cdot \frac{180^0}{p} = 300^0$.

Qo‘shni burchaklarga doir masalalar:

1) Ikki parallel to`g`ri chiziqni 3-to`g`ri chiziq kesib o`tganda hosil bo`lgan ichki bir tomonli burchaklardan biri ikkinchisidan 100^0 ga kichik bo`lsa, shu burchaklardan kattasini toping.

Yechish: *Qo‘shni burchaklarning yig‘indisi* $\alpha + \beta = 180^0$ *ga teng. Masala shartiga asosan* $\alpha = \beta - 100^0$ *yoki* $\alpha - \beta = -100^0$ *bularni birgalikda sistema qilib yechamiz:*

$$\begin{cases} \alpha + \beta = 180^0 \\ \alpha - \beta = -100^0 \end{cases}$$ *tengliklarni bir biridan ayirsak,* $2\beta = 280^0 \rightarrow \beta = 140^0$.

2) Qo‘shni burchaklardan biri ikkinchisidan 52^{o} ga katta bo‘lsa, shu burchaklardan kichigini toping.

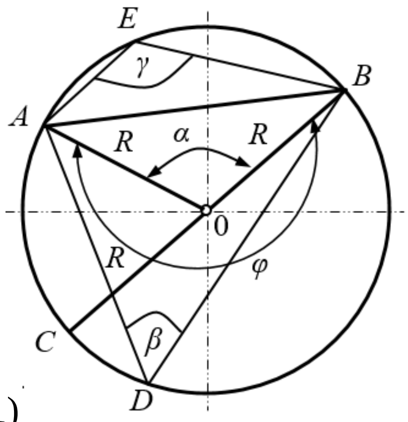
Yechish: *Qo‘shni burchaklarning yig‘indisi* $б + в = 180^0$ *ga teng. Masala shartiga asosan* $б = в + 52^0$ *yoki* $б - в = 52^0$ *bularni birgalikda sistema qilib yechamiz:*

$\begin{cases} б + в = 180^0 \\ б - в = 52^0 \end{cases}$ *tengliklarni bir biridan ayirsak,* $2в = 128^0 \rightarrow в = 64^0$.

3) Qo‘shni burchaklardan biri ikkinchisidan 4 marta katta bo‘lsa, shu burchaklardan kichigini toping.

Yechish: *Qo‘shni burchaklarning yig‘indisi* $б + в = 180^0$ *ga teng. Masala shartiga asosan* $б = 4в$ *buni α ni o‘rniga qo‘ysak,* $4в + в = 180^0$*yoki* $в = 36^0$.

3.2. Aylana va doira ularning asosiy ko‘rsatkichlari.



3.3-rasm.

Aylana deb, markaz deb ataluvchi bir (0) nuqtadan baravar uzoqlikda joylashgan nuqtalarning geometrik o‘rniga aytiladi (3.3-rasm). Aylananing markaziy burchagi $б + ц = 360^0$ ga teng.

Aylananing ixtiyoriy ikki *AB* nuqtasini birlashtiruvchi chiziq ***vatar*** deb ataladi.

Markazdan o‘tgan vatar $(BC = D)$ga ***diametr***, diametrning yarmiga $R = \frac{D}{2}$ ***radius*** deb ataladi.

Aylananing uzunligi $\ell = р \cdot D = 2рR$ ga teng. (3.1)

Har qanday aylana uzunligini, uning diametriga nisbati o‘zgarmas kattalik bo‘lib, u $р = \frac{\ell}{D} = 3{,}14$ ga teng.

Aylananing *AB* vatari uni ikkita б burchakka tiralgan kichik va ц burchakka tiralgan katta yoylarga ajratadi (yoy, hariflar ustida maxsus yoysimon belgi qo‘yish $(\widetilde{AB})$ orqali ifodalash).

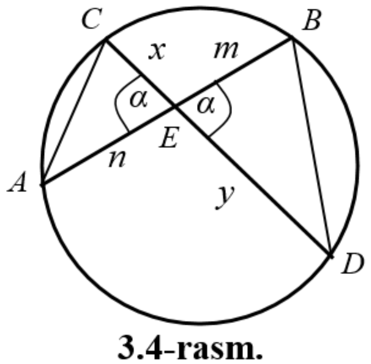
AB vatar, katta yoyning ixtiyoriy *D* nuqtasidan в burchak ostida ko‘rinadi.

Uni quyidagicha aniqlash mumkin, $в = \frac{б}{2}$, *AB* vatar, kichik yoyning ixtiyoriy *E* nuqtasidan г burchak ostida ko‘rinadi. Uni quyidagicha aniqlash mumkin,

$$г = \frac{360^0 - б}{2} = 180^0 - \frac{б}{2}.$$

Agar aylananing radiusi va yoy tiralgan burchak berilgan bo‘lsa, bu yoyning uzunligini quyidagicha aniqlash mumkin: $\widetilde{AB} = \check{\ell} = \frac{рR}{180^0} \cdot б$ (3.2)

Aylananing vatarlari kesishgan nuqtasidagi bo‘laklarining nisbati.



3.4-rasm.

Aylananig *AB* va *CD* vatarlari kesishgan *E* nuqtada tegishli ravishda uzunliklari *n*, *m* va *x*, *y* kesmalarga ajralgan bo‘lsa (3.4-rasm), bu kesmalarning ko‘paytmasi bir biriga teng bo‘ladi. $n \cdot m = x \cdot y$ (3.3)

Isbot: *3.4-rasidagi uchburchak* AEC, *uchburchak* BED *ga o‘xshashligidan foydalanib,* $\frac{m}{x} = \frac{y}{n}$ *bundan* $n \cdot m = x \cdot y$

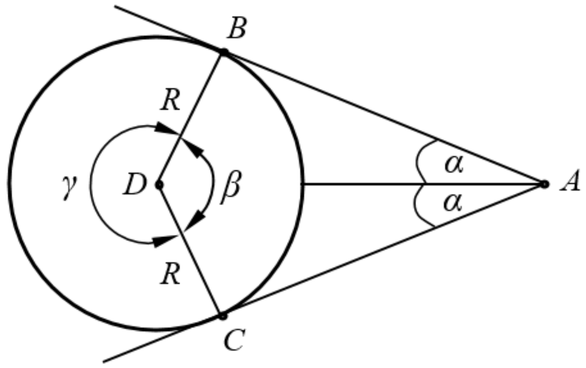
Masala: *AB vatarning uzunligi $\ell = 16\ sm$, CD vatar AB vatar bilan E nuqtada kesishadi va kesishish nuqtasi AB vatarni $m = 6\ sm$, $n = 8\ sm$ kesmalarga ajratadi (3.4-rasm). AB vatarni E nuqta ajratgan bo'laklari x va y uzunliklarini toping.*

<u>Yechish:</u> $\ell = x + y = 16\ sm$ *dan* $y = 16 - x$ *bu qiymatlarni (3.3) formulaga qo'yib, quyidagi tenlamani hosil vqilamiz:*

$8 \cdot 6 = x(16 - x)$ *yoki* $x^2 - 16x + 48 = 0$, *bu tenlamani yechib,* $x_1 = 4$ *va* $x_2 = 12.\ y_1 = 16 - 4 = 12\ va\ y_2 = 16 - 12 = 4.$ *Demak* $x = 4\ sm, y = 12\ sm.$

Aylanaga o'tkazilgan urunmalar.

R radiusli aylana tashqarida yotgan *A* nuq-tadan uning *B* va *C* nuqtalariga urunmalar o't-kazilgan bo'lsin (3.5-rasm). Aylananing marka-zi *D* nuqtada bo'lsin. Unda *DB* va *DC* kesma-lar, tegishli ravishda *AB* va *AC* kesmalarga per-pendikulyar bo'ladi.



3.5-rasm.

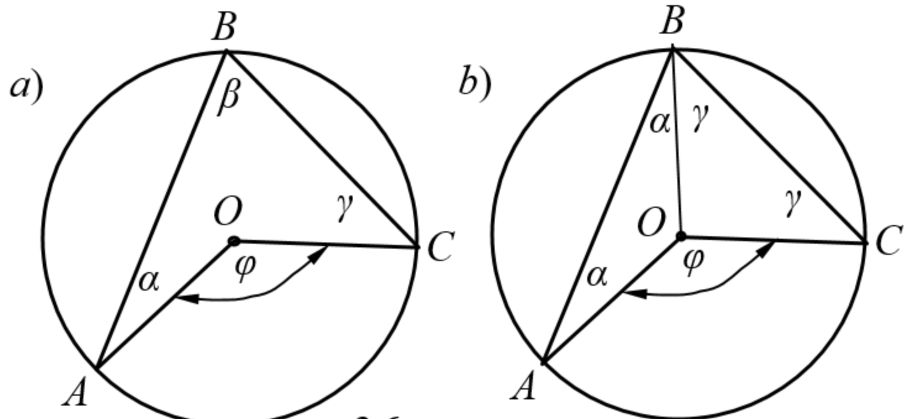
Chizmadan $AB = AC$. $2б + в + 90^0 + 90^0 = = 360^0 \to 2б = 180^0 - в$.

$в + г = 360^0 = 2б + в + 180^0 \to 180^0 = \frac{г+в}{2}$

$2б = 180^0 - в = \frac{г+в}{2} - в$ yoki $2б = \frac{г-в}{2}$ bundan $б = \frac{г-в}{4}$ (3.4)

Aylananing ikkita vatari aylanadagi *B* nuqtada uchrashib, qolgan uchlari aylananing *A* va *B* uchlarida yotgan bo'lsin. Agar aylana markazi *O* nuqtada bo'lsa, *ABCO* to'rt-burchakda б, в, г va ц burchaklarni hosil qilamnz (3.6*a*-rasm).

3.6*b*-rasmdan $AO = OB = OC = R$.



3.6-rasm.

Rasmdan $360^0 = 360^0 - 2(б + г) + ц$. Bundan $ц = 2(б + г)$. (3.5)

<u>Doira</u> deb, tekislikning aylana bilan chegaralangan qismiga aytiladi. Doiraning yuzi quyidagi formula orqali aniqlanadi: $S = \frac{pD^2}{4} = \frac{p(2R)^2}{4} = \frac{4pR^2}{4} = pR^2$. (3.6)

Doiraning bir bo'lagiga **sektor** deb ataladi. Sektorning yuzi quyidagi formula orqali aniqlanadi: $S_{sek} = \frac{pR^2}{360^0} \cdot б$. (3.7)

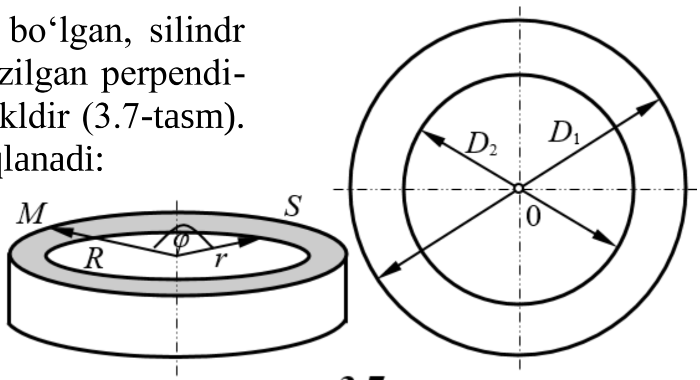
Bu yerda б ning qiymati gradusda qo'yiladi.

Halqa - bu ma’lum qalinlikka ega bo‘lgan, silindr simon materialning o‘qiga nisbatan o‘tkazilgan perpendikulyar kesim yuzasidan hosil bo‘lgan shakldir (3.7-tasm). Uning yuzasi quyidagi formula orqali aniqlanadi:

$S_h = \frac{\text{p}}{4}\cdot\ (D_1^2 - D_2^2).$ (3.8)

$S_h = \text{p}(R^2 - r^2).$ (3.9)

Strelkali soatnini soat va minut strelka-lari orasidagi burchakni aniqlash.



3.7-rasm.

$$ц = \frac{(60\cdot S + M)360^0}{12\cdot 60} - \frac{M\cdot 360^0}{1\cdot 60} = \frac{60\cdot S - 11M}{2}$$

3.3. To‘rtburchak, uning turlari va asosiy ko‘rsatkichlari. To‘rtburchak deb, istalgan uchta nuqtasi bir to‘g‘ri chiziqda yotmaydigan, to‘rtta nuqtani ketma-ket tutash-tiruvchi kesmadan tashkil topgan geometrik shaklga aytiladi.

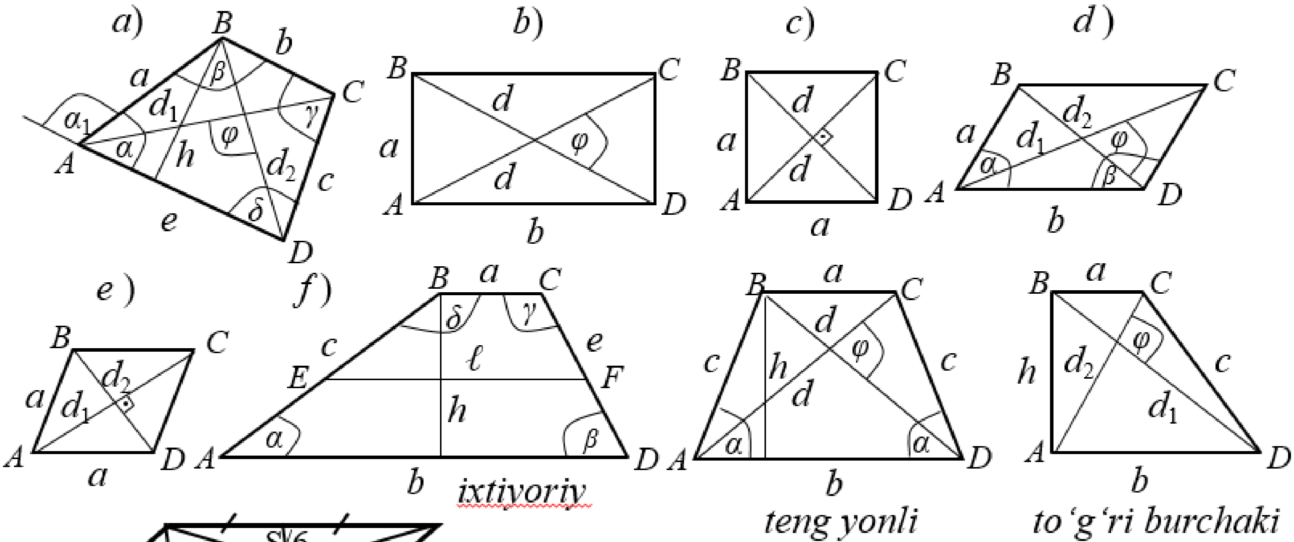
To‘rtburchakning quyidagi **turlari** mavjud: ixtiyoriy (3.8*a*-rasm), to‘g‘ri burchakli (3.8*b*-rasm), kvadrat (3.8*c*-rasm), parallelogramm (3.8*d*-rasm), romb (3.8*e*-rasm), trapetsiya (3.8*f*-rasm).

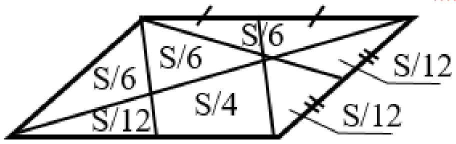
To‘rtburchakning qarama-qashi uchlarini tutashtiruvchi kesmaga **diaganali** deb ataladi va u *d* harfi bilan belgilanadi.

Ixtiyoriy to‘rtburchakning asosiy ko‘rsatkichlari (3.8*a*-rasm):

1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = a + b + c + e$.
2. Ichki burchaklarining yig‘ndisi $\Sigma б_i = б + в + г + д = 360^0$.
3. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = б_1 + в_1 + г_1 + д_1 = 360^0$.
4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.
5. To‘rtburchakning yuzi:

Asosi va balandligi berilganda $S = a \cdot h$; diagonallari d_1, d_2 va ular orasidagi burchak ц berilganda $S = \frac{1}{2} d_1 \cdot d_2 \cdot sinц$.





3.8-rasm.

6. To‘rtburchakka tashqi chizilgan aylananing radiusi R. Agar to‘rtburchak, qarama-qarshi burchaklarining yig‘indisi $б + г = 180^0$ yoki $в + д = 180^0$ bo‘lsa, unga tashqi aylana chizish mumkin. $a \cdot c + b \cdot e = d_1 \cdot d_2$ $(Ptolomey\ teoremasi)$.

To‘rtburchakning tomonlari va yuzasi berilganda uni tashqi chizilgan aylananing radiusi quyidagi formula orqali aniqlanadi $R = \frac{1}{4S} \cdot \sqrt{(ab + ce)(ac + be)(ae + bc)}$.

7. To‘rtburchakka ichki chizilgan aylananing radiusi r. Agar to‘rtburchak, qarama-qarshi tomonlarining yig‘indisi $a + c = b + d$ bo‘lsa, unga ichki aylana chizish mumkin.

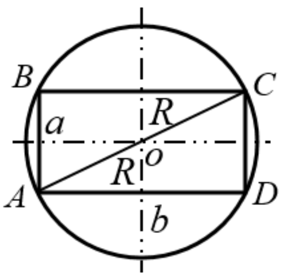
To‘rtburchakning perimetri va unga ichki chizilgan aylananing radiusi berilganda uning yuzi $S = p \cdot r$; tomonlari berilganda esa $S = \sqrt{p(p - a)(p - b)(p - c)(p - e)}$ formulalar orqali aniqlanadi. Bu yerda $p -$ to‘rtburchakning yarim perimetri.

To‘rtburchakning **bimedianasi** - bu to‘rtburchak tomonlarini o‘rtalarini tutashtiruv-chi kesmadir. Unga doir **Varinyon** teoremasi mavjud.

Teorema-qavariq ko‘pburchak tomonlarining o‘rtalarini birlashtiruvchi chiziqlardan hosil bo‘lgan shakl parallelogramm bo‘ladi va uning yuzi berilgan to‘rtburchak yuzasi-ning yarmiga tengdir.

To‘g‘ri burchakli to‘rtburchakning asosiy ko‘rsatkichlari (3.8*b*-rasm):

1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = 2a + 2b = 2(a + b)$.
2. Ichki burchaklarining yig‘ndisi $\Sigma б_i = 360^0$. Chunki $б = в = г = д = 90^0$.
3. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = 360^0$.
Chunki $б_1 = в_1 = г_1 = д_1 = 90^0$.
4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.
5. To‘g‘ri to‘rtburchakning yuzi:
Asosi va balandligi berilganda $S = a \cdot h$; diagonallari d_1, d_2 (bunda diagonallar biri briga teng bo‘ladi) va ular orasidagi burchak ц berilganda $S = \frac{1}{2} \cdot d^2 \cdot sinц$.
6. To‘g‘ri to‘rtburchakka tashqi chizilgan aylananing radiusi
$R = \frac{d}{2} = \frac{1}{2} \cdot \sqrt{a^2 + b^2}$.
7. To‘g‘ri to‘rtburchakka ichki chizilgan aylananing radiusi r.
Qarama-qarshi tomonlarining yig‘indisi teng bo‘lmagani uchun unga ichki aylana chizish mumkin emas.

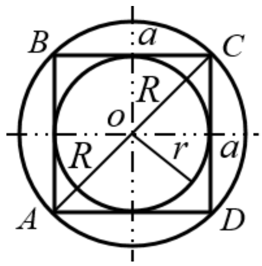


Kvadratning asosiy ko‘rsatkichlari (3.8*c*-rasm):

1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = 4a,\ \ a = \frac{p}{4}$.
2. Ichki burchaklarining yig‘ndisi $\Sigma б_i = 360^0$. Chunki $б = в = г = д = 90^0$.
3. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = 360^0$.
Chunki $б_1 = в_1 = г_1 = д_1 = 90^0$.
4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.
5. Kvadtatning yuzi: Asosi balandligiga teng bo‘lgani uchun $S = a^2$; diagonallari d_1, d_2 (bunda diagonallar biri biriga teng bo‘ladi) va ular orasidagi burchak, bunda $ц = 90^0$ bo‘lgani uchun $S = \frac{1}{2} \cdot d^2$ bo‘ladi. Perimetri berilganda $S = \frac{1}{16} \cdot p^2$.
6. Kvadratga tashqi chizilgan aylananing radiusi uning diagonalining yarmiga teng.

$$R=\frac{d}{2}=\frac{1}{2}\cdot\sqrt{a^2+a^2}=\frac{\sqrt{2}}{2}\cdot a.$$

7. Kvadratga ichki chizilgan aylananing radiusi $r=\frac{a}{2}$.



Parallelogrammning asosiy ko‘rsatkichlari (3.8*d*-rasm):

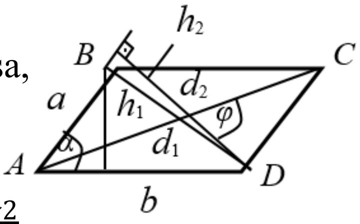
1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p=2a+2b=2(a+b)$.

2. Ichki burchaklarining yig‘ndisi $\Sigmaб_i=360^0$. Chunki $2б+2в=360^0$.

3. Tashqi burchaklarining yig‘ndisi $\Sigmaб_t=360^0$. Chunki $2б_1+2в_1=360^0$.

4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б+б_1=180^0$.

5. Parallelogrammning yuzi:



Agar asoslari va ularga tushirilgan balandligi berilgan bo‘lsa, $S=a\cdot h_2$; $S=b\cdot h_1$ bu yuzalarni tenglashtirsak, $\frac{a}{b}=\frac{h_1}{h_2}$.

$\frac{h_1}{a}=sinб$ bundan $a=\frac{h_1}{sinб}$ $S=S=a\cdot h_2=\frac{h_1}{sinб}h_2=\frac{h_1h_2}{sinб}$

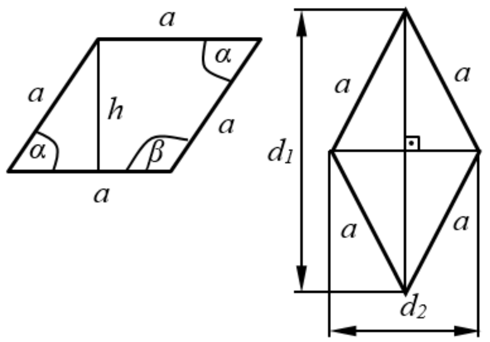
Diagonallari d_1, d_2 va ular orasidagi burchak ц berilganda $S=\frac{1}{2}d_1\cdot d_2\cdot sinц$. Ikki tomoni va ular orasidagi burchak berilganda $S=a\cdot b\cdot sinб$.

Parallelogrammning diagonali, uni yuzini teng ikkiga, diagonallari esa uni yuzini teng to‘rt bo‘lakka bo‘ladi. Parallelogrammning bissiktrisalari 90^0 li bur-chak ostida kesishadi.

6. Parallelogrammga tashqi aylana chizish mumkin emas, chunki, qarama-qarshi bur-chaklarining yig‘indisi 180^0 ga teng emas.

7. Parallelogrammga ichki aylana chizish mumkin emas, chunki, qarama-qarshi tomonlarining yig‘indisi teng emas.

Rombning asosiy ko‘rsatkichlari (3.8*e*-rasm):



1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p=4a$.

2. Ichki burchaklarining yig‘ndisi $\Sigmaб_i=360^0$. Chunki $2б+2в=360^0$.

3. Tashqi burchaklarining yig‘ndisi $\Sigmaб_t=360^0$. Chunki $2б_1+2в_1=360^0$.

4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б+б_1=180^0$.

5. Rombning yuzi: Asosi va balandligi berilganda $S=a\cdot h$; diagonallari d_1, d_2 va ular orasidagi burchak, bunda $ц=90^0$ bo‘lgani uchun $S=\frac{1}{2}d_1\cdot d_2$. Ikki tomoni va ular orasidagi burchak berilganda $S=a^2\cdot sinб$.

6. Rombga tashqi aylana chizish mumkin emas, chunki, qarama-qarshi burchaklari-ning yig‘indisi 180^0 ga teng emas.

7. Rombga ichki chizilgan aylananing radiusi $r=\frac{h}{2}=\frac{S}{2a}=\frac{1}{2}\cdot a\cdot sinб$.

Trapetsiya. Trapetsiya deb, ikki tomoni parallel, qolgan ikki tomoni parallel bo‘lma-gan to‘rtburchakka aytiladi (3.8*f*-rasm).

Uning ixtiyoriy, teng yonli va to‘g‘ri burchakli turlari bor.

Trapetsiyaning asosiy ko‘rsatkichlari:

Ixtiyoriy trapetsiya uchun.

1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = a + b + c + e$.
2. Ichki burchaklarining yig‘ndisi $\Sigma б_i = б + в + г + д = 360^0$.
3. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = б_1 + в_1 + г_1 + д_1 = 360^0$.
4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.
5. Trapetsiyaning yuzi ustki va ostki asoslari yig‘indisining yarmi bilan balandligining ko‘paytmasiga teng: $S = \frac{a+b}{2} \cdot h$. Bu yerda $\frac{a+b}{2} = \ell -$ trapetsiyaning o‘rta chizig‘i bo‘ladi. Demak, uning yuzini o‘rta chizig‘i orqali $S = \ell \cdot h$ deb yozish mumkin. Diagonallari d_1, d_2 va ular orasidagi burchak ц berilganda uning yuzi

$$S = \frac{1}{2} d_1 \cdot d_2 \cdot sinц.$$

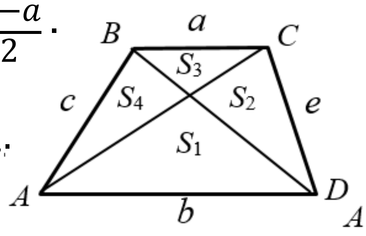
6. Trapetsiyaga tashqi chizilgan aylananing radiusi *R*. Agar trapetsiyaning qarama-qarshi burchaklarining yig‘indisi $б + г = 180^0$ yoki $в + д = 180^0$ bo‘lsa, unga tashqi aylana chizish mumkin.
7. Trapetsiyaga ichki chizilgan aylananing radiusi *r*. Agar trapetsiyaning qarama-qarshi tomonlarining yig‘indisi $a + b = c + e$ bo‘lsa, unga ichki aylana chizish mumkin.
8. Asoslarining o‘rtalarini tutashtiruvchi kesma uzunligi $\ell_{o‘r}$ berilganda, hamda asosidagi burchaklarning yig‘indisi $б + в = 90^0$ bo‘lsa, $a = \ell - \ell_{o‘r}$; $b = \ell + \ell_{o‘r}$ bo‘ladi. $EF = \ell -$ trapetsiyaning o‘rta chizig‘i bo‘lgani uchun $MF = \frac{b}{2}, EM = FN = \frac{a}{2}$ ga teng.

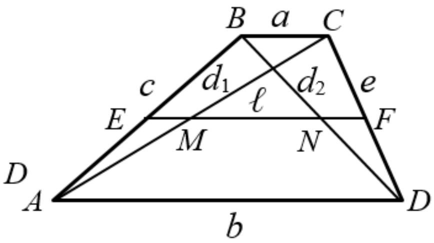
$MN = \ell - 2EM = \frac{a+b}{2} - 2 \cdot \frac{a}{2} = \frac{b-a}{2}$.

$d_1^2 + d_2^2 = c^2 + e^2 + 2ab$.

$S_2 = S_4 = \sqrt{S_1 \cdot S_3}$. $S_1 \cdot S_3 = S_2 \cdot S_4$.

$$S = \left(\sqrt{S_1} + \sqrt{S_2}\right)^2.$$





Teng yonli trapetsiya uchun.

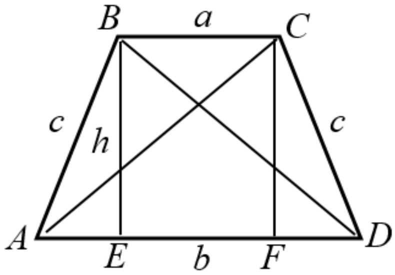
1. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = a + b + 2c$.
2. Ichki burchaklarining yig‘ndisi $\Sigma б_i = 2б + 2в = 360^0$.
3. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = 2б_1 + 2в_1 = 360^0$.
4. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.
5. Teng yonli trapetsiyaning asosidagi burchaklari teng bo‘ladi.
6. Teng yonli trapetsiyaning yuzi ustki va ostki asoslari yig‘indisining yarmi bilan balandligining ko‘paytmasiga teng: $S = \frac{a+b}{2} \cdot h$. Bu yerda $\frac{a+b}{2} = \ell -$ trapetsiyaning o‘rta chizig‘i bo‘ladi. Demak, uning yuzini o‘rta chizig‘i orqali $S = \ell \cdot h$ deb yozish mumkin.

Diagonallari d_1, d_2 (bunda diagonallar biri biriga teng bo‘ladi) va ular orasidagi bur-chak ц berilganda uning yuzi $S = \frac{1}{2} \cdot d^2 \cdot sinц$.

7. Teng yonli trapetsiyaga tashqi chizilgan aylananing radiusi *R*.

Agar uning qarama-qarshi burchaklarining yig‘indisi $б + в = 180^0$ bo‘lsa, unga tashqi aylana chizish mumkin.

8. Teng yonli trapetsiyaga ichki chizilgan aylananing radiusi *r.* Agar uning qarama-qarshi tomonlarining yig‘indisi $a + b = c + e$ bo‘lsa, unga ichki aylana chizish mumkin.



Teng yonli trapetsiyada:

$AE = DF = \frac{b-a}{2}$; $AF = DE = \frac{a+b}{2}$;

$h = \frac{1}{2} \cdot \sqrt{b^2 - a^2}$. $a \cdot b = d^2 - c^2$.

Teng yonli trapetsiyaga aylana ichki chizilgan bo‘lsa, $a + b = 2c$ va $h = 2r$ bo‘ladi.

Chizmadan $h = \sqrt{c^2 - (AE)^2} = \sqrt{\left(\frac{a+b}{2}\right)^2 - \left(\frac{b-a}{2}\right)^2} = \sqrt{a \cdot b}$.

Agar teng yonli trapetsiyaning diagonallari o‘zaro perpendkul-yar bo‘lsa, $h = \frac{a+b}{2} = \ell$; Yuzasi $S = \ell \cdot h = \frac{a+b}{2} \cdot \frac{a+b}{2} = \left(\frac{a+b}{2}\right)^2 = \ell^2$.

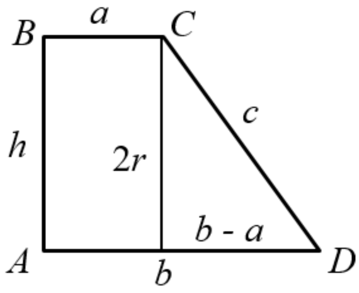
$S = \frac{1}{2} \cdot d^2 = h^2$ bo‘ladi.

Agar teng yonli trapetsiyaning diagonallari yon tomoniga perpendkulyar bo‘lsa, $h = \sqrt{\frac{a+b}{2} \cdot \frac{a+b}{2}} = \frac{1}{2}\sqrt{b^2 - a^2}$. Yuzasi $S = \frac{a+b}{2} \cdot \frac{1}{2}\sqrt{b^2 - a^2} = \frac{a+b}{4}\sqrt{b^2 - a^2}$.

To‘g‘ri burchakli trapetsiyga ichki chizilgan aylananing radiusi $r = \frac{ab}{a+b}$.

Isboti:

To‘g‘ri burchakli trapetsiyga aylana ichki chizilgani uchun $2r + c = a + b$ bundan $c = a + b - 2r$.



Chizmadadi to‘g‘ri burchakli uchburchakdan $c = \sqrt{(2r)^2 + (b - a)^2}$ buni c ning o‘rniga qo‘yib, uni ikkala tomonini kvadratga oshirib, quyidagi tenglikni hosil qilamiz:

$4r^2 + b^2 - 2ab + a^2 = a^2 + 2ab + b^2 - 4(a + b)r + 4r^2$

yoki $4(a + b)r = 4ab$ bundan $r = \frac{ab}{a+b}$.

3.4. Uchburchak, uning turlari va asosiy ko‘rsatkichlari. Uchburchak deb, istalgan uchta nuqtasi bir to‘g‘ri chiziqda yotmaydigan, uchta nuqtani ketma-ket tutashtiruvchi kesmadan tashkil topgan geometrik shaklga aytiladi.

Uchburchakning quyidagi **turlari** mavjud: ixtiyoriy (3.9*a*-rasm), teng yonli (3.9*b*-rasm), teng tomonli (3.9*c*-rasm), to‘gr‘i birchakli (3.9*d*-rasm).

Ixtiyoriy uchburchakning asosiy ko‘rsatkichlari (3.9*a*-rasm):

1. Uchburchak mavjud bo‘lishi uchun, uning ixtiyoriy ikki tomon uzunliklarining yig‘indisi uchunchi tomon uzunligidan katta bo‘lishi kerak:

$$\begin{cases} a + b > c > |a - b| \\ b + c > a > |b - c| \\ a + c > b > |a - c| \end{cases}$$

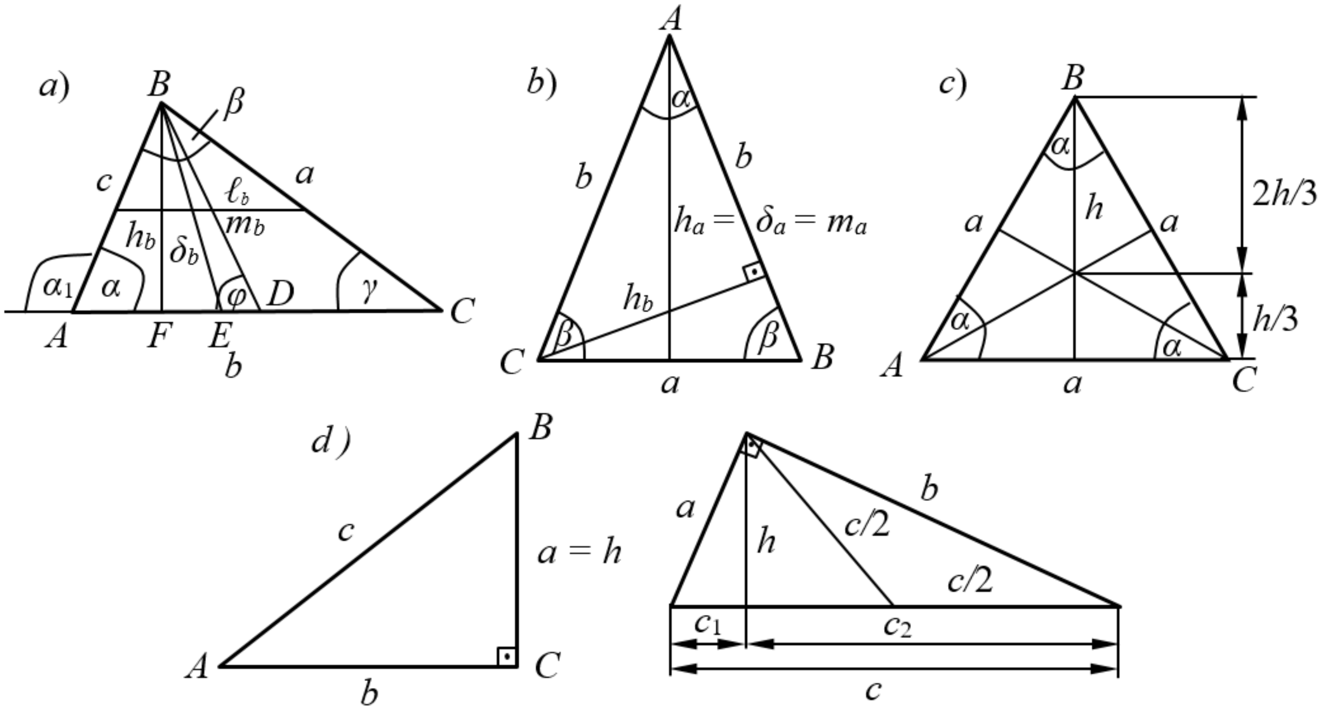
2. Perimetri (tomonlari uzunliklarining yig‘indisi) $p = a + b + c$.

3. Ichki burchaklarining yig‘ndisi $\Sigma б_i = б + в + г = 180^0$.

4. Tashqi burchaklarining yig‘ndisi $\Sigma б_t = б_1 + в_1 + г_1 = 360^0$.

5. Bitta ichki va bitta tashqi burchaklarining yig‘ndisi $б + б_1 = 180^0$.

6. Uchburchakning yuzi, tomonlari uzunligini ularga tegishli balandliklarga bo‘lgan ko‘paytmasining yarmiga teng: $S = \frac{1}{2} \cdot b \cdot h_b$; $S = \frac{1}{2} \cdot a \cdot h_a$; $S = \frac{1}{2} \cdot c \cdot h_c$.



3.9-rasm.

Agar uchburchakning ikki tomoni va ular orasidagi burchak berilgan bo‘lsa, uning yuzi quyidagi formula orqali aniqlanadi:

$$S = \frac{1}{2} \cdot b \cdot c \cdot sinб; \; S = \frac{1}{2} \cdot a \cdot c \cdot sinв; \; S = \frac{1}{2} \cdot a \cdot b \cdot sinг.$$

Agar uchburchakning barcha tomonlari berilgan bo‘lsa, uning yuzi Geron formulasi orqali aniqlanadi: $S = \sqrt{p(p-a)(p-b)(p-c)}$. Bu yerda $\frac{a+b+c}{2} = p -$ uchburchak perimetrining yarimi.

7. Uchburchakning o‘rta chizig‘i, uchburchak tomonlariga parallel bo‘lib, uning tegishli tomonini yarmiga teng bo‘ladi: $\ell_a = \frac{a}{2}$; $\ell_b = \frac{b}{2}$; $\ell_c = \frac{c}{2}$.

8. Uchburchakning bissiktrisalari. Bissiktrisa, uchburchak uchidagi burchakni teng ikkiga bo‘lib, shu uchi bilan unga qarshi tomonni birlashtiruvchi kesmadir:

$$д_b = \sqrt{a \cdot c - AE \cdot CE}. \quad д_b = \frac{2 \cdot a \cdot c \cdot cos\frac{в}{2}}{a+c}; \; д_a = \frac{2 \cdot b \cdot c \cdot cos\frac{б}{2}}{b+c}; \; д_c = \frac{2 \cdot a \cdot b \cdot cos\frac{г}{2}}{a+b}.$$

<u>Isbot:</u> *Uchburchakning B uchidagi* в *burchakni bissiktrisa teng ikkiga,* $\frac{в}{2}$ *bo‘ladi (3.9a-rasm). ABC uchburchakning yuzini quyidagicha aniqlaymiz:*

$$S = \frac{1}{2} \cdot a \cdot c \cdot sinв = \frac{1}{2} \cdot д_b \cdot c \cdot sin\frac{в}{2} + \frac{1}{2} \cdot д_b \cdot a \cdot sin\frac{в}{2} = \frac{1}{2} \cdot д_b \cdot sin\frac{в}{2} \cdot (a+c).$$

$$\frac{1}{2}\cdot a\cdot c\cdot 2sin\frac{\beta}{2}\cdot cos\frac{\beta}{2}=\frac{1}{2}\cdot \delta_b\cdot sin\frac{\beta}{2}\cdot\left(a+c\right).$$

Yoki $2\cdot a\cdot c\cdot cos\frac{\beta}{2}=\delta_b\cdot(a+c).$ *Bundan* $\delta_b=\frac{2\cdot a\cdot c\cdot cos\frac{\beta}{2}}{a+c}.$

9. Sinuslar teoremasi. ***Teorema***-uchburchak tomoni o‘z qarshisida yotgan burchak sinusiga proporsionaldir (3.9*a*-rasm). $\frac{a}{sin\text{б}}=\frac{b}{sin\text{в}}=\frac{c}{sin\text{г}}=2R.$

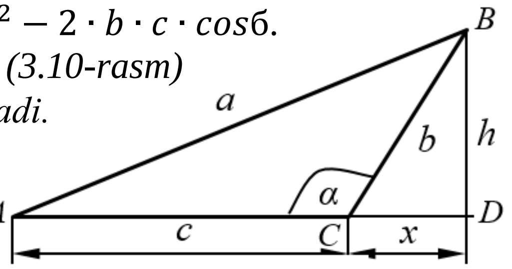
bu yerda *R*-uchburchakka tashqi shizilgan aylananing radiusi.

Isbot: *Uchburchakning berilgan tomonlari va burchaklaridan foydalanib, uni yuzini topamiz:*

$$S=\frac{1}{2}\cdot b\cdot c\cdot sin\text{б}=\frac{1}{2}\cdot a\cdot c\cdot sin\text{в}=\frac{1}{2}\cdot a\cdot b\cdot sin\text{г}$$

yoki $b\cdot sin\text{б}=a\cdot sin\text{в}=c\cdot sin\text{г}$ *bundan* $\frac{a}{sin\text{б}}=\frac{b}{sin\text{в}}=\frac{c}{sin\text{г}}.$

10. Kosinuslar teoremasi. ***Teorema*** - uchburchak tomonining kvadrati, qolgan ikki tomon kvadratlarining yig‘indisidan, shu ikki tomon va ular orasidagi burchak kosinusining ikkilanganligini ayrishga tengdir. $a^2=b^2+c^2-2\cdot b\cdot c\cdot cos\text{б}.$



3.10-rasm.

Isbot: *To‘g‘ri burchakli uchburchak ABD dan (3.10-rasm) Pifagor teoremasiga asosan* $a^2=(c+x)^2+h^2$ *bo‘ladi.*

BCD uchburchakdan $h=b\cdot\sin(180^0-\text{б})=$
$=b\cdot sin\text{б}$ *va* $x=b\cdot cos(180^0-\text{б})=-b\cdot cos\text{б}.$

$a^2=c^2+2cx+x^2+h^2=$
$=c^2-2\cdot b\cdot c\cdot cos\text{б}+b^2\cdot cos^2\text{б}+b^2\cdot sin^2\text{б}.$

Bunda $sin^2\text{б}=1-cos^2\text{б}$ bo‘lganini hisobga olsak, unda tenglik quyidagi ko‘rinishda bo‘ladi: $a^2=c^2-2\cdot b\cdot c\cdot cos\text{б}+b^2\cdot cos^2\text{б}+b^2(1-cos^2\text{б})$ buni soddlashtirib, quyidagiga ega bo‘lamiz: $a^2=b^2+c^2-2\cdot b\cdot c\cdot cos\text{б}.$

11. Uchburchakning medianalari. Mediana, uchburchak uchidan qarshi tomonining o‘rtasiga tushirilgan kesmadir:

$$m_a=\frac{1}{2}\sqrt{2(b^2+c^2)-a^2};\ m_b=\frac{1}{2}\sqrt{2(a^2+c^2)-b^2};\ m_c=\frac{1}{2}\sqrt{2(a^2+b^2)-c^2}.$$

Isbot: *Uchburchakning B uchidan AC tomoniga tushirilgan mediana unit eng ikkiga,* $\frac{b}{2}$ *bo‘ladi (3.9a-rasm). Kosinuslar teoremasidan foydalanib, quyidagilarni aniqlaymiz:* $c^2=m_b^2+\left(\frac{b}{2}\right)^2-2m_b\cdot\frac{b}{2}\cdot cos\text{ц};$ $a^2=m_b^2+\left(\frac{b}{2}\right)^2+2m_b\cdot\frac{b}{2}\cdot cos\text{ц}.$

Bularni hadma had qo‘shib, quyidagini hosil qilamiz: $2(a^2+c^2)-b^2=4m_b^2.$ *Bundan* $m_b=\frac{1}{2}\sqrt{2(a^2+c^2)-b^2}.$

Uchburchakning medianalari berilganda uni tomonlari va yuzasini topish:

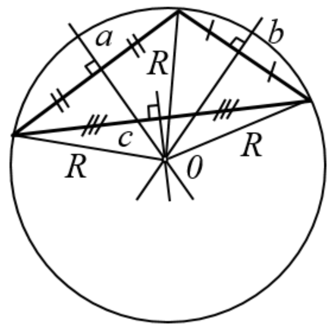
$$a=\frac{2}{3}\sqrt{2(m_b^2+m_c^2)-m_a^2};\ b=\frac{2}{3}\sqrt{2(m_a^2+m_c^2)-m_b^2};\ c=\frac{2}{3}\sqrt{2(m_a^2+m_b^2)-m_c^2}.$$

$S=\frac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}.$ Bu yerda $m=\frac{m_a+m_b+m_c}{2}$ yarim mediana.

Medianalar kesishgan nuqta, ularning mos uzunliklarini 1:2 kabi nisbatdagi kesmalarga ajratadi. Uchburchak medianalari o‘zaro kesishib, berilgan uchburchakni yuzalari bir biriga teng bo‘lgan 6 ta uchburchakka ajratadi, demak ularni yuzi berilgan uchburchak yuzining oltidan bir qismiga teng. $S_1 = S_2 = S_3 = S_4 = S_5 = S_6 = \frac{1}{6} \cdot S$.

Uchburchak medianalari kesishgan nuqtaning koordinatalari: $x_m = \frac{x_1+x_2+x_3}{3}$; $y_m = \frac{y_1+y_2+y_3}{3}$; $z_m = \frac{z_1+z_2+z_3}{3}$. Agar $m_b \perp m_c$ bo‘lsa, $m_a = \sqrt{m_b^2 + m_c^2}$.

12. Uchburchakka tashqi chizilgan aylana radiusi. Uchburchakka tashqi chizilgan aylana radiusini markazini topish uchun uning tomonlari o‘rtasidagi nuqtalardan shu tomonlarning o‘ziga perpendikulyar kesmalar o‘tkaziladi, ularning kesishish nuqtasi uchburchakka tashqi chizilgan aylananing markazi bo‘ladi (3.11-rasm). Tashqi chizilgan aylana radiusi quyidagi formuladan aniqlanadi $R = \frac{abc}{4S}$.



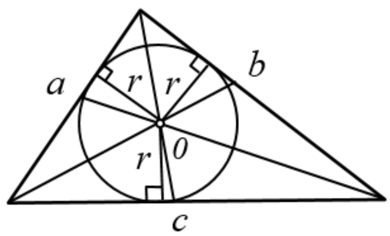
3.11-rasm.

Isbot: *ABC uchburchakning (a-rasm) yuzi* $S = 1/2 \cdot b \cdot c \cdot sin\text{б}$ *ga teng. Sinuslar teoremasiga asosan* $sin\text{б} = \frac{a}{2R}$ *buni formulaga qo‘yamiz:* $S = \frac{1}{2} \cdot b \cdot c \cdot \frac{a}{2R}$ *. Bundan:* $R = \frac{abc}{4S}$ ·

13. *ABC* uchburchak uchlarining koordinatalari $A(x_1;y_1)$, $B(x_2;y_2)$ va $C(x_3;y_3)$ berilganda unga tashqi chizilgan aylana markazining koordinatalarini $M(a;b)$ aniqlash uchun quyidagi sistemani yrechish kerak.

$$\begin{cases} 2a(x_2 - x_1) + 2b(y_2 - y_1) = (x_2 - x_1)(x_2 + x_1) + (y_2 - y_1)(y_2 + y_1) \\ 2a(x_3 - x_1) + 2b(y_3 - y_1) = (x_3 - x_1)(x_3 + x_1) + (y_3 - y_1)(y_3 + y_1) \end{cases}$$

14. Uchburchakka ichki chizilgan aylana radiusi. Uch-burchakka ichki chizilgan aylana radiusini markazini topish uchun uning uchidagi burchaklaridan bissiktrisalar o‘tkaziladi va ularning kesishish nuqtasi uchburchakka ichki chizilgan aylananing markazi bo‘ladi (3.12-rasm). Ichki chizilgan aylana radiusi quyidagi formuladan aniqlanadi $r = \frac{2S}{a+b+c}$.



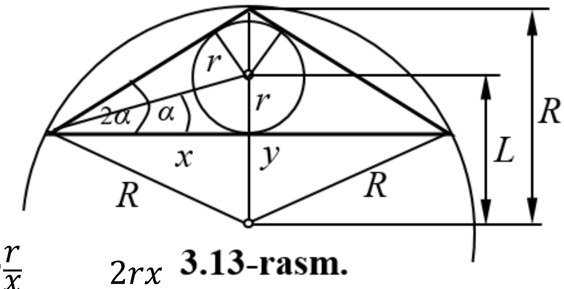
3.12-rasm.

Isbot: *Chizmada bissiktrisalarning kesishish nuqtasida uchburchak uchta uchburchakka ajralgan.*

Ularning yuzi $S = \frac{1}{2} \cdot a \cdot r + \frac{1}{2} \cdot b \cdot r + \frac{1}{2} \cdot c \cdot r = \frac{1}{2} \cdot r(a + b + c)$. $r = \frac{2S}{a+b+c}$ ·

15. Bir uchburchakka tashqi va ichki chizilgan aylanalar markazi orasidagi masofa. $L = \sqrt{R^2 - 2Rr}$.

Isbot: *Masala sharti va chizmaga asosan* $L = y + r$ *ni topish talab qilinadi (3.13-rasm). Chizmadan* $x^2 = R^2 - y^2 = (R - y)(R + y)$,



3.13-rasm.

$tg\text{б} = \frac{r}{x}$ *va* $tg2\text{б} = \frac{R-y}{x} = \frac{2tg\text{б}}{1-tg^2\text{б}} = \frac{2\cdot\frac{r}{x}}{1-\frac{r^2}{x^2}} = \frac{2rx}{x^2-r^2}$ ·

Buni so‘ldalashtirsak $(R-y)(x^2-r^2)=2rx^2=2r(R-y)(R+y)$;
$x^2-r^2=2r(R+y)$; $R^2-y^2-r^2=2Rr+2ry$; $R^2-2Rr=$
$=y^2+r^2+2ry; R^2-2Rr=(y+r)^2=L^2$. *Bundan* $L=\sqrt{R^2-2Rr}$.

16. Uchburchaklarning tenglik alomatlari.

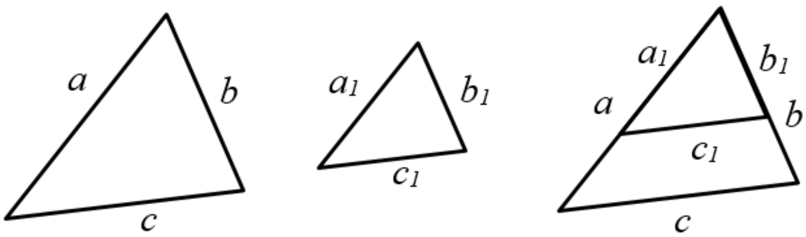
- Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi ikkinchi uchburchakning ikki tomoni va ular orasidagi burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar teng bo‘ladi.
- Agar bir uchburchakning bir tomoni va unga yopishgan burchaklari, ikkinchi uchburchakning mos tomoni va unga yopishgan burchaklariga teng bo‘lsa, bunday uchburchaklar teng bo‘ladi.
- Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda teng bo‘lsa, bunday uchburchaklar teng bo‘ladi.

16. Uchburchaklarning o‘xshashlik alomatlari.

- Agar uchburchakning uchta tomoni, ikkinchi uchburchakning mos tomonlariga proporsional bo‘lsa bunday uchburchaklar o‘xshash bo‘ladi.
- Agar bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi.
- Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proporsinal va bu tomonlar orasidagi burchagi teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi.

Agar tomonlari *a, b, c* bo‘lgan uchburchak, tomonlari a_1, b_1, c_1 bo‘lgan uchburchakka o‘xshash bo‘lsa, ularni birini ikkinchisi ustiga qo‘yganda ustma ust tushishi kerak (3.14-rasm). Shunda $\frac{a}{a_1}=\frac{b}{b_1}=\frac{c}{c_1}$.

Agar ular mos tushmasa bu uchburchaklar o‘xshash emasdir. O‘xshash shakllar yuzalarining nisbati, ularning mos chiziqli o‘lchovlari nisbatining kvadratiga neng: $\frac{S}{S_1}=k^2$.



3.14-rasm.

bu yerda k- proporsionallik koeffitsienti bo‘lib, chiziqli o‘lchamlar nisbatidir:

$$k=\frac{a}{a_1}=\frac{h}{h_1}=\frac{p}{p_1}\cdot$$

Teng yonli uchburchakning asosiy ko‘rsatkichlari (3.9*b*-rasm): Bunda uchburchakning asosidagi burchaklari va asosiga tushirilgan balandlik, bissiktrisa va medianalar bir biriga teng bo‘ladi: $h_a=д_a=m_a$; yon tomoniga tushirilgan balandlik, bissiktrisa va medianalar esa quyidagi tartibda bo‘ladi: $h_b<д_b<m_b$.

Teng tomonli (muntazam) uchburchakning asosiy ko‘rsatkichlari (3.9*c*-rasm): Uning barcha tomonlari va ichki burchaklari bir biriga teng bo‘lib, ular $б=60^0$ dan bo‘ladi. Unga ichki va tashqi chizilgan aylana markazlari bir nuqtada joylashgan bo‘ladi. Unga tashqi chizilgan aylana radiusi $R=\frac{2}{3}\cdot h=\frac{2}{3}\cdot\frac{\sqrt{3}}{2}\cdot a=\frac{\sqrt{3}}{3}\cdot a$, Unga ichki chizilgan aylana radiusi $r = \frac{1}{3} \cdot h = \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot a = \frac{\sqrt{3}}{6} \cdot a.$

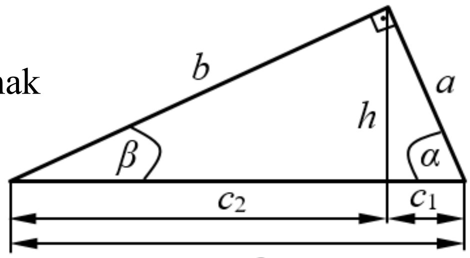
Barcha tomonlariga tushirilgan balandlik, bissiktrisa va medianalar bir biriga teng bo‘ladi. $h = д = m = \frac{\sqrt{3}}{2} \cdot a.$

Muntazam uchburchakning yuzi: $S = \frac{1}{2} \cdot a^2 \cdot sin60^0 = \frac{1}{2} \cdot a^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \cdot a^2.$

To‘g‘ri burchakli uchburchakning asosiy ko‘rsatkichlari (3.9*d*-rasm): Bunda uning bitta burchagi 90^0 bo‘lib, uning qarshisida gipotenuza yotadi, qolgan ikkitasi o‘tkir bur-chak bo‘ladi va ularning qarshisida katetlar joylashgan bo‘ladi. Uning gipotenuzasiga tushirilgan balandlik gipotenuzani c_1 va c_2 kesmalarga ajratadi. Agar bu kesmalar beril-gan bo‘lsa, gipotenuzaga tushirilgan balandlik quydagi formula orqali aniqlanadi:

$$h = \sqrt{c_1 \cdot c_2}.$$

To‘g‘ri burchakli uchburchakka tashqi chizilgan aylana radiusi gipotenizaning yar-miga teng bo‘ladi. $R = \frac{c}{2} \cdot$ Yoki $c = 2R$.



3.15-rasm.

Pifagor teoremasi faqat to‘g‘ri burchakli uchburchak uchun qo‘llaniladi. Pifagor teoremasi.

Teorema - to‘g‘ri burchakli uchburchak gipote-nuzasining kvadrati uning katetlari kvadratlarining yig‘indisiga teng. $c^2 = a^2 + b^2$.

Bundan $c = \sqrt{a^2 + b^2},\ a = \sqrt{c^2 - b^2},\ b = \sqrt{c^2 - a^2}.$

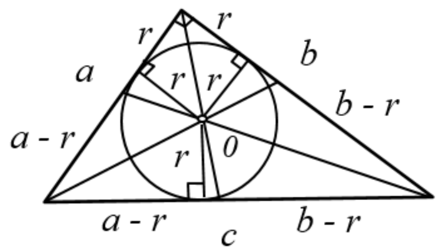
Isbot: *3.15-rasmdan* $c = c_1 + c_2;\ \frac{c_1}{a} = cosб = \frac{a}{c}.$

Bundan $c_1 = \frac{a^2}{c}.\ \ \frac{c_2}{b} = cosв = \frac{b}{c}.$ *Bundan* $c_2 = \frac{b^2}{c}.$ *Shunda*

$c = c_1 + c_2 = \frac{a^2}{c} + \frac{b^2}{c}.$ *Bundan* $c^2 = a^2 + b^2.$

To‘g‘ri burchakli uchburchakda katetlarining yi-g‘indisi, unga tashqi va ichki chizilgan aylana radius-lari yig‘indisining ikkilanganiga teng;

$a + b = 2(R + r)$ o‘rinlidir.



3.16-rasm.

Isbot; *3.16-rasmdan* $c = a + b - 2r$. *Bundan*

$a + b = c + 2r = 2R + 2r = 2(R + r).$

Aylana, to‘rtburchak va uchburchaklar qatnashgan shakillar bo‘yich qo‘shim-cha ma’limotlar:

1) R radiusli aylanaga ichki chizilgan kvadratning yuzi (3.17*a*-rasm) $S_{kv} = 2R^2$ va unga ichki chizilgan muntazam uchburchakning yuzi (3.17*a*-rasm) $S_\Delta = \frac{3\sqrt{3}}{4} \cdot R^2.$

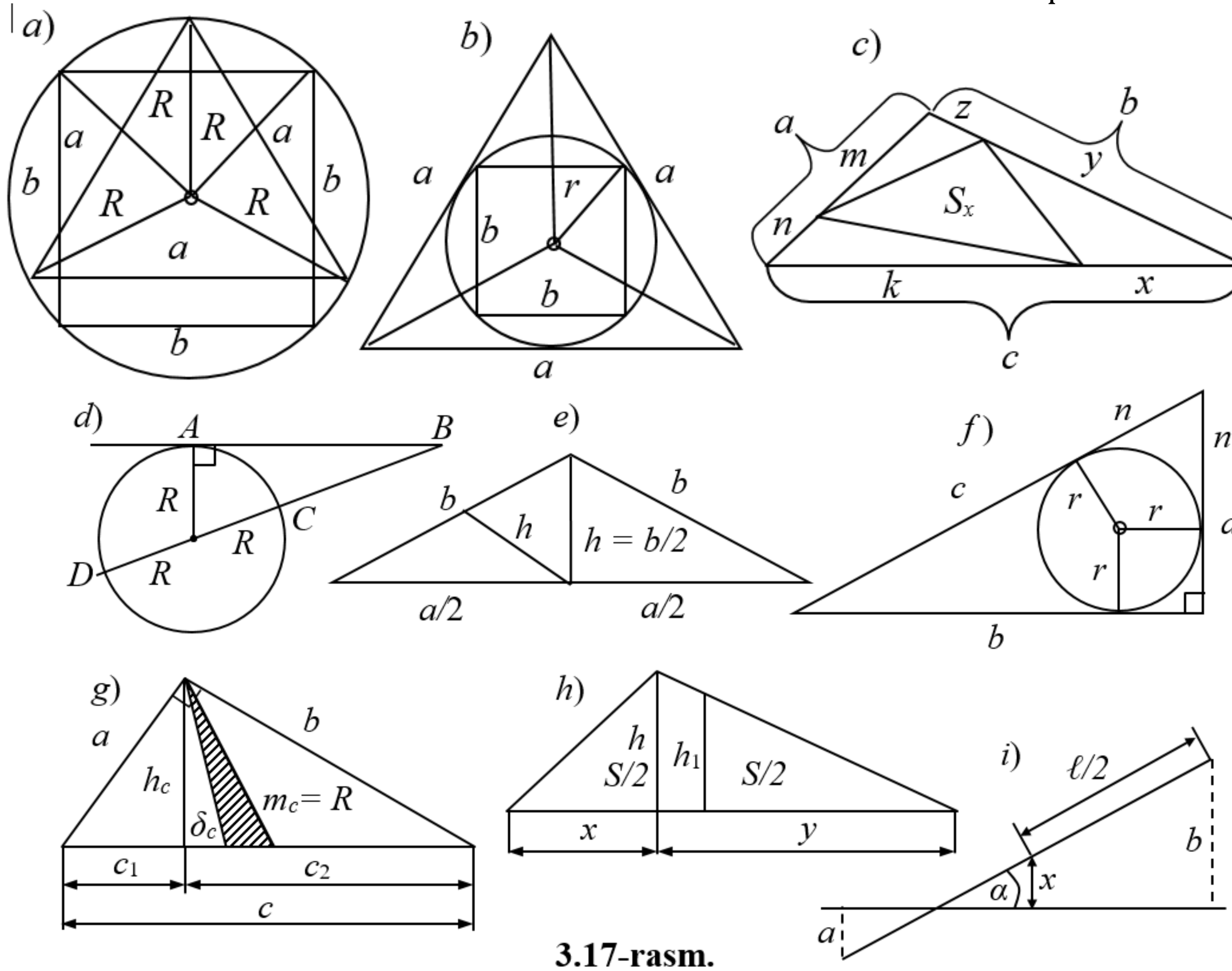
2) *r* radiusli aylanaga tashqi chizilgan muntazam uchburchakning yuzi (3.17*b*-rasm) $S_\Delta = 3\sqrt{3} \cdot r^2.$

3) Tomonlati *a, b* va *c* bo‘lgan uchburchakning uchlaridan boshlab uning tomonlar *k:x, y:z* va *n:m* kabi nisbatda bo‘lingan (3.17*c*-rasm), shu bo‘lingan nuqtalarni birlash-

tirishdan hosil bo‘lgan uchburchak yuzi S_x ni berilgan uchburchak yuziga bo‘lgan nisbati quyidagich aniqlanadi: $\frac{S_x}{S}=\frac{k\cdot y\cdot m+x\cdot z\cdot n}{a\cdot b\cdot c}$.

4) Aylanadan tashqaridagi nuqta *B* dan, unga *BA* urunma va uni kesib o‘tuvchi *BD* kesma o‘tkazilgan bo‘lsa (3.17d-rasm), $(AB)^2 = BD\cdot BC$ bo‘ladi.

5) Yon tomoni b ga teng bo‘lgan teng yunli uchburchakning uchidan tushirilgan balandlik va uni yon tomonlarini hamda asosining o‘rtalarini tutashtiruvch kesmalar teng bo‘lsa (3.17e-rasm), uni yuzasi quyidagi formula orqali aniqlanadi: $S=\frac{\sqrt{3}}{4}\cdot b^2$.



3.17-rasm.

6) To‘g‘ri burchakli uchburchakka *r* radiusli aylana ichki chizilgada (3.17*f*-rasm) uni yuzasi quyidagi formula orqali aniqlanadi: $S=\frac{n\cdot c\cdot(n+r)}{n-r}$.

Agar perimetri berilgan bo‘lsa, uni yuzasi quyidagi formula orqali aniqlanadi:

$$S=\frac{p\cdot(p-2c)}{4}.$$

7) To‘g‘ri burchakli uchburchakning bissiktrissa va medianasi orasidagi shtrixlangan yuza (3.17g-rasm).

$S_k=\frac{a\cdot b\cdot(b-a)}{4(a+b)}$, gippotenuzaga tushirilgan balandlik va medianalarning nisbati berilganda, katetlarining nisbati:

$$\frac{a}{b}=\frac{\frac{h}{m}}{1+\sqrt{1-\left(\frac{h}{m}\right)^2}}=\frac{r+1}{R-1}.$$

8) Uchburchakning balandligi h, uni asosini $x\!:y=k$ nisbatdagi kesmalarga ajratadi, uni yuzasini teng ikkiga ajratish uchun shu balandlikka parallel bo‘lgan qanday balandlikdagi kesma olish kerak (3.17h-rasm). $h_1=h\sqrt{\frac{k+1}{2}}$.

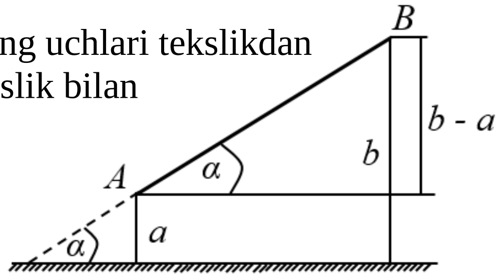
Agar (3.9a-rasm) $a^2+b^2=c^2$ bo‘lsa, uchburchak no‘g‘ri burchakli; $a^2+b^2>c^2$ bo‘lsa, uchburchak o‘tkir burchakli; $a^2+b^2<c^2$ bo‘lsa, uchburchak o‘tmas burchakli bo‘ladi.

Tekslikni kesib o‘tuvchi va unda yotmaydigan to‘g‘ri chiziq, nuqtadan tekslikka tushirilgan og‘ma va perpendikulyarga doir masalalar.

1) Tekslikni kesib o‘tuvch ℓ uzunlikdagi kesmaning uchlari tekslikdan a va b masofada joylashgan bo‘lsa (3.17i-rasm), kesmaning o‘rtasi tekislikdan qanday x masofada joylashgan bo‘ladi. $x=\frac{b-a}{2}$.

2) Tekslikni kesib o‘tmaydigan *AB* to‘g‘ri chiziqning uchlari tekslikdan a va b masofada joylashgan bo‘lsa (3.18-rasm), uni tekslik bilan tashkil qilgan burchagining sinusini toping.

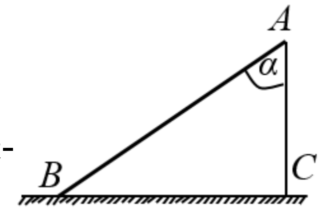
$$sinб=\frac{b-a}{AB}.$$



3.18-rasm.

3) *A* nuqtadan tekslikka *AB* og‘ma va *AC* perpendikulyar tushirilgan.

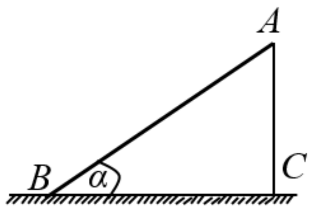
Og‘maning uzunligi 8, perpendikulyar va og‘ma orasidagi burchak $б=arcsin\,0{,}75$ bo‘lsa, perpendikulyarning uzunligini aniqlang (3.19-rasm).



3.19-rasm.

Yeshish: *Masala shartiga asosan* $sinб=0{,}75=\frac{3}{4}$. *Rasmdan* $\frac{AC}{AB}=sinб$. *Bundan* $AC=AB\cdot sinб=8\cdot\frac{3}{4}=6$.

4) *A* nuqtadan tekslikka *AB* og‘ma va *AC* perpendikulyar tu-shirilgan (3.20-rasm), og‘maning uzunligi 10, tekslik va og‘ma orasidagi burchak $б=arccos\,0{,}3$ bo‘lsa, perpendikulyarning uzunligini aniqlang.



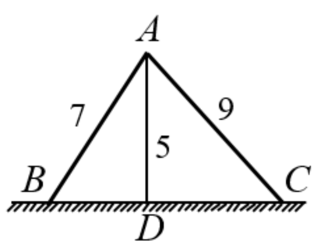
3.20-rasm.

Yeshish: *Masala shartiga asosan* $cosб=0{,}6=\frac{3}{5}$.

Chizmadan $\frac{AC}{AB}=sinб=\sqrt{1-cos^2б}==\sqrt{1-\frac{9}{25}}=\frac{4}{5}$.

Bundan $AC=AB\cdot sinб=10\cdot\frac{4}{5}=8$.

5) *A* nuqtadan tekslikka *AB* va *AC* og‘malar tushirilgan (3.21-rasm). Agar *AD* perpendikulyarning uzunligi 5, og‘malarning uzunliklari mos ravisha 7 va 9 ga teng bo‘lsa, og‘malarning tekslikdagi proeksiyalarini toping.



3.21-rasm.

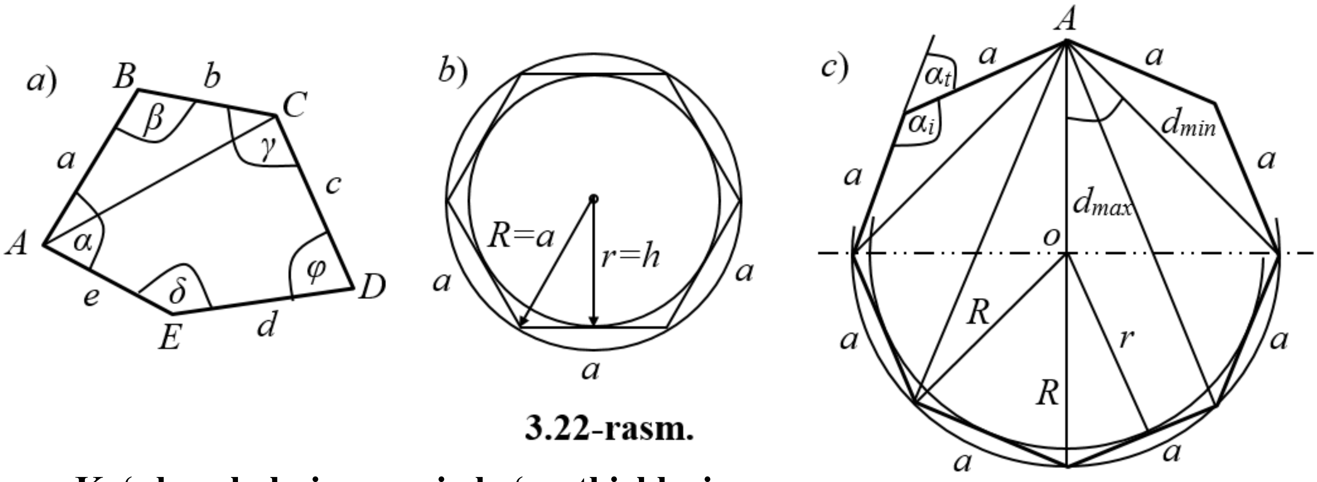
Yeshish: *Masala shartiga asosan AB va AC og'malarning tekslikdagi proeksiyalari BD va CD ni Pifagor teoremasidan foydalanib aniqlaymiz:*

$$BD = \sqrt{7^2 - 5^2} = \sqrt{49 - 25} = 2\sqrt{6}; \quad CD = \sqrt{9^2 - 5^2} = \sqrt{81 - 25} = 2\sqrt{14}.$$

3.5. Ko'pburchak, uning turlari va asosiy ko'rsatkichlari.

Ko'pburchak deb, istalgan uchta nuqtasi bir to'g'ri chiziqda yotmaydigan, besh va undan ortiq nuqtalarni ketma-ket tutashtiruvchi kesmadan tashkil topgan geometrik shaklga aytiladi. Ko'pburchakning quyidagi **turlari** mavjud: ixtiyoriy (3.22a-rasm), tomonlari teng bo'lgan yoki muntazam ko'pburchak (3.22b-rasmda muntazam oltiburchak; 3.22c-rasmda muntazam sakkizburchaklar ko'rsatilgan).

Ko'pburchak ichki burchaklaridan eng kattasi 180^0 dan kichik bo'lishi kerak. Bun-day ko'pburchaklarga ***qavariq*** ko'pburchak deb ataladi.



3.22-rasm.

Ko'pburchakning asosiy ko'rsatkichlari.

1. Perimetri, agar ixtiyoriy ko'pburchak bo'lsa (3.22a-rasm), $p = a + b + c + d + e$. Agar muntazam ko'pburchak bo'lsa (3.22b,c-rasm), $p = n \cdot a$. bu yerda n - ko'pburchak tomonlarining soni.

2. Ichki burchaklarining yig'indisi $\Sigma \text{б}_i = \text{p} \cdot (n - 2)$.

3. Tashqi burchaklarining yig'indisi $\Sigma \text{б}_t = 360^0$.

4. Diagonallarining soni $d_n = \frac{n \cdot (n-3)}{2}$.

5. Bir uchidan chiqqan diagonallarining soni (3.22c-rasm) $d_{uch} = n - 3$.

6. Bitta tashqi burchagi $\text{б}_t = \frac{360^0}{n}$.

7. Bitta ichkqi burchagi $\text{б}_i = \frac{\text{p} \cdot (n-2)}{n} = \frac{180^0 \cdot (n-2)}{n}$.

8. Eng katta va eng kichik diagolali orasidagi burchak (n juft bo'lganda)

$$d_{max} \wedge d_{min} = \frac{\text{p}}{2n} \cdot (n - 4).$$

9. Muntazam ko'pburchakning yuzi, asosining perimetri bilan, unga ichki chizilgan aylana radiusi ko'paytmasining yarmiga teng: $S = \frac{1}{2} \cdot p \cdot r = \frac{1}{2} \cdot a \cdot h \cdot n$.

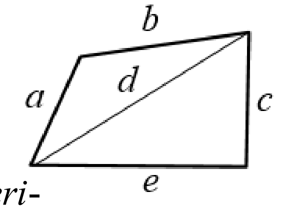
10. Ko'pburchakka tashqi chizilgan aylana radiusi $R = \frac{a}{2cos\frac{\text{б}_i}{2}}$.

11. Ko‘pburchakka ichki chizilgan aylana radiusi $r = \frac{a}{2} . tg \frac{б_i}{2}$.

AYLANA, DOIRA, TO‘RTBURCHAK, UCHBURCHAK VA KO‘P BURCHAKLARGA DOIR MASALALAR.

1) Aylananing MN varari $б = 110^0$ li yoyni tortib turibdi, bu varar o‘zi tortib turgan kichik yoyning ixtiyoriy nuqtasidan qanday burchak ostida ko‘rinadi.

Yechish: *Agar MN vatarning o‘zi tortib turgan kichik yoyning ixtiyoriy nuqtasidagi burchakni* г *deb belgilasak* $г = 180^0 - \frac{б}{2} = 180^0 - \frac{110^0}{2} = 180^0 - 55^0 = 125^0$.

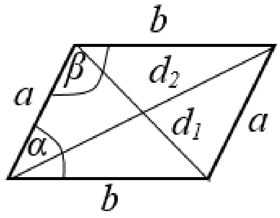


2) To‘g‘ri to‘rtburchakning diagonali uni perimetrlari 25 va 27 ga teng bo‘lgan ikkita uchburchakka ajratadi (rasmga qarang). Agar to‘rtburchakning perimetri 36 ga teng bo‘lsa, derilgan diagonal uzunligini toping.

Yechish: *Masala shartiga asosan berilgan to‘rtburchakning perimetri* $a + b + c + e = 36$.

Chizma va masala shartiga asosan $a + b + d = 25$; $c + e + d = 27$.

Bu tengliklarni hadma had qo‘shsak, $2d + a + b + c + e = 25 + 27 = 52$ *yoki* $2d + 36 = 52$*hosil bo‘ladi. Bundan* $d = 8$.



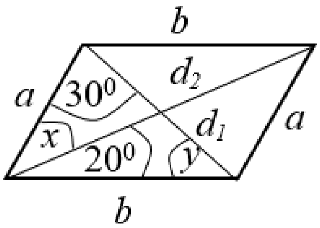
3) Parallelogrammning diagonallari $d_1 = 7$ va $d_2 = 24$ ga teng (rasm). Uning barcha tomonlari yig‘indisini toping.

Yechish: *Kosinuslar teoremasini qo‘llab, quyidagilarni aniqlaymiz:*

$a^2 + b^2 - 2 \cdot a \cdot b \cdot cosб = 7^2$; $a^2 + b^2 - 2 \cdot a \cdot b \cdot cosв = 24^2$.

$в = 180^0 - б$ *ekanligini hisobga olib, quyidagi sistemani tuzamiz:*

$\begin{cases} a^2 + b^2 - 2 \cdot a \cdot b \cdot cosб = 49 \\ a^2 + b^2 + 2 \cdot a \cdot b \cdot cosб = 576 \end{cases}$ *bularni hadma had qo‘shsak*$2a^2 + 2b^2 = 625$.



4) Parallelogrammning diagonallari tomonlari bilan $б = 20^0$ va $в = 30^0$ li burchak hosil qiladi (rasm). Parallelogrammning katta burchagini toping.

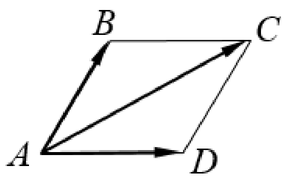
Yechish: *Parallelogramning qarama-qarsh burchaklarining yig‘indisi* $2б + 2в = 360^0$ *yoki* $б + в = 180^0$.

Chizmadan $б = x + 20^0, в = y + 30^0$ *yoki* $б + в = 180^0 = x + y + 50^0$.

Bundan; $x + y = 130^0$.

5) *ABCD* parallelogrammning uchlari $A(4; 1)$, $B(6; 3)$, $D(7; \sqrt{171} - 1)$ nuqtalar bo‘lsa, *AC* diagonalning uzunligini toping (rasmga qarang).

Yechish: *Parallelogramning A va B nuqtalari orqali* $\overrightarrow{AB}[(6 - 4); (3 - 1)] =$
$= \overrightarrow{AB}(2; 2)$ *hamda A va D nuqtalari orqali*
$\overrightarrow{AD}[(7 - 4); \ (\sqrt{171} - 1 - 1)] = \overrightarrow{AD}(3; \sqrt{171} - 2)$ *vektorlarni tuzamiz.*
$\overrightarrow{AB}$ *va* $\overrightarrow{AD}$ *vektorlarning skalyar ko‘paytmasini aniqlaymiz:*
$\overrightarrow{AB} \cdot \overrightarrow{AD} = 2 \cdot 3 + 2\sqrt{171} - 4 = 2(1 + \sqrt{171})$.

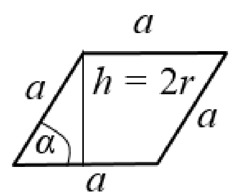


Bu vektorlarni uzunliklarini aniq-laymiz:

$$\left|\overrightarrow{AB}\right| = \sqrt{2^2+2^2} = 2\sqrt{2};\ \left|\overrightarrow{AD}\right| = \sqrt{3^2+\left(\sqrt{171}-2\right)^2} =$$
$$= 2\sqrt{46-\sqrt{171}}.$$

Parallelogramning AC diagonali, $\overrightarrow{AB}$ *va* $\overrightarrow{AD}$ *vektorlarning yig'ndisidan tashkil topgan* $\overrightarrow{AC}$ *vektorning uzunligiga teng bo'ladi,* $\left|\overrightarrow{AC}\right| = \sqrt{\left(\overrightarrow{AB}\right)^2 + 2\overrightarrow{AB}\cdot\overrightarrow{AD} + \left(\overrightarrow{AD}\right)^2} =$

$$= \sqrt{\left(2\sqrt{2}\right)^2 + 2\cdot 2\cdot\left(1+\sqrt{171}\right) + \left(2\sqrt{46-\sqrt{171}}\right)^2} = 14.$$



6) Rombning tomoni $a = 10$ ga, unga ichga aylananing radiusi $r = 3$ ga teng (rasm). Rombning o`tkir burchagi kosinusini toping.

Yechish: *Romning balandligi unga ichki chizilgan aylana radiusining ikkilanganiga teng.*

Chizmadan $sinб = \frac{h}{a} = \frac{2r}{a} = \frac{2\cdot 3}{10} = \frac{3}{5}\cdot$

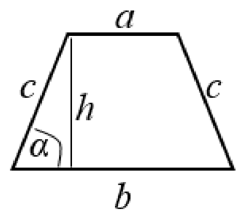
Masala shartiga ko'ra rombning o`tkir burchagi kosinusini topish talab etiladi.

$$cosб = \sqrt{1-sin^2б} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}.$$

7) Rombning yuzi 120, diagonallaridan biri 24 ga teng bo'lsa, uni tomonini toping.

Yechish: *Diagonallari orqali rombning yuzi* $S = (1/2)d_1\cdot d_2$ *bo'lgani uchun, bun-dan* $d_1 = \frac{2S}{d_2} = \frac{2\cdot 120}{24} = 10.\ a = \frac{1}{2}\cdot\sqrt{d_1^2+d_2^2} = \frac{1}{2}\cdot\sqrt{10^2+24^2} = 13.$

8) Teng yonli trapetsiyaning asoslari $a = 10$ va $b = 18$, asosidagi burchagi $б = 60^0$ bo'lsa, uni yuzini toping.



Yechish: *Chizmadan* $\frac{b-a}{2} = \frac{18-10}{2} = 4.$

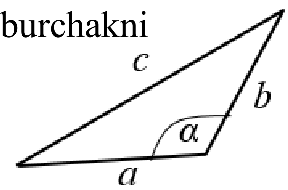
$h = tgб = tg60^0 = \sqrt{3}.$

Trapetsiyaning yuzi $S = \frac{a+b}{2}\cdot h = \frac{10+18}{2}\cdot\sqrt{3} = 14\sqrt{3}.$

9) Uchburchak burchklari o`zaro 1:1:2 nisbatda. Uchburchakning katta tomoni 12 ga teng. Katta tomonga tushirilgan balandlikni toping.

Yechish: *Uchburchak ichki bir burchagini* б *deb olsak, unda uning ichki burchak-larining yig'indisi* $б + б + 2б = 180^0 \to б = 45^0.$ *Demak, uchburchak teng tomonli va to'g'ri burchakli ekan.* $h = \frac{12}{2} = 6.$

10) Ikki tomonining yig'indisi 1,8 ga, ular orasidagi burchak $б = 120^0$ ga teng bo'lgan uchburchaklar ichida yuzasi eng katta bo'lgan uchburchakni yuzini toping.



Yechish: *Masala shartiga ko'ra* $a + b = 1{,}8.$

Bundan $b = 1{,}8 - a.$ *Uchburchak yuzi*

$$S=\frac{1}{2}\cdot a\cdot b\cdot sinб=\frac{1}{2}\cdot a\cdot(1{,}8-a)\cdot sin120^0=\frac{\sqrt{3}}{4}\cdot(1{,}8\cdot a-a^2).$$

Buni argumenti a bo‘lgan funksiya deb qaraymiz va bu funksiya o‘zining maksimum qiymatiga birinchi tartibli hosilasi nolga teng bo‘lganda erishadi.

$S'(a)=\frac{\sqrt{3}}{4}\cdot(1{,}8-2a)=0.$ *Bundan* $a=0{,}9.$ $b=1{,}8-0{,}9=0{,}9.$

$$S=\frac{\sqrt{3}}{4}\cdot a\cdot b=\frac{\sqrt{3}}{4}\cdot 0{,}9\cdot 0{,}9=\frac{81\sqrt{3}}{400}.$$

11) To`g`ri burchakli uchburchakning tomonlari 24; 32; 40 ga teng. Uchburchakka ichki chizilgan aylananing markazi bilan tashqi chizilgan aylananing markazi orasidagi masofani toping.

<u>Yechish:</u> *Geron formulasidan foydalanib berilgan uchburchakni yuzini aniqlaymiz: buni uchun yarim perimetrini topamiz* $p=\frac{P}{2}=\frac{24+32+40}{2}=48.$

$$S=\sqrt{p\cdot(p-a)\cdot(p-b)\cdot(p-c)}=\sqrt{48\cdot(48-24)\cdot(48-32)\cdot(48-40)}=$$
$$=384.$$

Uchburchakka tashqi chizilgan aylana radiusi $R=\frac{a\cdot b\cdot c}{4\cdot S}=\frac{24\cdot 32\cdot 40}{4\cdot 384}=20.$

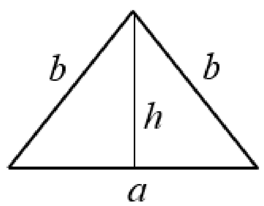
Uchburchakka ichki chizilgan aylana radiusi $r=\frac{2\cdot S}{a+b+c}=\frac{2\cdot 384}{24+32+40}=8.$

Uchburchakka ichki chizilgan aylananing markazi bilan tashqi chizilgan aylananing markazi orasidagi masofa $L=\sqrt{R^2-2Rr}=\sqrt{20^2-2\cdot 20\cdot 8}=$
$=\sqrt{400-320}=4\sqrt{5}.$

12) To`g`ri burchakli uchburchak katetining gipotenuzadagi proyeksiyasi 16 ga, gipotenuzasi $c=25$ ga teng. Shu uchburchakning yuzini toping.

<u>Yechish:</u> *Gipotenuzasiga tushirilgan balandlik* $h=\sqrt{(25-16)\cdot 16}=12.$ *Uchburchakning yuzini* $S=\frac{1}{2}\cdot c\cdot h=\frac{1}{2}\cdot 25\cdot 12=150.$

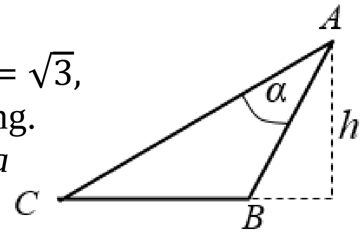
13) Teng yonli uchburchakning asosi $a=40$ sm, balandligi $h=21$ sm bo‘lsa (rasm), uni yon tomonini toping.



<u>Yechish:</u> *Teng yonli uchburchakning balandligi uni asosini teng ikkiga bo‘lishini hisobga olib, Pifagor teoremasidan foydalanib,uchburchakni yon tomonini topamiz:*

$$b=\sqrt{h^2+\left(\frac{a}{2}\right)^2}=\sqrt{21^2+\left(\frac{40}{2}\right)^2}=29.$$

14) ABC uchburchakda (rasm) $\angle A=б=30^0$, $AB=\sqrt{3}$, $AC=6$ ga teng bo‘lsa, A uchidan tushirilgan balandlikni toping.



<u>Yechish:</u> *ABC uchburchakning yuzini quyidagi formula orqali aniqlaymiz:*

$$S=\frac{1}{2}\cdot AC\cdot AB\cdot sinб=\frac{1}{2}\cdot 6\cdot\sqrt{3}\cdot sin30^0=\frac{3\sqrt{3}}{2}.$$

Kosinuslar teoremasidan $BC=\sqrt{(AC)^2+(AB)^2-2\cdot AC\cdot AB\cdot cosб}=$

$$=\sqrt{(6)^2+\left(\sqrt{3}\right)^2-2\cdot 6\cdot\sqrt{3}\cdot cos30^0}=\sqrt{21}.$$

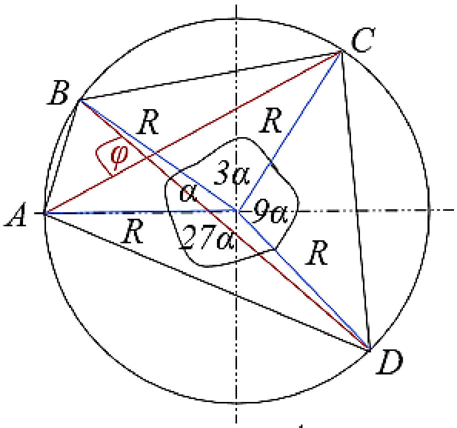
Uchburchak asosiga tushirilgn balandlik $h = \frac{2S}{BC} = \frac{2\cdot 3\sqrt{3}}{2\cdot\sqrt{21}} = \frac{3\sqrt{7}}{7}$.

15) Aylananing *ABCD* nuqtalaridan tashkil topgan yoylarning uzunliklari mazraji 3 ga teng bo‘lgan geometrik progressiyadan iborat bo‘lgan bo‘laklarga ajratilgan (rasm).

Shu nuqtalarni ketma-ket tutashtirishdan hosil bo‘lgan to‘rtburchak diagonallari ora-sidagi kichik burchakni toping.

Yechish: *ABCD nuqtalaridan tashkil topgan yoy burchaklarining* $б + 3б + 9б + +27б = 360^0$. *Bundan* $б = 9^0$.

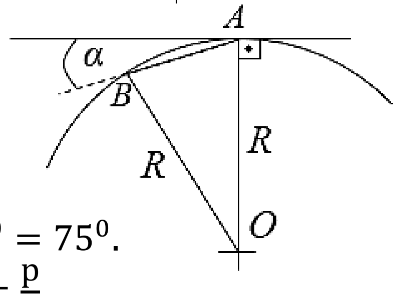
Markaziy burchagi $3б$ *bo‘lgan* $\overset{\frown}{BC}$ *kichik yoy, katta yoyning ixtiyoriy D nuqtasidan* $\angle D = \frac{3}{2}\cdot б$ *burchak ostida ko‘rinadi. Markaziy burchagi* $27б$ *bo‘lgan* $\overset{\frown}{AD}$ *kichik yoy, katta yoyning ixtiyoriy C nuqtasidan* $\angle C = \frac{27}{2}\cdot б$ *burchak ostida ko‘rinadi.*



Izlanayotgan burchakni chizmadan foydalanib, quyidagicha aniqlaymiz: $ц = 180^0 - (\angle D + \angle C) =$
$= -180^0 - \left(\frac{3}{2}\cdot б + \frac{27}{2}\cdot б\right) = 180^0 - 15б.$

$ц = 180^0 - 15\cdot 9^0 = 45^0.$

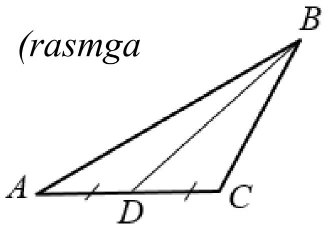
16) Aylananing *A* nuqtasiga o‘tkazilgan urunma va *AB* vatar orasidagi burchak $б = 15^0$ ga teng (rasmga qarang). Agar aylananing radiusi $R = 3$ bo‘lsa, vatar ajratgan kichik yoyning uzunligini toping.



Yechish: *Chizmadan* $\angle A = \angle B = 90^0 - б = 90^0 - 15^0 = 75^0$.

$\angle O = 180^0 - 2\cdot 75^0 = 30^0 = \frac{р}{6}$. $\overset{\frown}{AD} = б\cdot R = \frac{р}{6}\cdot 3 = \frac{р}{2}$.

17) Uchlari $A(4;5;1), B(2;2;0)$ va $C(2;1;-3)$ nuqtalarda joylashgan uchburchak-ning *BD* medianasi uzunligini toping.



Yechish: *D nuqta AC nuqtalarning o‘rtasi bo‘ladi (rasmga qarang). D(x;y) nuqtaning koordinatalarini aniqlaymiz:*

$x = \frac{4+2+2}{2} = 4;$

$y = \frac{5+2+1}{2} = 4;\quad z = \frac{1+0-3}{2} = -1.\quad D(4;4;-1).$

BD mediana uzunligi B va D nuqtalar orasidagi masofaga teng bo‘lgani uchun shu masofani topamiz: $|BD| = \sqrt{(4-2)^2 + (4-2)^2 + (-1-0)^2} = \sqrt{4+4+1} = 3.$

18) Uchburchakning tomonlari *a, b* va *c* ga teng bo‘lib, $a^2 = b^2 + c^2 + \sqrt{2}bc$ tenglikni qanoatlantirsa, uzunligi *a* ga teng bo‘lgan tomoni qarshisidagi burchakni toping.

Yechish: *Uzunligi a ga teng bo‘lgan tomoni qarshisidagi burchakni* $б$ *deb olamiz va kosinuslar teremasidan* $a^2 = b^2 + c^2 - 2bc\cdot cosб$ *ekanligini bilgan holda* a^2 *ni o‘rniga berilgan qiymatni qo‘ysak, quyidagi tenglik hosil bo‘ladi:*

$$a^2 = b^2 + c^2 - 2bc\cdot cosб = b^2 + c^2 + \sqrt{2}bc.$$

Bundan $cos б = -\frac{\sqrt{2}}{2}\cdot$ *Yoki* $б = 135^0$.

19) To‘g‘ri to‘rtburchakning bir tomoni kvadratning tomonidan 8 *sm* ga kichik, ikkinchi tomno esa kvadrat tomonidan ikki marta uzun. Agar ularning yuzalari teng bo‘lsa, to‘g‘ri to‘rtburchakning perimetrini toping.

Yechish: *Kvadratning tomonini a desak, uni yuzi* $S = a^2$ *bo‘ladi.*

Unda masala shartiga asosan to‘g‘ri to‘rtburchakning yuzi $S = (a - 8)\cdot 2a = a^2$.

Bundan $a = 16\ sm$.

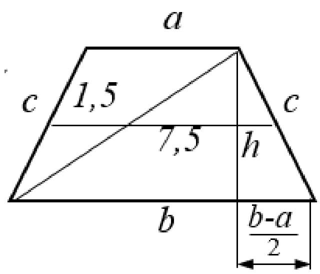
To‘rtburchakning perimetri $p = 2(a - 8) + 2\cdot 2a = 2a - 16 = 80\ sm$.

20) To‘g‘ri burchakli uchburchakka ichki chizilgan aylananing uzunligi 10p ga, gipotenuzasi esa 25 ga teng bo‘lsa, uni yuzini toping.

Yechish: *To‘g‘ri burchakli uchburchakka ichki chizilgan aylananing uzunligidan, unga ichki chizilgan aylananing radiusini topamiz:* $\ell = 10\text{p} = 2\text{p}r$. *Bundan* $r = 5$. *To‘g‘ri burchakli uchburchakda gipotenuza* $c = 2R = 25$. *Bundan* $R = \frac{25}{2}$. *To‘g‘ri burchakli uchburchak katetlarining yig‘indisi* $a + b = 2(R + r) = 2\left(\frac{25}{2} + 5\right) = 35$.

Uchburchakni yuzi $S = \frac{r(a+b+2R)}{2} = \frac{5\cdot(35+25)}{2} = 150$.

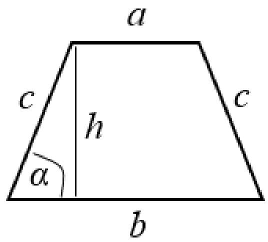
21) Aylanaga tashqi chizilgan teng yonli trapetsiyaning o‘rta chizig‘i $\ell = 8$ ga teng bo‘lsa, uni yon tomonini uzunligini toping.



Yechish: *Aylanaga tashqi chizilgan teng yonli trapetsiyaning yon tomonlarining yig‘indisi uning asoslari yig‘indisiga teng bo‘ladi.* $2c = a + b = 2\ell$.

Bundan $c = \ell = 8$.

22) Teng yonli trapetsiyaning diagonali uning o‘rta chizi-g‘ini 1,5 va 7,5 ga teng kesmalarga ajratadi. Agar trapetsiyaning yuzi 72 ga teng bo‘lsa, uni yon tomonini toping (rasmga qarang).



Yechish: *Chizmadan* $a = 2\cdot 1{,}5 = 3$; $b = 2\cdot 7{,}5 = 15$.

Trapetsiyaning yuzi $S = \frac{a+b}{2}\cdot h = 72 = \frac{3+15}{2}\cdot h$.

Bundan $h = 8$. $\frac{b-a}{2} = \frac{15-3}{2} = 6$ $c = \sqrt{8^2 + 6^2} = 10$.

23) Teng yonli trapetsiyaning asosidagi burchagining sinusi 0,6 ga, asoslarining ayirmasi esa $b - a = 4$ ga teng bo‘lsa, uni yon tomonini toping (rasmga qarang).

Yechish: *Masala shartidan* $sin б = 0{,}6$. *Bundan* $cos б = 0{,}8$.

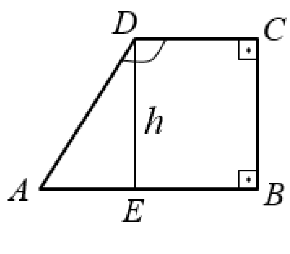
Chizmadan $b - a = 2\cdot c\cdot cos б$.

Bundan $c = \frac{b-a}{2\cdot cos б} = \frac{4}{2\cdot 0{,}8} = 2{,}5$.

24) *ABCD* to‘g‘ri burchakli trapetsiyaning yuzi $S = 14 sm^2$ga, $AB = 8\ sm$; $DC = 6\ sm$ bo‘lsa, uning $\angle ADC$ ni toping.

Yechish: *Chizmadan* $AE = AB - DC = 8 - 6 = 2\ sm$. *Trapetsiyaning yuzi* $S = \frac{DC+AB}{2}\cdot h = 14 = \frac{6+8}{2}\cdot h$. *Bundan* $h = 2$.

Demak, trapetsiyaning asosidagi o‘tkir burchagi 45^0 *bo‘ladi.*



Unda $\angle ADC = 45^0 + +90^0 = 135^0$ *ga teng bo'ladi.*

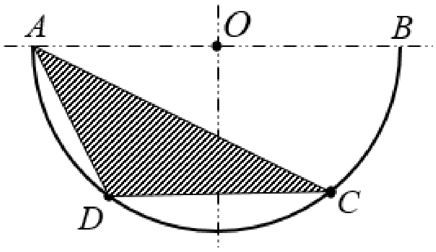
25) Teng yonli uchburchakning yon tomoni $b = 20\ sm$.

Agar uning asosi yon tomo-nining 60% ni tashkil qilsa, unga tashqi chizilgan aylana radiusini toping.

<u>Yechish:</u> *Teng yonli uchburchakning asosi* $a = 0{,}6 \cdot 20 = 12\ sm$ *ga teng bo'ladi.*

Uchburchakning balandligi $h = \sqrt{20^2 - 6^2} = 2\sqrt{91}$ *bo'ladi. Uchburchakning yuzi* $S = \frac{1}{2} \cdot a \cdot h = \frac{1}{2} \cdot 12 \cdot 2\sqrt{91} = 12 \cdot \sqrt{91}$. *Teng yonli uchburchakka tashqi chizilgan ayla-na radiusi* $R = \frac{ab^2}{4S} = \frac{12 \cdot 20^2}{4 \cdot 12 \cdot \sqrt{91}} = \frac{100}{\sqrt{91}} = \frac{100 \cdot \sqrt{91}}{91}$.

26) Rasmda berilgan yarim doira ichidagi shtrixlan-gan yuzani toping. $AO = OB = 2$. $\overset{\frown}{AD} = \overset{\frown}{DC} = \overset{\frown}{BC}$.



<u>Yechish:</u> $\overset{\frown}{AD} = \overset{\frown}{DC} = \overset{\frown}{BC}$ *yoylar teng bo'lgani uchun, ulardan tuzilgan* $AD = DC = BC$ *vatarlar ham teng bo'la-di.Masala shartiga ko'ra, shrixlangan yuza yarim doira ichida joylashgan, hamda vatarlarning bir biriga tengligi, doiraga ichki chizilgan muntazam oltiburchak hisoblanadi.*

Shuningdek, OA *doiraning radiusi. Demak, vatar, radiusga* $AD = OA$ *teng bo'ladi. Shunda* $\angle ADC = б = 120^0$ *ni tyashkil qiladi.*

Shrixlangan yuza $S = \frac{1}{2} \cdot AD \cdot CD \cdot sin б$. $S = \frac{1}{2} \cdot 2 \cdot 2 \cdot sin 120^0 = \sqrt{3}$.

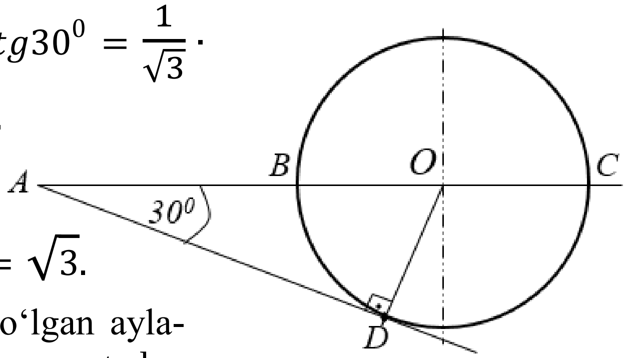
27) Rasmda berilganiga ko'ra, *AD* kesma uzunligini *AB* kesma uzunligiga bo'lgan nisbatini toping.

<u>Yechish:</u> *Chizmadan* $OB = OD$. $\frac{OD}{AD} = tg 30^0 = \frac{1}{\sqrt{3}}$.

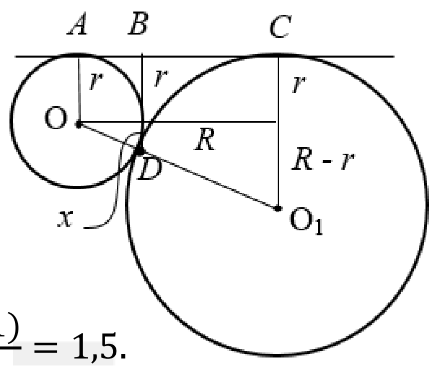
$AD = \sqrt{3} \cdot OD$. $\frac{OD}{AB + OD} = ain 30^0 = \frac{1}{2}$.

$2OD = AB + OD$; $AB = OD$.

Masala shartiga asosan $\frac{AD}{AB} = \frac{\sqrt{3} \cdot OD}{OD} = \sqrt{3}$.



28) Radiuslari $r = 1\ sm$ va $R = 3\ sm$ bo'lgan ayla-nalat tashqi ravishda *D* nuqtada urinadi. Shu nuqtadan ularning umumiy urunmalari bo'lmish *AC* to'g'ri chiziq-qacha bo'lgan masofani toping (rasmga qarang).

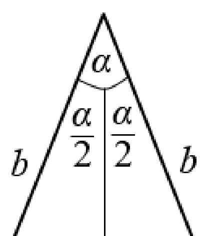


<u>Yechish:</u> *Masala shartiga asosan* $BD = r + x$ *masofani topish kerak. Chizmadan* $\frac{x}{r} = \frac{R-r}{R+r}$.

Bundan $x = \frac{r(R-r)}{R+r}$, *bo'ladi.*

Izlanayotgan masofa $BD = r + \frac{r(R-r)}{R+r} = 1 + \frac{1 \cdot (3-1)}{3+1} = 1{,}5$.

29) Teng yonli uchburchakning uchidagi burchagi б ga teng (rasmga qarang). Shu uchburchakka ichki va tashqi chizildan doiralar radiuslari-ning nisbatini toping.



Yechish: *Uchburchakning yuzi* $S = \frac{1}{2} \cdot b^2 \cdot sin б$. *Chizmadan uchburchak asosini aniqlaymiz:* $a = 2 \cdot b \cdot sin\frac{б}{2} \cdot$ *Uchburchakka ichki chizilgan doiraning radiusi*

$$r = \frac{2S}{b+b+a} = \frac{2 \cdot 1/2 \cdot b^2 \cdot sin б}{b+b+2 \cdot b \cdot sin\frac{б}{2}} = \frac{b \cdot sin б}{2(1+sin\frac{б}{2})} \cdot$$

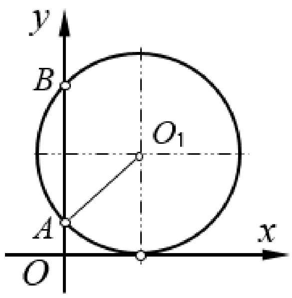
Uchburchakka tashqi chizilgan doiraning radiusi:

$$R = \frac{b \cdot b \cdot a}{4S} = \frac{b^2 \cdot 2 \cdot b \cdot sin\frac{б}{2}}{4 \cdot 1/2 \cdot b^2 \cdot sin б.} = \frac{b \cdot sin\frac{б}{2}}{sin б} \cdot \quad \frac{r}{R} = \frac{b \cdot sin б}{2(1+sin\frac{б}{2})} \cdot \quad \frac{sin б}{b \cdot sin\frac{б}{2}} = \frac{2 \cdot sin\frac{б}{2} \cdot cos\frac{б}{2} \cdot sin б}{2(1+sin\frac{б}{2}) \cdot sin\frac{б}{2}} ==$$

$$\frac{cos\frac{б}{2} \cdot sin б}{1+sin\frac{б}{2}} = \frac{\frac{1-tg^2\frac{б}{4}}{1+tg^2\frac{б}{4}} \cdot sin б}{1+\frac{2tg\frac{б}{4}}{1+tg^2\frac{б}{4}}} = \frac{\frac{1-tg^2\frac{б}{4}}{1+tg^2\frac{б}{4}} \cdot sin б}{\frac{tg^2\frac{б}{4}+2tg\frac{б}{4}+1}{1+tg^2\frac{б}{4}}} = \frac{\frac{1-tg^2\frac{б}{4}}{1+tg^2\frac{б}{4}} \cdot sin б}{\frac{\left(1+tg\frac{б}{4}\right)^2}{1+tg^2\frac{б}{4}}} = \frac{(1-tg^2\frac{б}{4}) \cdot sin б}{\left(1+tg\frac{б}{4}\right)^2} =$$

$$= \frac{(1-tg\frac{б}{4}) \cdot sin б}{1+tg\frac{б}{4}} = tg\,\frac{\text{p}-б}{4} \cdot sin б.$$

30) *XOY* teksligida markazi $O_1(2;3)$ nuqtada bo‘lgan aylana rasmdagidek chizilgan. Aylana *OY* o‘qini *A* va *B* nuqtalarida kesib o‘tadi. *A* nuqtaning ordinatasi *y* ni toping.



Yechish: *Chizmada aylana abssissa o‘qiga uringanligi uchun uning radiusi* $R = 3$ *ga teng bo‘ladi. A nuqtaning koordinatalari A(0;y). Aylana markazining koordinatalari* $O_1(2;3)$.

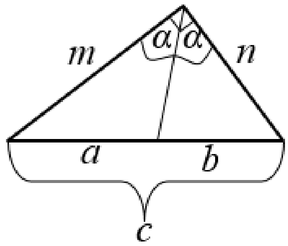
Chizmadan $AO_1 = R$.

A va O_1 *nuqtalar orasidagi masofani topish formulasini qo‘l-lab, quyidagi muno-sabatni yozamiz:*

$3^2 = (2-0)^2 + (3-y)^2$.

Buni so‘ldalashtirib, $y^2 - 6y + 4 = 0$ *tenglamani hosil qilamiz, bundan* $y_1 = 3 - \sqrt{5}$ *va* $y_2 = 3 + \sqrt{5}$ *larga teng. Bunda birinchi yechim A nuqtaga tegishli bo‘lgani uchun masalaning javobi* $3 - \sqrt{5}$ *bo‘ladi.*

31) To‘g‘ri burchakli uchburchakning to‘g‘ri burchagidan gipotenuzaga tushirilgan bissektrissa uni *a* va *b* teng bo‘lgan kes-malarga ajratadi (rasmga qarang). Uchburchakning yuzini toping.



Yechish: *Bissektrissa to‘g‘ri burchakni teng ikkiga ajratib, berilgan uchburchakni o‘xshash uchburchaklarga ajratadi.*

O‘xshash uchburchaklarning qoidasidan foydalanib,quyidagi tenglikni tuzamiz:

$\frac{m}{n} = \frac{a}{b} \cdot$ *Bundan* $m = \frac{a}{b} \cdot n$.

Pifagor teoremasidan foydalanib, $(a+b)^2 = \left(\frac{a}{b} \cdot n\right)^2 + n^2$ *tenglikni hosil qilamiz.*

Bundan $n = \frac{b(a+b)}{\sqrt{a^2+b^2}}$ *bo'ladi. Unda* $m = \frac{a}{b} \cdot n = \frac{a}{b} \cdot \frac{b(a+b)}{\sqrt{a^2+b^2}} = \frac{a(a+b)}{\sqrt{a^2+b^2}}$.

Berilgan uchburchakning yuzi $S = \frac{1}{2} \cdot m \cdot n = \frac{1}{2} \cdot \frac{a(a+b)}{\sqrt{a^2+b^2}} \cdot \frac{b(a+b)}{\sqrt{a^2+b^2}} = \frac{ab(a+b)^2}{2(a^2+b^2)}$.

32) Agar $\angle ACD = 70^0$, $\angle DEF = 30^0$ va $\angle BOC = 110^0$ bo'lsa, $\angle CAF$ ni toping (rasmga qarang).

Yechish: *Chizmada* $OB = OC = OD$, *hamda* $\angle BOC = 110^0$ *bo'lgani uchun* $\angle OCB = \angle OBC = 35^0$, $\angle OCD = \angle OCD = 70^0 - 35^0 = 35^0$.

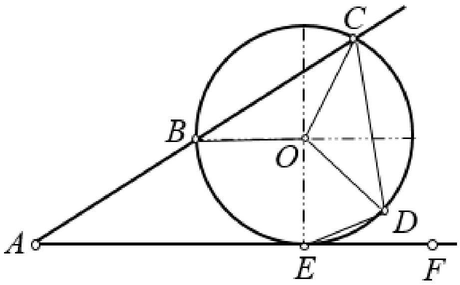
Demak $\angle COD = 110^0$, $\angle DEF = 30^0$ *bo'lgani uchun*

$\angle OED = \angle ODE = \angle DOE = 90^0 - 30^0 = 60^0$.

$\angle BOE = 360^0 - (\angle BOC + \angle COD + \angle DOE) =$
$360^0 - (110^0 + 110^0 + 60^0) = 80^0$.

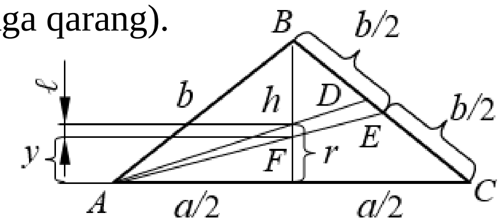
$\angle OBA = 180^0 - \angle OBC = 180^0 - 35^0 = 145^0$.

$\angle CAF = 360^0 - (90^0 + \angle BOE + \angle OBA) =$
$= 360^0 - (90^0 + 80^0 + 145^0) = 45^0$.



33) Yon tomoni $b = 20$ ga teng bo'lgan teng yonli uchburchak *ABC* ning asosai $a = 24$ ga teng.

Uning medianalarining kesishish nuqtasi bilan, unga ichki chizilgan aylana markazi orasidagi masofa ℓ ni toping (rasmga qarang).



Yechish: *Uchburchakning balandligi*

$h = \sqrt{b^2 - (a/2)^2} = \sqrt{20^2 - (24/2)^2} = 16$.

Uchburchakning yuzi $S = \frac{1}{2} \cdot a \cdot h = \frac{1}{2} \cdot 24 \cdot 16 = 192$.

Uchburchakka ichki chizilgan aylana radiusi $r = \frac{2S}{b+b+a} = \frac{2 \cdot 192}{20+20+24} = 6$.

AE mediana uzunligini aniqlaymiz:

$AE = \frac{1}{2} \cdot \sqrt{2b^2 + 2a^2 - b^2} = \frac{1}{2} \cdot \sqrt{2a^2 + b^2} = \frac{1}{2} \cdot \sqrt{2 \cdot (24)^2 + 20^2} = 2\sqrt{97}$.

AE mediana katta bo'lagining uzunligi $AF = \frac{2}{3}AE = \frac{2}{3} \cdot 2\sqrt{97} = \frac{4}{3} \cdot \sqrt{97}$.

Chizmadan $y = \sqrt{(AF)^2 - (a/2)^2} = \sqrt{\left(\frac{4}{3} \cdot \sqrt{97}\right)^2 - (24/2)^2} = \frac{16}{3}$.

Markazi orasidagi masofa $\ell = r - y = 6 - \frac{16}{3} = \frac{2}{3}$.

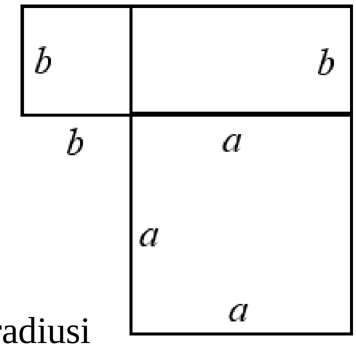
34) Uchburchaning bir burchagi 60^0 ga eng bo'lib, unga tashqi chizilgan aylananing radiusi $\frac{7}{\sqrt{3}}$ ga, ichki chizilgan aylananing radiusi $\sqrt{3}$ ga teng bo'lsa, uni yuzini toping.

Yechish: 60^0 *burchak qrshisidagi tomonni c desak, sinuslar teoremasidan* $c = 2Rsin60^0 = 2 \cdot \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 7$. *Cosinuslar teoremasiga asosan* $7^2 = a^2 + b^2 -$ $-2abcos60^0$ *yuki* $a^2 + b^2 = 49 + ab$. *Uchburchakka tashqi chizilgan aylananing* *radiusi* $\frac{7}{\sqrt{3}} = \frac{7ab}{4S}$ *; uchburchakka ichki chizilgan aylananing radiusi* $\sqrt{3} = \frac{2S}{a+b+7}$ *bulardan* $\frac{ab}{2} = a + b + 7$ *bundan* $a + b = \frac{ab}{2} - 7$ *bu tenlikni ikkala tomonini kvadratga oshirib. Quyidagini hosil qilamiz:*

$a^2 + b^2 = \left(\frac{ab}{2}\right)^2 - 7ab + 49 - 2ab = 49 + ab.$ *Bundan* $\left(\frac{ab}{2}\right)^2 = 10ab.$ *Yuki* $ab \cdot \left(\frac{ab}{4} - 10\right)$. *Bundan* $ab \neq 0$ *va* $ab = 40$. *Berilgan uchburchakning yuzi*

$$S = \frac{1}{2}absin60^0 = \frac{1}{2}40 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}.$$

34) To‘g‘ri to‘rtburchakning perimetri 30 sm, uning ikki qo‘shni tomonlariga chizilgan kvadratlar yuzalarining yig‘indisi 137 sm^2 ga teng bo‘lsa (rasmga qarang), berilgan to‘g‘ri to‘rtburchakning yuzini toping.



Yechish: *Chizmada hosil qilingan kvadratlar yuzalarining yi-g‘indisi* $a^2 + b^2 = 137$ *ga, masala shartiga asosan berilgan to‘g‘ri to‘rtburchakning perimetri* $p = 2(a + b) = 30$ *yoki* $a + b = 15$. *Bundan* $b = 15 - a$. *Shunda* $a^2 + (15 - a)^2 = = 137$. *Bu tenglamani yechib,* $a = 11\, sm$ *ekanligini aniqlaymiz.* $b = 15 - 11 = 4\, sm$. *Berilgan to‘g‘ri to‘rtburchakning yuzi* $S = a \cdot b = 11 \cdot 4 = 44\, sm^2$.

35) Muntazam oltiburchakka tashqi chizilgan aylananing radiusi $R = 4\sqrt{3}$ ga teng bo‘lsa, uning kichik diagonali d_{ki} ni toping.

Yechish: *Ko‘pburchak muntazam oltiburchak bo‘lgani uchun uning bitta ichki burchagi, ya’ni qo‘shni tomonlari orasidagi burchak* $б_i = 120^0$ *ga va uning tomoni unga tashqi chizilgan aylana radiusiga* $a = R$ *ga teng bo‘ladi. Kichik diagonal esa shu burchak qarshisidasgi kesma uzunligidir. Kichik diagonalni kosinuslsr teoremasidan foydalanib aniqlaymiz:*

$$d_{ki} = \sqrt{R^2 + R^2 - 2 \cdot R \cdot R \cdot cosб} = \sqrt{2R^2(1 - cosб)} =$$
$$= \sqrt{2\left(4\sqrt{3}\right)^2(1 - cos120^0)} = 12.$$

36) Diagonallarining soni tomonlarining sonidan 1,5 marta ko‘p bo‘lgan qavariq muntazam ko‘pburchakning ichki burchaklari va bitta tashqi burchagining yig‘indisini toping.

Yechish: *Ko‘pburchak diagonallarining soni* $d_n = \frac{n(n-3)}{2} \cdot$ *Masala shartiga ko‘ra* $\frac{n(n-3)}{2} = \frac{3n}{2} \cdot$ *Bundan* $n = 6$. *Ya’ni berilgan ko‘pburchak muntazam oltiburchak ekan. Uning bitta ichki burchagi* $б_i = 120^0$ *ga, bitta tashqi burchagi* $б_t = 60^0$ *ga teng bo‘ladi. Muntazam oltiburchakning ichki burchaklarining yig‘indisi*

$\Sigma б_i = 180^0(6 - 2) = 720^0$.

Masala shartiga ko‘ra $\Sigma б_i + б_t = 720^0 + 60^0 = 780^0$.

37) Muntazam oltiburchakning tomoni $a = \sqrt{108}\, sm$ ga teng. Unga tashqi va ichki chizilgan aylana radiuslarining nisbatini toping.

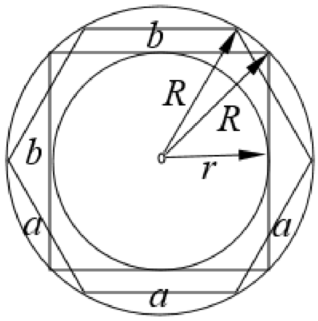
Yechish: *Muntazam oltiburchakning tomoni unga tashqi chizilgan aylana radiusiga* $a = R$ *va radiuslari hamda tomonidan tuzilgan uchburchak muntazam bo'lib ular orasidagi burchak* $б = 60^0$ *ga teng bo'ladi.*

Unga ichki chizilgan aylana radiusi $r = R \cdot sinб = R \cdot sin60^0 = \sqrt{108} \cdot \frac{\sqrt{3}}{2} = 9 \cdot \frac{R}{r} = \frac{\sqrt{108}}{9} = \frac{6\sqrt{3}}{9} = \frac{2\sqrt{3}}{3}$.

38) Aylanaga ichki chizilgan muntazam oltiburchakning tomoni 12 ga teng. Shu aylanaga kvadrat ham ichki chizilgan (rasmga qarang). Shu kvadratga ichki chizilgan doiraning yuzini toping.

Yechish: *Masala shartiga asosan, muntazam oltiburchakka aylana tashqi chizilgan, demak,* $a = R = 12.$ *Aylanaga ichki chizilgan kvadratning diagonali aylana diametriga teng bo'ladi* $d = 2R$.

Bu diagonalning uzunligi $(2R)^2 = b^2 + b^2$.



Bundan kvadratning tomoni $b = \sqrt{2}R = 2r$. *Kvadratga ichki chizilgan doiraning radiusi* $r = \frac{\sqrt{2}}{2} \cdot R$ *ga teng bo'ladi. Kvadratga ichki chizilgan doiraning yuzi*

$$S = p \cdot \left(\frac{\sqrt{2}}{2} \cdot R\right)^2 = p \cdot \left(\frac{\sqrt{2}}{2} \cdot 12\right)^2 = 72p.$$

3.6. Fazoviy jismlar.

3.6.1. Prizma, uning turlari va asosiy ko'rsatkichlari. Prizma deb, asosi ko'pburchakdan (*ABCDE*) qolgan yon yoqlari parallelgramlardan (A_1A E_1E) tashkil topgan geometrik shaklga aytiladi. Ikki yoqli tomonini kesishgan qismiga qirra (*AB*, A_1A) deb ataladi.

To'g'ri prizmalarda yon qirra, prizmaning balandligiga $AA_1 = H$ teng bo'ladi.

a-rasmda $DB_1 = d -$ diaganolali, $AC = d_{as} -$ asosining diagonali.

Uning quyidagi turlari mavjud:

1) Ixtiyoriy (3.23*a*-rasm).

2) Uchburchakli (3.23*b*-rasm).

3) To'rtburchakli, uni o'latda parallelepiped deb ham ataladi (3.23*c*-rasm), barcha qirralari teng bo'lgan parallelepipedga kub deb ataladi (3.23*d*-rasm).

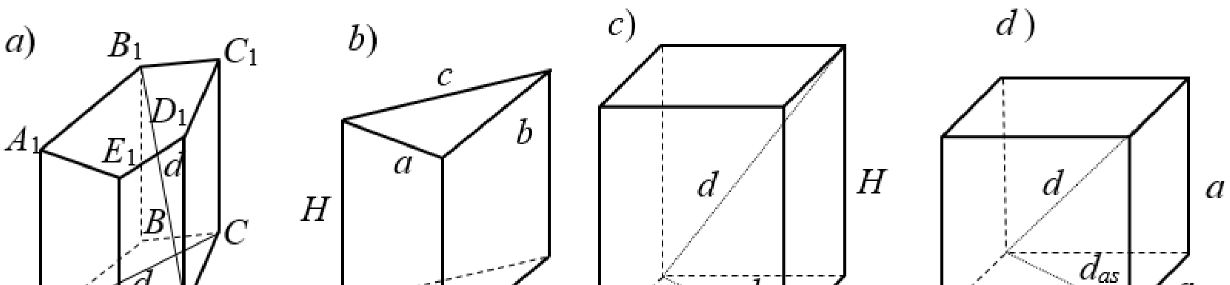
4) Muntazam prizma, ya'ni asosi muntazam uchburchakli, muntazam to'rtburchakli, muntazam beshburchakli (3.23*e*-rasm), muntazam oltiburchakli (3.23*f*-rasm).

5) Og'ma prizma (3.23*g*-rasm).

3) To'rtburchakli, uni o'latda parallelepiped deb ham ataladi (3.23*c*-rasm), barcha qirralari teng bo'lgan parallelepipedga kub deb ataladi (3.23*d*-rasm).

4) Muntazam prizma, ya'ni asosi muntazam uchburchakli, muntazam to'rtburchakli, muntazam beshburchakli (3.23*e*-rasm), muntazam oltiburchakli (3.23*f*-rasm).

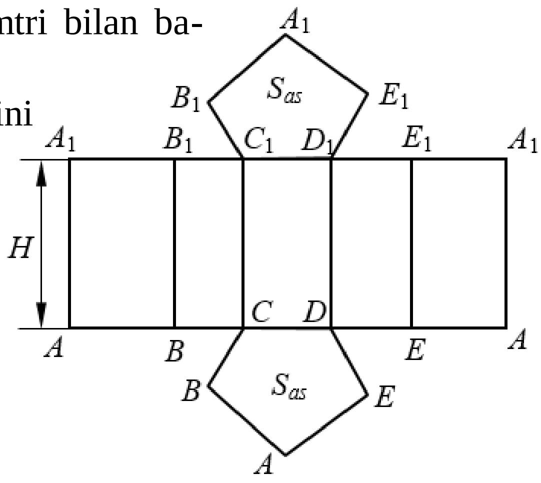
5) Og'ma prizma (3.23*g*-rasm).



Ixtiyoriy prizmaning asosiy ko'rsatkichlari:

1. Prizmaning hajmi, asosining yuzi bilan balandligining ko'paytmasiga teng: $V = S_{as} \cdot H,$ kub birlik.

2. Prizma yon sirtining yuzi, asosining perimtri bilan balandligining ko'paytmasiga teng: $S_{yon} = p \cdot H.$



Buni isbotlash uchun prizmaning AA_1 qirrasini qirqib (3.23a-rasm) yoyamiz (rasmga qarang).

Prizma yon sirtining yuzi chizmadagi beshta to'g'ri
Burchakli to'rtburchakning yuziga teng:
$S_{yon} = (AB + BC + CD + DE + EA) \cdot H = p \cdot H.$

3. Prizmaning to'la sirti, uning yon sirti yuzi bilan ikkita asosi yuzalarining yig'indisiga teng:

$S_{t.srt} = S_{yon} + 2S_{as}.$

4. Prizmaning diagonallari soni quyidagicha aniqlanadi $n(n-3)$.

Parallelepipedning asosiy ko'rsatkichlari:

1. Parallelepipedning hajmi, asosining yuzi bilan balandligining ko'paytmasiga teng: $V = S_{as} \cdot H,$ kub birlik.

2. Parallelepiped yon sirtining yuzi, asosining perimtri bilan balandligining ko'-paytmasiga teng: $S_{yon} = p \cdot H.$

3. Parallelepipedning to'la sirti, uning yon sirti yuzi bilan ikkita asosi yuzalarining yig'indisiga teng: $S_{t.srt} = S_{yon} + 2S_{as}.$

Og'ma prizmaning hajmi, ko'ndalang kesm yuzasini yon qirrasi ko'paytmasiga teng.

$V = S_k \cdot \ell$. Bu yerda S_k - og‘ma prizma o‘qiga perpendikulyar o‘tkazilgan tekislikdan hosil bo‘lgan yuza; ℓ - og‘ma prizmaning qirrasi. Agar prizma gorizontga nisbatan б *burchak tashkil qilsa (g-rasm), $\ell = \frac{H}{sin б}$ bo‘ladi.*

Unda og‘ma prizmaning hajmi $V = S_k \cdot \frac{H}{sin б}$.

Kubning asosiy ko‘rsatkichlari:

1. Kubning hajmi, asosining yuzi bilan balandligining ko‘paytmasiga teng:

$$V = S_{as} \cdot H = a^2 \cdot a = a^3, \text{ kub birlik.}$$

2. Kub yon sirtining yuzi, asosining perimtri bilan balandligining ko‘paytmasiga teng: $S_{yon} = p \cdot H = 4 \cdot a^2$.

3. Kubning to‘la sirti, uning yon sirti yuzi bilan ikkita asosi yuzalarining yig‘indisiga teng:

$$S_{t.srt} = S_{yon} + 2S_{as} = 4 \cdot a^2 + 2 \cdot a^2 = 6 \cdot a^2.$$

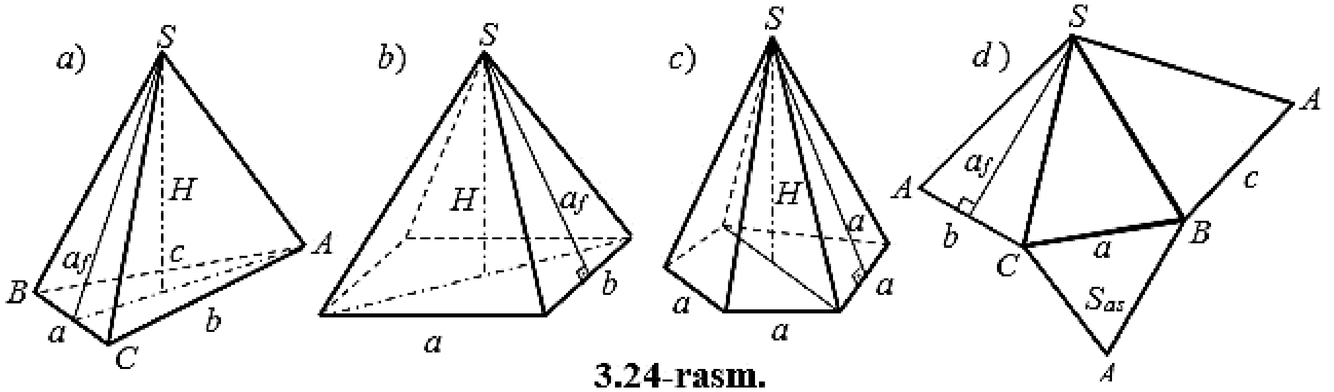
3.6.2. Piramida, kesik piramida.

Piramida deb, asosi ko‘pburchakdan, qolgan yon yoqlari uchburchakdan iborat bo‘lib, umumiy bir uchga birlashgan geometrik shaklga aytiladi (3.24*a, b, c-rasmlar*).

Piramida uchidan asosining tomoniga tishirilgan perpendikulyar kesmaga apofema deb ataladi (3.24*a*-rasm). Agar piramida muntazam bo‘lsa, uning asosini tomonlari teng bo‘ladi (3.24*c*-rasm).

Piramidaning asosiy ko‘rsatkichlari:

1. Piramidaning hajmi, asosining yuzi bilan balandligining ko‘paytmasining uchdan bir qismiga teng: $V = \frac{1}{3} \cdot S_{as} \cdot H,$ kub birlik.



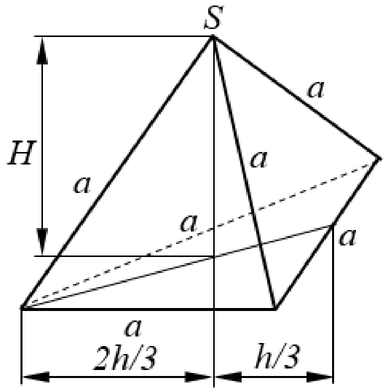
3.24-rasm.

2. Piramidaning yon sirtining yuzi, asosining perimtri bilan apofemasi ko‘paytmasining yarmiga teng (3.24*d*-rasm): $S_{yon} = \frac{1}{2} \cdot p \cdot a_f$.

3. Piramidaning to‘la sirtining yuzi, uning yon sirti yuzi bilan asosi yuzasining yig‘indisiga teng (3.24*d*-rasm): $S_{t.srt} = S_{yon} + S_{as}$.

4. Uchburchakli piramida qirralari ℓ o‘zaro perpendikulyar bo‘lsa, uni hajmi quyidagicha aniqlanadi: $V = \frac{1}{6} \cdot \ell_1 \cdot \ell_2 \cdot \ell_3$.

Tetraedr: Barcha qirralari teng bo‘lgan muntazam pira-midaga **tetraedr** deyiladi (rasmga qarang).



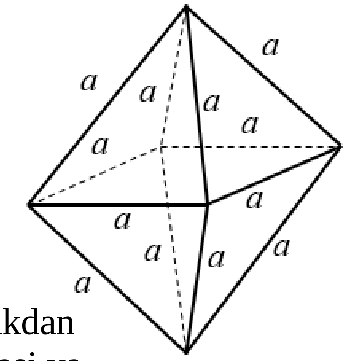
1. To‘la sirti: $S_{t.s} = \sqrt{3}a^2$. 2. Balandligi: $H = \frac{\sqrt{6}}{3} \cdot a$.

3. Hajmi: $V = \frac{\sqrt{2}}{12} \cdot a^3$. 4. Tashqi chizilgan aylana radiusi:

$R = \frac{\sqrt{6}}{4} \cdot a$. 5. Ichki chizilgan aylana radiusi:

$r = \frac{\sqrt{6}}{12} \cdot a$.

Oktaedr: Barcha sakkizta yoqlari teng bo‘lgan muntazam piramidaga **oktaedr** deyiladi (rasmga qarang). Uning 6 ta uchi va 12 ta qirrasi bo‘ladi.



1. To‘la sirti: $S_{t.s} = 2\sqrt{3}a^2$. 2. Hajmi: $V = \frac{\sqrt{2}}{3} \cdot a^3$.

3. Tashqi chizilgan aylana radiusi: $R = \frac{\sqrt{2}}{2} \cdot a$.

4. Ichki chzilgan aylana radiusi: $r = \frac{\sqrt{6}}{6} \cdot a$.

Tetraedr va oktaedrning tomonlari muntazam uchburchakdan tashkil topgan bo‘la-di, uning yon tomoni balandligi, medianasi va bissiktrissasi teng bo‘ladi: $h = m = д = \frac{\sqrt{3}}{2} \cdot a$.

Kesik piramida: Piramidaning uchidan ma’lum masofada asosiga parallel ravishda qirqilsa, kesik piramida hosil bo‘ladi, shunda uning yon yoqlari trapetsiyalardan tashkil topgan bo‘lib, uning ostki Q va ustki q asoslari bo‘ladi (3.25a, b-ramlar). Agar kesik piramida muntazam bo‘lsa, uning asoslarining tomonlari teng bo‘ladi (3.25b-rasm), diagonal kesimi teng yonli trapetsiyadan (3.25c-rasm) tashkil topadi.

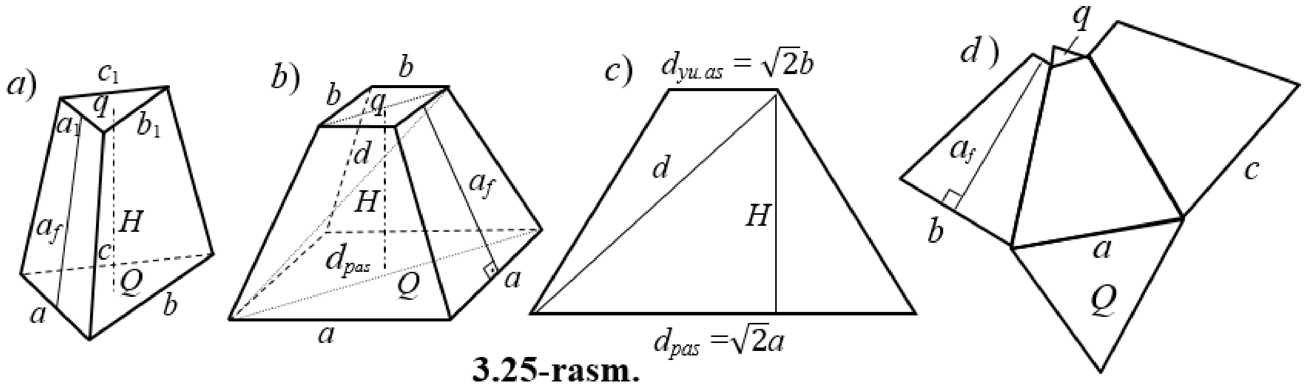
Kesik piramidaning asosiy ko‘rsatkichlari:

1. Kesik piramidaning hajmi: $V = \frac{1}{3} \cdot H\left(Q + \sqrt{Qq} + q\right)$, kub birlik.

2. Kesik piramidaning yon sirtining yuzi ostki va ustki asoslari perimetrlari yig‘indisi bilan apofemasi ko‘paytmasining yarmiga teng (3.25d-rasm):

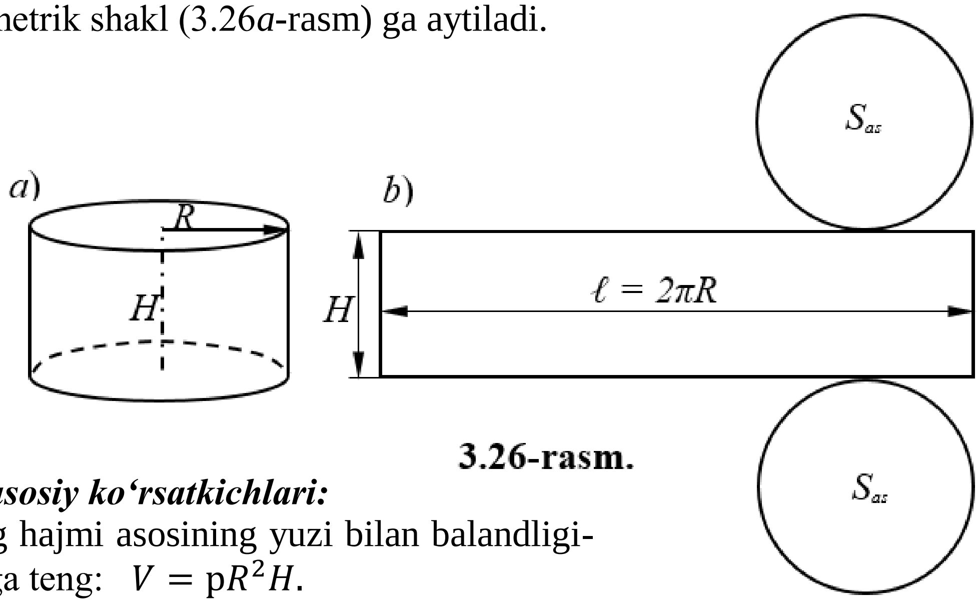
$$S_{yon} = \frac{1}{2} \cdot (p_{osa} + p_{uas}) \cdot a_f.$$

3. Kesik piramidaning to‘la sirtining yuzi, uning yun sirti yuziga ostki va ustki asoslari yuzalari qo‘shiladi (3.25d-rasm): $S_{t.srt} = S_{yon} + 2S_{as}$.



3.25-rasm.

3.6.3. Silindr. Silindr deb, to‘g‘ri to‘rtburchani yon tomoni atrofida aylantirishdan hosil bo‘lgan geometrik shakl (3.26a-rasm) ga aytiladi.



3.26-rasm.

Silindrning asosiy ko‘rsatkichlari:

1. Silindrning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng: $V = \text{p}R^2H$.

2. Silindrning yon sirti asos aylanasi uzunligi bilan balandligining ko‘paytmasiga teng (3.26b-rasm): $S_{yon} = 2\text{p}RH$.

3. Silindrning to‘la sirti yon sirti yuzi bilan asoslari yuzalarining yig‘indisiga teng:

$$S_{t.srt} = S_{yon} + 2S_{as} = 2\text{p}RH + 2\text{p}R^2 = 2\text{p}R(H + R).$$

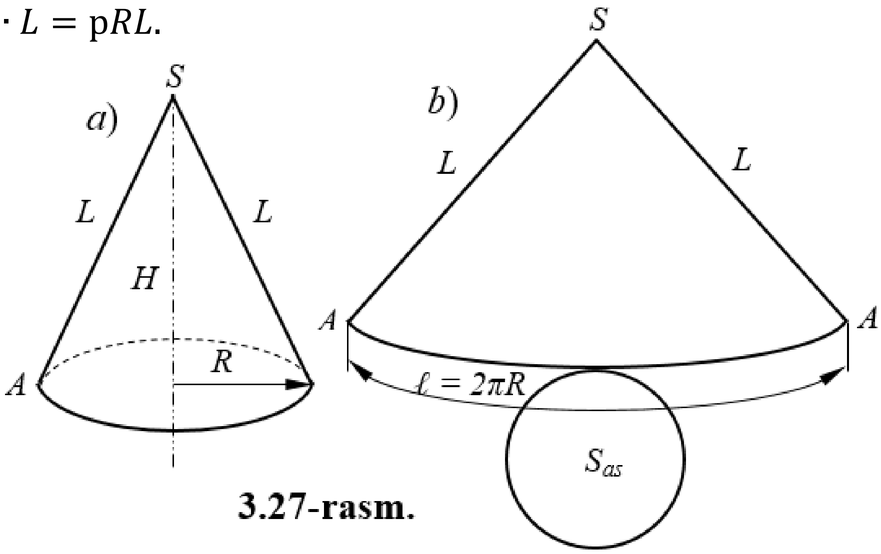
3.6.4. Konus, kesik konus. Konus deb, to‘g‘ri burchakli uchburchani biror kateti atrofida aylantirishdan hosil bo‘lgan geometrik shakl (3.27a-rasm) ga aytiladi.

Konusning asosiy ko‘rsatkichlari:

1. Konusning hajmi asosining yuzi bilan balandligi ko‘paytma-sining uchdan bir qismiga teng: $V = \frac{1}{3} \cdot \text{p}R^2 \cdot H$.

2. Konusning yon sirti asos aylanasi uzunligi bilan yasovchisi ko‘paytmasining yarmiga teng (3.27b-rasm):

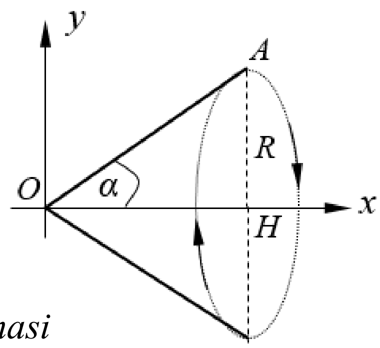
$S_{yon} = \frac{1}{2} \cdot 2\text{p}R \cdot L = \text{p}RL$.



3.27-rasm.

3. Konusning to‘la sirti, yon sirti va asosi yuzasining yig‘indisiga teng:

$$S_{t.srt} = S_{yon} + S_{as} = \text{p}RL + \text{p}R^2 = \text{p}R(L + R).$$



Konusning hajmi keltirib chiqarish: *Buning uchun, tekslikdan biror bir A nuqtani olib, uni koordinata boshi bilan tutashtiramiz, bunda hosil bo‘lgan OA to‘g‘ri chiziq abssissa o‘qi bilan* б *burchakni tashkil qiladi.*

Agar OA to‘g‘ri chiziqni OX o‘qi atrofida aylantirsak, konus hosil bo‘ladi (chizmaga qarang). Shunda H konusning balandligi, R konus asosining radiusi.

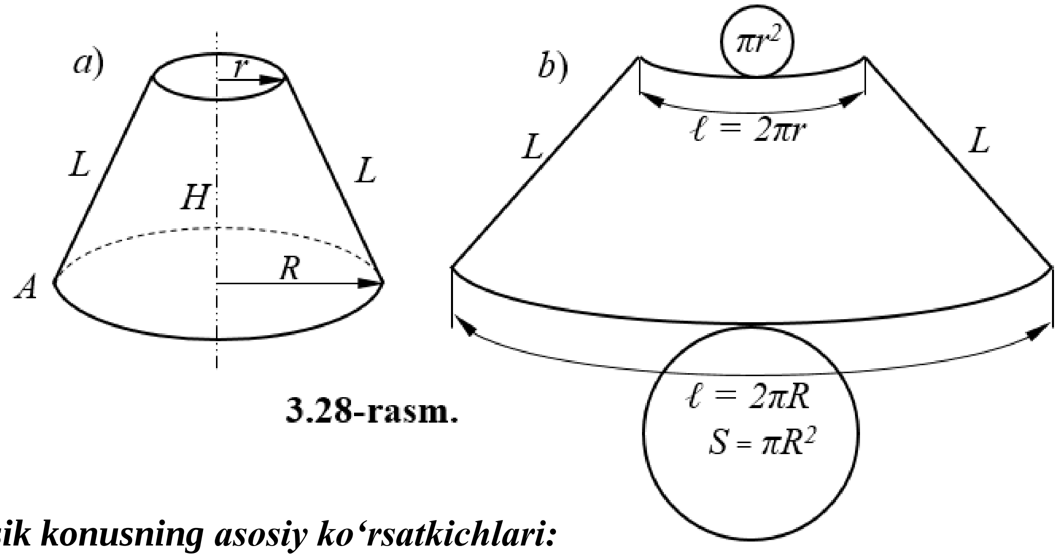
Koordinata boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasi $y = kx$ *ga teng. Bu yer-da k, to‘g‘ri chiziqning burchak koeffiitsiyenti bo‘lib, u* $k = \frac{R}{H}$ *ga teng.*

Konusning hajmini aniq integral formulasidan foydalanib aniqlash mumkin:

$$V = \text{p}\int_a^b f^2(x)dx = \text{p}\int_0^H (kx)^2 dx = \text{p}\frac{R^2}{H^2}\int_0^H (x)^2 dx = \text{p}\frac{R^2}{H^2}\cdot\frac{x^3}{3} =$$

$$= \text{p}\frac{R^2 \cdot H^3}{3H^2} = \frac{1}{3}\cdot \text{p}R^2H.$$

Kesik konus: Kesik konus deb, ten yonli trapetsiyaning vertikal o‘qi atrofida aylan-tirishdan hosil bo‘lgan geometrik shaklga aytiladi (3.28a-rasm).



3.28-rasm.

Kesik konusning asosiy ko‘rsatkichlari:

1. Kesik konusning hajmi: $V = \frac{\text{p}}{3}\cdot H(R^2 + Rr + r^2)$, kub birlik.

2. Kesik konusning yon sirti, ostki va ustki aylana uzunliklarining yarmiga yasov-chisining ko‘paytmasiga teng (3.28b-rasm):

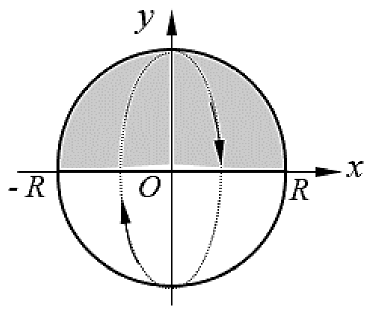
$$S_{yon} = \frac{1}{2}\cdot(2\text{p}R + 2\text{p}r)\cdot L = \text{p}L(R + r).$$

3. Kesik konusning to‘la sirti, yon sirti va ostki hamda ustki asoslari yuzalarining yig‘indisiga teng (b-rasm): $S_{t.srt} = S_{yon} + S_{oas} + S_{uas} = \text{p}L(R + r) + \text{p}R^2 + \text{p}r^2$.

3.6.5. Shar va uning bo‘laklari.

Shar, sfera deb ham yuritiladi.

Yarim doirani diametri atrofidan aylantirishdan hosil bo‘lgan geometrik shaklga shar deb ataladi.



Uni doiradan farq qilishi uchun uning belbog‘i ko‘rsatiladi (3.29*a*-rasm).

Uning bo‘laklariga segment (3.29*b*-rasm) va sektor (3.29*c*-rasm) lar kiradi.

Sharning asosiy ko‘rsatkichlari:

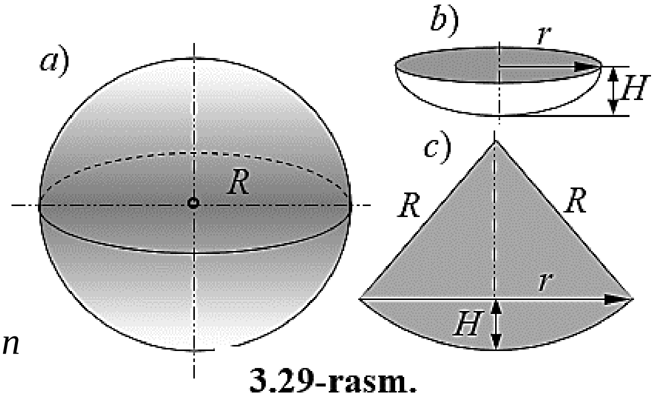
1. Sharning hajmi: $V = \frac{4}{3} \cdot \text{p}R^3$.

2. Sharning sirti: $S = 4\text{p}R^2$.

Shar hajmi formulasini keltirib chiqarish:

Buning uchun, yarim doirani markazini koordinata markaziga qo‘yib, uni OX o‘qi atrofida aylantirsak, shar hosil bo‘ladi (chizmaga qarang).

Aylana tenglamasi $y = \sqrt{R^2 - x^2}$ *ga teng. Sharning hajmini aniq integral formulasidan foydalanib aniqlash mumkin:*



3.29-rasm.

$$V = \text{p}\int_a^b f^2(x)dx = \text{p}\int_{-R}^{R}\left(\sqrt{R^2 - x^2}\right)^2 dx = \text{p}\int_{-R}^{R}(R^2 - x^2)dx = R^2x - \frac{x^3}{3} = \frac{4}{3} \cdot \text{p}R^3.$$

Shar segmentining ko‘rsatkichlari:

1. Segmentning radiusi: $r = \sqrt{H(2R - H)}$.

bu yerda *R*-sharning radiusi, *H*-segmentning balandligi (3.29*b*-rasm).

2. Segmentning yon sirti: $S_{yon} = 2\text{p}RH = \text{p}(r^2 + H^2)$.

3. Segmentning to‘la sirti: $S_{t.srt} = \text{p}(r^2 + 2RH)$.

4. Segmentning hajmi: $V_{seg} = \text{p}H^2\left(R - \frac{H}{3}\right)$.

Shar sektorining ko‘rsatkichlari:

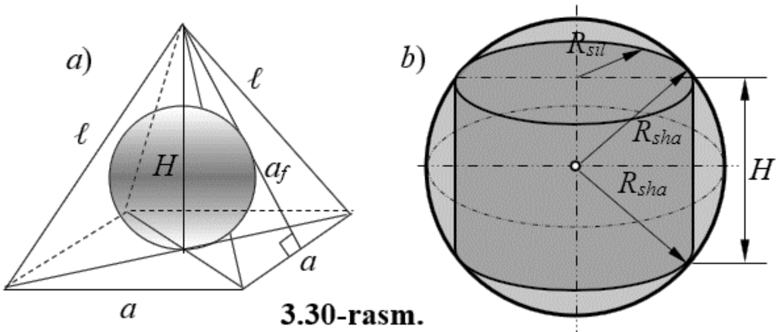
1. Sektorning yon sirti: $S_{yon} = \text{p}Rr$.

2. Sektorning to‘la sirti: $S_{t.srt} = \text{p}Rr + 2\text{p}RH = \text{p}R(2H + r)$.

3. Sektorning hajmi: $V_{sek} = \frac{2}{3} \cdot \text{p}R^2 \cdot H$.

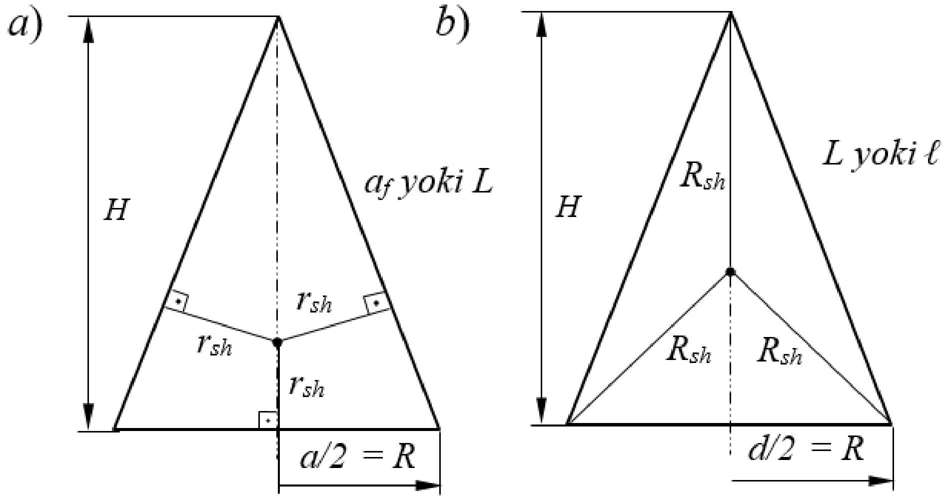
O‘xshash bo‘lgan ikki jism hajimlarining nisbati ularning mos chiziqli o‘lchamlari nisbatining kubiga teng: $\frac{V_1}{V_2} = \left(\frac{a_1}{a_2}\right)^3 = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{p_1}{p_2}\right)^3$.

3.6.6. Fazoviy shakllarga ichki (3.30*a*-rasm) va tashqi (3.30*b*-rasm) chizilgan sharlar.



3.30-rasm.

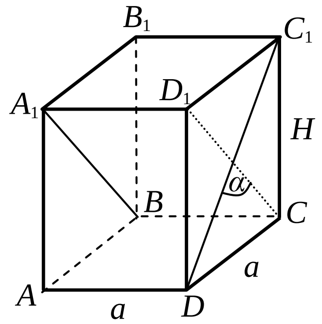
Fazoviy jismlarga shar ichki chizilgan bo‘lsa, uni o‘q kesimi bo‘yicha qirqilsa, u teng yonli uchburchakka ichki chizilgan aylana radiusi *r*, shar ichki chizilgan shar radiusi r_{sh} ga teng bo‘ladi (3.31a-rasm). Fazoviy jismlarga shar tashqi chizilgan bo‘lsa, uni o‘q kesimi bo‘yicha qirqilsa, u teng yonli uchburchakka tashqi chizilgan aylana radiusi *R*, shar ichki chizilgan shar radiusi R_{sh} ga teng bo‘ladi (3.31b-rasm).



3.31-rasm.

FAZOVIY SHAKLLARGA DOIR MASALALAR.

1) Muntazam to‘rtburchakli prizmaning asosi $a = 8$ ga, balandligi $H = 12$ ga teng (rasmga qarang). Prizma parallel yon yoqlarining o‘zaro ayqash diagonallari orasidagi o‘tkir burchakni toping.



<u>Yechish:</u> *Yon yoqlarining* ***ayqash*** *diagonallar A_1B va C_1D lardir. A_1B ni parallel ko‘chirib, D_1C ni hosil qilamiz. Masala shartiga asosan* б *burchakni topish talab qilinadi. Buni uchun D_1C va C_1D diagonallar orasidagi burchakni topish kerak.*

Chizmadan $C_1D = \sqrt{H^2 + a^2} = \sqrt{12^2 + 8^2} = 4\sqrt{13}$.

Kosinuslar teoremasidan foydalanib;

$$a^2 = \left(\frac{C_1D}{2}\right)^2 + \left(\frac{C_1D}{2}\right)^2 - 2\left(\frac{C_1D}{2}\right)^2 cos\text{б};$$

yoki $8^2 = 2\left(\frac{4\sqrt{13}}{2}\right)^2 (1 - cos\text{б})$: *Bundan* $cos\text{б} = \frac{5}{13}\cdot H$. $\text{б} = arccos\,\frac{5}{13}$.

2) Qirrasi $a = 12$ ga teng bo‘lgan kub yoqlarining markazlarini tutashtirishdan hosil bo‘lgan jismni hajmini toping.

<u>Yechish:</u> *Kub yoqlarining markazlarini tutashtirilganda qirrasi b ga teng bo‘lgan oktaedr hosil bo‘ladi.* $b = \frac{\sqrt{2}}{2}\cdot a = \frac{\sqrt{2}}{2}\cdot 12 = 6\sqrt{2}$. *Oktaedrning hajmi:*

$$V = \frac{\sqrt{2}}{3}\cdot b^3 = \frac{\sqrt{2}}{3}\cdot\left(6\sqrt{2}\right)^3 = 288.$$

3) Barcha qirralari teng bo‘lgan muntazam uchburchakli prizma asosining medianasi $m = 6\sqrt{3}$ ga teng. Shu prizmaning hajmini toping.

Yechish: *Prizmaning qirralari teng bo‘lgani uchun uning balandligi asosining tomoniga teng bo‘ladi* $a = H$. *Prizmaning asosi muntazam uchburchakdan tashkil topgani uchun asosining medianasi, asosining balandligiga teng bo‘ladi*

$$m = h = \frac{\sqrt{3}}{2} \cdot a.$$

Bundan $a = \frac{2}{\sqrt{3}} \cdot m = \frac{2}{\sqrt{3}} \cdot 6\sqrt{3} = 12.$ *Prizma asosining yuzi:* $S_{as} = \frac{\sqrt{3}}{4} \cdot a^2.$

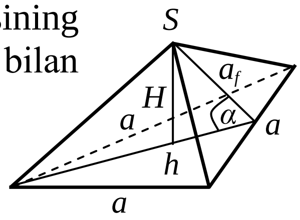
Prizmaning hajmi: $V = S_{as} \cdot H = \frac{\sqrt{3}}{4} \cdot a^2 \cdot a = \frac{\sqrt{3}}{4} \cdot a^3 = \frac{\sqrt{3}}{4} \cdot (12)^3 = 432\sqrt{3}.$

4) Uchburchakli piramida qirralari o‘zaro perpendikulyar bo‘lib, ularning uzunliklari mos ravishda $\sqrt{70}$, $\sqrt{99}$ va $\sqrt{126}$ ga teng bo‘lsa, uni hajmini toping.

Yechish: *Qirralari o‘zaro perpendikulyar bo‘lgan uchburchakli piramidaning hajmi:*

$$V = \frac{1}{6} \cdot \ell_1 \cdot \ell_2 \cdot \ell_3 = \frac{1}{6} \cdot \sqrt{70} \cdot \sqrt{99} \cdot \sqrt{126} = 21\sqrt{55}.$$

5) Muntazam uchburchakli piramidaning balandligi asosining tomonidan olti marta kichik. Piramidaning yon yog‘i asos tekisligi bilan qanday burchak tashkil qiladi.



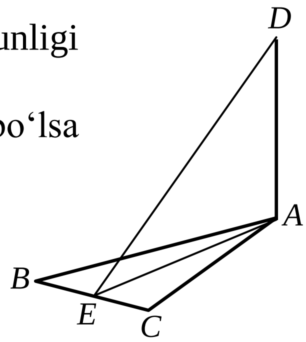
Yechish: *Masala shartiga asosan* $H = \frac{1}{6} \cdot a.$

Chizmadan $h = \frac{\sqrt{3}}{2} \cdot a.$ $\frac{1}{3} \cdot h = \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot a = \frac{\sqrt{3}}{6} \cdot a.$

$$tgб = \frac{\frac{1}{6}a}{\frac{\sqrt{3}}{6}a} = \frac{1}{\sqrt{3}} \cdot \quad б = 30^0.$$

6) Teng yonli *ABC* uchburchakning $AB = AC$. *A* uchidan uzunligi $AD = 32$ ga teng bo‘lgan *AD* perpendikulyar o‘tkazildi.

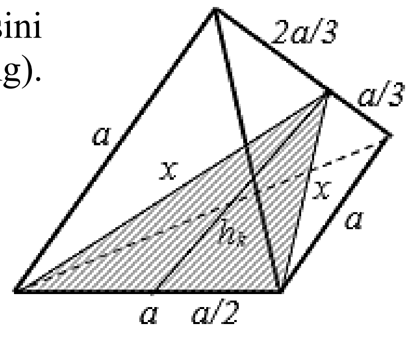
D nuqtadan *BC* tomongacha bo‘lgan masofa $DE = 40$ bo‘lsa (rasmga qarang), uchburchak asosining balandligini toping.



Yechish: *Chizmadan DAE uchburchak to‘g‘ri burchakli uchburchak.*

$$AE = \sqrt{(DE)^2 - (AD)^2} = \sqrt{(40)^2 - (32)^2} = 24.$$

7) Tetraedrning qirrasi *a* ga teng. Uning yon qirrasini 2:1 nisbatda bo‘luvchi nuqta va shu qirra qarshisidagi asosini tomo-nini kesib o‘tuvchi tekislik o‘tkazildi (rasmga qarang). Kesim yuzini toping.



Yechish: *Tekslik kesib o‘tgan kesim yon tomoni x ga teng bo‘lgan teng yonli uchburchakdir. Tetraedrning tomonlari muntazam uchburchaklarni va tomonlari orasidagi burchak,* 60^0*ni tashkil qiladi. Kesimni yon tomonini kosinuslar teoremasidan:*

$$x = \sqrt{a^2 + \left(\frac{a}{3}\right)^2 - 2a \cdot \frac{a}{3} \cdot cos60^0} = \frac{\sqrt{7}}{3} \cdot a.$$

Kesimni balandligi: $h_k = \sqrt{x^2 - \left(\frac{a}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{7}}{3}a\right)^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{19}}{6} \cdot a.$

Kesimning yuzi: $S_k = \frac{1}{2} \cdot б \cdot h_k = \frac{1}{2} \cdot б \cdot \frac{\sqrt{19}}{6} \cdot a = \frac{\sqrt{19}}{12} \cdot a^2.$

8) Muntazam to‘rtburchakli piramidaning hajmi $V = 19200$ ga, balandligi $H = 9$ ga teng bo‘lsa, uni apofemasini toping.

Yechish: *Piramida muntazam to‘rtburchakli bo‘lgani uchun uning asosi kvadratdan tashkil topgan bo‘ladi, shunda uni hajmi* $V = \frac{1}{3} \cdot a^2 \cdot H.$

Demak, $19200 = \frac{1}{3} \cdot 9 \cdot a^2$. *Bundan* $a^2 = 6400.$

Yoki $a = 80$. Piramidaning apofemasi $a_f = \sqrt{H^2 + \left(\frac{a}{2}\right)^2} = \sqrt{9^2 + \left(\frac{80}{2}\right)^2} = 41.$

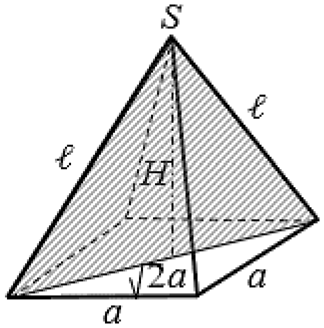
9) Muntazam to‘rtburchakli piramidaning yon sirti $S_{yon} = 324\sqrt{2}$, asosidagi ikki yoqli burchak $p/4$ ga teng bo‘lsa, piramida hajmini toping.

Yechish: *Piramida muntazam to‘rtburchakli bo‘lgani uchun uning asosi kvadratdan tashkil topgan bo‘ladi. Masala shartiga asosan asosidagi ikki yoqli burchak* 45^0*ga teng bo‘lgani uchun piramidaning balandligi asosi tomonining yarmiga* $H = a/2$ *ga teng bo‘ladi. Piramidaning yon sirti:*

$S_{yon} = \frac{1}{2} \cdot 4a \cdot a_f$. *Apofema* $a_f = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{2}}{2} \cdot a.$

Shunda $324\sqrt{2} = \sqrt{2}a^2$. *Bundan* $a = 18.$

Piramidaning hajmi $V = \frac{1}{3} \cdot a^2 \cdot H = \frac{1}{6} \cdot a^3 = \frac{1}{6} \cdot (18)^3 = 972.$



10) Muntazam to‘rtburchakli piramidaning balandligi $H = 9$ ga, diagonal kesmi-ning yuzi $S_k = 54$ ga teng bo‘lsa (rasmga qa-rang), uni hajmini toping.

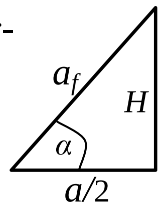
Yechish: *Piramida muntazam to‘rtburchakli bo‘lgani uchun uning asosi kvadratdan tashkil topgan bo‘ladi.*

Asosining diagonali $d_{as} = \sqrt{a^2 + a^2} = \sqrt{2}a$ *ga teng bo‘ladi.*

Kesimning yuzi: $S_k = \frac{1}{2} \cdot d_{as} \cdot H = \frac{1}{2} \cdot \sqrt{2}a \cdot H$. *Bundan* $a = \frac{\sqrt{2}S}{H}.$

Piramidaning hajmi: $V = \frac{1}{3} \cdot a^2 \cdot H = \frac{1}{3} \cdot \frac{2S^2}{H^2} \cdot H = \frac{2}{3} \cdot \frac{S^2}{H} = \frac{2}{3} \cdot \frac{54^2}{9} = 216.$

11) Muntazam to‘rtburchakli piramida asosining $a = 5$ ga, to‘la sirti esa $S_{t.s} = 65$ ga teng bo‘lsa, uni yon yog‘i asos tekisligi bilan qanday bur-chak hosil qiladi (rasm).

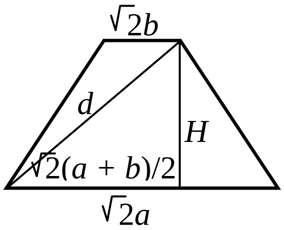


Yechish: *Piramidaning to‘la sirti* $S_{t.s} = 2a \cdot a_f + a^2$ *ga teng.*

Yoki $65 = 2 \cdot 5 \cdot a_f + 25$. *Bundan* $a_f = 4.$

Chizmadan $\frac{a/2}{a_f} = cosб$. *Yoki* $cosб = 5/8$. *Bundan* $б = arccos\, 5/8.$

12) Muntazam to‘rtburchakli kesik piramida asoslarining tomonlari 3 va 5 *sm* ga, diagonali esa $2\sqrt{17}$ ga teng bo‘lsa, uni balandligi necha *sm*.



Yechish: *Kesik piramidaning diagonal kesimidan unig balandligini (rasmga qarang) topamiz:*

$$H = \sqrt{d^2 - \left(\sqrt{2}(a+b)/2\right)^2}.$$

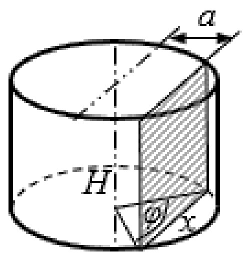
$$H = \sqrt{\left(2\sqrt{17}\right)^2 - \left(\sqrt{2}(5+3)/2\right)^2} = 6.$$

13) Silindrning balandligi va asosining radiusi $H = R = 8$ ga teng. Yuzi silindrning to‘la sirtiga teng bo‘lgan doiraning radiusini toping.

Yechish: *Silindrni to‘la sirti* $S_{t.s} = 2\text{p}R^2 + 2\text{p}RH = 4\text{p}R^2 = \text{p}R_d^2$.

Bundan $R_d = 16$.

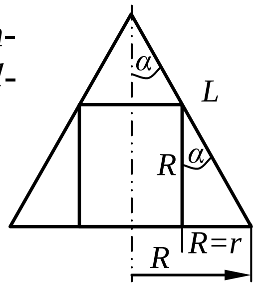
14) Balandligi *H* bo‘lgan silindr o‘qidan *a* masofada unga parallel ravishda o‘tkazilgan tekislik uni asosidan ц kattalikdagi yoyni ajratdi. Tekislikning silindrdan ajratgan kesim yuzini toping (rasmga qarang).



Yechish: *Tekislikning silindrdan ajratgan kesim yuzi* $S_k = H \cdot x$.

Chizmadan $x = 2 \cdot a \cdot tg\frac{\text{ц}}{2}$. *Demak,* $S_k = 2 \cdot a \cdot H \cdot tg\frac{\text{ц}}{2}$.

15) Konus ichiga, balandligi konus asosining radiusiga teng silindr chizilgan. Silindr to‘la sirtining konus asosining yuzasiga nisbati 3:2 bo‘lsa, konus o‘qi bilan yasovchisi orasidagi burchakni toping.



Yechish: *Agar konus va silindrlarni o‘q kesimini qarasak (rasmga qarang),teng yonli uchburchakka to‘g‘ri to‘rtburchak ichki chizilgan. Silindrning to‘la sirti:*

$S_{t.s} = 2\text{p}rR + 2\text{p}r^2 = 2\text{p}r(R + r)$.

Konus asosining yuzi: $S_{as} = \text{p}R^2$.

Masala shartiga asosan $\frac{S_{t.s}}{S_{as}} = \frac{3}{2} = \frac{2\text{p}r(R+r)}{\text{p}R^2} = \frac{r(R+r)}{R^2}$.

Bundan $r = \frac{1}{2} \cdot R$. *Talab qilinayotgan burchak:* $tg\text{б} = \frac{R-r}{R} = \frac{R-0{,}5R}{R} = 0{,}5$.

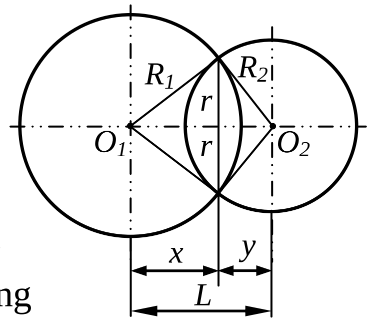
Yoki $\text{б} = arctg0{,}5$.

16) Asoslarining radiuslari $R = 22$ va $r = 4$ *sm* bo‘lgan kesik konus, hamda unga tengdosh bo‘lgan silindrning balandliklari bir xil. Silindr asosining radiusini toping.

Yechish: *Kesik konus va silindr tengdosh bo‘lgani uchun ularning hajmlari teng bo‘ladi.* $V = \frac{\text{p}}{3} \cdot H \cdot (R^2 + Rr + r^2) = \text{p}R_{sl}^2 \cdot H$. *Bundan silindrning radiusi:*

$$R_{sl} = \sqrt{\frac{1}{3} \cdot (R^2 + Rr + r^2)} =$$

$$= \sqrt{\frac{1}{3} \cdot ((22)^2 + 22 \cdot 4 + (4)^2)} = 14.$$



17) Radiuslari $R_1 = 3\ dm$ va $R_2 = 25\ sm$ bo‘lgan sharlar bir ikkinchisining ichiga kirgan (rasmga qarang) va ularning

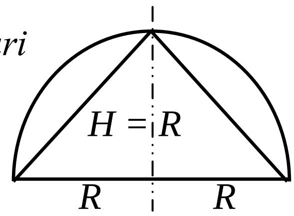
kesishish chizig‘ning uzunligi $\ell = 48$ p ga teng bo‘lsa, sharlar markazi orasidagi masofa *L* ni toping.

<u>Yechish:</u> *Kesishish chizig‘ning uzunligi* $2\text{p}r = 48$p.

Bundan $r = 24.$ $x = \sqrt{(R_1)^2 - r^2} = \sqrt{(30)^2 - (24)^2} = 18.$

$x = \sqrt{(R_2)^2 - r^2} = \sqrt{(25)^2 - (24)^2} = 7.$ $L = x + y = 18 + 7 = 25.$

18) Sharga ichki chizilgan konusning asosi, sharning katta dia-gonaliga teng. Konus o‘q kesimining yuzi $S_k = 36$ ga teng bo‘lsa, sharning hajmini toping.

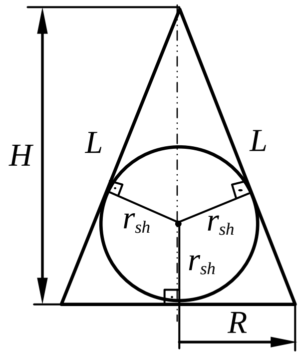


<u>Yechish:</u> *Masala shartiga ko‘ra, shar va konusning radiuslari hamda konusning balandligi bir biriga teng (rasmga qarang).*

$R_{sh} = R_k = H = R.$ *O‘q kesimni yuzi:* $S_k = \frac{1}{2} \cdot 2R \cdot H = R^2 = 36.$

Bundan $R = 6.$ *Sharning hajmi:* $V = \frac{4}{3} \cdot \text{p}R^3 = \frac{4}{3} \cdot \text{p}6^3 = 288\text{p}.$

19) Balandligi $H = 9$, yasovchisi $L = 15$ ga teng bo‘lgan konusga ichki chizilgan shar sirtining yuzini toping.



<u>Yechish:</u> *Chizmadan konusning radiusini topamiz:*

$R = \sqrt{L^2 - H^2} = \sqrt{15^2 - 9^2} = 12.$

Masalani yechishda konus va sharning o‘q kesimini qara-sak, teng yonli uchburchakka ichki chizilgan aylana radiusi, ich-ki chizilgan shar radiusiga teng ekanligini ko‘rish mumkin

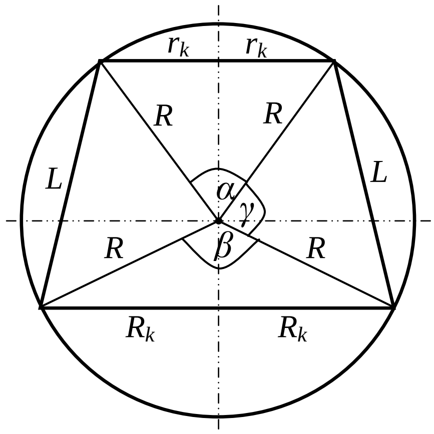
Kesmning yuzasi: $S_k = \frac{1}{2} \cdot 2R \cdot H = R \cdot H = 12 \cdot 9 = 108.$

Sharni radiusi $r_{sh} = \frac{2S_k}{2(L+R)} = \frac{S_k}{L+R} = \frac{108}{15+12} = 4.$

Shar sirtining yuzi $S_{sr} = 4\text{p} \cdot r_{sh}^2 = 64\text{p}.$

20) Radiusi *R* ga teng bo‘lgan sharga kesik konus ichki chizilgan.

Konusning yuqori asosai, shar segmenti б burchakli yoyni tortib turadi, konusning pastki asosi, shar segmenti в burchakli yoyni tortib turadi (rasmda o‘q kesimi ko‘r-satilgan). Kesik konusning yasovchisini aniqlang.



<u>Yechish:</u> *Chizmadan* $\text{г} = 180^0 - \frac{\text{б}+\text{в}}{2} \cdot$ *Kosinuslar teore-masidan* $L = \sqrt{2R^2\left[1 - cos\left(180^0 - \frac{\text{б}+\text{в}}{2}\right)\right]} =$

$= R\sqrt{2\left[1 + cos\frac{\text{б}+\text{в}}{2}\right]} = R\sqrt{2 \cdot 2cos^2\left(\frac{\text{б}+\text{в}}{4}\right)} =$

$= 2Rcos\left(\frac{\text{б}+\text{в}}{4}\right).$

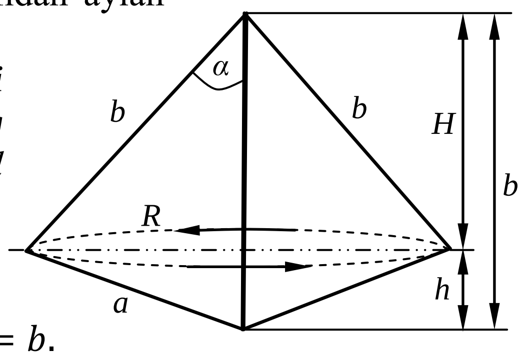
$R_k = Rsin\frac{\text{в}}{2};$ $r_k = Rsin\frac{\text{б}}{2}.$ *Kesik konusning yon sirti* $S_{yo} = \text{p}(R_k + r_k) \cdot L.$

Topilgan qiymatlarni bu formulaga qo‘ysak.

$$S_{yo} = \text{p}R\left(sin\frac{\text{б}}{2} + sin\frac{\text{в}}{2}\right) \cdot 2Rcos\left(\frac{\text{б}+\text{в}}{4}\right) =$$

$$= 2pR^2 \cdot 2sin\left(\frac{б+в}{4}\right) \cdot cos\left(\frac{б-в}{4}\right) \cdot cos\left(\frac{б+в}{4}\right) = 2pR^2 sin\left(\frac{б+в}{2}\right) \cdot cos\left(\frac{б+в}{4}\right).$$

21) Yon tomoni b ga, ular orasidagi burchak б ga teng bo‘lgan teng yonli uchburchakni yon tomoni atrofidan aylantirishdan hosil bo‘lgan shaklni hajmini toping.



<u>Yechish:</u> *Teng yonli uchburchakni yon tomoni atrofida aylantirsak, balandliklari H va h ga teng bo‘lgan ikkita radiusi R ga teng bo‘lgan konus hosil bo‘ladi.*

Chizmadan $H = b \cdot cos$б,

$R = b \cdot sin$б, $h = b - H$.

Yoki $h = b - b \cdot cos$б $= b(1 - cos$б$)$. $H + h = b$.

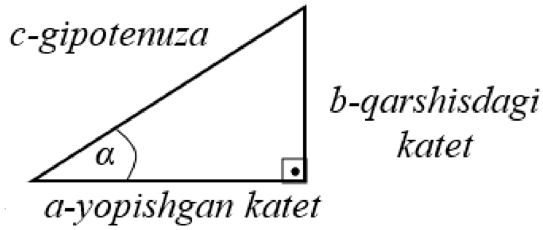
Shunday qilib,hosil bo‘lgan shaklning hajmi:

$$V = V_1 + V_2 = \frac{p}{3} \cdot R^2 \cdot H + \frac{p}{3} \cdot R^2 \cdot h.$$

Yoki $V = \frac{p}{3} \cdot R^2(H + h) = \frac{p}{3} \cdot R^2 \cdot b = \frac{p}{3} \cdot (b \cdot sin$б$,)^2 \cdot b = \frac{p}{3} \cdot b^3 \cdot sin^2$б.

4. TRIGONOMETRIYA

4.1. O‘tkir burchak trigonometriyasi.



4.1-rasm.

Buni aniqlash uchun togri burchakli uch-burchak chizamiz (4.1-rasm).

1. O‘tkir burchakning sinusi deb, o‘tkir burchak qarshisidagi katetning gipotenuzaga bo‘lgan nisbatiga aytiladi:

$sin\text{б} = \frac{b}{c} \rightarrow b = c \cdot sin\text{б}.$

2. O‘tkir burchakning kosinusi deb, o‘tkir burchakka yopishgan katetning gipotenuzaga bo‘lgan nisbatiga aytiladi: $cos\text{б} = \frac{a}{c} \rightarrow a = c \cdot cos\text{б}.$

3. O‘tkir burchakning tangensi deb, o‘tkir burchak qarshisidagi katetni, o‘tkir burchakka yopishgan katetga bo‘lgan nisbatiga aytiladi: $tg\text{б} = \frac{b}{a} = \frac{c \cdot sin\text{б}}{c \cdot cos\text{б}} = \frac{sin\text{б}}{cos\text{б}}.$

4. O‘tkir burchakning kotangensi deb, o‘tkir burchakka yopishgan katetning, o‘tkir burchak qarshisidagi katetga bo‘lgan nisbatiga aytiladi:

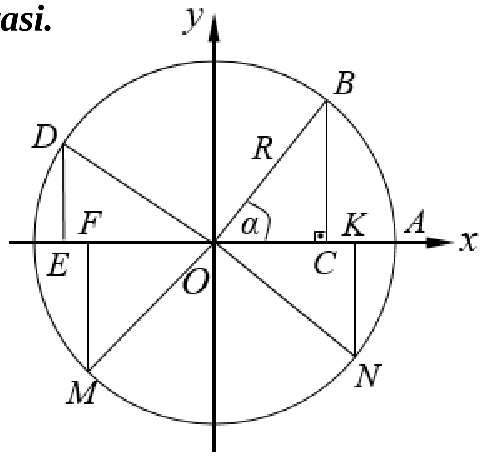
$$ctg\text{б} = \frac{a}{b} = \frac{c \cdot cos\text{б}}{c \cdot sin\text{б}} = \frac{cos\text{б}}{sin\text{б}}.$$

5. O‘tkir burchakning sekansi deb, gipotenizaning o‘tkir burchakka yopishgan katetga bo‘lgan nisbatiga aytiladi: $sec\text{б} = \frac{c}{a} = \frac{c}{c \cdot cos\text{б}} = \frac{1}{cos\text{б}}.$

6. O‘tkir burchakning kosekansi deb, gippotenizaning o‘tkir burchak qarshisidagi katetga bo‘lgan nisbatiga aytiladi: $cosec\text{б} = \frac{c}{b} = \frac{c}{c \cdot sin\text{б}} = \frac{1}{sin\text{б}}.$

4.2. Trigonometrik ifodalarni chorakdagi ishorasi va qiymatlari.

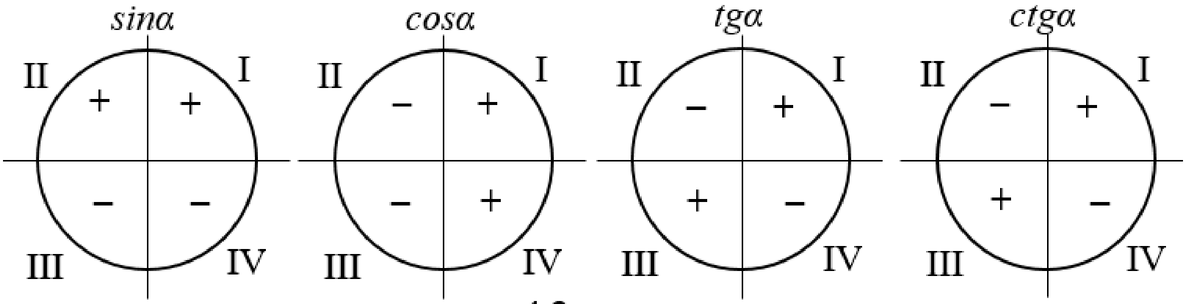
4.2.1. Trigonometrik ifodalarni chorakdagi ishorasi.



4.2-rasm.

Markazi koordinata boshida, radiusi $OA = R$ ga teng bo‘lan aylana chizamiz (rasmga qarang). *OA* radiusni *α* burchakka burib, katetlari *OC* va *BC* bo‘lgan *OCB* to‘g‘ri burchakli uchburchakni hosil qilamiz. б burchak qarshisidagi *BC* va *DE* katetlarni shartli ravishda sinus chizig‘i deb olamiz. Bu chiziqlar *OY* o‘qiga proeksiyalanadi, demak, $sin\text{б}$ ning I va II chorakdagi qiymatlari musbat, III va IV chorakdagi qiymatlari manfiy bo‘ladi (4.2-rasm). б burchakka yopishgan *OC* va *OK* katetlarni shartli ravishda kosinus chizig‘i deb olamiz.

Bu chiziqlar *OX* o‘qiga proeksiyalanadi, demak, cosб ning I va IV chorakdagi qiymatlari musbat, II va III chorakdagi qiymatlari manfiy bo‘ladi. Tangens va katangenslarni chorakdagi ishoralarini, sinus va kosinuslarning tegishli chorakdagi ishoralarini bir biriga bo‘lish orqali aniqlanadi (4.3-rasm).



4.3-rasm.

4.2.2. Trigonometrik ifodalarni chorakdagi qiymatlari.

Radusi $OA = OB = OC = OD = R = 1$ ga teng bo‘lgan aylana chizamiz (4.4-rasm): Chizmadan ΔOBE teng tomonli uchburchak bo‘lib, tomonlari $R = 1$ va ichki burchaklarining har biri 60^0 ga teng bo‘lishini ko‘rish mumkin.

Bundan $BB_1 = y_1 = \frac{R}{2} = \frac{1}{2}$.

ΔOCB_1teng yonli to‘go‘ri burchakli uchburchak bo‘lgani uchun $x_2 = y_2$ bo‘ladi. Bundan Pifagor teoremasiga asosan, $R^2 = y_2^2 + y_2^2 = 2y_2^2$ bo‘ladi.

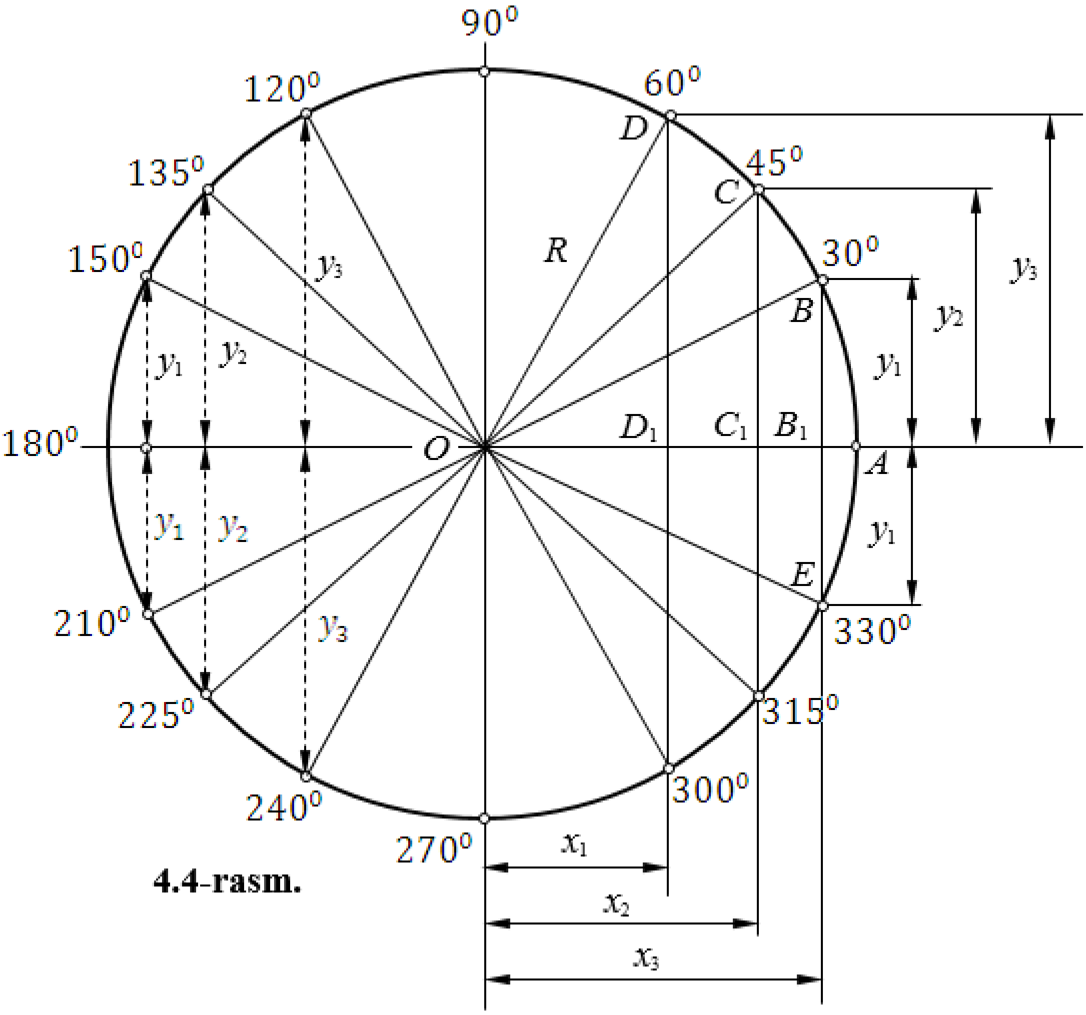
Yoki $y_2 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

ΔODD_1 to‘go‘ri burchakli uchburchak bo‘lgani uchun $\angle D_1 = 90^0, \angle DOD_1 = 60^0$, $\angle ODD_1 = 30^0$ bo‘ladi. Bundan $x_1 = \frac{R}{2} = \frac{1}{2}$. ΔODD_1 dan Pifagor teoremasiga asosan, $R^2 = y_3^2 + x_1^2$ bo‘ladi, bundan $y_3 = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$. ΔOBB_1 to‘go‘ri burchakli uchburchakdan $R^2 = y_1^2 + x_3^2$ bo‘lgani uchun va bundan $x_3 = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$.

Bu aniqlangan qiymatlar sinusning birinchi choragi uchun bo‘lib, uning cosines, tangens va kotangens uchun ham qiymatlarini topamiz va ularni quyidagi jadval ko‘rinishda yozamiz.

α	0°	30° π/6	45° π/4	60° π/3	90° π/2	180° π	270° 3π/2	360° 2π
sinα	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
cosα	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1	0	1
tgα	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	- ∞	0
ctgα	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	- ∞	0	∞

Bu qiymatlar sinus, cosinus, tangens va kotangenslarning chorakdagi ishoralariga qarab o‘zgaradi.



4.4-rasm.

Keltirish formulalari:

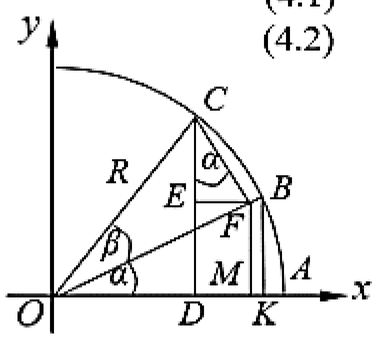
α	$90^0 \pm б$	$270^0 \pm б$	$180^0 \pm б$	$360^0 \pm б$
$sin\alpha$	$cos\alpha$	$-cos\alpha$	$\mp sin\alpha$	$\pm sin\alpha$
$cos\alpha$	$\mp sin\alpha$	$\pm sin\alpha$	$-cos\alpha$	$cos\alpha$
$tg\alpha$	$\mp ctg\alpha$	$\mp ctg\alpha$	$\pm tg\alpha$	$\pm tg\alpha$
$ctg\alpha$	$\mp tg\alpha$	$\mp tg\alpha$	$\pm ctg\alpha$	$\pm ctg\alpha$

4.3. Ikki burchak yig'indisi va ayirmasining *sin α, cos α, tg α* va *ctg α*

4.3.1. Ikki burchak yig'indisi va ayirmasining sinusi.

$$sin(\alpha+\beta) = sin\alpha \cdot cos\beta + sin\beta \cdot cos\alpha. \quad (4.1)$$

$$sin(\alpha-\beta) = sin\alpha \cdot cos\beta - sin\beta \cdot cos\alpha. \quad (4.2)$$



4.5-rasm.

<u>Isbot:</u> *4.5-rasmda* $OA = OB = OC = R;\ DE = MF;$
$EF = DM. CD = DE + CE = MF + CE.$
$OF = R \cdot cos\beta.\ CF = R \cdot sin\beta. MF = OF \cdot sin\alpha =$
$= R \cdot sin\alpha \cdot cos\beta.\ CE = CF \cdot cos\alpha = R \cdot sin\beta \cdot cos\alpha.$

$$sin(\alpha+\beta) = \frac{CD}{R} = \frac{MF+CE}{R} =$$

$$= \frac{R(sin\alpha \cdot cos\beta + sin\beta \cdot cos\alpha)}{R} = sin\alpha \cdot cos\beta + sin\beta \cdot cos\alpha.$$

4.3.2. Ikki burchak yig'indisi va ayirmasining kosinusi.

$cos(б + в) = cosб \cdot cosв - sinб \cdot sinв.$ (4.3)

$cos(б - в) = cosб \cdot cosв + sinб \cdot sinв.$ (4.4)

Isbot: *Chizmada* $OA = OB = OC = R;\ EF = DM. OD = OM - DM = OM - EF.$

$OF = R \cdot cosв.\ CF = R \cdot sinв.\ EF = CF \cdot sinб = R \cdot sinб \cdot sinв.$

$OM = OF \cdot cosб = R \cdot cosб \cdot cosв.$

$$cos(б + в) = \frac{OD}{R} = \frac{OM - EF}{R} = \frac{R(cosб \cdot cosв - sinб \cdot sinв)}{R} =$$

$$= cosб \cdot cosв - sinб \cdot sinв.$$

4.3.3. Ikki burchak yig'indisi va ayirmasining tangensi.

$$tg(б + в) = \frac{tgб + tgв}{1 - tgб \cdot tgв};\qquad (4.5)$$

$$tg(б - в) = \frac{tgб - tgв}{1 + tgб \cdot tgв}.\qquad (4.6)$$

Isbot: $tg(б + в) = \frac{sin(б+в)}{\cos(б+в)} = \frac{sinб \cdot cosв + sinв \cdot cosб}{\cos б \cdot cosв - sinб \cdot sinв}$. *Kasrni surat va maxrajini* $cosб \cdot cosв$ *ga bo'lsak,* $\frac{tgб + tgв}{1 - tgб \cdot tgв}$ *hosil bo'ladi.*

4.3.4. Ikki burchak yig'indisi va ayirmasining katangensi.

$$ctg(б + в) = \frac{ctgб \cdot ctgв - 1}{ctgб + ctgв};\qquad (4.7)$$

$$ctg(б - в) = \frac{ctgб \cdot ctgв + 1}{ctgб - ctgв}.\qquad (4.8)$$

Isbot: $ctg(б + в) = \frac{cos(б+в)}{\sin(б+в)} = \frac{\cos б \cdot cosв - sinб \cdot sinв}{sinб \cdot cosв + sinв \cdot cosб}$. *Kasrni surat va maxra-jini* $sinб \cdot sinв$ *ga bo'lsak,* $\frac{ctgб \cdot ctgв - 1}{ctgб + ctgв}$ *hosil bo'ladi.*

4.4. Trigonometrik ifodalar ko'paytmasini yig'indi va ayirmaga almashtirish.

(4.1) va (4.2) formulalarni hadma had qo'shib, quyidagi formulani hasil qilamiz:

1. $sinб \cdot cosв = \frac{1}{2}[sin(б + в) + sin(б - в)].$

(4.1) formuladan (4.2) formulalarni hadma had ayirib, quyidagi formulani hasil qilamiz:

2. $sinв \cdot cosб = \frac{1}{2}[sin(б + в) - sin(б - в)].$

(4.3) va (4.4) formulalarni hadma had qo'shib, quyidagi formulani hasil qilamiz:

3. $cosб \cdot cosв = \frac{1}{2}[cos(б + в) + cos(б - в)].$

(4.3) formuladan (4.4) formulalarni hadma had ayirib, quyidagi formulani hasil qilamiz:

4. $sin\alpha \cdot sin\beta = -\frac{1}{2}[cos(\alpha + \beta) - cos(\alpha - \beta)].$

4.5. Ikkilangan burchak trigonometriyasi.

4.5.1. Ikkilangan burchak sinusi. $sin2\alpha = 2sin\alpha \cdot cos\alpha;\quad sin\alpha = 2sin\frac{\alpha}{2} \cdot cos\frac{\alpha}{2}.$

Isbot: *Agar* $sin(\alpha + \beta) = sin\alpha \cdot cos\beta + sin\beta \cdot cos\alpha$ *formulada* $\beta = \alpha$ *bo'lsa,* $sin(\alpha + \alpha) = sin\alpha \cdot cos\alpha + sin\alpha \cdot cos\alpha$ *bo'ladi, bundan*

$$sin2б = 2sinб \cdot cosб.$$

4.5.2.Ikkilangan burchak kosinusi.

$$cos2б = cos^2б - sin^2б;\ cosб = cos^2\frac{б}{2} - sin^2\frac{б}{2}.$$

Isbot: *Agar* $cos(б + в) = cosб \cdot cosв - sinб \cdot sinв$ *formulada* $в = б$ *bo'lsa,* $cos(б + б) = cosб \cdot cosб - sinб \cdot sinб$ *bo'ladi, bundan*

$$cos2б = cos^2б - sin^2б.$$

4.5.3. Ikkilangan burchak tangensi. $tg2б = \frac{2tgб}{1-tg^2б}$; $tgб = \frac{2tg\frac{б}{2}}{1-tg^2\frac{б}{2}}.$

Isbot: *Agar* $tg(б + в) = \frac{tgб+tgв}{1-tgб \cdot tgв}$ *formulada* $в = б$ *bo'lsa,*

$tg(б + б) = \frac{tgб+tgб}{1-tgб \cdot tgб}$ *bo'ladi, bundan* $tg2б = \frac{2tgб}{1-tg^2б}.$

4.6. Uchlangan burchak trigonometriyasi.

1) $sin3б = 3sinб - 4sin^3б.$

Isboti: $sin3б = sin(2б + б) = sin2б \cdot cosб + sinб \cdot cos2б =$

$$= 2sinб \cdot cos^2б + sinб \cdot (cos^2б - sin^2б) =$$
$$= 2sinб \cdot (1 - sin^2б) + sinб \cdot (1 - sin^2б - sin^2б) =$$
$$= 2sinб - 2sin^3б + sinб - 2sin^3б = 3sinб - 4sin^3б.$$

2) $cos3б = 4cos^3б - 3cosб.$

Isboti: $cos3б = cos(2б + б) = cos2б \cdot cosб - sinб \cdot sin2б =$

$$= (cos^2б - sin^2б) \cdot cosб - 2sin^2б \cdot cosб =$$
$$= 2cos^3б - cosб - 2cosб + 2cos^3б = 4cos^3б - 3cosб.$$

3) $tg3б = \frac{3tgб-tg^3б}{1-3tg^2б}.$

Isboti: $tg3б = tg(2б + б) = \frac{tg2б+tgб}{1-tg2б \cdot tgб} = \frac{\frac{2tgб}{1-tg^2б}+tgб}{1-\frac{2tgб}{1-tg^2б} \cdot tgб} = \frac{3tgб-tg^3б}{1-3tg^2б}.$

4.7. To'rtlangan burchak trigonometriyasi.

1) $cos4б = 8cos^4б - 8cos^2б + 1.$

2) $sin4б = 4cosб(sinб - 2sin^3б).$ 3) $tg4б = \frac{4tgб(1-tg^2б)}{1-6tg^2б+tg^4б}.$

4.8. Darajani pasaytirish va yarim burchak trigonometryasi.

4.8.1. Darajani pasaytirish.

$sin^2б = \frac{1-cos2б}{2}$; $sinб = \pm\sqrt{\frac{1-cos2б}{2}}.$

Isbot: $cos2б = cos^2б - sin^2б$ *formulaning ikkala tomonidan 1 ni ayirib, quyidagi tenglikni hosil qilamiz:*

$1 - cos2б = 1 - cos^2б + sin^2б = 2sin^2б.$ *Bundan:* $sin^2б = \frac{1-cos2б}{2}.$

$cos^2 б = \frac{1+cos2б}{2}$; $cosб = \pm\sqrt{\frac{1+cos2б}{2}}$.

Isbot: $cos2б = cos^2б - sin^2б$ *formulaning ikkala tomoniga 1 ni qo'shib, quyidagi tenglikni hosil qilamiz:*

$1 + cos2б = 1 + cos^2б - sin^2б = 2cos^2б$. *Bundan:* $cos^2б = \frac{1+cos2б}{2}$.

$tg^2б = \frac{1-cos2б}{1+cos2б}$. $1 - tg^2б = \frac{cos2б}{cos^2б}$. $1 - ctg^2б = -\frac{cos2б}{sin^2б}$.

$sin^3б = \frac{3sinб - sin3б}{4}$.

Isbot: $sin^3б = sinб \cdot sin^2б = sinб \cdot \frac{1-cos2б}{2} = \frac{sinб - sinб \cdot cos2б}{2} =$

$$= \frac{sinб - \frac{1}{2}(sin3б - sinб)}{2} = \frac{3sinб - sin3б}{4}.$$

$cos^3б = \frac{3cosб + cos3б}{4}$.

Isbot: $cos^3б = cosб \cdot cos^2б = cosб \cdot \frac{1+cos2б}{2} = \frac{cosб + cosб \cdot cos2б}{2} =$

$$= \frac{cosб + \frac{1}{2}(cos3б + cosб)}{2} = \frac{3cosб + cos3б}{4}.$$

4.8.2. Yarim burchak trigonometryasi.

$sin\frac{б}{2} = \pm\sqrt{\frac{1-cosб}{2}}$. $cosб\frac{б}{2} = \pm\sqrt{\frac{1+cosб}{2}}$. $tg\frac{б}{2} = \pm\sqrt{\frac{1-cosб}{1+cosб}}$.

4.9. Trigonometrik ifodalar yig'indi va ayirmasini ko'paytmaga almashtirish.

1. $sinб + sinв = 2sin\frac{б+в}{2} \cdot cos\frac{б-в}{2}$.

Isbot: $\begin{cases} б = x + y \\ в = x - y \end{cases}$ *deb belgilash kiritaniz:*

$sinб + sinв = sin(x + y) + sin(x - y) =$

$= sinx \cdot cosy + siny \cdot cosx + sinx \cdot cosy - siny \cdot cosx = 2sinx \cdot cosy.$

Yuqoridagi sistemani hadlab qo'shib, $x = \frac{б+в}{2}$ *ni, ayirib,* $x = \frac{б+в}{2}$ *ni hosil qilamiz. Bularni o'rniga qo'ysak:* $sinб + sinв = 2sin\frac{б+в}{2} \cdot cos\frac{б-в}{2}$.

2. $sinб - sinв = 2sin\frac{б-в}{2} \cdot cos\frac{б+в}{2}$.

3. $cosб + cosв = 2cos\frac{б+в}{2} \cdot cos\frac{б-в}{2}$.

4. $cosб - cosв = -2sin\frac{б+в}{2} \cdot sin\frac{б-в}{2}$. Bularni isboti ham yuqoridagi kabi amalga oshiriladi.

5. $tgб + tgв = \frac{\sin(б+в)}{cosб \cdot cosв}$.

Isbot: $tgб + tgв = \frac{\sinб}{cosб} + \frac{\sin\beta}{cosв} = \frac{\sinб \cdot cosв + \sin\beta \cdot cosб}{cosб \cdot cosв} = \frac{\sin(б+в)}{cosб \cdot cosв}$.

6. $tgб - tgв = \frac{\sin(б-в)}{cosб \cdot cosв}$.

4.10. Qo'shimcha trigonometrik formulalar.

1. $sinб = \frac{2tg\frac{б}{2}}{1+tg^2\frac{б}{2}}$. 2. $cosб = \frac{1-tg^2\frac{б}{2}}{1+tg^2\frac{б}{2}}$. 3. $tgб = \frac{2tg\frac{б}{2}}{1-tg^2\frac{б}{2}}$.

4. $16 \cdot sin10^0 \cdot sin30^0 \cdot sin50^0 \cdot sin70^0 = 1$.

5. $16 \cdot cos10^0 \cdot cos30^0 \cdot cos50^0 \cdot cos70^0 = 3$

6. $16 \cdot sin20^0 \cdot sin40^0 \cdot sin60^0 \cdot sin80^0 = 3$

7. $16 \cdot cos20^0 \cdot cos40^0 \cdot cos60^0 \cdot cos80^0 = 1$

8. $tg20^0 \cdot tg40^0 \cdot tg60^0 \cdot tg80^0 = 3$

9. $4cos\frac{р}{5} \cdot cos\frac{3р}{5} = -1$. 10. $8cos\frac{р}{7} \cdot cos\frac{3р}{7} \cdot cos\frac{5р}{7} = 1$.

11. $8cos\frac{р}{7} \cdot cos\frac{2р}{7} \cdot cos\frac{4р}{7} = -1$; $8cos\frac{р}{7} \cdot cos\frac{4р}{7} \cdot cos\frac{5р}{7} = 1$.

12. $sin\ \frac{р}{4n} \cdot sin\ \frac{3р}{4n} \cdot sin\ \frac{5р}{4n} \cdots sin\ \frac{(2n-1)р}{4n} = \frac{\sqrt{2}}{2^n}$.

13. $cos\frac{р}{5} + cos\frac{3р}{5} = \frac{1}{2}$; $cos\frac{2р}{7} + cos\frac{4р}{7} + cos\frac{6р}{7} = -\frac{1}{2}$.

14. $sinб \cdot sin(60^0 - б) \cdot sin(60^0 + б) = \frac{1}{4}sin3б$.

15. $cosб \cdot cos(60^0 - б) \cdot cos(60^0 + б) = \frac{1}{4}cos3б$.

16. $tgб \cdot tg(60^0 - б) \cdot tg(60^0 + б) = tg3б$.

17. $sinб + sinв + sinг = 4cos\frac{б}{2}\,cos\frac{в}{2}\,cos\frac{г}{2}$.

18. $tgб + tgв + tgг = tgб \cdot tgв \cdot tgг$.

4.11.Teskari trigonometrik funksiyalar uchun asosiy formulalar.

1. $sin(arcsinx) = x$. 2. $cos(arccosx) = x$. 3. $tg(arctgx) = x$.

4. $ctg(arcctgx) = x$. 5. $sin(arccosx) = \sqrt{1-x^2}$. 6. $cos(arcsinsx) = \sqrt{1-x^2}$.

7. $sin(arctgx) = \frac{x}{\sqrt{1+x^2}}$. 8. $sin(arcctgx) = \frac{1}{\sqrt{1+x^2}}$. 9. $cos(arctgx) = \frac{1}{\sqrt{1+x^2}}$.

10. $cos(arcctgx) = \frac{x}{\sqrt{1+x^2}}$. 11. $tg(arcsinx) = \frac{x}{\sqrt{1-x^2}}$.

12. $tg(arccosx) = \frac{\sqrt{1+x^2}}{x}$. 13. $tg(arcctgx) = \frac{1}{x}$. 14. $c\,tg(arcsinx) = \frac{\sqrt{1+x^2}}{x}$.

15. $ctg(arccosx) = \frac{x}{\sqrt{1-x^2}}$. 16. $ctg(arctgx) = \frac{1}{x}$.

17. $arcsinx = \arccos\sqrt{1-x^2} = \frac{р}{2} - arccosx$. 18. $arcsinx = arctg\frac{x}{\sqrt{1-x^2}}$.

19. $arccosx = -arcsinx = arctg\frac{x}{\sqrt{1-x^2}}$.

20. $arctgx = -arctgx = arcsin\frac{x}{\sqrt{1+x^2}}$.

21. $arcsinx + arcsiny = arccos\left(\sqrt{1-x^2} \cdot \sqrt{1-y^2} - xy\right)$.

22. $arccosx + arccosy = arccos\left(xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}\right)$.

23. $arcsinx - arcsiny = arcsin\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$.

24. $arccosx - arccosy = arcsin\left(y\sqrt{1-x^2} - x\sqrt{1-y^2}\right).$

25. $arctgx + arctgy = arctg\,\frac{x+y}{1-xy}.$ 26. $arctgx - arctgy = arctg\,\frac{x-y}{1+xy}.$

27. $arctgx + arctgy + arctgz = arctg\,\frac{x+y+z-xyz}{1-(xy+xz+yz)}.$

28. $arcctgx + arcctgy = arctg\,\frac{xy-1}{x+y}.$

29. $arcctgx - arcctgy = arcctg\,\frac{xy+1}{y-x}.$

TRIGONOMETRIK HISOBLASHGA DOIR MISALLAR

1) $sin\frac{\text{p}}{6} + cos\frac{2\text{p}}{3} + tg\frac{\text{p}}{4}$ ni hisoblang.

Yechish: $sin\frac{\text{p}}{6} + cos\frac{2\text{p}}{3} + tg\frac{\text{p}}{4} = \frac{1}{2} - \frac{1}{2} \pm = 1.$

2) Agar $cos\text{б} = \frac{4}{5}$ va $0 < \text{б} < \frac{\text{p}}{2}$ bo‘lsa, $tg\text{б}$ ni qiymatini hisoblang.

Yechish: *Misol shartiga asosan* б *I chorakda joylashgan.*

$$sin\text{б} = \sqrt{1 - cos^2\text{б}} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}. \quad tg\text{б} = \frac{\sin\text{б}}{cos\text{б}} = \frac{3/5}{4/5} = \frac{3}{4}.$$

3) Agar $sin\text{б} = \frac{5}{13}$ va $\frac{\text{p}}{2} < \text{б} < \text{p}$ bo‘lsa, $ctg\text{б}$ ni qiymatini hisoblang.

Yechish: *Misol shartiga asosan* б *II chorakda joylashgan. Bu chorakda kosinus manfiy qiymatlarni qabul qiladi:* $cos\text{б} = -\sqrt{1 - sin^2\text{б}} = -\sqrt{1 - \left(\frac{5}{13}\right)^2} = -\frac{12}{13}.$

$$ctg\text{б} = \frac{cos\text{б}}{\sin\text{б}} = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5} = -2{,}4.$$

4) Agar $tg\text{б} = -\frac{3}{4}$ va $\frac{\text{p}}{2} < \text{б} < \text{p}$ bo‘lsa, $sin\text{б} - cos\text{б}$ ni qiymatini hisoblang.

Yechish: *Misol shartiga asosan* б *II chorakda joylashgan. Agar* $sin\text{б} - cos\text{б} = a$ *bo‘lsa, tenglikni ikkala tomonini kvadratga oshirib,* $a^2 = (sin\text{б} - cos\text{б})^2 = 1 - sin2\text{б}$ *ni hosil qilamiz.*

$$\textit{Shunda } a = \sqrt{1 - sin2\text{б}} = \sqrt{1 - \frac{2tg\text{б}}{1+tg^2\text{б}}} = \sqrt{1 - \frac{2\cdot(-3/4)}{1+(-3/4)^2}} = \frac{7}{5}.$$

5) Agar $cos2\text{б} = \frac{1}{4}$ bo‘lsa, $sin^2\text{б}$ ni hisoblang.

Yechish: *Darajani pasaytirish formulasidan foydalanib,*

$$sin^2\text{б} = \frac{1-cos2\text{б}}{2} = \frac{1-1/4}{2} = \frac{3}{8}.$$

6) Agar $tg\left(\frac{\text{p}}{4} - \text{б}\right) = -11$ bo‘lsa, $ctg\text{б}$ ni hisoblang.

Yechish: *Ikki burchak ayirmasining tangensi formulasidan foydalanib,*

$$tg\left(\frac{\text{p}}{4} - \text{б}\right) = \frac{tg(\text{p}/4) - tg\text{б}}{1 + tg(\text{p}/4)\cdot tg\text{б}} = \frac{1 - tg\text{б}}{1 + tg\text{б}} = -11. \textit{ Bundan } tg\text{б} = -6/5.$$

$ctgб = \frac{1}{tgб} = \frac{1}{-6/5} = -\frac{5}{6}.$

7) $sin75^0 \cdot sin15^0$ ni hisoblang.

Yechish: $sin75^0 \cdot sin15^0 = -\frac{1}{2}[cos(75^0 + 15^0) - cos(75^0 - 15^0)] =$

$$= -\frac{1}{2}(cos90^0 - cos60^0) = \frac{1}{2}cos60^0 = \frac{1}{4}.$$

8) $cos105^0 + cos15^0$ ni hisoblang.

Yechish: $cos105^0 + cos15^0 = 2cos\frac{105^0+15^0}{2} \cdot cos\frac{105^0-15^0}{2} =$

$$= 2cos\frac{120^0}{2} \cdot cos\frac{90^0}{2} = 2cos60^0 \cdot cos45^0 = \frac{\sqrt{2}}{2}.$$

9) $\frac{sin\alpha - 2sin2\alpha + sin3\alpha}{cos\alpha - 2cos2\alpha + cos3\alpha} - tg2\alpha$ ni so‘ldalashtiring.

Yechish: $\frac{sin\alpha - 2sin2\alpha + sin3\alpha}{cos\alpha - 2cos2\alpha + cos3\alpha} - tg2\alpha = \frac{sin3\alpha + sin\alpha - 2sin2\alpha}{cos3\alpha + cos\alpha - 2cos2\alpha} - \frac{sin2\alpha}{cos2\alpha} =$

$$= \frac{2sin2\alpha \cdot cos\alpha - 2sin2\alpha}{2cos2\alpha \cdot cos\alpha - 2cos2\alpha} - \frac{sin2\alpha}{cos2\alpha} = \frac{2sin2\alpha \cdot (cos\alpha - 1)}{2cos2\alpha \cdot (cos\alpha - 1)} - \frac{sin2\alpha}{cos2\alpha} = 0.$$

10) $sin^2 7^0 30' \cdot sin45^0 - cos45^0 \cdot cos^2 52^0 30'$ ni hisoblang.

Yechish: *Darajani pasaytirish formulasidan foydalanib,* $sin^2б = \frac{1-cos15^0}{2}$*;* $cos^2б = \frac{1+cos105^0}{2}$ *ekanligini aniqlaymiz. Shunda*

$$\frac{1-cos15^0}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1+cos105^0}{2} = \frac{\sqrt{2}}{4}\left(1 - cos15^0 - 1 - cos105^0\right) =$$

$$= -\frac{\sqrt{2}}{4}(cos105^0 + cos15^0) - 2 \cdot \frac{\sqrt{2}}{4} \cdot cos60^0 \cdot cos45^0 = -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{4}.$$

11) $tg^2\frac{11p}{4} + ctg(-7{,}25p) + 4cos^2\frac{31p}{6}$ ni hisoblang.

Yechish: *Quyidagi o‘zgartirishlarni kiritamiz:* $\frac{11p}{4} = 3p - \frac{p}{4}$; $\frac{31p}{6} = 5p + \frac{p}{6}$ *va* $7{,}25p = 7p + \frac{p}{4}$. $\left(tg\left(3p - \frac{p}{4}\right)\right)^2 - ctg\left(7p + \frac{p}{4}\right) + 4\left(cos\left(5p - \frac{p}{6}\right)\right)^2 =$

$$= \left(-tg\left(\frac{p}{4}\right)\right)^2 - ctg\frac{p}{4} + 4\left(-cos\left(\frac{p}{6}\right)\right)^2 = 1 - 1 + 4 \cdot \frac{1}{4} = 1.$$

12) $\frac{cos70^0 \cdot cos10^0 + cos80^0 \cdot cos20^0}{cos68^0 \cdot cos8^0 + cos82^0 \cdot cos22^0}$ ni hisoblang.

Yechish: *Ko‘paytmani yig‘indiga keltirish formulasidan foydalanib, berilgan ifodani quyidagicha yozamiz:*

$$\frac{\frac{1}{2}(cos80^0 + cos60^0 + cos100^0 + cos60^0)}{\frac{1}{2}(cos76^0 + cos60^0 + cos104^0 + cos60^0)} = \frac{1 + cos100^0 + cos80^0}{1 + cos104^0 + cos76^0} =$$

$$= \frac{1 + 2cos90^0 cos10^0}{1 + 2cos90^0 cos14^0} = 1$$

13) $\dfrac{sin91^0 - sin1^0}{9\sqrt{2}cos46^0 + \sqrt{2}sin44^0}$ ni hisoblang.

Yechish:

$$\frac{sin91^0 - sin1^0}{9\sqrt{2}cos46^0 + \sqrt{2}sin44^0} = \frac{2sin45^0 \cdot cos46^0}{\sqrt{2}(9cos46^0 + sin(90^0 - 46^0))} = \frac{2 \cdot \sqrt{2}/2 \cdot cos46^0}{\sqrt{2}10 \cdot cos46^0} = \frac{1}{10}.$$

14) $6cos80^0 - \dfrac{3\sqrt{3}}{2cos50^0}$ ni hisoblang.

Yechish:

$$6cos80^0 - \frac{3\sqrt{3}}{2cos50^0} = \frac{12cos80^0 \cdot cos50^0 - 3\sqrt{3}}{2cos50^0} = \frac{12 \cdot \frac{1}{2}(cos130^0 + cos30^0) - 3\sqrt{3}}{2cos50^0} =$$

$$= \frac{6\left(\cos\left(90^0 + 40^0\right) + \frac{\sqrt{3}}{2}\right) - 3\sqrt{3}}{2cos50^0} = \frac{6(-sin40^0 + \sqrt{3}/2) - 3\sqrt{3}}{2cos50^0} = \frac{-6sin40^0 + 3\sqrt{3} - 3\sqrt{3}}{2cos50^0} =$$

$$= \frac{-6sin40^0}{2cos50^0} = -3\frac{sin(90^0 - 50^0)}{cos50^0} = -3\frac{cos50^0}{cos50^0} = -3.$$

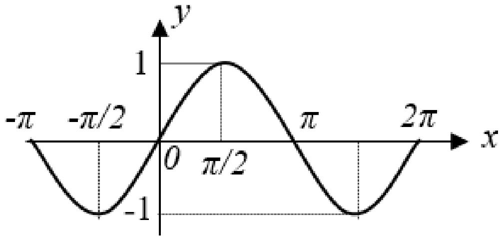
15) $\dfrac{cos^2 37^0 - sin^2 23^0}{cos14^0}$ ni hisoblang.

Yechish: $\dfrac{cos^2 37^0 - sin^2 23^0}{cos14^0} = \dfrac{(1 + cos74^0)/2 - (1 - cos46^0)/2}{cos14^0} =$

$$= \frac{1 + cos74^0 - 1 + cos46^0}{2cos14^0} = \frac{cos74^0 + cos46^0}{2cos14^0} = \frac{2cos60^0 \cdot cos14^0}{2cos14^0} = \frac{1}{2}.$$

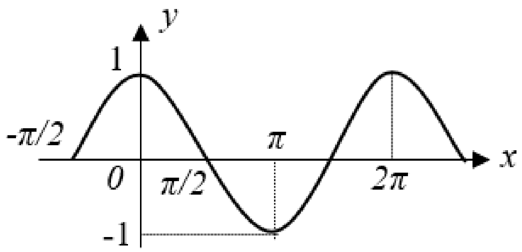
4.12. Trigonometrik funksiyalar.

$\boldsymbol{y = sinx}$ funksiya va uning xossalari.



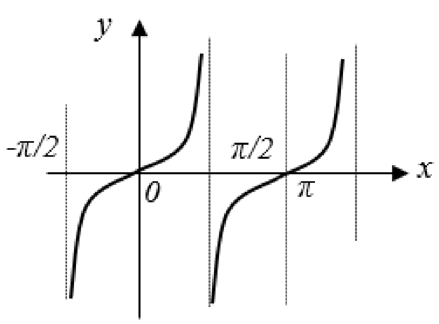
1) Grafigi.
2) Aniqlanish sohasi: $D(f) = R \to (-\infty; \infty)$.
3) Qiymatlar sohasi: $E(f) \to [-1; 1]$.
4) Toq funksiya: $sin(-x) = -sinx;\ x \in R$.
5) Teskari funksiya: $y = arcsinx$.
6) Davriy funksiya. Eng kichik davri: $T_0 = 2$p.

$\boldsymbol{y = cosx}$ funksiya va uning xossalari.



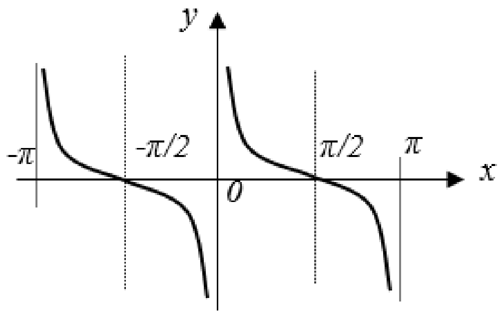
1) Grafigi.
2) Aniqlanish sohasi: $D(f) = R \to (-\infty; \infty)$.
3) Qiymatlar sohasi: $E(f) \to [-1; 1]$.
4) Juft funksiya: $cos(-x) = cosx;\ x \in R$.
5) Teskari funksiya: $y = arccosx$.
6) Davriy funksiya. Eng kichik davri: $T_0 = 2$p.

$\boldsymbol{y = tgx}$ funksiya va uning xossalari.



1) Grafigi.
2) Aniqlanish sohasi: $D(f) \to \left(-\frac{\text{p}}{2}; \frac{\text{p}}{2}\right)$.
3) Qiymatlar sohasi: $E(f) \to (-\infty; \infty)$.

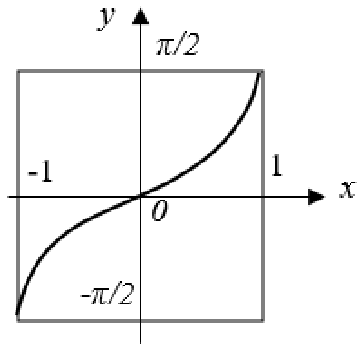
4) Toq funksiya: $tg(-x) = -tgx;\ x \in R$.
5) Teskari funksiya: $y = arctgx$.
6) Davriy funksiya. Eng kichik davri: $T_0 =$ p.
$\boldsymbol{y = ctgx}$ funksiya va uning xossalari.
1) Grafigi.



2) Aniqlanish sohasi: $D(f) \rightarrow$ (0; p).
3) Qiymatlar sohasi: $E(f) \rightarrow (-\infty; \infty)$.
4) Toq funksiya: $ctg(-x) = -ctgx;\ x \in R$.
5) Teskari funksiya: $y = arcctgx$.
6) Davriy funksiya. Eng kichik davri: $T_0 =$ p.

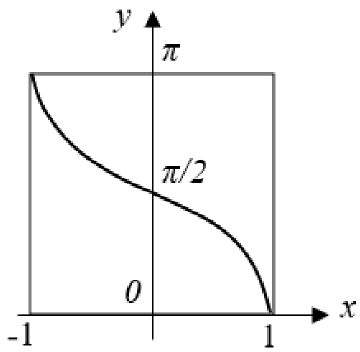
4.13. Teskari trigonometrik funksiyalar.

$\boldsymbol{y = arcsinx}$ funksiya va uning xossalari.
1) Grafigi.



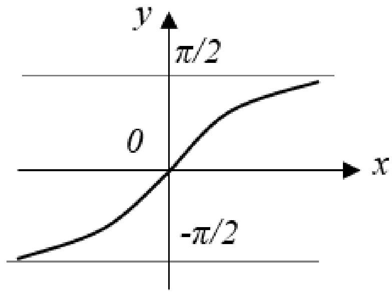
2) Aniqlanish sohasi: $D(f) \rightarrow [-1; 1]$.
3) Qiymatlar sohasi: $E(f) \rightarrow \left[-\frac{p}{2}; \frac{p}{2}\right]$.
4) Toq funksiya: $arcsin(-x) = -arcsinx$.
5) Teskari funksiya: $y = sinx$.
6) O‘suvchi funksiya.

$\boldsymbol{y = arccosx}$ funksiya va uning xossalari.
1) Grafigi.



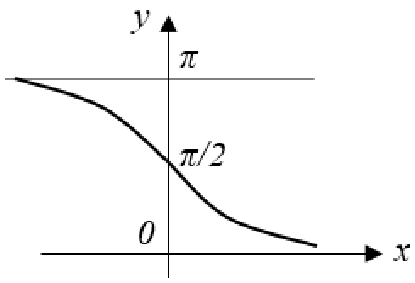
2) Aniqlanish sohasi: $D(f) \rightarrow [-1; 1]$.
3) Qiymatlar sohasi: $E(f) \rightarrow$ [0; p].
4) Funksiya toq ham juft ham emas.
5) Teskari funksiya: $y = cosx$.
6) Kamayuvchi funksiya.

$\boldsymbol{y = arctgx}$ funksiya va uning xossalari.
1) Grafigi.



2) Aniqlanish sohasi: $D(f) \rightarrow (-\infty; \infty)$.
3) Qiymatlar sohasi: $E(f) \rightarrow \left(-\frac{p}{2}; \frac{p}{2}\right)$.
4) Toq funksiya: $arctg(-x) = -arctgx$.
5) Teskari funksiya: $y = tgx$.
6) O‘suvchi funksiya.

$\boldsymbol{y = arcctgx}$ funksiya va uning xossalari.
1) Grafigi.



2) Aniqlanish sohasi: $D(f) \rightarrow (-\infty; \infty)$.
3) Qiymatlar sohasi: $E(f) \rightarrow$ (0; p).
4) Funksiya toq ham juft ham emas.
5) Teskari funksiya: $y = ctgx$.
6) Kamayuvchi funksiya.

Agar funksiya argument oldida koeffitsiyent berilgan bo‘lsa, uni k deb belgilab, bu funksiyaning eng kichik davri quyidagicha aniqlanadi: $T = \frac{T_0}{k}$.

Misol uchun: $y = sin3x,\ T = \frac{T_0}{k} = \frac{2p}{3}$. $y = cos\frac{p}{2},\ T = \frac{T_0}{k} = \frac{2p}{1/2} = 4p$.

4.14. Trigonometrik tenglama va tengsizliklar.

4.14.1.Trigonometrik tenglamalar:

1. $sinx = a;\ \ 0 < a < 1\ da \rightarrow\ x = (-1)^n arcsina + pn;$

$-1 < a < 0\ da \rightarrow x = (-1)^{n+1}\ arcsina + pn;$

$sin^2x = a,\ 0 \leq a \leq 1\ da \rightarrow \pm arcsin\sqrt{a} + pn;\ sinx = 0\ da \rightarrow x = pn;$

$sinx = 1\ da \rightarrow x = p/2 + 2pn;\ sinx = -1\ da\ \rightarrow x = -p/2 + 2pn;$

2. $cosx = a;\ 0 < a < 1\ da \rightarrow\ x = \pm arccosa + 2pn;$

$-1 < a < 0\ da \rightarrow x = \pm(p - arccosa) + 2pn;\ \ cos^2x = a, 0 \leq a \leq 1\ da \rightarrow$
$x = \pm arccos\sqrt{a} + pn;\ cosx = 0\ da \rightarrow x = p/2 + pn;\ cosx = 1\ da \rightarrow x = 2pn;$
$cosx = -1\ da \rightarrow x = p + 2pn;$

3. $tgx = a;\ \ a > 0\ da \rightarrow x = arctga + pn;\ \ a < 0\ da \rightarrow x = -arctga + pn;$

$tg^2x = a, x = arctg\sqrt{a} + pn;\ \ tgx = 0\ da \rightarrow x = pn;$
$tgx = 1\ da \rightarrow x = p/4 + pn;\ \ tgx = -1\ da \rightarrow x = -p/4 + pn;$

4. $ctgx = a;\ a > 0\ da \rightarrow x = arcctga + pn;\ a < 0\ da \rightarrow x = p - arcctga + pn;$

$ctg^2x = a, x = arcctg\sqrt{a} + pn;\ ctgx = 0\ da \rightarrow\ \ x = p/2 + pn;$
$ctgx = 1\ da \rightarrow x = p/4 + pn;\ \ ctgx = -1\ da \rightarrow x = -p/4 + pn.$

4.14.2.Ayrim trigonometrik tenglamalarni yechish usullari:

***a*)** $a \cdot sinx + b \cdot cosx = 0$ ko‘rinishdagi tenglamalarni yechish.

Tenglikni ikkala tomonini cosб ga bo‘lib, quyidagi tenglamani hosil qilaaamiz:
$tgx = -\frac{b}{a}$

b) $a \cdot sin^2x + b \cdot sinx \cdot cosx + c \cdot cos^2x = 0$ ko‘rinishdagi tenglamalarni yrchish uchun, tenglikni ikkala tomonini cos^2б ga bo‘lib, quyidagi tenglamani hosil qilaaamiz: $a \cdot tg^2x + b \cdot tgx + c = 0$. Bu tenglikni tgx ga nisbatan to‘la kvadrat tenglama qilib yechiladi.

c) $a \cdot sinx + b \cdot cosx = c$ ko‘rinishdagi tenglamalarni yrchish. Buni ikki usulda yechish mumkin:

1) Qoshimcha argument kiritish. Bunda $a \cdot sinx + b \cdot cosx$ ifoda $R \cdot \sin(x + ц)$ ifoda bilan almashtiriladi. $R = \sqrt{a^2 + b^2}$, $sinц = \frac{a}{\sqrt{a^2+b^2}}$ va $cosц = \frac{b}{\sqrt{a^2+b^2}}$. Bunda burchak ц −qoshimcha argumet deb ataladi.

Misol: $4 \cdot sinx + 3 \cdot cosx = 5$ *tenglamani yeching.*

Bunda $\sqrt{4^2 + 3^2} = 5,\ sinц = \frac{4}{5}$, bundan $ц = arcsin\frac{4}{5}$ va $cosц = \frac{3}{5}$, bundan $ц = arccos\frac{3}{5}$ *bo‘ladi. Demak,* $\frac{4}{5} \cdot sinx + \frac{4}{5} \cdot cosx = 1$ *bo‘ladi.*

Shunda $sinц \cdot cosx + sinx \cdot cosц = 1$ *yoki* $\sin(x + ц) - 1$. *Yoki* $x + ц = \frac{p}{2} + 2pn$.

Bundan $x = \frac{p}{2} - ц + 2pn = \frac{p}{2} - arcsin\frac{4}{5} + 2pn$.

2) O‘rniga qo‘yish usuli. Bunda $sinx$ va $cosx$ lar 4.10. ning 1 va 2 formulalari yordamida tangensga almashtiriladi.

Misol: $4 \cdot sinx + 3 \cdot cosx = 5$ *tenglamani yeching.*

Bunda $4 \cdot \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} + 3 \cdot \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = 5$ *b‘oladi.*

Bundan $4 \cdot tg^2\frac{x}{2} - 4 \cdot tg\frac{x}{2} + 1 = 0$ *yoki* $\left(2 \cdot tg\frac{x}{2} - 1\right)^2 = 0.$

Bundan $tg\frac{x}{2} = \frac{1}{2}$, $\frac{x}{2} = arctg\frac{1}{2} + \text{p}n$, *yoki* $x = 2arctg\frac{1}{2} + 2\text{p}n.$

d) $a \cdot sin^2x + b \cdot sinx \cdot cosx + c \cdot cos^2x = d$ ko‘rinishdagi tenglamalarni yrchish uchun, $1 = cos^2x + sin^2x$ ayniyatdan foydalanamiz. $d \cdot 1 = d \cdot (cos^2x + sin^2x)$.

Shunda $a \cdot sin^2x + b \cdot sinx \cdot cosx + c \cdot cos^2x = d \cdot (cos^2x + sin^2x)$ bo‘ladi.

$(a - d) \cdot sin^2x + b \cdot sinx \cdot cosx + (c - d) \cdot cos^2x = 0.$

Bu tenglikni ikkala tomonini $cos^2x \neq 0$ ga bo‘lib,

$(a - d) \cdot tg^2x + b \cdot tgx + (c - d) = 0$ tenglamani hosil qilamiz va uni tgx ga nisbatan kvadrat tenglama qilib yechib ildizlarini aniqlaymiz.

4.14.3.Trigonometrik tengsizliklar:

1. $sinx \geq a$; $0 < a < 1\ da \rightarrow\ x \in [arcsina + 2\text{p}n;\ \text{p} - arcsina + 2\text{p}n]$.
2. $sinx \leq a$; $-1 < a < 0\ da \rightarrow\ x \in [-\text{p} - arcsina + 2\text{p}n;\ arcsina + 2\text{p}n]$.
3. $|sinx| \geq a$; $0 \leq a \leq 1\ da \rightarrow\ x \in [arcsina + \text{p}n;\ \text{p} - arcsina + \text{p}n]$.
4. $|sinx| \leq a$; $0 \leq a \leq 1\ da \rightarrow\ x \in [-arcsina + \text{p}n;\ arcsina + \text{p}n]$.
5. $cosx \geq a$; $0 < a < 1\ da \rightarrow\ x \in [-arccosa + 2\text{p}n;\ arccosa + 2\text{p}n]$.
6. $cosx \leq a$; $-1 < a < 0\ da \rightarrow\ x \in [arcsina + 2\text{p}n;\ 2\text{p} - arcsina + 2\text{p}n]$.
7. $|cosx| \geq a$; $0 \leq a \leq 1\ da \rightarrow\ x \in [-arccosa + \text{p}n;\ arccosa + \text{p}n]$.
8. $|cosx| \leq a$; $0 \leq a \leq 1\ da \rightarrow\ x \in [arccosa + \text{p}n;\ \text{p} - arccosa + \text{p}n]$.
9. $tgx \geq a$; $x \in [arctga + \text{p}n;\ \text{p}/2 + \text{p}n]$.

10 $tgx \leq a$; $x \in [-\text{p}/2 + \text{p}n;\ arctga + \text{p}n]$.

11. $ctgx \geq a$; $x \in [arcctga + \text{p}n;\ \text{p} + \text{p}n]$.
12. $ctgx \leq a$; $x \in [\text{p}n;\ arcctga + \text{p}n]$.

TRIGONOMETRIK TENGLAMA VA TENGSIZLIKLARGA DOIR MASHQLAR

Quyidagi trigonometrik tenglamalarni yeching.

1) $2sin6x - \sqrt{2} = 0$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko‘rinishda yozamiz:* $sin6x = \frac{\sqrt{2}}{2}$.

Bundan $6x = (-1)^n\frac{\text{p}}{4} + \text{p}n$. *Yoki* $x = (-1)^n\frac{\text{p}}{24} + \frac{\text{p}n}{6}$.

2) $2cos3x - 1 = 0$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko‘rinishda yozamiz:* $cos3x = \frac{1}{2}$.

Bundan $3x = \pm\frac{\text{p}}{3} + 2\text{p}n$. *Yoki* $x = \pm\frac{\text{p}}{9} + \frac{2\text{p}n}{3}$.

3) $2sin^2x - 5\sin(0{,}5\text{p} - x) = -5$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko‘rinishda yozamiz:*

$2-2cos^2x-5cosx+5=0$. *Yoki* $2cos^2x+5cosx-7=0$. *Buni* $cosx$ *ga nisbatan kvadrat tenglama qilib yechamiz:* $(cosx)_{1,2}=\frac{-5\pm\sqrt{25+4\cdot2\cdot7}}{2\cdot2}=\frac{-5\pm9}{4}$.

$(cosx)_1\neq\frac{-14}{4}\neq-3,5$. $(cosx)_2=\frac{4}{4}=1$. *Bundan* $x=2\text{p}n$.

4) $cos3x\cdot sinx-cos3x=0$ tenglamani yeching.

Yechish: *Berilgan tenglamadan* $cos3x$ *ni qavusdan chiqarib, quyidagi tenglamani hosil qilamiz:* $cos3x\cdot(sinx-1)=0$.

Buni quyidagi sistema qo'rinishiga keltirib, yechamiz:

$$\begin{cases}cos3x=0\\ sinx-1=0\end{cases}\Rightarrow\begin{cases}cos3x=0\Rightarrow x=\frac{\text{p}}{6}+\frac{\text{p}n}{3}.\\ sinx=1\Rightarrow x=\frac{\text{p}}{2}+2\text{p}n.\end{cases}$$

5) $tgx-tg\frac{\text{p}}{3}-tgx\cdot tg\frac{\text{p}}{3}=1$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*

$tgx-tg\frac{\text{p}}{3}=1+tgx\cdot tg\frac{\text{p}}{3}$ *tenglamani ikkala tomonini* $1+tgx\cdot tg\frac{\text{p}}{3}$ *ga bo'lib, quyidagi tenglikni hosil qilamiz:* $\frac{tgx-tg\frac{\text{p}}{3}}{1+tgx\cdot tg\frac{\text{p}}{3}}=1;\ tg\left(x-\frac{\text{p}}{3}\right)=1$.

Bundan $x-\frac{\text{p}}{3}=\frac{\text{p}}{4}+\text{p}n;\ x=\frac{7\text{p}}{12}+\text{p}n$.

6) $1-sin3x=\left(sim\frac{x}{2}-cos\frac{x}{2}\right)^2$ tenglamani yeching.

Yechish: *Berilgan tenglamani o'ng tomonini kvadratini ochsak:* $1-sinx$ *ga teng bo'ladi. Shunda* $1-sin3x=1-sinx$ *tenglik hosil bo'ladi.*

Bundan $sin3x-sinx=0$.

$2sinx\cdot cos2x=0$. *Buni quyidagi sistema qo'rinishiga keltirib, yechamiz:*

$$\begin{cases}2sinx=0\quad\Rightarrow x=\text{p}n.\\ cos2x=0\Rightarrow x=\frac{\text{p}}{4}+\frac{\text{p}n}{2}.\end{cases}$$

7) $sin3x+sinx=sin2x$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*

$2sin2x\cdot cosx-sin2x=0$. *Bundan* $sin2x\cdot(2cosx-1)=0$. *Buni quyidagi siste-ma qo'rinishiga keltirib, uni yechamiz:* $\begin{cases}sin2x=0\Rightarrow x=\frac{\text{p}n}{2}.\\ cosx=\frac{1}{2}\Rightarrow x=\pm\frac{\text{p}}{3}+2\text{p}n.\end{cases}$

8) $sin3x\cdot sinx+sin8x\cdot sin4x=0$ tenglamani yeching.

Yechish: *Berilgan tenglamadagi ko'paytmalarni yig'indiga keltiramiz:*

$\frac{1}{2}\left(cos2x-cos4x+cos4x-cos12x\right)=0$. *Yoki*$cos12x-cos2x=0$. *Ayirmani ko'payitmaga keltirish formulasini qo'llab quyidagi tenglikni hosil qilamiz:*

$-2sin7x\cdot sin5x=0$. *Yoki* $sin7x\cdot sin5x=0$. *Buni quyidagi sistema ko'rinishiga keltirib, uni yechamiz:* $\begin{cases}sin7x=0\Rightarrow x=\frac{\text{p}n}{7}.\\ sin5x=0\Rightarrow x=\frac{\text{p}n}{5}.\end{cases}$

9) $sin^2 2x + sin^2 3x + sin^2 4x + sin^2 5x = 2\ \ 0$ tenglamani yeching.

Yechish: *Darajani pasaytirish formulasini qo'llab, quyidagi tenglikni hosil qilamiz:*

$$\frac{1-cos4x}{2} + \frac{1-cos6x}{2} + \frac{1-cos8x}{2} + \frac{1-cos10x}{2} = 2.$$

Buni so'ldalashtirib, quyidagi tenglamani hosil qilamiz:

$cos10x + cos8x + cos6x + cos4x = 0.$ *Yig'indini ko'paytmaga keltirish formulasini qo'llab, quyidagi tenglamani hosil qilamiz:*

$2cos9x \cdot cosx + 2cos5x \cdot cosx = 0.$ *Bundan* $2cosx(cos9x + cos5x) = 0.$

Yoki $cosx \cdot cos7x \cdot cos2x = 0.$ *Buni quyidagi sistema ko'rinishiga keltirib, uni yechamiz:*

$$\begin{cases} cosx = 0 \Rightarrow x = \frac{p}{2} + pn. \\ cos2x = 0 \Rightarrow x = \frac{p}{4} + \frac{pn}{2}. \\ cos7x = 0 \Rightarrow x = \frac{p}{14} + \frac{pn}{7}. \end{cases}$$

10) $2cos2x + 2tg^2x = 5$ tenglamani yeching.

Yechish: *Kosinusni tangens orqali ifodalash formulasini qo'llab, berilgan tenglamani quyidagicha yozamiz:* $2\frac{1-tg^2x}{1+tg^2x} + 2tg^2x = 5.$ *Bundan quyidagi tenglamani hosil qilamiz:*

$2tg^4x - 5tg^2x - 3 = 0.$ *Bundan* $(tg^2x)_{1,2} = \frac{5\pm\sqrt{25+4\cdot2\cdot3}}{2\cdot2} = \frac{5\pm7}{4}.$

$(tg^2x)_1 \neq \frac{-1}{2} \neq -3{,}5.$ $(tg^2x)_2 = 3.$ *Bundan* $x = \pm arctg\sqrt{3} + pn.$

$x = \pm\frac{p}{3} + pn.$

11) $\frac{sinx}{1-cosx} = 0$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*

$\frac{sinx}{1-cosx} = \frac{2sin\frac{x}{2}\cdot cos\frac{x}{2}}{2sin^2\frac{x}{2}} = 0.$ *Yoki* $ctg\frac{x}{2} = 0.$ *Bundan* $\frac{x}{2} = \frac{p}{2} + pn.$

$x = p + 2pn.$

12) $|tgx + ctgx| = \frac{4}{\sqrt{3}}$ tenglamani yeching.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*

$\left|tgx + \frac{1}{tgx}\right| = \frac{4}{\sqrt{3}}$ *yoki*

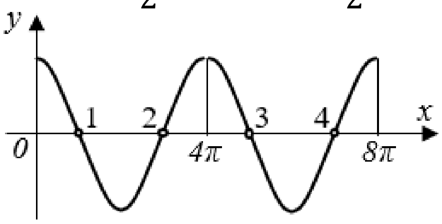
$$\left|\frac{1+tg^2x}{tgx}\right| = \left|\frac{2(1+tg^2x)}{2tgx}\right| = \left|\left(\frac{2tgx}{2(1+tg^2x)}\right)^{-1}\right| = \left|\left(\frac{1}{2}sin2x\right)^{-1}\right| = \left|\frac{2}{sin2x}\right| = \frac{4}{\sqrt{3}}.$$

$\left|\frac{1}{sin2x}\right| = \frac{2}{\sqrt{3}}$ $sin^2 2x = \frac{3}{4}$ *bundan* $2x = \pm arcsin\sqrt{\frac{3}{4}} + pn$ $x = \pm\frac{p}{6} + \frac{pn}{2}$

13) $4cos\frac{x}{2} + cosx + 1 = 0$ tenglamani [0;8p] oraliqda nechta ildizga ega.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*
$4cos\frac{x}{2}+2cos^2\frac{x}{2}=0.$ *Yoki* $2cos\frac{x}{2}\left(2+cos\frac{x}{2}\right)=0.$ *Bundan* $cos\frac{x}{2}\neq-2;\ cos\frac{x}{2}=0.$

Buni grafik usulda yechamiz; bunda kosinusning eng kichik davri $T=\frac{T_0}{k}=\frac{2\text{p}}{1/2}=4\text{p}$ *ga teng bo'ladi. Grafikdan* [0;8 p] *oraliqda y 4 marta 0 ga teng bo'lishini ko'rish mumkin.*



14) *k* ning quiydagi qo'rsatilgan qiymatlaridan qaysi birida $coskx\cdot cos4x-sinkx\cdot sin4x=\frac{\sqrt{3}}{2}$ tenglamaning ildizlari $\pm\frac{\text{p}}{30}+\frac{2\text{p}n}{5}$ bo'ladi.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:*
$\cos(kx+4x)=\frac{\sqrt{3}}{2}.$ *Bundan* $(k+4)x=\pm\frac{\text{p}}{6}+2\text{p}n.$ $x=\pm\frac{\text{p}}{6(k+4)}+\frac{2\text{p}n}{k+4}.$
Agar $k=1$ *bo'lsa,* $x=\pm\frac{\text{p}}{6(1+4)}+\frac{2\text{p}n}{1+4}=\pm\frac{\text{p}}{30}+\frac{2\text{p}n}{5}$ *bo'ladi.*

15) Nechta butun son $sin\frac{16\text{p}}{x}=0$ tenglamani qanoatlantiradi.

Yechish: *Tenglamani yechimi:* $\frac{16\text{p}}{x}=\text{p}n.$

Bundan $x=\frac{16}{n}$ $n=\pm1;\ \pm2;\ \pm4;\ \pm8;\ \pm16.$ *Demak x ning 10 ta qiymati tenglamani qanoatlantiradi.*

Quyidagi trigonometrik tengsizliklarni yeching.

1) $2sinx\geq\sqrt{3}$ tengsizlikni yeching.

Yechish: *Berilgan tengsizlikni quyidagi ko'rinishda yozamiz:* $sinx\geq\frac{\sqrt{3}}{2}.$
Buni trigonometrik tengsizliklarning (1) formulasidan foydalanib, yechamiz:
$\left[arcsin\frac{\sqrt{3}}{2}+2\text{p}n;\ \ \text{p}-arcsin\frac{\sqrt{3}}{2}+2\text{p}n\right].$ *Yoki* $\left[\frac{\text{p}}{3}+2\text{p}n;\frac{2\text{p}}{3}+2\text{p}n\right].$

2) $tg\left(x-\frac{\text{p}}{4}\right)\geq-1$ tengsizlikni yeching.

Yechish: *Berilgan tengsizliknini yechimi* $-\frac{\text{p}}{4}+\text{p}n\leq x-\frac{\text{p}}{4}<\frac{\text{p}}{2}+\text{p}n.$

$\begin{cases}x-\frac{\text{p}}{4}\geq-\frac{\text{p}}{4}+\text{p}n\ \Rightarrow x\geq\text{p}n.\\ x-\frac{\text{p}}{4}<\frac{\text{p}}{2}+\text{p}n\Rightarrow x<\frac{3\text{p}}{4}+\text{p}n.\end{cases}$ *Yoki* $\text{p}n\leq x<\frac{3\text{p}}{4}+\text{p}n.$

3) $2sin\frac{x}{2}\leq ctg\frac{\text{p}}{4}$ tengsizlikni yeching.

Yechish: *Berilgan tengsizlik* $2sin\frac{x}{2}\leq1.$ *Yoki* $sin\frac{x}{2}\leq\frac{1}{2}.$ *Buni yechimi*
$-\text{p}-arcsin\frac{1}{2}+2\text{p}n\leq\frac{x}{2}\leq\ \ arcsin\frac{1}{2}+2\text{p}n.$
Bundan $-\text{p}-\frac{\text{p}}{6}+2\text{p}n\leq\frac{x}{2}\leq\frac{\text{p}}{6}+2\text{p}n.$ *Yoki* $-\frac{7\text{p}}{3}+4\text{p}n\leq x\leq\frac{\text{p}}{3}+4\text{p}n.$

4) $2cos3x+8cosx<0$ tengsizlikni yeching.

Yechish: *Berilgan tengsizlikni 2 ga bo'lib, quyidagi ko'rinishda yozamiz:*

$cos3x + cosx + 3cosx < 0$. *Yig'indini ko'paytmaga keltirish formulasini qo'llab, tengsizlikni quyidagi ko'rinishda yozamiz:* $2cos2x \cdot cosx + 3cosx < 0$. *Bundan* $cosx$ *ni qavsdan chiqarsak,* $cosx(2cos2x + 3) < 0$ *bo'ladi. Bunda* $cosx < -1{,}5$ *bo'lishi mumkin emas va bu qiymat har doim manfiy bo'ladi. Shunda* $cosx > 0$ *bo'lishi kerak. Buni trigonometrik tengsizliklarning (5) formulasidan foydalanib, yechamiz:*

$(-arccos0 + 2\pi n;\ arccos0 + 2\pi n)$. *Yoki* $\left(-\frac{\pi}{2} + 2\pi n;\ \frac{\pi}{2} + 2\pi n\right)$.

5) $4cos^2x - 3 \geq 0$ tengsizlikni yeching.

Yechish: *Darajani pasaytirish formulasini qo'llab, berilgan tengsizlikni quyidagi ko'rinishda yozamiz:*

$2 + 2cos2x - 3 \geq 0$ *yoki* $cos2x \geq \frac{1}{2}$. *Bundan* $-\frac{\pi}{3} + 2\pi n \leq 2x \leq \frac{\pi}{3} + 2\pi n$

Yoki $-\frac{\pi}{6} + \pi n \leq x \leq \frac{\pi}{6} + \pi n$.

6) $1 - cos2x > sin^2 2x$ tengsizlikni yeching.

Yechish: *Darajani pasaytirish formulasini qo'llab, berilgan tengsizlikni quyidagi ko'rinishda yozamiz:*

$1 - cos2x > \frac{1 - cos4x}{2}$ *yoki* $2 - 2cos2x - 1 + cos4x > 0$. *Buni quyidagi ko'rinishda yozamiz:* $cos4x - cos2x + 1 - cos2x > 0$. *Ayirmani ko'paytmaga keltirish formulasini qo'llab, tengsizlikni quyidagi ko'rinishda yozamiz:*

$-2sin3x \cdot sinx + 2sin^2x > 0$ *buni -2 ga bo'lib,* $sinx$ *ni qavsdan chiqaramiz:*

$sinx(sin3x - sinx) < 0$ *yoki* $2sinx \cdot cos2x \cdot sinx < 0$. $sin^2x \cdot cos2x < 0$ *bunda* $sin^2x > 0$; $cos2x < 0$. *Bundan* $\frac{\pi}{2} + 2\pi n < 2x < \frac{3\pi}{2} + 2\pi n$.

Yoki $\frac{\pi}{4} + \pi n < x < \frac{3\pi}{2} + \pi n$.

7) $|sinx| \geq \frac{\sqrt{3}}{2}$ tengsizlikni yeching.

Yechish: *Buni trigonometrik tengsizliklarning (3) formulasidan foydalanib, yechamiz:* $\left[arcsin\frac{\sqrt{3}}{2} + \pi n;\ \pi - arcsin\frac{\sqrt{3}}{2} + \pi n\right]$ *yoki*

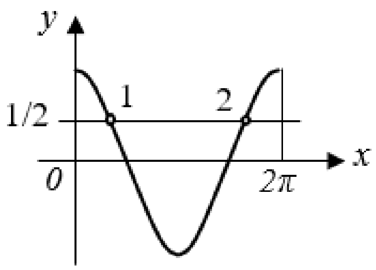
$\left[\frac{\pi}{3} + \pi n;\ \pi - \frac{\pi}{3} + \pi n\right]$; $\left[\frac{\pi}{3} + \pi n;\ \frac{2\pi}{3} + \pi n\right]$.

8) $cos^2x - \frac{5}{2}cosx + 1 \leq 0$ tengsizlik $[0; 2\pi]$ kesimda nechta butun yechimga ega.

Yechish: *Berilgan tengsizlikni quyidagi ko'rinishda yozamiz:*

$2cos^2x - 5cosx + 2 \leq 0$.

Bu kvadrat uch hadni ko'paytuvchilarga ajratamiz: $(2cosx - 1)(cosx - 2) \leq 0$. *Bunda* $cosx - 2 < 0$; $2cosx - 1 \geq 0$. *Buni* $cosx \geq \frac{1}{2}$ *deb yozib, uni* $[0; 2\pi]$ *kesimdagi butun yechimini grafik usulda aniqlaymiz:*



Bunda kosinusning eng kichik davri $T = \frac{T_0}{1} = \frac{2\pi}{1} = 2\pi$ *ga teng bo'ladi. Demak, berilgan* $[0; 2\pi]$ *kesimda 2 ta butun yechim bor ekan.*

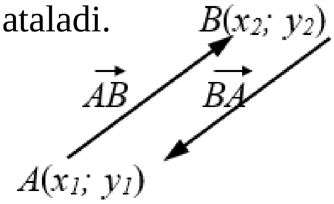
5. ANALITIK GEOMETRIYA

5.1. Vektorlar.

Ayrim fizik kattaliklar (masalan: ko‘chish, tezlik, tezlanish, kuch va boshqalar) son qiymatlaridan tashqari yo‘nalishlari bilan ham tavsiflanadi.

Yo‘nalishga ega bo‘lgan fizik kattalikka ***vektor*** kattalik deb ataladi.

Agar ikkita $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar koordinatalari bilan berilgan bo‘lsa, ulardan A (vektorni boshi) nuqtadan B (vektorning oxiri) nuqtaga yo‘nalgan $\overrightarrow{AB}$ va aksincha B nuqtadan A nuqtaga yo‘nalgan $\overrightarrow{BA}$ vektorlarni hosil qilish mumkin (5.1-rasm).



5.1-rasm.

5.1.1.Vektorlar koordinatalarini aniqlash uchun ularning yo‘nalishiga e’tibor beriladi. Bunda vektorning oxirgi nuqtasi koordinatasidan, vektorning boshidagi nuqtasining koordinatasi ayriladi. Musol uchun $\overrightarrow{AB}(x_2 - x_1;\ y_2 - y_1)$.

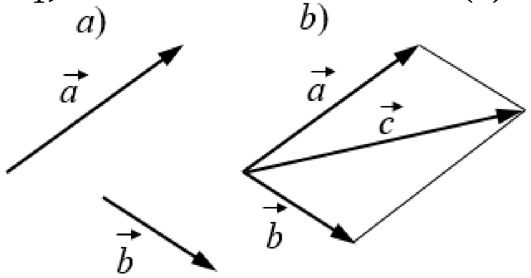
Agar fazo‘la $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ bo‘lsa $\overrightarrow{AB}(x_2 - x_1;\ y_2 - y_1;\ z_2 - z_1)$. (1)

5.1.2. Vektorlarning mo‘luli (absalyut qiymati,uzunligi).

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (2)$$

5.1.3. Vektorlarni qo‘shish. Agar vektorlar ma’lum burchak ostida bir tomonga yo‘nalgan bo‘lsa (5.2a-rasm), ularni biri ikkinchisiga parallel ko‘chirilib, parallelogram hosil qilinadi (5.2b-rasm) va parallelogramning diagonali uzunligi vektorlar uzunligining yig‘indisiga teng bo‘ladi.

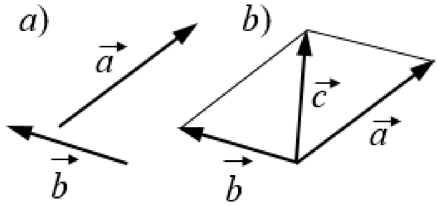
$$\vec{c} = \vec{a} + \vec{b}$$



5.2-rasm.

5.1.4. Vektorlarni ayrish. Agar vektorlar ma’lum burchak ostida turli tomonga yo‘nalgan bo‘lsa (5.3a-rasm), ularni biri ikkinchisiga parallel ko‘chirilib, parallelogram hosil qilinadi (5.3b-rasim) va parallelogramning diagonali uzunligi vektorlar uzunligining ayirmasiga teng bo‘ladi.

$$\vec{c} = \vec{a} - \vec{b}$$



5.3-rasm.

Vektorlarni qo‘shish va ayrishning umumiy formulasini quyidagicha yozish mumkin: $\vec{c} = \vec{a} \pm \vec{b}.$ Bu vektorlarni mo‘luli yoki uzunligi

$$|\vec{c}\,| = \left|\vec{a} \pm \vec{b}\right| = \sqrt{\vec{a}^2 \pm 2\vec{a}\cdot\vec{b} + \vec{b}^2}. \qquad (3)$$

5.1.5. Vektorlarni songa ko‘paytirish.Agar vektor biror songa ko‘paytirilsa, bu vektorning koordinatalarining har biri shu songa ko‘paytiriladi:

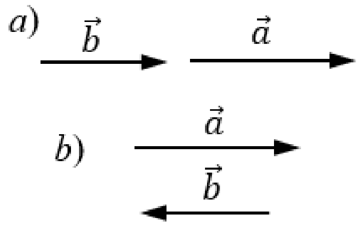
$$\text{л}\left(\vec{a} \pm \vec{b}\right) = \text{л}\vec{a} \pm \text{л}\vec{b}. \qquad (4)$$

5.1.6. Vektorlarni skalyar ko‘paytmasi. $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z.$ (5)

5.1.7. Parallel (kollinear) vektorlar. Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotgan, noldan farqli ikkita vektor ***kollinear (parallel)*** vektorlar deb ataladi

$\vec{a} \parallel \vec{b}$. Ular bir (5.4*a*-rasm, orasidagi burchak $ц = 0^0$) yoki qarama-qarshi yo‘nalishda (5.4*b*-rasm, orasidagi burchak $ц = 180^0$) bo‘lishi mumkin. Parallel vektorlar koordinatalarining nisbatlari teng bo‘ladi.

$$\frac{a_x}{b_z} = \frac{a_y}{b_y} = \frac{a_z}{b_z}. \qquad (6)$$

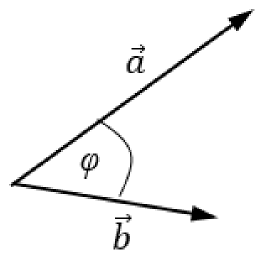


5.4-rasm.

5.1.8. Perpendikulyar vektorlar. Vektorlar perpendikulyar $\vec{a} \perp \vec{b}$ bo‘lish uchun ularning skalyar ko‘paytmasi nolga teng bo‘lishi kerak: $a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z = 0.$ (7)

5.1.9. Vektorlar orasidagi burchak.

$$cosц = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_x\cdot b_x + a_y\cdot b_y + a_z\cdot b_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}. \qquad (8)$$



Bundan $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot cosц$. (9)

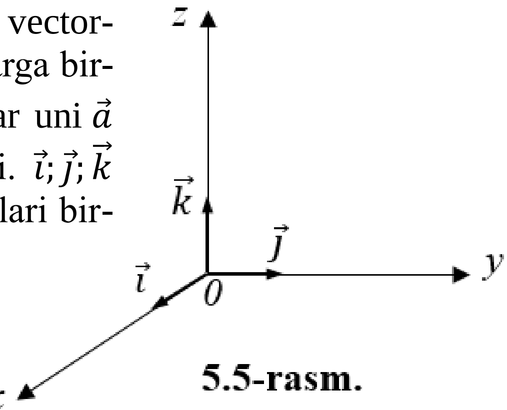
Bu qiymatni (1) formulaga qo‘yib, quyidagi formulani hosil qilamiz:

$$|\vec{c}\,| = |\vec{a} \pm \vec{b}| = \sqrt{\vec{a}^2 \pm 2|\vec{a}| \cdot |\vec{b}| \cdot cosц + \vec{b}^2}. \qquad (10)$$

5.1.10. Fazo‘lagi vektorlar. Ular, ortli yoki birlik vectorlari ham deb yuritiladi. Bunda fazo‘lagi *x*, *y* va *z* o‘qlarga birlik vektorlar $\vec{i}; \vec{j}; \vec{k}$ joylashgan bo‘ladi (5.5-rasm). Agar uni $\vec{a}$ vektor desak, uning koordinatalari $\vec{a}(\vec{i}; \vec{j}; \vec{k})$ bo‘ladi. $\vec{i}; \vec{j}; \vec{k}$ vektorlar birlik vektor bo‘lgani uchun ularning mo‘lullari birga teng bo‘ladi:

$|\vec{i}| = 1;\ |\vec{j}| = 1;\ |\vec{k}| = 1.$

Birlik vettorlarning skaryar ko‘paytmasi nolga teng bo‘ladi: $\vec{i} \cdot \vec{j} = 0;\ \vec{i} \cdot \vec{k} = 0;\ \vec{k} \cdot \vec{j} = 0.$



5.5-rasm.

Bunga sabab, birlik vektorlari orasidagi burchaklarning bir biri bilan $ц = 90^0$ li burchakni tashkil qilishidir. Chunki $cos90^0 = 0$.

Birlik vektorning mo‘luli:

$$|\vec{a}| = \sqrt{(\vec{i})^2 + (\vec{j})^2 + (\vec{k})^2} = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}.$$

VEKTORLARGA DOIR MASHQLAR

1) Agar $\vec{a}(-2; 3)$ va $\vec{b}(4; 1)$ bo‘lsa, $\vec{m} = 2\vec{a} - 3\vec{b}$ vektorning koordinatalarini toping.

Yechish: *$2\vec{a}$ va $3\vec{b}$ vektorlarning koordinatalarini aniqlaymiz:*

$2\vec{a}(-4; 6),\ 3\vec{b}(12; 3).\quad \vec{m}[(-4-12); (6-3)]$ *yoki* $\vec{m}(-16; 3)$.

2) *A(2;-1;0)* va *B(0;3;2)* nuqtalardan hosil bo‘lgan $\overrightarrow{AB}$ vektorning uzunligini toping.

Yechish: *Bu vektorning uzunligini* (2) *formuladan foydalanib aniqlaymiz:*

$$|\overrightarrow{AB}| = \sqrt{(0-2)^2 + (3-(-1))^2 + (2-0)^2} = 2\sqrt{6}.$$

3) $\vec{a}(-3;5)$ va $\vec{b}(2;-4)$ berilgan bo‘lsa, $4\vec{a}+3\vec{b}$ vektorni mo‘lulini toping.

Yechish: *$4\vec{a}+3\vec{b}$ vektorni $\vec{m}$ vektor bilan belgilab,$4\vec{a}$ va $3\vec{b}$ vektorlarning koordinatalarini aniqlaymiz: $4\vec{a}(-12;20)$, $3\vec{b}(6;-12)$. $\vec{m}$ vektorning koordinatalari $(-6;8)$ ga teng.* $|\vec{m}| = \sqrt{(-6)^2+8^2} = 10.$

4) Agar $|\vec{a}| = \sqrt{85}$, $|\vec{a}+\vec{b}| = 20$ va $|\vec{a}-\vec{b}| = 9\sqrt{2}$ bo`lsa, $|\vec{b}|$ ni toping.

Yechish: *Vektorlar yig‘indisi va ayirmasini mo‘lullarini kvadratga oshirib, quyidagi sistemani tizamiz:*

$$\begin{cases}\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2 = 400\\ \vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2 = 162\end{cases}$$ *bu sistema hadlarini bir biriga qo‘shib va uni ikkiga bo‘lib, quyidagi tenglikni hosil qilamiz:*

$\vec{a}^2+\vec{b}^2 = 281$ *yoki* $\sqrt{85}^2+\vec{b}^2 = 281$ *bundan* $|\vec{b}| = \sqrt{281-85} = \sqrt{196} = 14.$

5) $|\vec{a}| = 2$ va $|\vec{b}| = 4$, bu vektorlar orasidagi burchak $\pi/3$ ga teng bo‘lsa, $3\vec{a}-2\vec{b}$ va $5\vec{a}-2\vec{b}$ vektorlarning skalyar ko`paytmasini toping.

Yechish: *$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi:*

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot cos60^0 = 2\cdot4\cdot0{,}5 = 4.$ $3\vec{a}-2\vec{b}$ *va* $5\vec{a}-2\vec{b}$ *vektorlarning skalyar ko‘paytmasi* $15\vec{a}^2-16\vec{a}\vec{b}+4\vec{b}^2 = 15\cdot2^2-16\cdot4+4\cdot4^2 = 60.$

6) $\vec{a}(1;2)$ va $\vec{b}(1;-0{,}5)$ vektorlar orasidagi burchakni toping.

Yechish: *Bu vektorlarning skalyar ko‘paytmasini topamiz:*

$\vec{a}\cdot\vec{b} = 1\cdot1+2\cdot(-0{,}5) = 0.$ *Demak vektorlar orasidagi burchak* ц $= 90^0$ *ekan.*

7) n ning qanday qiymatlarida $\vec{a}(n;\ -2;\ 1)$ va $\vec{b}(n;\ 1;\ -3n+2)$ vektorlar $\vec{a}\perp\vec{b}$ bo`ladi.

Yechish: *Bu vektorlar perpendicular bo`lish uchun ularning skalyar ko‘paytmasi nolga teng bo‘lishi kerak. Ya’ni* $n^2-2-3n+2 = 0$. *Yoki* $n^2-3n = 0$.

Bundan $n = 0$ *va* $n = 3$.

8) $\vec{a}(-2;1;4)$ vector va $M(2;-\frac{23}{21};-\frac{19}{3})$ nuqta berilgan. Agar $2\vec{a}+3\overrightarrow{MN}=0$ bo‘lsa, N nuqtaning koordinatalarini toping.

Yechish: *N nuqtaning koordinatalari $N(x;y;x)$. $2\vec{a}$ va $3\overrightarrow{MN}$ vektorlarlarni koordinatalarini aniqlaymiz: $2\vec{a}(-4;2;8)$, $3\overrightarrow{MN}[(3x-6);(3y+23/7);(3z+19)]$.*

$$\begin{cases}-4+3x-6 = 0 \Rightarrow x = 10/3\\ 2+3y+23/7 = 0 \Rightarrow y = -3/7\\ 8+3z+19 = 0 \Rightarrow z = -11/3\end{cases} \quad N\left(\frac{10}{3};\ -\frac{3}{7};\ -\frac{11}{3}\right)$$

1) $\vec{a}(x;1;-1)$ va $\vec{b}(1;0;1)$ vektorlar berilgan. x ning qanday qiymatida $(\vec{a}+3\vec{b})^2 = (\vec{a}-2\vec{b})^2$ tenglik o‘rinli bo‘ladi.

Yechish: $\vec{a}^2+6\vec{a}\cdot\vec{b}+9\vec{b}^2 = \vec{a}^2-4\vec{a}\cdot\vec{b}+4\vec{b}^2$. *Bundan* $5\vec{b}^2+10\vec{a}\cdot\vec{b} = 0$

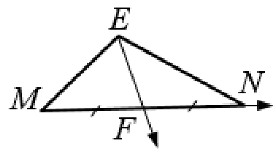
$\vec{a}\cdot\vec{b} = x+0-1 = x-1.$ $|\vec{b}| = \sqrt{1^2+0^2+1^2} = \sqrt{2}.$ $5\cdot2+10x-10 = 0 \Rightarrow x = 0.$

10) $\vec{a}=5\vec{i}+\sqrt{2}\,\vec{j}-3\vec{k}$ va $\vec{i}$ vektorlar orasidagi burchakning kosinusini toping.

Yechish: *$\vec{a}$ verktorning koordinatalari $\vec{a}(5;\ \sqrt{2};\ -3)$,*

mo'luli $|\vec{a}|=\sqrt{5^2+\left(\sqrt{2}\right)^2+(-3)^2}=6.\ \vec{i}(1;0;0).\ |\vec{i}|=1.\ cos\text{ц}=\frac{5+0+0}{6\cdot1}=\frac{5}{6}.$

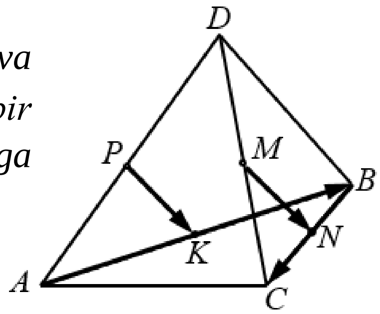
11) Uchlari *M*(-3;3;1); *N*(3;-5;1) va *E*(-4;-1;-2) nuqtalarda bo'lgan uchburchakning *MN* tomoni va *EF* medianasi orasidagi burchakni toping.



Yechish: $\overrightarrow{MN}\ (6;\ -8;0).\ \left|\overrightarrow{MN}\right|=10\ F(0;-1;1).\ \overrightarrow{EF}\ (4;0;3).$

$\left|\overrightarrow{EF}\right|=5.\ cos\varphi=\frac{24+0+0}{10\cdot5}=\frac{12}{25}.\quad \varphi=arccos0{,}48.$

12) *DABC* tetraedr *M, N* , *K* va *P* nuqtalar mos ravishda *DC, BC, AB* va *DA* qirralarining o'rtalari. Tetraedrning qirrasi 4 ga teng bo'lsa, $\overrightarrow{MN}\cdot\overrightarrow{PK}+\overrightarrow{AB}\cdot\overrightarrow{BC}$vektorlar skalyar ko'paytmasini yig'indisini toping.



Yechish: *Masala shartiga asosan $AB=BC=DA=4$ va $\left|\overrightarrow{MN}\right|=\left|\overrightarrow{PK}\right|=2$. Cizmadan $\overrightarrow{MN}\parallel\overrightarrow{PK}$ bo'lgani va bir tomonga yo'langani uchun ular orasidagi burchak* $\text{ц}=0^0$*ga teng. $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlar orasidagi burchak* $\text{ц}=120^0$.

$\overrightarrow{MN}\cdot\overrightarrow{PK}=\left|\overrightarrow{MN}\right|\cdot\left|\overrightarrow{PK}\right|cos0^0=2\cdot2\cdot1=4.$

$\overrightarrow{AB}\cdot\overrightarrow{BC}=\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{BC}\right|cos120^0=4\cdot4\cdot(-0{,}5)=-8.$

$$\overrightarrow{MN}\cdot\overrightarrow{PK}+\overrightarrow{AB}\cdot\overrightarrow{BC}=4-8=-4.$$

6. FUNKSIYANING LIMITI

Funksiya ***limiti*** tushunchasiga quyidagicha tushuntirish mumkin: agar $x\neq a$ bo'lib, uning qiymatlari *a* soniga yaqinlashsa, $f(x)$ funksiyaning mos qiymqtlari *A* soniga yaqinlashsin. Bu holda *A* soni *x a* ga yaqinlashganda $f(x)$ funksiyaning ***limiti*** deyiladi va uquyidagicha belgilanadi: $\lim_{x\to a}f(x)=A$

Limitlarni hisoblashga doir misollar.

1) $\lim_{x\to2}(x^2+3x-5)=5.$ 2) $\lim_{x\to3}\frac{x+5}{x-1}=4.$ 3) $\lim_{x\to2}\sqrt{3x^2+x+2}=4.$

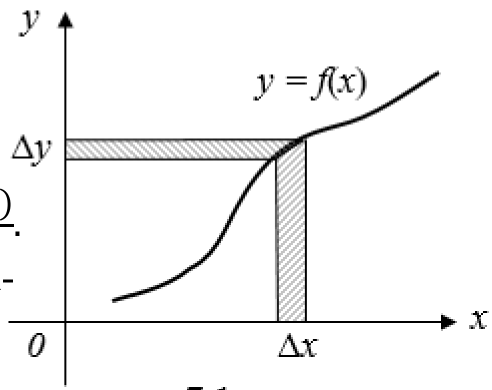
$\frac{0}{0}$ ko'rinishdagi limitlarni yechishga doir misollar.

1) $\lim_{x\to3}\frac{x^2-9}{x-3}=\lim_{x\to3}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to3}(x+3)=6.$

2) $\lim_{x\to0}\frac{2x^2-x}{x}=\lim_{x\to0}\frac{x(2x-1)}{x}=\lim_{x\to0}(2x-1)=-1.$

3) $\lim_{x\to3}\frac{x^2-x-6}{x-3}=\lim_{x\to3}\frac{(x-3)(x+2)}{x-3}=\lim_{x\to3}(x+2)=4.$

7. MATEMATIK ANALIZ ASOSLARI



7.1-rasm.

7.1. Hosila. Hosila deb, funksiya orttirmasi Δy dan argument orttirmasi Δx ni (7.1-rasm) nolga intiltirib olingan limitga aytiladi.

$$f'(x) = lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = lim_{\Delta x \to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

$f(x)$ funksiyaning $f'(x)$ hosilasini aniqlashga funksiyani ***diffrensiyalash*** deyiladi.

Hosila formulalari:

O‘ldiy funksiya	*Murakkab funksiya*
1. $(c)' = 0$; 2. $(cx)' = c$. 3. $(x^n)' = n \cdot x^{n-1}$. 4. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$. 5. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$. 6. $(e^x)' = e^x$. 7. $(a^x)' = a^x \cdot lna$. 8. $(log_a x)' = \frac{1}{x \cdot lna}$. 9. $(lnx)' = \frac{1}{x}$. 10. $(sinx)' = cosx$. 11. $(cosx)' = -sinx$. 12. $(tgx)' = \frac{1}{cos^2x}$. 13. $(ctgx)' = -\frac{1}{sin^2x}$.	1. $(u \pm \vartheta)' = (u)' \pm (\vartheta)'$. 2. $(u \cdot \vartheta)' = (u)' \cdot \vartheta + (\vartheta)' \cdot u$. 3. $\left(\frac{u}{\vartheta}\right)' = \frac{(u)' \cdot \vartheta - (\vartheta)' \cdot u}{\vartheta^2}$. 4. $(u^\vartheta)' = \vartheta \cdot u^{\vartheta-1} \cdot (u)'$. 5. $\left(\frac{1}{u}\right)' = -\frac{(u)'}{u^2}$. 6. $(\sqrt{u})' = \frac{(u)'}{2\sqrt{u}}$. 7. $(a^u)' = a^u \cdot lna \cdot (u)'$. 8. $(e^u)' = e^u \cdot (u)'$. 9. $(log_a u)' = \frac{(u)'}{u \cdot lna}$. 10. $(lnu)' = \frac{(u)'}{u}$. 11. $(sinu)' = (u)' \cdot cosu$. 12. $(cosu)' = -(u)' \cdot sinu$. 13. $(tgu)' = \frac{(u)'}{cos^2u}$. 14. $(ctgu)' = -\frac{(u)'}{sin^2u}$.

Agar $a \le x \le y \le z \le t \le b$ bo‘lsa, $\frac{x}{y} + \frac{z}{t}$ ifodaning eng kichik va eng katta qiymatini toping. Ifodalarning bir biriga tengligidan foydalanib, quyidagi funksiyani tuzamiz: $f(x) = \frac{a}{x} + \frac{x}{b}$. Bu funksiyaning hosilasini olamiz va uni nolga tenglab, x ni qiymatini topamiz: $f'(x) = -\frac{a}{x^2} + \frac{1}{b} = 0 \Rightarrow x = \sqrt{ab}$.

$$f(x) = \frac{a}{\sqrt{ab}} + \frac{\sqrt{ab}}{b} = \frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{a}}{\sqrt{b}} = 2\sqrt{\frac{a}{b}}.$$

7.1.1. Hosilaning tadbig‘i:

1. Funksiyaning kritik nuqtalarini aniqlashda. Funksiyaning chekki nuqtalariga ktitik nuqta deyiladi. Kritik nuqtalarini aniqlash uchun berilgan funksiyadan hosila olib, u nolga tenglanadi. $f'(x) = y' = 0$.

2. Funksiyaning o‘sish va kamayish oralig‘ini aniqlashda. Agar kritik nuqtagacha bo‘lgan barcha qiymatda funksiya hosilasi $y' > 0$ bo‘lsa, shu oraliqda funksiya o‘sad $y\uparrow$. Agar kritik nuqtagacha bo‘lgan barcha qiymatda funksiya hosilasi $y' < 0$ bo‘lsa, shu oraliqda funksiya kamyadi $y\downarrow$.

3. Agar funksiya kritik nuqtada ishorasini $(-)$ dan $(+)$ ga o‘zgartirsa, funksiya shu nuqtada minimum (***min***), agar funksiya kritik nuqtada ishorasini $(+)$ dan $(-)$ ga o‘z-gartirsa, funksiya shu nuqtada maksimum (***max***) bo‘lgan ***ekstrimum*** nuqtalarga ega bo‘ladi.

4. Berilgan oraliqda eng katta va eng kichik qiymatini topishda. Agar berilgan oraliqda ekstrimum nuqta bo‘lsa, uni ham hisobga olish kerak.

5. Egri chiziqni x_0 nuqtasiga o‘tkazilgan urinma tenglamasini tuzishda. Ma’lumki urinma to‘g‘ri chiziqdan iborat bo‘ladi. Bunda to‘g‘ri chiziqning tenglamasi $y = kx + b$ dagi k (urinmaning burchak koffitsiyenti $k = f'(x_0)$.) va b larni aniqlash talab etiladi. Urinma tenglamasi quyidagi formula orqali aniqlanadi $y = f(x_0) + f'(x_0)(x - x_0)$.

6. Harakatlanayotgan mo‘ldiy nuqtani tezligi va tezlanishini aniqlashda.

Harakat tenglamasidan olingan birinchi tartibli hosila shu harakatni tezligini aniqlaydi $S'(t) = \vartheta(t)$.

Harakat tenglamasidan olingan ikkinchi tartibli yoki tezlikdan olingan birinchi tartibli hosila shu harakatni tezlanishini aniqlaydi $S''(t) = \vartheta'(t) = a(t)$.

Funksiyani tekshirish va uni grafigini qurishga doir misol.

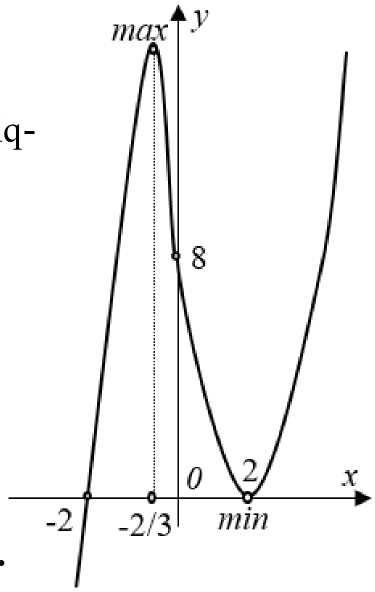
$y = x^3 - 2x^2 - 4x + 8$ funksiyani tekshirish va uni grafigini qurish.

1. Absissa va ordinata oqlari bilan kesishish nuqtalarini aniqlaymiz: $x = 0$ da $y = 8$, $y = 0$ da $x_1 = -2$; $x_2 = 2$.

2. Kritik nuqtalarini aniqlaymiz:

$$3x^2 - 4x - 4 = 0 \Rightarrow x_1 = -\frac{2}{3};\ x_2 = 2.$$

3. O‘sish, kamayish oraliqlarini va ekstrimum nuqtalarini aniqlovchi jadval tuzamiz:



x	$(-\infty; -2/3)$	$-2/3$	$(-2/3; 2)$	2	$(2;\ \infty)$
$f'(x)$	*o‘sadi* $y\uparrow (+)$		*Kamayadi* $y\downarrow (-)$		*o‘sadi* $y\uparrow (+)$
$f(x)$		$4\frac{22}{27}$		0	
Ekstri-mumlari		*max*		*min*	

7.1.2. Funksiya grafigiga o‘tkazilgan urinma tenglamasi.

$y = f(x)$ funksiya grafigining $(x_0; f(x_0))$ nuqtasidan o‘tuvchi urunma tenglamasini tuzamiz (7.2-rasm).

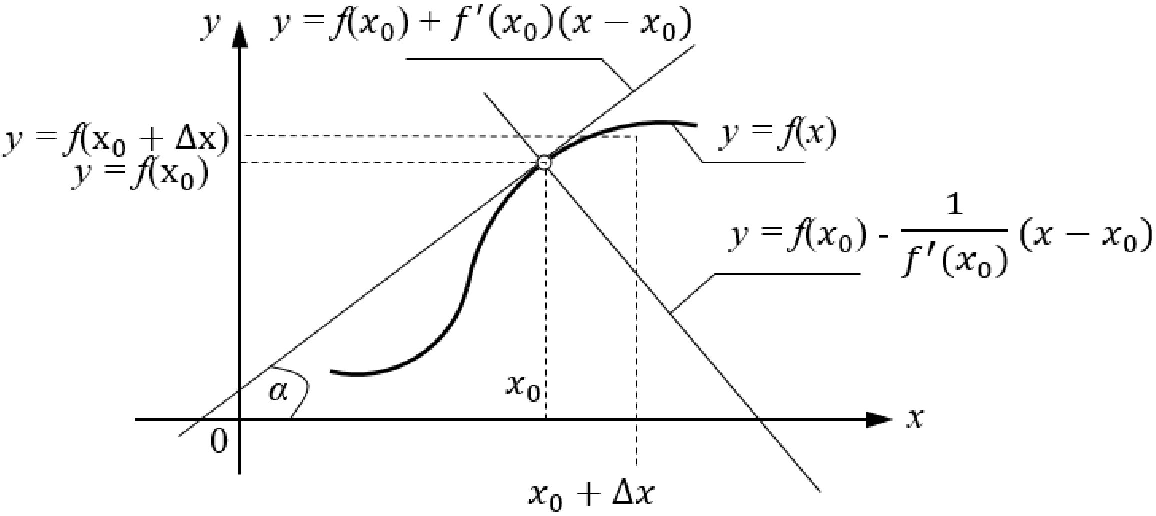
Urunma to‘g‘ri chiziqdan iborat bo‘lgani uchun uning tenglamasi $y = kx + b$ bo‘la-di. Hosilaning geometrik ma’nisiga ko‘ra urinmaning burchak koffitsiyenti $k = f'(x_0)$. Urinma tenglamasi $y = f'(x_0)x + b$ ko‘rinishga ega bo‘ladi. Bu urunma $(x_0; f(x_0))$ nuqtasidan o‘tgani uchun $f(x_0) = f'(x_0)x_0 + b$ bo‘ladi. Bundan $b =$

$f(x_0) - f'(x_0)x_0$ bo‘ladi. Shunda urinma tenglamasi $y = f(x_0) + f'(x_0)(x - x_0)$ korinishda bo‘ladi.

7.1.3. Funksiya grafigiga o‘tkazilgan normal tenglamasi.

$y = f(x)$ funksiya grafigining $(x_0; f(x_0))$ nuqtasidan o‘tuvchi urunma $x = x_0$ nuqtada perpendikulyar bo‘lgani uchun (7.2-rasm), funksiya grafigiga o‘tkazilgan normal tenglamasi quyidagi ko‘rinishda bo‘ladi.

$$y = f(x_0) - \frac{1}{f'(x_0)}(x - x_0). \qquad (7.2)$$



7.2-rasm.

7.1.4. Taqribiy hisoblashlar.

Agar 7.2-rasmdagi $x - x_0 = \Delta x$ deb belgilasak (7.1) formulani quyidagi ko‘rinishda yozish mumkin:

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x. \qquad (7.3)$$

Taqribiy hisoblashga doir misollar.

1) $f(x) = x^4 - 3x^3 + 2x^2 - x + 5$ fugksiyani $x = 2{,}02$ nuqtadagi qiymatini taqribiy hisoblang.

<u>Yechish:</u> *Funksiyaning* $x = 2{,}02$ *nuqtasiga yaqin bo‘lgan* $x_0 = 2$ *nuqtani olsak, bu nuqtada funksiyaning qiymati* $f(x_0) = f(2) = 2^4 - 3 \cdot 2^3 + 2 \cdot 2^2 - 2 + 5 = 3.$

Bu funksiyaning hosilasi $f'(x) = 4x^3 - 3 \cdot x^2 + 2 \cdot 2x - 1.$

U holda $f'(2) = 4 \cdot 2^3 - 3 \cdot 2^2 + 2 \cdot 2 - 1 = 32 - 12 + 4 - 1 = 23.$

$\Delta x = x - x_0 = 2{,}02 - 2 = 0.02.$ *Bularni (7.3) formulaga qo‘ysak*

$$f(2{,}02) \approx 3 + 23 \cdot 0{,}02 = 3 + 0{,}46 = 3{,}46..$$

2) $\sqrt{4{,}02}$ ildizning qiymatini taqribiy hisoblang.

<u>Yechish:</u> *Buni* $f(x) = \sqrt{x}$ *funksiya sifatida qarab, uni hosilasini topamiz:*

$f'(x) = \frac{1}{2\sqrt{x}}$ *agar* $x_0 = 4$ *deb olsak,* $f(x_0) = f(4) = \sqrt{4} = 2,$

$f'(x_0) = f'(4) = \frac{1}{2\sqrt{4}} = \quad = \frac{1}{2 \cdot 2} = 0{,}25. \quad \Delta x = x - x_0 = 4{,}02 - 4 = 0.02.$

Bularni (7.3) formulaga qo‘ysak $\sqrt{4{,}02} \approx 2 + 0{,}25 \cdot 0{,}02 \approx 2 + 0{,}005 = 2{,}005.$

3) $\sqrt[3]{26{,}998}$ ildizning qiymatini taqribiy hisoblang.

Yechish: *Buni* $f(x) = \sqrt[3]{x}$ *funksiya sifatida qarab, uni hosilasini topamiz:*

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$ *agar* $x_0 = 27$ *deb olsak,* $f(x_0) = f(27) = \sqrt[3]{27} = 3,$

$$f'(x_0) = f'(27) = \frac{1}{3\sqrt[3]{27}} = \frac{1}{3\cdot 3} = \frac{1}{9}. \quad \Delta x = x - x_0 = 26{,}998 - 27 = -0.002.$$

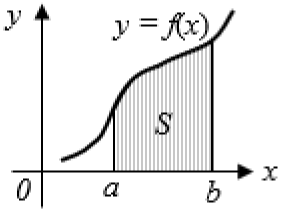
Bularni (7.3) formulaga qo'ysak $\sqrt[3]{26{,}998} \approx 3 - \frac{1}{9}\cdot 0{,}002 \approx 3 - 0{,}000222 =$
$= 2{,}999778.$

7.2. Boshlang'ich funksiya (aniq integral): $F(x) = \int_a^b f(x)dx.$

Egri chiziqli yuzalarni hisoblash:

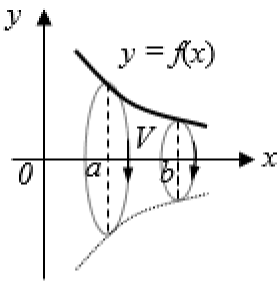
$F(x) = S = \int_a^b f(x)dx = F(b) - F(a).$

Nyuton-Leybnis formulasi.



Egri chiziqli fazoviy jismlarni hajmini hisoblash:

$V = \text{p}\int_a^b f^2(x)dx = \text{p}[F(b) - F(a)].$



Konus va sharni hajmini hisoblashda ushbu formuladan foydalanilgan.

Aniqmas integral formulalari:

1. $\int dx = x + C$; 2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$; 3. $\int e^x\,dx = e^x + C$; 4. $\int a^x\,dx = \frac{a^x}{lna} + C$;
5. $\int sinxdx = -cosx + C$; 6. $\int cosxdx = sinx + C$; 7. $\int \frac{dx}{cos^2x} = tgx + C$; 8. $\int \frac{dx}{sin^2x} = -ctgx + C$;
9. $\int \frac{dx}{x} = ln|x| + C$; 10 $\int \frac{(u)'dx}{u} = ln|u| + C.$ 11. $\int \frac{dx}{1+x^2} = arctgx + C$;
12. $\int \frac{dx}{\sqrt{1-x^2}} = arcsinx + C$; 13. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}arctg\frac{x}{a} + C.$

Bo'laklab integrallash: Agar biror X oraliqda $f(x)$ va $g(x)$ funksiyalar uzluksiz $f'(x)$ va $g'(x)$ hosilaga ega bo'lsa, u holda

$$\int f(x)\cdot g'(x)dx = f(x)\cdot g(x) - \int g(x)\cdot f'(x)dx \qquad (7.4)$$

formula o'rinli bo'ladi. Bu formulaga bo'laklab integrallash formulasi deyiladi.

Bo'laklab integrallashga doir misollar:

1) $\int lnxdx$ *integralni hisoblang.*

Yechish: Integral ostidagi lnx funksiyani $f(x) = lnx$ va $g'(x) = 1$ larning ko'paytmasi hisoblaymiz va natijada $lnx = f(x)\cdot g'(x)$ bo'ladi.

U holda $f'(x) = \frac{1}{x}$ va $g(x) = \int dx = x + C$ bo'ladi. (7.1) formulaga ko'ra

$$\int lnxdx = x\cdot lnx - \int x\cdot\frac{1}{x}dx = x\cdot lnx - \int dx = x\cdot lnx - x + C =$$
$$= x\cdot(lnx - 1) + C = x\cdot(lnx - lne) + C = x\cdot ln\frac{x}{e} + C.$$

2) $\int xcosxdx$ *integralni hisoblang.*

Yechish: Integral hisoblash uchun $f(x) = x$ va $g'(x) = cosx$ deb olamiz.

U holda $f'(x) = 1$ va $g(x) = \int cosxdx = sinx + C$ bo'ladi. (7.4) formulaga ko'ra

$\int xcosxdx = x \cdot sinx - \int sinx \cdot 1dx = x \cdot sinx + cosx + C.$

Aniq integralni yangi o'zgartirish kiritish yo'li bilan yechish:

$$\textbf{Misol: } \int_{\frac{p}{6}}^{\frac{p}{2}} \frac{sinx}{sinx+2}dx = \left| \begin{matrix} sinx = t \\ sinxdx = dt \\ x = \frac{p}{2} \Rightarrow t = 1 \\ x = \frac{p}{6} \Rightarrow t = \frac{1}{2} \end{matrix} \right| = \int_{\frac{1}{2}}^{1} \frac{dt}{t+2} = \ln(t+2) =$$

$$= \ln(1+2) - \ln\left(\frac{1}{2} + 2\right) = ln3 - ln\frac{5}{2} = ln\frac{6}{5}.$$

HOSILA VA BOSHLANG'ISH FUNKSIYALARGA DOIR MISOL VA MASALALAR

Hosilaga doir:

1) $f(x) = \dfrac{2-x}{x}$ funksiyani hosilasini aniqlang.

Yechish: *Berilgan funksiyani quyidagi ko'rinishda yozamiz:* $f(x) = \frac{2}{x} - 1.$

Bu funksiyani hosilasi $f'(x) = -\frac{2}{x^2}.$

2) $f(x) = \sqrt[3]{2x + x^2}$ funksiyani hosilasini aniqlang.

Yechish: *Berilgan funksiyani quyidagi ko'rinishda yozamiz:* $f(x) = (2x + x^2)^{\frac{1}{3}}.$

Bu funksiyani hosilasi $f'(x) = \frac{1}{3}(2x + x^2)^{\frac{1}{3}-1}(2 + 2x) = \dfrac{2(x+1)}{3\sqrt[3]{\left(2x+x^2\right)^2}}.$

3) $f(x) = \cos^2(2x + \frac{\pi}{3})$ funksiyani hosilasini aniqlang.

Yechish: *Darajani pasaytirish formulasidan foydalanib, berilgan funksiyani quyidagi ko'rinishda yozamiz:* $f(x) = \frac{1}{2} + \frac{1}{2}cos\left(4x + \frac{2p}{3}\right).$

Bu funksiyani hosilasi $f'(x) = -2\,sin\left(4x + \frac{2p}{3}\right).$

4) $f(x) = 2x^3 - 3x^2 + 5x - 8$ funksiya hosilasini $x = 2, f'(2)$ da hisoblang.

Yechish: *Berilgan funksiyani hosilasini olib, undagi x ni o'rniga 2 ni qo'yib hisoblaymiz:* $f'(2) = 6x^2 - 6x + 5 = 6 \cdot 2^2 - 6 \cdot 2 + 5 = 17.$

5) $f(x) = 3\sqrt[3]{x^2} + 4\sqrt{2x} + x - 3$ funksiya hosilasini $x = 8, f'(8)$ da hisoblang.

Yechish: *Berilgan funksiyani quyidagi ko'rinishda yozamiz:*

$f(x) = 3x^{\frac{2}{3}} + 4\sqrt{2x} + x - 3.$ *Bu funksiyani hosilasini olib, undagi x ni o'rniga 8 ni qo'yib hisoblaymiz:*

$f'(8) = 3 \cdot \frac{2}{3}x^{\frac{2}{3}-1} + 4 \cdot \frac{2}{2\sqrt{2x}} + 1 = \frac{2}{\sqrt[3]{x}} + \frac{4}{\sqrt{2x}} + 1 = \frac{2}{\sqrt[3]{8}} + \frac{4}{\sqrt{2 \cdot 8}} + 1 = 3.$

6) $f(x) = sinx \cdot cosx$ funksiya hosilasini $x = \frac{p}{3}, f'(\frac{p}{3})$ da hisoblang.

Yechish: *Berilgan funksiyani quyidagi ko‘rinishda yozamiz:* $f(x) = \frac{1}{2}sin2x.$

Bu funksiyani hosilasini olib, undagi x ni o‘rniga $\frac{p}{3}$ *ni qo‘yib hisoblaymiz:*

$f'\left(\frac{p}{3}\right) = \frac{1}{2} \cdot 2 \cdot cos2x = cos\left(2 \cdot \frac{p}{3}\right) = -\frac{1}{2}.$

7) $f(x) = 2^{x-3} \cdot (2x-5)$ funksiya hosilasini $x = 3, f'(3)$ da hisoblang.

Yechish: *Funksiyani hosilasini olib, undagi x ni o‘rniga 3 ni qo‘yib hisoblaymiz:*

$f'(3) = 2^{x-3} \cdot ln2 \cdot (2x-5) + 2 \cdot 2^{x-3} = 2^{3-3} \cdot ln2 \cdot (2 \cdot 3 - 5) + 2 \cdot 2^{3-3} =$
$= ln2 + 2.$

8) $f(x) = e^{x^2-5x}$ funksiya hosilasini $x = 0, f'(0)$ da hisoblang.

Yechish: *Funksiyani hosilasini olib, undagi x ni o‘rniga 0 ni qo‘yib hisoblaymiz:*

$f'(0) = (2x-5) \cdot e^{x^2-5x} = (2 \cdot 0 - 5) \cdot e^{0^2-5 \cdot 0} = -5.$

9) $f(x) = \sqrt{3^x + x^3}$ funksiya hosilasini $x = 1, f'(1)$ da hisoblang.

Yechish: *Funksiyani hosilasini olib, undagi x ni o‘rniga 1 ni qo‘yib hisoblaymiz:*

$f'(1) = \frac{3^x \cdot ln3 + 3x^2}{2\sqrt{3^x + x^3}} = \frac{3^1 \cdot ln3 + 3 \cdot 1^2}{2\sqrt{3^1 + 1^3}} = \frac{3 \cdot (ln3+1)}{4}.$

10) $f(x) = cos^4x - sin^4x$ funksiya hosilasini $x = \frac{p}{4}, f'(\frac{p}{4})$ da hisoblang.

Yechish: *Berilgan funksiyani quyidagi ko‘rinishda yozamiz:*

$f(x) = (cos^2x - sin^2x) \cdot (cos^2x + sin^2x) = cos2x.$

Bu funksiyani hosilasini olib, undagi x ni o‘rniga $\frac{p}{4}$ *ni qo‘yib hisoblaymiz:*

$f'\left(\frac{p}{4}\right) = -2 \cdot sin2x = -2\,sin\left(2 \cdot \frac{p}{4}\right) = -2.$

11) $f(x) = 4cos^2 2x$ funksiya hosilasini $x = \frac{p}{24}, f'(\frac{p}{24})$ da hisoblang.

Yechish: *Funksiyani hosilasini olib, undagi x ni o‘rniga* $\frac{p}{24}$ *ni qo‘yib hisoblaymiz:*

$f'\left(\frac{p}{24}\right) = -4 \cdot 2 \cdot 2cos2x \cdot sin2x = -8sin4x = -8sin\left(4 \cdot \frac{p}{24}\right) = -4.$

12) $f(x) = \frac{18}{x^2} + 1\frac{7}{18}x^2$ funksiyani eng kichik qiymatini toping.

Yechish: *Funksiyani hosilasini olamiz:*

$$f'(x) = -\frac{2 \cdot 18}{x^3} + 2 \cdot \frac{25}{18}x = 0 \Rightarrow x^2 = \frac{18}{5}.$$

$$f(x) = \frac{18}{\frac{18}{5}} + \frac{25}{18} \cdot \frac{18}{5} = 10.$$

13) $f(x) = 3x - x^2$ funksiyani [-1;3] oraliqdagi eng kichik qiymatini toping.

Yechish: *Funksiyani kritik nuqtasini aniqlaymiz:*

$$f'(x) = 3 - 2x = 0 \Rightarrow x = 1,5.$$

Demak berilgan oraliqda kritik nuqta mavjud, shuni uchun uni qiymatini ham hisoblaymiz: $f(1{,}5) = 3 \cdot 1{,}5 - 1{,}5^2 = 2{,}25;\ f(-1) = 3 \cdot (-1) - (-1)^2 = -4.$
$f(3) = 3 \cdot 3 - (3)^2 = 0.$ *Bu qiymatlarni ichida eng kichigi* -4.

14) $f(x) = -\frac{1}{3}x^2 - \frac{1}{6}x + \frac{1}{3}$ funksiyani [-1;1] oraliqdagi eng kichik va eng katta qiymatlari yig‘indisini toping.

Yechish: *Funksiyani kritik nuqtasini aniqlaymiz:*

$$f'(x) = -\frac{2}{3}x - \frac{1}{6} = 0 \Rightarrow x = -\frac{1}{4}.$$

Demak berilgan oraliqda kritik nuqta mavjud, shuni uchun uni qiymatini ham hisoblaymiz: $f\left(-\frac{1}{4}\right) = -\frac{1}{3} \cdot \left(-\frac{1}{4}\right)^2 - \frac{1}{6} \cdot \left(-\frac{1}{4}\right) + \frac{1}{3} = \frac{13}{48}.$

$f(-1) = -\frac{1}{3} \cdot (-1)^2 - \frac{1}{6} \cdot (-1) + \frac{1}{3} = \frac{1}{6}.$ $\quad f(1) = -\frac{1}{3} \cdot (1)^2 - \frac{1}{6} \cdot (1) + \frac{1}{3} = -\frac{1}{6}.$

Masala shartiga asosan $-\frac{1}{6} + \frac{13}{48} = \frac{5}{48}.$

15) $f(x) = 2cos\frac{x}{3}$ funksiya grafigining $M\left(\frac{3p}{2}; 0\right)$ nuqtasiga o‘tkazilgan urunma tenglamasini tuzing.

Yechish: *Masala shartiga ko‘ra* $x_0 = \frac{3p}{2}$ *ekanligini hisobga olib, urunma tenglamasidagi* $f\left(\frac{3p}{2}\right) = 2cos\frac{3p}{2 \cdot 3} = 2cos\frac{p}{2} = 0$ *va*
$f'\left(\frac{3p}{2}\right) = -2 \cdot \frac{1}{3}sin\frac{x}{3} = -\frac{2}{3}sin\frac{3p}{2 \cdot 3} = -\frac{2}{3}$ *larni aniqlaymiz va ularni urunma tenglamasiga qo‘yamiz:* $y = 0 - \frac{2}{3}\left(x - \frac{3p}{2}\right) = -\frac{2}{3}x + p.$

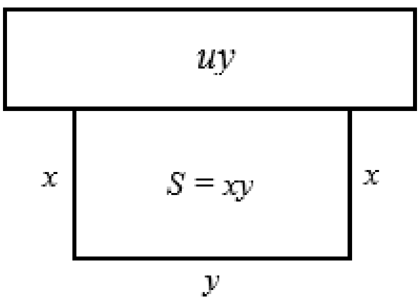
16) $f(x) = \frac{\sqrt{3}}{3}x^3 - 1$ funksiya grafigining $x_0 = 1$ nuqtasiga o‘tkazilgan urunmaning *OX* o‘qi bilan tashkil qilgan burchagini toping.

Yechish: *Masala shartiga asosan urunmaning burchak koeffitsientini k topish talab qilinadi.* $k = f'(1) = \frac{\sqrt{3}}{3} \cdot 3x^2 = \sqrt{3} \cdot 1 = \sqrt{3} = tgб \rightarrow б = 60^0.$

17) $y = 5 - \sqrt{8x + 1}$ funksiyani $x_0 = 1$ nuqtasiga o‘tkazilgan urunma va koordinata o‘qlari bilan chegaralangan uchburchakni yuzini toping.

Yechish: *Urunma tenglamasidagi* $f(1) = 5 - \sqrt{8 \cdot 1 + 1} = 2$ *va*
$f'(1) = -\frac{4}{\sqrt{8x+1}} = = -\frac{4}{\sqrt{8 \cdot 1 + 1}} = -\frac{4}{3}$ *larni aniqlaymiz va ularni urunma tenglamasiga qo‘yamiz:*

$$y = 2 - \frac{4}{3}(x - 1) = -\frac{4}{3}x + 2 + \frac{4}{3} = -\frac{4}{3}x + \frac{10}{3}.$$



18) Uy oldidan uzunligi $\ell = 120\,m$ ga teng bo‘lgan sim to‘r yordamida yuzasi eng katta maydon hosil qilish uchun uning o‘lchamlari qanday bo‘lishi kerak (rasmga qarang).

Yechish: *Rasmdan* $\ell = 2x + y = 120.$

Bundan $y = 120 - 2x$.

Maydonning yuzasi $S = xy$ *bundagi y ni o'rniga uni qiymatini qo'yib, yuza funksiyasini tuzamiz:* $S(x) = x(120 - 2x) = 120x - 2x^2$.

Bu funksiyaning hosilasini olb, nolga tenlaymiz:

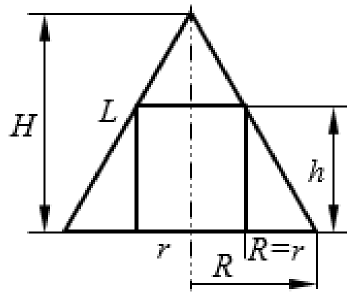
$$120 - 4x = 0 \Rightarrow x = 30;\ y = 60.$$

19) Tomonlarining yig'indisi $a + b = 18\,m$, ular orasidagi burchak $б = 150^0$ bo'lgan uchburchaklar ichida yuzasi eng katta bo'lganini toping.

Yechish: *Ikki tomoni va ular orasidagi burchak berilganda uchburchakni yuzi quyidagi formula orqali aniqlanadi:* $S = \frac{1}{2}absinб$. *Berilgan shartdan* $b = 18 - a$ *ekanligini topib, yuza funksiyasini tuzamiz:*

$S(x) = \frac{1}{2}a(18 - a)sin150^0 = \frac{1}{4}(18a - a^2)$. *Bu funksiyaning hosilasini olb, nolga tenlaymiz:* $18 - 2a = 0 \Rightarrow a = 9;\ b = 9.\ S = \frac{1}{4}\cdot 9\cdot 9 = 20\frac{1}{4}m^2$.

20) Yasovchisi $L = 10$, asosining diametri $D = 12$ bo'lgan konus ichiga eng katta hajimli silindrni sig'dirish uchun uni balandligi qancha bo'lishi kerak (rasimda o'q kesimi ko'rsatilgan).



Yechish: *Chizmadan quyidagilarni aniqlaymiz:*

$H = \sqrt{L^2 - R^2};\ \frac{R}{r} = \frac{H}{H-h}$ *bundan* $h = \frac{H(R-r)}{R}$.

Silindrni hajmi $V = \text{p}\cdot r^2\cdot h$. *h ni qiymatini bu formulaga qo'yib, hajim funksiyasini tuzamiz:* $V(r) = \text{p}\cdot r^2\cdot\frac{H(R-r)}{R} = \text{p}\cdot\frac{H}{R}\cdot(Rr^2 - r^3)$. *Bu funksiyani hosilasini olib, nolga tenglaymiz:*

$$V'(r) = \text{p}\cdot r^2\cdot\frac{H(R-r)}{R} = \text{p}\cdot\frac{H}{R}\cdot(2Rr - 3r^2) = 0 \Rightarrow r \neq 0;\ r = \frac{2}{3}\cdot R.$$

$$h = \frac{H(R-r)}{R} = \frac{H(R-\frac{2}{3}\cdot R)}{R} = \frac{H}{3} = \frac{8}{3} = 2\frac{2}{3}.$$

Boshlang'ich funksiya va integralga doir:

1) $f(x) = e^{7x} + sin6x$ funksiyaning boshlang'ichini toping.

Yechish: $F(x) = \int e^{7x}dx + \int sin6xdx$: *Bu funksiyani quyidagicha yozamiz:*

$$F(x) = \frac{1}{7}\int e^{7x}d(7x) + \frac{1}{6}\int sin6xd(6x) = \frac{1}{7}e^{7x} - \frac{1}{6}cos6x + C.$$

2) $f(x) = sin\left(\frac{x}{4} + 5\right)$ funksiyaning boshlang'ichini toping.

Yechish: $F(x) = \int sin\left(\frac{x}{4} + 5\right)dx$: *Bu funksiyani quyidagicha yozamiz:*

$$F(x) = 4\int sin\left(\frac{x}{4} + 5\right)d\left(\frac{x}{4} + 5\right) = -4cos\left(\frac{x}{4} + 5\right) + C.$$

3) $y = \frac{5}{\cos^2 x} + 3$ funksiyaning boshlang'ichini toping.

Yechish: $F(x) = \int\frac{5}{cos^2x}dx + \int 3dx = 5tgx + 3x + C$.

4) $f(x) = \frac{1-sin^2x}{1+sinx} - \frac{1-cos^2x}{1-cosx}$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$f(x) = 1 - sinx - 1 - cosx = -sinx - cosx.$

$F(x) = -\int sinxdx - \int cosxdx = cosx - sinx + C.$

5) $f(x) = 18sin4x \cdot sin5x$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$f(x) = 9(cosx - \cos(9x)).$

$F(x) = 9\int cosxdx - \frac{1}{9}\int \cos(9x)\, d(9)x = 9sinx - \sin(9x) + C.$

6) $f(x) = -tg^2x$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$f(x) = -\frac{sin^2x}{cos^2x} = -\frac{1-cos^2x}{cos^2x} = 1 - \frac{1}{cos^2x}.$

$F(x) = \int dx - \int \frac{1}{cos^2x}dx = x - tgx + C.$

7) $f(x) = (x-1)x^3 + e^{3x} + \frac{1}{3x}$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$f(x) = x^4 - x^3 + e^{3x} + \frac{1}{3x}$ $F(x) = \int x^4dx - \int x^3dx + \frac{1}{3}\int e^{3x}d(3x) +$
$+\int \frac{1}{3x}dx = \frac{x^5}{5} - \frac{x^4}{4} + \frac{1}{3}e^{3x} + \frac{1}{3}lnx + C.$

8) $f(x) = x + 1 + ctg^2x$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:* $f(x) = x + \frac{1}{sin^2x}.$

$F(x) = \int xdx + \int \frac{1}{sin^2x}dx = \frac{x^2}{2} - ctgx + C.$

9) $f(x) = \frac{2}{\sqrt[4]{2x+5}}$ funksiyaning boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:* $f(x) = 2(2x+5)^{-\frac{1}{4}}$

$F(x) = \int (2x+5)^{-\frac{1}{4}}d(2x+5) = \frac{4}{3}(2x+5)^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{(2x+5)^3} + C.$

10) $f(x) = -x + \frac{x^2}{2}$ funksiyani $(2; 3)$ nuqtadan o'tuvchi boshlang'ichini toping.

Yechish: $F(x) = -\int xdx + \frac{1}{2}\int x^2dx = -\frac{x^2}{2} + \frac{1}{2}\cdot\frac{x^2}{3} + C = \frac{x^3}{6} - \frac{x^2}{2} + C.$ *C ni toppish uchun bu funksiyaga nuqtaning koordinatalarini qo'yamiz:*

$3 = \frac{2^3}{6} - \frac{2^2}{2} + C.$ *Bundan* $C = -\frac{11}{3}.$ *Demak* $F(x) = \frac{x^3}{6} - \frac{x^2}{2} - \frac{11}{3}.$

11) $y = \frac{1}{\sqrt[3]{x^2}} + (2x-3)^2$ funksiyani $\left(1; \frac{1}{6}\right)$ nuqtadan o'tuvchi boshlang'ichini toping.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$f(x) = x^{-\frac{2}{3}} + 4x^2 - 12x + 9.$ $F(x) = \int x^{-\frac{2}{3}}dx + 4\int x^2dx - 12\int xdx +$
$+9\int dx = 3\sqrt[3]{x} + \frac{4}{3}x^3 - 6x^2 + 9x + C.$

C ni topish uchun bu funksiyaga nuqtaning koordinatalarini qo'yamiz:

$$\frac{1}{6} = 3\sqrt[3]{1} + \frac{4}{3}\cdot 1 - 6\cdot 1 + 9\cdot 1 + C. \ \ \textit{Bundan}\ C = -\frac{43}{6}.$$

$$\textit{Demak}\ F(x) = 3\sqrt[3]{x} + \frac{4}{3}x^3 - 6x^2 + 9x - \frac{43}{6}.$$

12) $\int\left(\sqrt{x} - \sqrt[3]{x^2} + 4^x + 5\right)dx$ ni hisoblang.

Yechish: $\int\left(\sqrt{x} - \sqrt[3]{x^2} + 4^x + 5\right)dx = \int\left(x^{\frac{1}{2}} - x^{\frac{2}{3}} + 4^x + 5\right)dx =$

$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{4^x}{ln4} + 5x + C = \frac{2}{3}\sqrt{x^3} - \frac{3}{5}\sqrt[3]{x^5} + \frac{4^x}{ln4} + 5x + C.$

13) $\int \frac{2+\sqrt{x}+x^2}{\sqrt[3]{x}}dx$ ni hisoblang.

Yechish: $\int \frac{2+\sqrt{x}+x^2}{\sqrt[3]{x}}dx = 2\int x^{-\frac{1}{3}}dx + \int x^{\frac{1}{6}}dx + \int x^{\frac{5}{3}}dx =$

$= 2\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1} + \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + C = 3\sqrt[3]{x^2} + \frac{6}{7}\sqrt[6]{x^7} + \frac{3}{8}\sqrt[3]{x^8} + C.$

14) $\int tg^2xdx$ ni hisoblang.

Yechish: $tg^2x = \frac{1}{cos^2x} - 1$ *bo'lganidan foydalanib, berilgan integralni quyidagicha yozamiz va hisoblaymiz:*

$$\int\left(\frac{1}{cos^2x} - 1\right)dx = \int\frac{1}{cos^2x}dx - \int dx = tgx - x + C.$$

15) $\int \frac{dx}{\sqrt{x}\cdot(1+\sqrt[3]{x})}$ ni hisoblang.

Yechish: $x = u^6$ *o'zgartirishni kiritib, berilgan integralni quyidagicha yozamiz:* $\int\frac{dx}{\sqrt{x}\cdot(1+\sqrt[3]{x})} = \int\frac{d(u^6)}{u^3\cdot(1+u^2)} = \int\frac{6u^5du}{u^3\cdot(1+u^2)} = \int\frac{6u^2du}{1+u^2} =$

$= 6\int\left(1 - \frac{1}{1+u^2}\right)du == 6u - 6arctgu + C = 6\sqrt[6]{x} - 6arctg\sqrt[6]{x} + C.$

16) $\int_{\frac{p}{6}}^{\frac{p}{2}} cosxdx$ ni hisoblang.

Yechish: $\int_{\frac{p}{6}}^{\frac{p}{2}} cosxdx = (sinx)\ /_{\frac{p}{6}}^{\frac{p}{2}} = \left(sin\frac{p}{2} - sin\frac{p}{6}\right) = 1 - \frac{1}{2} = \frac{1}{2}.$

17) $8\int_0^{\frac{p}{12}} sinx\cdot cosx\cdot cos2xdx$ ni hisoblang.

Yechish: *Bu funksiyani quyidagicha so'ldalashtiramiz:*

$8\cdot\frac{1}{2}\int_0^{\frac{p}{12}} sin2x\cdot cos2xdx = 2\int_0^{\frac{p}{12}} sin4xdx =$

$= \frac{1}{2}\int_0^{\frac{p}{12}} sin4xd(4x) = -\frac{1}{2}\left[\cos\left(4\cdot\frac{p}{12}\right) - -\cos(4\cdot 0)\right] = \frac{1}{4}.$

18) $\int_{-1}^{2}(x+1)^2dx$ ni hisoblang.

Yechish: $\int_{-1}^{2}(x+1)^2dx=\int_{-1}^{2}(x+1)^2d(x+1)=\frac{(2+1)^3}{3}-\frac{(-1+1)^3}{3}=9.$

19) $\int\limits_{1}^{4e}\frac{dx}{0,25x+1}$ ni hisoblang.

Yechish: $\int\limits_{1}^{4e}\frac{dx}{0,25x+1}=\int\limits_{1}^{4e}\frac{4dx}{x+4}=4(ln(4e+4)-ln(14)=4ln[\frac{4}{5}(e+1)].$

20) $\int\limits_{\frac{\pi}{2}}^{\frac{\pi}{3}}\frac{3\mathrm{dx}}{2\sin^2\frac{\mathrm{x}}{2}}$ ni hisoblang.

Yechish: $\int\limits_{\frac{\pi}{2}}^{\frac{\pi}{3}}\frac{3\mathrm{dx}}{2\sin^2\frac{\mathrm{x}}{2}}=2\cdot\frac{3}{2}\int\limits_{\frac{\pi}{2}}^{\frac{\pi}{3}}\frac{\mathrm{d}\left(\frac{\mathrm{x}}{2}\right)}{\sin^2\frac{\mathrm{x}}{2}}=-3\left(\mathrm{ctg}\frac{\pi}{6}-\mathrm{ctg}\frac{\pi}{4}\right)=-3\left(\sqrt{3}-1\right)=3-3\sqrt{3}.$

21) $\int\limits_{0}^{\frac{\pi}{2}}\sin 2x\,dx$ ni hisoblang.

Yechish: $\int\limits_{0}^{\frac{\pi}{2}}sin2x\,d\,x=\frac{1}{2}\int\limits_{0}^{\frac{\pi}{2}}sin2x\,d(2x)=-\frac{1}{2}\left(cos(2\cdot\frac{\pi}{2})-cos(2\cdot0)\right)=1.$

22) $\int_0^{\frac{p}{4}}cos2xsin2xdx$ ni hisoblang.

Yechish: $\int_0^{\frac{p}{4}}cos2xsin2xdx=\frac{1}{8}\int_0^{\frac{p}{4}}sin4xd(4x)=-\frac{1}{8}\left(cos\left(4\cdot\frac{p}{4}\right)-cos(4\cdot0)\right)=\frac{1}{4}.$

23) Agar $a=1$ bo‘lsa, $\int_a^{a+1}[ln(sin^2 2x+cos^2 2x)+1]$ ni hisoblang.

Yechish:

24) $\int_4^6\frac{0,5x}{x-3}dx$ ni hisoblang.

Yechish: $\int_4^6\frac{0,5x}{x-3}dx=\int_4^6\frac{x}{2(x-3)}dx=\int_4^6\left(\frac{1}{2}+\frac{3}{2(x-3)}\right)dx=\frac{1}{2}\int_4^6dx+\frac{3}{2}\int_4^6\frac{dx}{x-3}=$
$=\left(\frac{1}{2}x+\frac{3}{2}l\,n(x-3)\right)=\left(\frac{1}{2}\cdot6+\frac{3}{2}l\,n(6-3)\right)-\left(\frac{1}{2}\cdot4+\frac{3}{2}l\,n(4-3)\right)=$
$=1+1,5ln3.$

25) $f(x)=sin2x,\ x_1=\frac{p}{6},\ x_2=\frac{p}{4}vay=0$ chiziqlar bilan chegaralangan shaklni yuzasini toping.

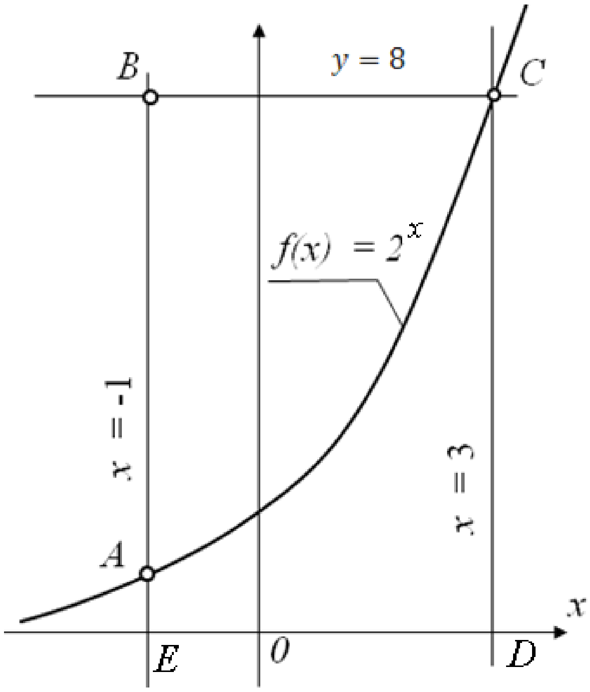
Yechish: $S=\int_{\frac{p}{6}}^{\frac{p}{4}}sin2xdx=\frac{1}{2}\int_{\frac{p}{6}}^{\frac{p}{4}}sin2xd(2x)=-\frac{1}{2}\left[\cos\left(2\cdot\frac{p}{4}\right)-\cos\left(2\cdot\frac{p}{6}\right)\right]=\frac{1}{4}.$

26) $f(x)=2^x,\ x_1=-1,$ va $y=8$ chiziqlar bilan chegaralangan shaklni yuzasini toping.

Yechish: *Berilganlardan foydalanib, ularning chizmasini chizamiz: Talab qilina-yotgan yuza ABC egri chiziqli uchburchakni yuzi. Buning uchun BCDE to‘rtburchakni yuzidan ACDE egri chiziqli trapetsiyani yuzini olish kerak.*

$S_{ABC} = S_{BCDE} - S_{ABDE} \quad S_{BCDE} = 4 \cdot 8 = 32.$

$S_{ABC} = 32 - \int_{-1}^{3} 2^x dx = 32 - \frac{2^x}{ln2} \Big/_{-1}^{3} = 32 - \frac{2^3}{ln2} + \frac{2^{-1}}{ln2} = 32 - \frac{8}{ln2} + \frac{\frac{1}{2}}{ln2} = 32 -$
$- \frac{15}{2ln2} = 32 - \frac{15}{ln4}$



27) $f(x) = \sqrt{3-x}$, $x_1 = 1$, $x_2 = 2$ va $y = 0$ chiziqlar bilan chegaralangan shakl-ni absissa o‘qi atrofida aylantirishdan hosil bo‘lgan jismni hajmini toping..

Yechish: $V = \text{p} \int_1^2 (3-x)dx = \text{p} \left[\left(3 \cdot 2 - \frac{2^2}{2}\right) - (3 \cdot 1 - \frac{1^2}{2}) \right] = \frac{3\text{p}}{2}.$

8. TO‘PLAMLAR NAZARIYASI VA MATEMATIK MANTIQ ELEMENTLARI

8.1. To‘plam haqida tushuncha.

To‘plam tushunchasi matematikaning boshlang‘inch (tariflanmaydigan) tushuncha-laridan biridir. U chekli yoki cheksiz ko‘p obyektlar (narsalar, buyumlar, shaxslar va h.k.) ni birgalikda bir butun deb qarash natijasida vujudga keladi.

To‘plamni tashkil etgan obyektlar uning *elementlari* deyiladi.

To‘plamlar o‘latda lotin alifbosining bosh harflari bilan, uning elementlari esa shu alifboning kichik harflari bilan belgilanadi. Masalan, $A = \{a, b, c, d\}$ yozuvida A to‘p-lam, a, b, c, d lar esa uning elementlari hisoblanadi.

x elementning X to‘plamga tegishli ekanligi $x \in X$ ko‘rinishda, tegishli emasligi esa $x \notin X$ ko‘rinishda belgilanadi.

Sonli to‘plam deyilganda, barcha elementlari sonlardan tashkil topgan har qanday to‘plam tushuniladi. Bunda N – natural sonlar to‘plami, Z - butun sonlar to‘plami, Q – rat-sional sonlar to‘plami, R-haqiqiy sonlar to‘plami deb qabul qilingan.

Elementlar soniga bog‘liq holda to‘plamlar *chekli* va *chekziz* to‘plamlarga ajraladi.

1-misol. $A = \{x | x \in N, x^2 > 5\}$ to'plam 2 dan katta bo'lgan barcha natural sonlardan tuzilgan, ya'ni $A = \{3, 4, 5, 6, 7, 8, 9 \dots\}$. Bu to'plam cheksiz to'plamdir.

Birorta ham elementga ega bo'lmagan to'plam *bo'sh to'plam* deyiladi va u $\emptyset$ orqali belgilanadi. Bush toplam ham chekli toplam hisoblanadi.

2-misol. $x^2 + 4x - 5 = 0$ tenglamaning ildizlari $x_1 = -5$ va $x_2 = 1$ bo'lgani uchun chekli to'plamni tashkil etadiu. $x^2 + 2x + 3 = 0$ tenglama ildizlari haqiqiy emas, ya'ni uning haqiqiy yechimlar to'plami $\emptyset$ dir.

Ayni bir xil elementlardan tuzilgan to'plamlar *teng to'plamlar* deyiladi.

3-misol. $X = \{x | x \in N, x \leq 3\}$va $Y = \{x | (x-1)(x-2)(x-3) = 0\}$ to'plamlarning har biri faqat 1, 2, 3 sonlardan tuzilgan. Shuning uchun bu to'plamlar teng, ya'ni $X = Y$.

Agar B to'plamning har bir elementi A to'plamning har bir elementi bolsa, B to'plam A to'plamning *qism – to'plami* deyladi va $B \subset A$ ko'rinishda belgilanadi.

Bunda $\emptyset \subset A$ va $A \subset A$ hisoblanadi. Bu qism to'plamlar *xosmas qism-to'plamlar* deyladi. A to'plamning qolgan barcha qism-to'plamlari *xos qism-to'plamlar* deyladi. Masalan: $N \subset Z \subset Q \subset R$.

Agar $A = \{3, 4, 5\}$, $B = \{x | x^2 - 7x + 12 = 0\}$ bo'lsa, $B \subset A$ yoki $A \subset B$ bo'ladi.

4-misol. $A = \{1, 2, 3, 4\}$ va $B = \left\{1, \frac{4}{2}, \sqrt{9}, 2^2\right\} = \{1, 2, 3, 4\}$.

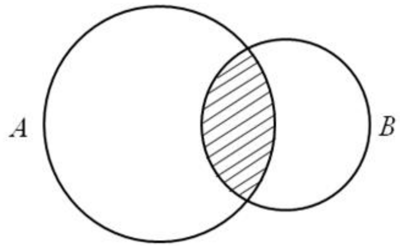
Bundan ko'rinadiki, $B \subset A$ yoki $A \subset B$ bo'ladi.

8.2. Toplamlar ustida amallar.

A va B to'plamlarning ikkalasida ham mavjud bo'lgan x elementga shu toplamning *umumiy* elementi deyiladi. A va B to'plamlarning *kesishmasi* (yoki *ko'paytmasi*) deb, ularning barcha umumiy elementlar idan tuzilgan to'plamga aytiladi.

A va B to'plamlarning kesishmasi $A \cap B$ ko'rinishda belgilanadi. $A \cap B = \{x | x \in A \text{ va } x \in B\}$.

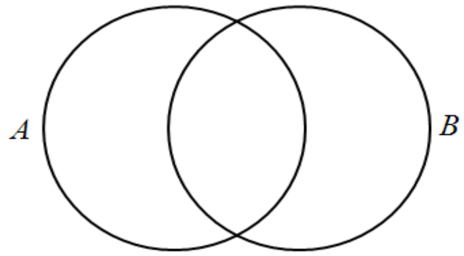
8.1-rasmda Eyler-Venn diagrammasi nomi bilan atalgan chizmada A va B shakillarning kesishmasi $A \cap B$ ni beradi (chizmada shtrixlanib ko'rsatilgan).



8.1-rasm.

A va B to'plamlarning *birlashmasi* (yoki *yig'indisi*) deb, ularning kamida bittasi mavjud bo'lgan barcha elementlardan tuzilgan to'plamga aytiladi.

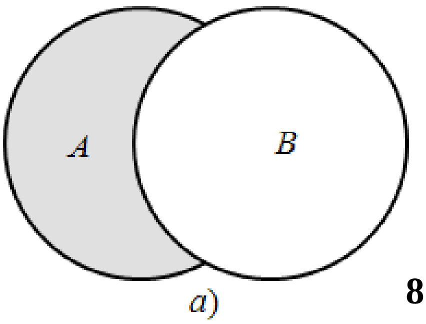
A va B to'plamlarning birlashmasi $A \cup B$ ko'rinishda belgilanadi. $A \cup B = \{x | x \in A \text{ va } x \in B\}$ (8.2-rasmga qarang).

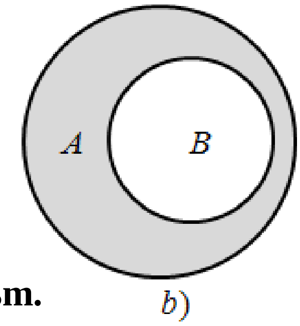


8.2-rasm.

A va B to'plamlarning *ayirmasi* deb, A to'plamning B to'plada mavjud bo'lmagan barcha elementlaridan tuzilgan to'plamga aytiladi.

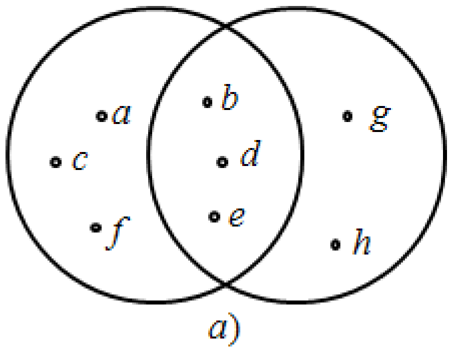
A va B to'plamlarning ayirmasi $A \setminus B$ ko'rinishda belgilanadi. $A \setminus B = \{x | x \in A \text{ va } x \notin B\}$ (8.3-rasmga qarang).

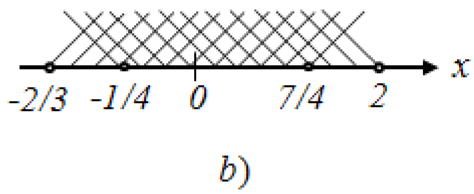




8.3-rasm.

1-misol. $A = \{a, b, c, d, e, f\}$ va $B = \{ b, d, e, g, h\}$ to‘plamlar berilgan. Ularning kesishmasi, birlashmasini tuzamiz va Eyler-Venn diagrammasida talqin etamiz. b, d, e elementlari A va B to‘plamlar uchun umumiy, shunga ko‘ra $A \cap B = \{b, d, e\}$. Bu to‘plamlarning birlashmasi esa $A \cup B = \{a, b, c, d, e, f, g, h\}$ dan iborat bo‘ladi (8.4a-rasm).





8.4-rasm.

2-misol. $A = \{x| -\frac{2}{3} \leq x \leq \frac{7}{4}\}$ va $B = \{x| -\frac{1}{4} \leq x \leq 2\}$ to‘plamlarning kesishmasi, birlashmasi va ayirmasini toping. Buning uchun sonlar o‘qida $-\frac{2}{3}, -\frac{1}{4}, \frac{7}{4}, 2$ nuqtalarni belgilaymiz (7.4b-rasm).

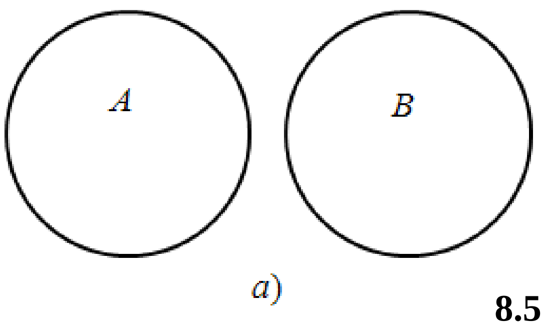
$$A \cap B = \left\{x| -\frac{1}{4} \leq x \leq \frac{7}{4}\right\},\quad A \cup B = \{x| -\frac{2}{3} \leq x \leq 2\} \text{ va}$$

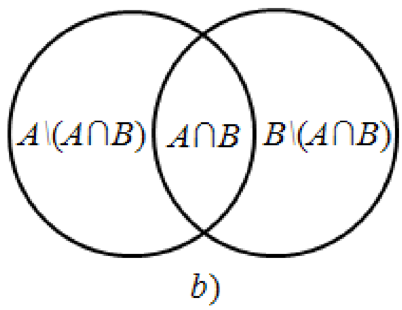
$$A \setminus B = \{x| -\frac{2}{3} \leq x \leq -\frac{1}{4}\}.$$

8.3. Toplam elementlarining soni bilan bog‘liq ayrim masalalar.

Toplamlar nazariyasining muhim qoidalaridan biri-jamlash qoidasidir. Bu qoida kesishmaydigan to‘plamlar birlashmasidagi elementlar sonini toppish imkonini beradi.

1-teorema (jamlash qoidasi). ***Kesishmaydigan A va B chekli to‘plamlarning (8.5a-rasm) birlashmasidagi elementlar soni A va B to‘plamlar elementlari sonining yig‘indisiga teng.*** $n(A \cup B) = n(A) + n(B)$.





8.5-rasm.

2-teorema. ***Ixtiyoriy A va B chekli to'plamlar uchun ushbi tenglik o'rinli (8.5b-rasm).***

$$n(A\cup B)=n(A)+n(B)-n(A\cap B) \qquad (8.1)$$

1-masala. 120 kishidan iborat sayyohlarning 80 tasi ingliz tilini, 55 tasi fransuz tilini, 33 tasi ikkala tilni ham bilmaydi. Sayyohlar ichida mecha kishi ingliz va fransuz tillarini umuman bilmaydi.

Yechish: *Ingliz tilini biladigan sayyohlar toplamini A bilan, fransuz tilini biladigan sayyohlar to'plamini B bilan belgilaymiz. U holda ham ingliz, ham fransuz tilini biladigan sayyohlar to'plami* $A\cap B$ *dan, shu ikki tildan hech bo'lmasa bittasini biladigan sayyohlar to'plam esa* $A\cup B$ *bo'ladi.* (*8.1*) *formuladan foydalanib, sayyohlarning hech bo'lmaganda ingliz yoki fransuz tillaridan bittasini biladiganladini topamiz:*

Shartga ko'ra, $n(A)=80,\ n(B)=55, n(A\cap B)=33.$

$n(A\cup B)=80+55-33=102$

Ikkala tilni ham bilmaydigan sayyohlarning soni $120-102=18$ *kishi ekan.*

9. KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR

Kompleks sonlarni tushuntirish uchun bir misolni ko'rib chiqamiz: $x^2+9=0$. Bu tenglamaning ildizlari haqiqiy son emas.

Bunday holatdan qutilish uchun uni ildizlarini $x_1=-2\sqrt{-1}$ va $x_1=2\sqrt{-1}$ deb qarash zarurati paydo bo'ladi. $\sqrt{-1}$ soni hech qanday real kattalikning olchamini yoki o'zgarishini ifodalamaydi. Shu sababli uni *mavhum* (xayoliy, haqiqatda mavjud bo'lmagan) *birlik* deb atash va maxsus belgilash qabul qilingan: $\sqrt{-1}=i$. Mavhum birlik uchun $i^2=-1$ tenglik o'rinli bo'ladi.

$a+bi$ ko'rinishdagi ifodani ko'rib chiqamiz. Buyerda a va b lar istalgan haqiqiy sonlar, i esa mavhum birlik. $a+bi$ ifoda *haqiqiy* son a va *mavhum* son bi lar «kompleksi» dan iborat bo'lgani uchun uni kompleks son deb atash qabul qilingan.

Kompleks sonlarni bitta harf bilan belgilash qulay. Masalan, $z=a+bi$ kompleks sonning *haqiqiy* qismi a ni $Re(z)$ (fransuzcha reelehaqiqiy) bilan, *mavhum* qismi b ni esa $Im(z)$ (fransuzcha imaginaire-mavhum) bilan belgilash qabul qilingan.

1-misol. $z_1=2+5i$; $z_2=3-2i$; $z_3=3{,}5$; $z_4=7i$. kompleks sonlarning haqiqiy va mavhum qismlarini toping.

Yechish: $Re(z_1)=2$; $Re(z_2)=3$; $Re(z_3)=3{,}5$; $Re(z_4)=0$.

$Im(z_1)=5$; $Im(z_2)=-2$; $Im(z_3)=0$; $Im(z_4)=7$.

Kompleks sonlar ustida arifmetik amallar quyidagicha aniqlanadi:

$$(a+bi)+(c+di)=(a+c)+(b+d)i; \qquad (9.1)$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i; \qquad (9.2)$$

$$(a+bi)\cdot(c+di)=(ac-bd)+(ad+bc)i; \qquad (9.3)$$

$$\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac+bd-(ab-bc)i}{c^2+d^2} \qquad (9.4)$$

2-misol. $(4+3i)\cdot\left(\frac{3}{5}-2i\right)=4\cdot\frac{3}{5}-2\cdot 4i+3i\cdot\frac{3}{5}-2i\cdot 3i=$

$$=\frac{12}{5}-8i+\frac{9}{5}i-6i^2=\frac{42}{5}-\frac{31}{5}i.$$

3-misol. $\frac{3-i}{-4+3i}=\frac{(3-i)(-4-3i)}{(-4+3i)(-4-3i)}=\frac{-12-9i+4i+3i^2}{(-4)^2+(3)^2}=\frac{-15-5i}{16+9}=\frac{-3-i}{5}=$
$=-\frac{3}{5}-\frac{i}{5}$.

10. KOMBINATORIKA ELEMENTLARI.

Turli diskret ob’ektlar to‘plamini turlicha joylashtirish, ular ichidan ixtiyoriy bir nechtasini takroriy, takrorsiz yoki hammasini turli usullar bilan tanlash va tanlash usullari sonini matematikaning ***kombinatorika*** bo‘limi o‘rgatadi.

10.1. Elementlari takrorlanmaydigan to‘plamni tanlash.

1-qoida. *n* ta elementdan tuzilgan elementlari takrorlanmaydigan to‘plamni ***takrorsiz o‘rin almashtirish*** deb, *n* ta elementidan tashlil topgan va faqat elementlarining joylashuvi bilan farq qiluvchi to‘plamlarga aytiladi.

Bunda takrorsiz o‘rin almashtirishlar soni P_n (fransuzcha *permutation* - o‘rin almash-tirish) quyidagicha topiladi:

$$P_n = n! \qquad (10.1)$$

1-misol. 4 ta detalni 4 ta qutiga necha xil tartibda joylashtirish mumkin?

<u>Yechish:</u> *Shartga asosan* $n = 4$ *ga tengligini hisobga olgan holda* (10.1) *formuladan foydalanamiz:* $P_4 = 4! = 1\cdot2\cdot3\cdot4 = 24.$

2-misol. 1, 2, 3, 4, 5 raqamlarini o‘rin almashtirish orqali nechta besh xonali son hosil qilish mumkin?

<u>Yechish:</u> *Shartga asosan* $n = 5$ *ga tengligini hisobga olgan holda* (10.1) *formuladan foydalanamiz:* $P_5 = 5! = 1\cdot2\cdot3\cdot4\cdot5 = 120.$ *Demak berilgan raqamlar yordamida* 120 *ta raqamlari almashgan besh xonali son hosil qilqsh mumkin ekan.*

2-qoida. *n* ta elementdan tuzilgan elementlari takrorlanmaydigan to‘plamni *m* tadan tanlab ***o‘rinalmashtirish*** deb, *n* ta element ichidan tanlab olingan *m* ta shunday elementga aytiladiki, ular bir-biridan elementlar tarkibi yoki tarkibdagi elementlar joylashuvi bilan farq qiladi. Bunday o‘rinalmashtirishlar soni quyidagi formula orqali topiladi:

$$A_n^m = n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot\big(n-(m-1)\big) = \frac{n!}{(n-m)!} \qquad (10.2)$$

3-misol. 1, 2, 4, 6 raqamlardan ixtiyoriy ikkitasini tanlab, nechta ikki xonali son hosil qilish mumkin?

<u>Yechish:</u> *Shartga asosan* $n = 4$ *ga va* $m = 2$ *tengligini hisobga olgan holda* (10.2) *formuladan foydalanamiz:* $A_4^2 = \frac{4!}{(4-2)!} = \frac{1\cdot2\cdot3\cdot4}{1\cdot2} = 12.$ *Demak berilgan raqamlar yordamida* 12 *ta ikki xonali son hosil qilqsh mumkin ekan. Bular 12, 14, 16, 21, 24 va h.k.*

4-misol. O‘zbek, ingliz, fransuz, nemis, rus va arab tillariningbiridan ikkinchisiga tarjima qila oladigan nechta lug‘at kitoblarini tuzish mumkin.

<u>Yechish:</u> *Umumiy tillar soni* $n = 6$ *ta, iug‘at kitobi uchun tanlanadigan tillar* $m = 2$ *ta.* $A_6^2 = \frac{6!}{(6-2)!} = \frac{4!\cdot5\cdot6}{4!} = 30.$

3-qoida. *n* ta elementdan tuzilgan elementlari takrorlanmaydigan to‘plamni *m* tadan tanlab ***takrorsiz kombinatsiyalar tuzish*** deb, *n* ta element ichidan tanlab olingan *m* ta shunday elementga aytiladiki, ular bir-biridan faqat elementlar tarkibi bilan farq qiladi. Bunda takrorsiz kombinatsiyalar soni quyidagi formula orqali topiladi:

$$C_n^m = \frac{n!}{m!(n-m)!}. \qquad (10.3)$$

5-misol. Yashikda 17 ta har xil detal bor. Ulardan har biri 6 tadan iborat necha xil detallar to‘plamini hosil qilish mumkin?

Yechish: *Har xil detallar soni* $n = 17$ *ta, har biri* $m = 6$ *tadan iborat bo‘lgan detall.*

$$C_{17}^6 = \frac{17!}{6!(17-6)!} = \frac{11!\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17}{6!\cdot 11!} = \frac{12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} = 12376.$$

10.2. Elementlari takrorlanadigan to‘plamni tanlash.

1-qoida. 1-elementi m_1 marta, 2-elementi m_2 marta, 3-elementi m_3 marta, …, k-elementi m_k marta takrorlanadigan va jami elementlar soni $m_1 + m_2 + m_3 + \cdots + m_k$ bo‘lgan to‘plamga ***elementlari takrorlanadigan to‘plam*** deyiladi. Bunda takrorli o‘rin almashtirishlar soni quyidagi formula orqali aniqlanadi:

$$P_{m_1,m_2,m_3,\dots m_k} = \frac{(m_1+m_2+m_3+\cdots+m_k)!}{m_1!\cdot m_2!\cdot m_3!\dots m_k!} \qquad (10.4)$$

1-misol. «matematika» so‘zidagi harflarni o‘rnini almashtirib, nechta so‘z hosil qilish mumkin?

Yechish: *«matematika» so‘zida* ***m*** *harfi ikki marta* $(m_1 = 2)$ *,**a** arfi uch marta* $(m_2 = 3)$, ***t*** *harfi ikki marta* $(m_3 = 2)$, ***e*** *harfi bir marta* $(m_4 = 1)$, ***i*** *harfi bir marta* $(m_5 = 1)$ *va* ***k*** *harfi bir marta* $(m_6 = 1)$. *Bularni (9.4) formulaga qo‘yamiz:*

$$P_{2,3,2,1,1,1.} = \frac{(2+3+2+1+1+1)!}{2!\cdot 3!\cdot 2!\cdot 1!\cdot 1!\cdot 1!} = \frac{10!}{2!\cdot 3!\cdot 2!} = \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10}{1\cdot 2\cdot 1\cdot 2\cdot 3\cdot 1\cdot 2\cdot 1\cdot 2} = 151200.$$

2-qoida. *n* ta elementdan tuzilgan elementlari takrorlanmaydigan to‘plamni *m* tadan tanlab ***takrorli o‘rinalmashtirish*** deb, *n* ta element ichidan uning har bir elementini bir marta yoki bir necha marta tanlab *m* ta shunday element hosil qilishga aytiladiki, ular bir-biridan elementlar tarkibi yoki tarkibidagi elementlar joylashuvi bilan farq qiladi. Bunda o‘rin almashtirishlar soni quyidagi formula orqali topiladi:

$$\bar{A}_n^m = n^m. \qquad (10.5)$$

2-misol. 1, 3, 4, 6 raqamlaridan foydalanib, nechta ikki xonali son hosil qilish mumkin?

Yechish: $n = 4$, *va* $m = 2$ *ekanligini bilgan holda ikki xonali sonlarning sonini (10.5) formuladan foydalanib aniqlaymiz:* $\bar{A}_4^2 = 4^2 = 16$. *Bu sonlar* 11, 22, 33, 44, 13, 31, 14, 41, 16, 61, 34, 43, 36, 63, 46, 64 *jami* 16 *tita.*

3-qoida. *n* ta elementdan tuzilgan elementlari takrorlanmaydigan to‘plamni *m* tadan tanlab ***takrorli kombinatsiyalar tuzish*** deb, *n* ta element ichidan uning har bir elementini bir marta yoki bir necha marta tanlab *m* ta shunday element hosil qilishga aytiladiki, ular bir-biridan faqat elementlar tarkibi bilan farq qiladi. Bunda takroriy kombinatsiyalar soni quyidagi formula orqali topiladi:

$$\bar{C}_n^m = C_{m+n-1}^m = \frac{(m+n-1)!}{m!\cdot(n-1)!} \qquad (10.6)$$

3-misol. Archa bayramida bolalarga sovg‘a tyyorlash uchun 6 xil meva tanlandi. Bu mevalardan 9 ta mevadan iborat bo‘lgan necha xil sovg»a tyyorlash mumkin.

Yechish: $n = 6$, *va* $m = 9$ *ekanligini bilgan holda necha xil sovg‘a tayyorlashni aniqlash uchun (10.6) formuladan foydalanib aniqlaymiz:*

$$\bar{C}_6^9 = C_{9+6-1}^9 = \frac{(9+6-1)!}{9!\cdot(6-1)!} = \frac{14!}{9!\cdot 5!} = \frac{9!\cdot 10\cdot 11\cdot 12\cdot 13\cdot 14}{9!\cdot 1\cdot 2\cdot 3\cdot 4\cdot 5} = \frac{10\cdot 11\cdot 12\cdot 13\cdot 14}{1\cdot 2\cdot 3\cdot 4\cdot 5} = 2002.$$

11. EHTIMOLLIK NAZARIYASI VA MATEMATIK STATISTIKA ELEMENTLARI

Matematikaning tasodifiy holisalarni o‘rganadigan bo‘limi ***ehtimollik nazariyasi*** deb ataladi. Bu nazariya yetarlicha ko‘p sonli sinashlar natijasi, ya’ni ommaviy tasodifiy hodisalarning qonuniyatlarini o‘rganish bilan shug‘ullanadi.

Ehtimollar nazariyasida hodisalar uch turga ajratiladi: ***muqarrar***, ***ro‘y bermaydigan*** va ***tasodifiy*** hodisalar.

A hodisaning ehtimoli deb, bu hodisa ro‘y berishiga qulaylik tugdiruvchi elementlar natijalari sonining tajribaning yagona mumkin bo‘lgan va teng imkoniyatli elementlar natijalari jami soniga bo‘lgan nisbatiga aytiladi va quyidagi formula orqali aniqlanadi:

$$P(A) = \frac{m}{n} \qquad (11.1)$$

bu yerda $P(A)$ - *A* hodisaning ehtimoli, *m*-tajribaning yagona mumkin bo‘lgan va teng imloniyatli elementlar natijalarining jami soni, *n*-elementlar natijalarii soni.

1-misol. Idishda 20 ta yashil va 8 ta qizil sharchalar joylashgan. Undan tavakkaliga olingan qizil sharchalar bo‘lish ehtimolini toping.

Yechish: *Sarchalardan bittadan olish imkoniyati* $n = 28$ *ta, shundan qizil chiqa-digan sharchalar* $n = 8$ *ta. Tanlangan qizil sharchalarning chiqish ehtimolini (11.1) formula orqali aniqlanadi:* $P(A) = \frac{8}{28} = \frac{2}{7}$

2-misol. Tavakkaliga 30 dan katta bo‘lmagan natural son tanlanganda uning 3 ga karrali bolish ehtimolini toping.

Yechish: 30 *dan katta bo‘lmagan sonlardan* 3 *ga karralisi* 9 *ta: Bular* 3, 6, 9, 12, 15, 18, 21, 24, 27. *Demak* $n = 30$ *ta,* $n = 9$ *ta. (11.1) formulaga asosan:*

$$P(A) = \frac{9}{30} = \frac{3}{10} = 0{,}3.$$

3-misol. Idishda 120 ta lampochka bo‘lib, ularning 12 tasi yaroqsiz. Tavakkaliga 6 ta lampochka olindi. Olingan lampochkalar ichida yaroqsizi yoq bolish ehtilolini toping.

Yechish: 120 *ta lampochkadan* 6 *tasini tanlashdagi kombinatsiyalar jami soni* C_{120}^6 . 100 *ta yaroqli lampochkadan* 6 *ta yaroqli lampochkani olishimiz kerak, bunday tanlash soni* C_{100}^6 *.Olingan lapochkalar ichida yaroqsizi yo‘q bo‘lish ehtimoli quyidagi formula orqali aniqlanadi:*

$$P(A) = \frac{m}{n} = \frac{C_{100}^6}{C_{120}^6} = \frac{\frac{100!}{6!\cdot(100-6)!}}{\frac{120!}{6!\cdot(120-6)!}} = \frac{\frac{100!}{6!\cdot(100-6)!}}{\frac{120!}{6!\cdot(120-6)!}} = \frac{\frac{100!}{94!}}{\frac{120!}{114!}} =$$

$$= \frac{\frac{94!\cdot 95\cdot 96\cdot 97\cdot 98\cdot 99\cdot 100}{94!}}{\frac{114!\cdot 115\cdot 116\cdot 117\cdot 118\cdot 119\cdot 120}{114!}} = \frac{95\cdot 96\cdot 97\cdot 98\cdot 99\cdot 100}{15\cdot 116\cdot 117\cdot 118\cdot 119\cdot 120} \approx 0{,}3.$$

11.1. Ehtimollarni qo‘shish va ko‘paytirish qoidalari.

Ikkita birgalikda bo‘lmagan hodisadan istalgan birining ro‘y berish ehtimoli bu hodisalar ehtimollarining yig‘indisiga teng boladi:

$$P(A+B) = P(A) + P(B). \qquad (11.2)$$

1-misol. Yashikda 12 ta ko‘k va 10 ta sariq sharchalar bor. Tavakkaliga 2 ta sharcha olindi. Olingan ikkita sharchaning bir xil rangli bo‘lish ehtimolini toping.

Yechish: *A hodisa olingan ikkala sharchaning ko‘k bo‘lish, B hodisa esa olingan ikkala sharchaning sariq bo‘lish hodisasi bo‘lsin. A hodisaning ro‘y berishiga* C_{12}^2 *ta natija imkoniyat yaratadi. B hodisaning ro‘y berishiga* C_{10}^2 *ta natija imkoniyat yaratadi.*

Olingan ikkita sharchaning bir xil rangli bo‘lish ehtimolini (11.2) formula orqali aniqlaymiz:

$$P(A+B) = \frac{C_{12}^2}{C_{22}^2} + \frac{C_{10}^2}{C_{22}^2} = \frac{C_{12}^2 + C_{10}^2}{C_{22}^2} = \frac{\frac{11\cdot 12}{2} + \frac{9\cdot 10}{2}}{\frac{21\cdot 22}{2}} = 0{,}48.$$

Bir nechta erkli hodisalarning birgalikda ro‘y berish ehtimoli, bu hodisalar ehtimollari ko‘paytmasiga temg.

$$P(A_1A_2A_3 \dots A_n) = P(A_1)\cdot P(A_2)\cdot P(A_3)\cdot P(A_4)\cdot \dots \cdot P(A_n) \qquad (11.3)$$

Birgalikda bogliq bolmagan $A_1, A_2, A_3, \dots, A_n$hodisalardan kamida bittasining ro‘y berishidan iborat A hodisaning ehtimoli 1dan $\bar{A}_1, \bar{A}_2, \bar{A}_3, \dots, \bar{A}_n$ qarama-qarshi hodisalar ehtimollari ko‘paytmasining ayirmasiga teng:

$$P(A) = 1 - P(\bar{A}_1)\cdot P(\bar{A}_2)\cdot P(\bar{A}_3)\cdot \dots \cdot P(\bar{A}_n) \qquad (11.4)$$

2-misol. Uchta ovchi bir vaqtda va bir-birlaridan mustaqil ravishda bir quyonga qarab faqat bittadan o‘q uzishgan. Agar ovchilardan aqalli bittasi quyonga o‘q tekkizgan bo‘lsa, quyon otilgan bo‘ladi. Birinchi, ikkinchi va uchinchi ovchilarning quyonga o‘q tekkizish ehtimolliklari mos ravishda 0,4; 0,7; 0,8 ga teng. Quyonning otilish ehtimolini toping.

Yechish:

1) Birinchi ovchining quyonga o‘q tekkizish ehtimoli $P(A_1) = 0{,}4$; *o‘q tekkiza olmaslik ehtimoli* $P(\bar{A}_1) = 1 - P(A_1) = 1 - 0{,}4 = 0{,}6$.

2) Ikkinchi ovchining quyonga o‘q tekkizish ehtimoli $P(A_2) = 0{,}7$; *o‘q tekkiza olmaslik ehtimoli* $P(\bar{A}_2) = 1 - P(A_2) = 1 - 0{,}7 = 0{,}3$.

3) Uchinchi ovchining quyonga o‘q tekkizish ehtimoli $P(A_3) = 0{,}8$; *o‘q tekkiza olmaslik ehtimoli* $P(\bar{A}_3) = 1 - P(A_3) = 1 - 0{,}8 = 0{,}2$.

4) Kamida ovchilardan birining quyonga o‘q tekkizish ehtimoli ya’ni quyonning otilish ehtimolini (11.4) formula orqali aniqlash mumkin:

$$P(A) = 1 - P(\bar{A}_1)\cdot P(\bar{A}_2)\cdot P(\bar{A}_3) = 1 - 0{,}6\cdot 0{,}3\cdot 0{,}2 = 0{,}964.$$

11.2. Shartli ehtimollik.

Agar ikkita A va B hodisalardan birining ro‘y berishi ikkinchisining ro‘y berish yoki bermasligiga bog‘loq bo‘lmasa, bunday hodisalar *o‘zaro bog‘liq emas* deyiladi.

Aks holda ular *o‘zaro bog‘liq* deyiladi.

$P(A/B)$ *sartli ehtimollik* deb, B hodisa ro‘y berganligi aniq bo‘lganligida A hodisa ro‘y berish ehtimolligiga aytiladi: $P(A/B) = \frac{P(AB)}{P(B)}$ (11.5)

Agar A va B hodisalar ozaro bog‘liq bo‘lmasa, u holda

$P(A/B) = P(A)$ va $P(B/A) = P(B)$ tengliklar bajariladi.

Ba’zan A hodisaning ehtimolini B hodisa ro‘y bergandan so‘ng hisoblashga to‘g‘ri keladi.

A hodisaning B hodisa ro‘y berganligi shartida hisoblangan ehtimolga shartli ehtimol deyiladi va $P_B(A)$ yoki $P_B(A/B)$ kabi belgilanadi.

Ikkita bog‘liq hodisalarning birgalikda ro‘y berishi ehtimoli ulardan birining ehtimolini ikkinchisining shartli ehtimoliga ko‘paytmasiga teng:

$$P(AB) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B) \quad (11.6)$$

1-misol. Firma 2 ta yirik A va B korxonalardan 2 ta buyurtma olishga harakat qilmoqda. Ekspertlarning xulosasiga ko‘ra, A korxonadan buyurtma olish ehtimoli 0,8 ga teng. Agar firma A korxonadan buyurtma olsa, u holda B korxonadan ham buyurtma olish ehtimoli 0,92 ga teng bo‘ladi. Firmaning ikkala buyurtmadan olish ehtimolini toping.

<u>Yechish:</u> *Shartga ko‘ra* $P(A) = 0{,}8$ *va* $P(B/A) = 0{,}92$; *ekanligini bilgan holda firmaning ikkala buyurtmadan olish ehtimolini (11.6) formuladan topamiz:*

$$P(AB) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B) = 0{,}8 \cdot 0{,}92 = 0{,}736.$$

11.3. To‘la ehtimol va Bayes formulasi.

$H_1, H_2, H_3, \dots, H_n$ hodisalar to‘la guruhni tashkil qilsin, ya’ni sinov natijasida ularning faqat bittasi ro‘y berishi mumkin va ular birgalikda emas. A hodisa ana shu hodisalardan bittasi ro‘y bergandagina ro‘y berishi mumkin bo‘lsin. $H_1, H_2, H_3, \dots, H_n$ hodisalarning qaysi biri ro‘y berishi oldindan ma’lum bo‘lmagani uchun ular ***gipotezalar*** deb ataladi.

A hodisa ro‘y berishi ehtimoli *to‘la ehtimollik* deyiladi va u quyidagi fopmula orqali aniqlanadi: $P(A) = \sum_{i=1}^{n} P(H_i) \cdot P(A/H_i)$ (11.7)

1-misol. Kelasi yilda mamlakat iqtiso‘liyoti ko‘rsatkichlari yuqori bo‘lsa, ma’lum bir kompaniya aksiyalari narxining oshish ehtimoli 0,86 ga, past bo‘lsa oshish ehtimoli 0,25 ga teng ekan. Shu bilan birga kelasi yilda mamlakat iqtiso‘liyoti ko‘rsatkichlari yuqori bo‘lish ehtimoli 0,8 ga teng ekan. Kelasi yilda kompaniya aksiyalari narxining oshish ehtimolini toping.

<u>Yechish:</u> *Shartga ko‘ra* $P(H_1) = 0{,}8$ *va* $P(H_2) = 1 - P(H_1) = 1 - 0{,}8 = 0{,}2$; $P(A/H_1) = 0{,}86$ *va* $P(A/H_2) = 0{,}2$ *ekanligini bilgan holda kelasi yilda kompaniya aksiyalari narxining oshish ehtimolini (11.7) formuladan topamiz:*

$$P(A) = P(H_1) \cdot P(A/H_1) + P(H_2) \cdot P(A/H_2) = 0{,}8 \cdot 0{,}86 + 0{,}2 \cdot 0{,}2 = 0{,}729.$$

B’zan, A hodisa ro‘y bergani ma’lum bo‘lgandan so‘ng H_k gipotezalarning $P(H_k/A)$ shartli ehtimolligini hisoblash zaruriyati tug‘iladi. Bu ehtimolliklar **Bayes** formulasidan aniqlanadi: $(H_k/A) = \frac{P(H_i) \cdot P(A/H_i)}{P(A)} = \frac{P(H_i) \cdot P(A/H_i)}{\sum_{i=1}^{n} P(H_i) \cdot P(A/H_i)}$ (11.8)

2-misol. Yuqori iqtiso‘liy o‘sish davrida Amerika dollori kursning o‘sish ehtimolligi 0,8; ortacha o‘sish davrida 0,5; past ko‘rsatkichli o‘sish davri esa 0,3 ga teng. Iqtiso‘liy o‘sish davri ko‘rsatkichlari yuqori, o‘rtacha va past bo‘lishi ehtimolliklari mos ravishda 0,4; 0,6; 0,2 ga teng. Hozir dollorning narxi o'smoqda, u holda bu davr yuqori ko'rsat-kichli o'sish davri bo'lishi ehtimolligi qancha bo'ladi?

Yechish: *Shartga ko'ra* $P(H_1) = 0{,}4$; $P(H_2) = 0{,}6$; *va* $P(H_3) = 0{,}2$. *Shuningdek,* $P(A/H_1) = 0{,}8$, $P(A/H_2) = 0{,}5$ *va* $P(A/H_3) = 0{,}3$ *ekanligini bilgan holda talab qilinayotgan qymatni (11.8) formuladan topamiz:*

$$P(H_k/A) = \frac{P(H_1)\cdot P(A/H_1)}{P(H_1)\cdot P(A/H_1)+P(H_2)\cdot P(A/H_2)+P(H_3)\cdot P(A/H_3)} =$$

$$= \frac{0{,}4\cdot 0{,}8}{0{,}4\cdot 0{,}8+0{,}6\cdot 0{,}5+0{,}2\cdot 0{,}3} \approx 0{,}48.$$

11.4. Bernulli tenlamasi. Eng ehtimolli son.

Aytaykil, biror *A* hodisaning ketma-ket o'tkazilayotgan bog'liqsiz tajribalarning har birida ro'y berish ham, bermasligi ham mumkin bo'lsin. Har bir tajribada *A* hodisaning ro'y berish ehtimolligi p ga teng va bu ehtimollik tajriba nomeriga bog'liq bo'lmagan o'zgarmas son.

Tabiiyki, har bir tajriba uchun *A* hodisaning ro'y berimaslik ehtimoli $q = 1 - p$ ga teng.

Yuqoridagi shartlarni qnoatlqntiruvchi tajribalat ketma-ketligiga **Bernulli** sxemasi deyiladi. Bernulli sxemasi 2 ta parameter uchun *n*-tajribalar soni va *p*-har bir tajribada *A* hodisaning ro'y berish ehtimolligi bilan aniqlanadi.

Bernulli sxemasi *A* hodisaning *m* marta ro'y berish ehtimolligi quyidagi Bernulli formulasi deb nom olgan formula yordamida aniqlanadi:

$$P(m) = C_n^m \cdot p^m \cdot q^{n-m} = \frac{n!}{m!(n-m)!} \cdot p^m \cdot q^{n-m}. \qquad (11.9)$$

1-misol. Har bir otilgan o'qning nishonga tegish ehtimoli $\frac{3}{4}$ bo'lsin. Otilgan 20 ta o'qdan beshtasining nishonga tegish ehtimolini toping.

Yechish: *Shartga ko'ra* $n = 20$; $m = 5$; $p = \frac{3}{4}$; $q = 1 - p = 1 - \frac{3}{4} = \frac{1}{4}$ *ekanligini bilgan holda talab qilinayotgan qymatni (11.9) formuladan topamiz:*

$$P(5) = C_{20}^5 \cdot \left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^{15} = \frac{20!}{5!\cdot 15!}\cdot\frac{3^5}{4^5}\cdot\frac{1^5}{4^{15}} = \frac{15!\cdot 16\cdot 17\cdot 18\cdot 19\cdot 20}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 15!}\cdot\frac{243}{4^{20}} \approx 0.0016.$$

12. MATRITSALAR VA DETERMINATLAR.

Matritsa deb, biror tartibda joylashtirilgan sonlarning to'g'ri to'rtburchak ko'rinishidagi jadvalga aytiladi. Bu sonlar shu matritsaning *elementlari* deyiladi. O'latda matritsalar qavs yoki ikkita vertical chiziq ichiga olib yoziladi. Masalan 20 dan kichik barcha tub sonlardan quyidagi matritsani tuzish mumkin:

$$\begin{pmatrix} 2 & 3 & 5 & 7 \\ 11 & 13 & 17 & 19 \end{pmatrix} \text{ yoki } \begin{vmatrix} 2 & 3 & 5 & 7 \\ 11 & 13 & 17 & 19 \end{vmatrix}$$

Bu matritsa 2 ta satr va 4 ta ustundan iborat bo'lganligi uchun 2×4 o'lchamli matritsa deyiladi.

Umuman *m* ta satr va *n* ta ustunli to'g'ri to'rtburchakli matritsa $m \times n$ *o'lchamli matritsa* deyiladi.

$n \times n$ o‘lchamli matritsa *kvadrat matritsa*, n son esa shu kvadrat matritsaning *tartibi* deyiladi.

Masalan bir xonali natural sonlardan tuzilgan 3×3 o‘lchamli matritsa *uchinchi tartibli kvadrat matritsa* deb ataladi:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Bir xonali juft natural sonlardan tuzilgan 2×2 o‘lchamli matritsa *ikkinchi tartibli kvadrat matritsa* deb ataladi:

$$\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

Matritsani umumiy holda yoziz uchun uning elementlari ikki indeksli biror harfdan, masalan, a_{ij} bilan belgilanadi; indeksdagi $i -$ satr nomerini; $j -$ ustun nomerini ko‘rsatadi.

Masalan 4×5 o‘lchamli matritsa umumiy holda quyidagicha yoziladi:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{pmatrix}$$

Kvadrt matritsalar quyidagi xarakteristikalarga ko‘ra o‘zaro taqqoslanadi:

1) Aniqlovchisi yoki ***determinanti***. 2) Normasi. 3) Rangi.

Uchinchi tartibli determinant Δ lar Sarrks qoidasi yoki «Uchburchak usuli» bilan aniqlanadi.

Uchinchi tartibli determinant Δ ni Sarrks qoidasi yordamida aniqlash:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} =$$

$$\begin{matrix} - & - & - & + & + & + \end{matrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}. \quad (12.1)$$

Uchinchi tartibli determinant Δ ni «Uchburchak usuli» da aniqlash:

$$\Delta = +\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}. \quad (11.2)$$

Kvadrat matritsaning ***normasi*** deb, quyidagi N soniga aytiladi va u quyidagi formula orqali aniqlanadi:

$$N = \sqrt{\sum_{i=1}^{n} \cdot \sum_{j=1}^{n} \cdot a_{ij}^2} \quad (12.3)$$

Ixtiyoriy to‘g‘ri burchakli matritsaning ***rangi*** deb, 0 dan farqli matritsa osti ustunlarning eng katta tartibiga aytiladi va u r harfi bilan belgianadi.

Quyida berilgan kvadrat matritsalarning determinant, normal va ranglarini aniqlang:

1) $\bar{\bar{A}}_2 = \begin{pmatrix} 1 & 2 \\ -2 & 4 \end{pmatrix}$; 1) $\Delta = \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = 1 \cdot 4 - 2(-2) = 8.$

2) $N(\bar{\bar{A}}_2) = \sqrt{1^2 + 2^2 + (-2)^2 + 4^2} = \sqrt{25} = 5.$

3) $\Delta \neq 0$ bo‘lgani uchun $r(\bar{\bar{A}}_2) = 2.$

1) $\bar{\bar{A}}_3 = \begin{pmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{pmatrix}$; 1) $\Delta = \begin{vmatrix} -1 & 0 & 8 \\ 5 & 9 & 0 \\ 0 & 4 & 3 \end{vmatrix} \begin{matrix} -1 & 0 \\ 5 & 9 \\ 0 & 4 \end{matrix} = -27 + 160 = 133 \neq 0.$

2) $N(\bar{\bar{A}}_2) = \sqrt{(-1)^2 + 8^2 + 5^2 + 9^2 + 4^2 + 3^2} = \sqrt{196} = 14.$ 3) $r(\bar{\bar{A}}_3) = 3.$

Quyudagi chiziqli tenglamalar sistemasini Gauss usuli hamda Kramer formulasidan foydalanib yeching.

1) $\begin{cases} 4x_1 + x_2 = 6 \\ 2x_1 + 3x_2 = -1 \end{cases}$ 2) $\begin{cases} x_1 + 2x_2 - 4x_3 = 8 \\ 3x_1 - x_2 + x_3 = 4 \\ 2x_1 + x_2 + 5x_3 = 0 \end{cases}$ 3) $\begin{cases} x + y - 3z = -1 \\ 2x - y + z = 2 \\ 3x + 2y - 4z = 1 \end{cases}$

1-misolni **Gauss usulida** yechamiz: sistemaning 2-tenglamasini -2 ga ko‘paytiramiz va natijani 1-tenglamaga qo‘shib, quyidagi tenglikni hosil qilamiz:

$$-5x_2 = 8.$$

Bundan $x_2 = -1{,}6.$ Buni sistemaning 1-tenlamasiga qo‘ysak, $4x_1 - 1{,}6 = 6$ tenglik hosil bo‘ladi. Bundan $x_1 = 1{,}9$ ekanligini aniqlaymiz.

1-misolni **Kramer** formulasidan foydalanib yechamiz: Buning uchun Δ, Δ_{x_1} va Δ_{x_2}larni aniqlaymiz:

$\Delta = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 4 \cdot 3 - 1 \cdot 2 = 10.$ $\Delta_{x_1} = \begin{vmatrix} 6 & 1 \\ -1 & 3 \end{vmatrix} = 6 \cdot 3 - 1(-1) = 19.$

$\Delta_{x_2} = \begin{vmatrix} 4 & 6 \\ 2 & -1 \end{vmatrix} = 4 \cdot (-1) - 6 \cdot 2 = -16.$ Bularni bilgan holda $x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{19}{10} = 1{,}9.$

$$x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{-16}{10} = -1{,}6.$$

2-misolni **Gauss usulida** yechamiz: sistemaning 1-tenglamasini -3 ga ko‘paytiramiz va natijani 2-tenglamaga qo‘shib, quyidagi tenglikni hosil qilamiz:

$$-7x_2 + 13x_3 = -20. \ (a)$$

Sistemaning 1-tenglamasini -2 ga ko‘paytiramiz va natijani 3-tenglamaga qo‘shib, quyidagi tenglikni hosil qilamiz: $-3x_2 + 13x_3 = -16.$ (*b*). (*a*) va (*b*) tengliklarni birni biridan ayirsak, $-4x_2 = -4$ bundan $x_2 = 1$. Buni (*b*) tenglikka qo‘yib, $x_3 = -1$. Bu aniqlangan qiymatlarni sismemaning 1-tenglamasiga qo‘ysak, $x_1 + 2 \cdot 1 - 4 \cdot (-1) = 8$ bundan $x_1 = 2$.

Demak javoblar: (2;1; -1).

2-misolni **Kramer** formulasidan foydalanib yechamiz:

Buning uchun Δ, Δ_{x_1}, Δ_{x_2}va Δ_{x_3} larni aniqlaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & -4 \\ 3 & -1 & 1 \\ 2 & 1 & 5 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & -1 \\ 2 & 1 \end{matrix} = -5 + 4 - 12 - 8 - 1 - 30 = -52.$$

$$\Delta_{x_1}=\left|\begin{array}{ccc}8 & 2 & -4\\ 4 & -1 & 1\\ 0 & 1 & 5\end{array}\right|\begin{array}{cc}8 & 2\\ 4 & -1\\ 0 & 1\end{array}=-40-16-8-40=-104.$$

$$\Delta_{x_2}=\left|\begin{array}{ccc}1 & 8 & -4\\ 3 & 4 & 1\\ 2 & 0 & 5\end{array}\right|\begin{array}{cc}1 & 8\\ 3 & 4\\ 2 & 0\end{array}=20+16+32-120=-52.$$

$$\Delta_{x_3}=\left|\begin{array}{ccc}1 & 2 & 8\\ 3 & -1 & 4\\ 2 & 1 & 0\end{array}\right|\begin{array}{cc}1 & 2\\ 3 & -1\\ 2 & 1\end{array}=16+24+16-4=52.$$

Bularni bilgan holda $x_1=\frac{\Delta_{x_1}}{\Delta}=\frac{-104}{-52}=2.$ $x_2=\frac{\Delta_{x_2}}{\Delta}=\frac{-52}{-52}=1.$

$x_3=\frac{\Delta_{x_3}}{\Delta}=\frac{52}{-52}=-1.$

3-misolni **Gauss usulida** yechamiz: sistemaning 1-tenglamasini -2 ga ko‘paytiramiz va natijani 2-tenglamaga qo‘shib, quyidagi tenglikni hosil qilamiz:

$$-3y+7z=4.\ (a)$$

Sistemaning 1-tenglamasini -3 ga ko‘paytiramiz va natijani 3-tenglamaga qo‘shib, quyidagi tenglikni hosil qilamiz: $-y+5z=4.$ (b). (a) va (b) tengliklarni birni biridan ayirsak, $-2y+2z=0$ bundan $z=y$. Buni (b) tenglikka qo‘yib, $-y+5y=4.$

Bundan $y=z=1$. Bu aniqlangan qiymatlarni sismemaning 1-tenglamasiga qo‘ysak, $x+1-3=-1$ bundan $x=1.$ Demak javoblar: (1; 1; 1).

3-misolni **Kramer** formulasidan foydalanib yechamiz: Buning uchun Δ, Δ_x, $\Delta_y$va Δ_z larni aniqlaymiz:

$$\Delta=\left|\begin{array}{ccc}1 & 1 & -3\\ 2 & -1 & 1\\ 3 & 2 & -4\end{array}\right|\begin{array}{cc}1 & 1\\ 2 & -1\\ 3 & 2\end{array}=4+3-12-9-2+8=-8.$$

$$\Delta_x=\left|\begin{array}{ccc}-1 & 1 & -3\\ 2 & -1 & 1\\ 1 & 2 & -4\end{array}\right|\begin{array}{cc}-1 & 1\\ 2 & -1\\ 1 & 2\end{array}=-4+1-12+3+2+8=-2.$$

$$\Delta_y=\left|\begin{array}{ccc}1 & -1 & -3\\ 3 & 2 & 1\\ 2 & 1 & 4\end{array}\right|\begin{array}{cc}1 & 8\\ 3 & 4\\ 2 & 0\end{array}=20+16+32-120=-52.$$

$$\Delta_z=\left|\begin{array}{ccc}1 & 1 & -1\\ 3 & -1 & 2\\ 2 & 2 & 1\end{array}\right|\begin{array}{cc}1 & 2\\ 3 & -1\\ 2 & 1\end{array}=16+24+16-4=52.$$

Bularni bilgan holda $x=\frac{\Delta_x}{\Delta}=\frac{-104}{-52}=2.$ $y=\frac{\Delta_y}{\Delta}=\frac{-52}{-52}=1.$

$z=\frac{\Delta_z}{\Delta}=\frac{52}{-52}=-1.$

13. BOBLARGA DOIR ARALASH MISOL VA MASALALAR

1) $\dfrac{1+\frac{1+\frac{\cdots}{5}}{5}}{5}+1$ ni hisoblang.

Yechish: *Berilgan ifodada* $1+\frac{\cdots}{5}$ *soni takrorlanishini hisobga olib, uni x deb olsak, unda berilgan ifoda quyidagi tenglamaga ega bo'ladi:* $\frac{x}{5}+1=x$ *bundan* $x=\frac{5}{4}$.

2) $\dfrac{20}{1+\frac{20}{1+\frac{20}{\cdots}}}+1$ ni hisoblang.

Yechish: *Berilgan ifodada* $1+\frac{20}{\cdots}$ *soni takrorlanishini hisobga olib, uni x deb olsak, unda berilgan ifoda quyidagi tenglamaga ega bo'ladi:* $\frac{20}{x}+1=x$ *bundan* $x=5$.

3) Agar $a=1\cdot2+2\cdot3+3\cdot4+\cdots+40\cdot41.$ va

$b=5\cdot4+10\cdot6+15\cdot8+\cdots+200\cdot82$ bo'lsa, $\frac{a}{b}$ ning qiymatini toping.

Yechish: *Ikkinchi tenglikdan* 10 *ni qavisdan chiqarib quyidagiga ega bo'lamiz:*
$b=10\cdot(1\cdot2+2\cdot3+3\cdot4+\cdots+40\cdot41)$ $\frac{a}{b}=\frac{1\cdot2+2\cdot3+3\cdot4+\cdots+40\cdot41}{10\cdot(1\cdot2+2\cdot3+3\cdot4+\cdots+40\cdot41)}=\frac{1}{10}$

4) $\frac{1}{2}+\frac{2}{3}+\frac{3}{2}+\frac{4}{3}+\cdots+\frac{15}{2}+\frac{16}{3}$ ni hisoblang.

Yechish: *Maxraji 2 bo'lgan kasrlar ayirmasi 1 ga teng bo'lgan arifmetik progressiyani hosil qiladi va uni hadlarining sonini aniqlab, progressiyaning yig'indisini aniqlaymiz:* $\frac{15}{2}=\frac{1}{2}+(n-1)\cdot1$ *yoki* $7=n-1$ *bundan* $n=8;$ $S_1=\frac{\frac{1}{2}+\frac{15}{2}}{2}\cdot8=32.$ *Maxraji 3 bo'lgan kasrlar ayirmasi 2/3 ga teng bo'lgan arifmetik progressiyani hosil qiladi va uni hadlarining sonini aniqlab, progressiyaning yig'indisini aniqlaymiz:*
$\frac{16}{3}=\frac{2}{3}+(n-1)\cdot\frac{2}{3}$ *bundan* $n=8;$ $S_2=\frac{\frac{2}{3}+\frac{16}{3}}{2}\cdot8=24.$ *Berilgan ifodaning yig'indisi* $S=S_1+S_2=32+24=56.$

5) Ishchi har kunlik rejada ko'rsatilganidan 25 ta detall ortiq yasab, uch kunda rejadagi 7 kunga mo'ljallanganidan 15 ta ortiq yasadi. Ishchi har kuni nechtadan detall yasagan.

Yechish: *Ishchi bajargan ish kunini x deb belgilab, masala shartidan kelib chiqib, quyidagi tenglamani tuzamiz:* $3(x+25)=7x+15.$ *Bundan* $x=15.$

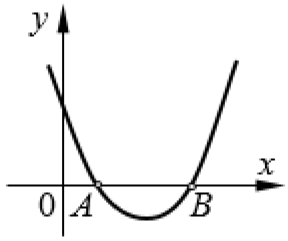
6) Oltin va durdan yasalgan bezakning og'irligi 3 misqol, narxi 24 dinor. Agar 1 misqol oltin 5 dinor, 1 misqol dur 15 dinor tursa, bezakda qancha misqol oltin bor.

Yechish: *Oltinning bir misqolini x, durning bir misqolini y deb belgilab, quyidagi tenglamalar sistemasini tuzamiz:*

$\begin{cases} 5x + 15y = 24 \\ x + y = 3 \end{cases}$ *bu sistemani yechib, bezakda* $x = 2{,}1$ *misqol oltin borligi aniqlanadi.*

7) *A* va *B* shaharlar orasidagi masofa 180 km. *A* shahardan *B* shahar tomonga qarab yuk poyezdi yo‘lga chiqdi. 30 daqiqadan co‘ng *B* shahardan *A* shahar tomonga qarab yo‘lovchi poyezd yo‘lga chiqdi. Agar yo‘lovchi poyezdning tezligi yuk poyezdning tezligidan 6 km/soat ga ko‘p bo‘lib, ular yo‘lning yarmida uchrashgan bo‘lsa, yuk poyezdning tezligini aniqlang.

Yechish: *Yuk poyezdning tezligini x deb olsak, unda masala shartiga ko‘ra yo‘lovchi poyezdning tezligi* $x + 6$ *bo‘ladi. Ular yo‘lning yarmida uchrashganini hamda yuk poyezdi 30 daqiqa yoki 0,5 soat davomida yo‘l bosganligini hisobga olib, quyidagi tenglikni tuzamiz:* $S/2 = x \cdot (t + 0{,}5) = (x + 6) \cdot t$ *bundan uchrashish vaqti* $t = \frac{x}{12}$ *ni topamiz. Shunda* $S/2 = 180/2 = (x + 6) \cdot \frac{x}{12}$ *yoki* $x^2 + 6x - 90 \cdot 12 = 0$ *tenglamani hosil qilamiz buni yechib,* $x = 30\ km/soat$ *ekanligini aniqlaymiz.*



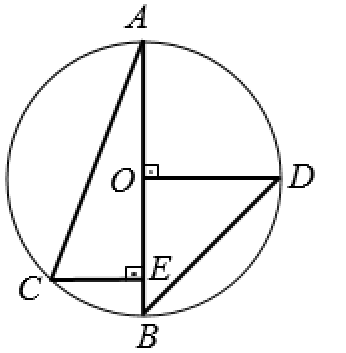
8) Rasmdagi parabola $y = x^2 - 5x - 2m + 2$ tenglama bilan berilgan. Bunda $OB - OA = 3$ ga teng bo‘lsa, nomalum son *m* ni toping.

Yechish: *Chizmadan* $OB = x_2$ *va* $OA = x_1$ *masofalarni deb, ola-miz, shunda* $x_2 - x_1 = 3$ *bo‘ladi.*

Biyeta teoremasiga asosan $x_1 + x_2 = 5,\ x_1 \cdot x_2 = 2 - 2m$. *Bu tengliklarni birgalikda sistema qilib yechsak,* $x_1 = 1,\ x_2 = 4.\ x_1 \cdot x_2 = 1 \cdot 4 = 4.\ \ 2 - 2m = 4.$

Bundan $m = -1.$

9) Rasmda ifodalangan $O‘L$ va CE kesmalar AB diametrga perpendikulya bo‘lib. $OA = 2CE$ bo‘lsa, uchburchak ΔACB yuzasining, uchburchak ΔADB yuzasiga bo‘lgan nisbatini toping.



Yechish: *Chizmadan* $OA = OB = OD = R,\ AB = 2R.$ ΔACB *va* ΔADB *lar to‘g‘ri burchakli uchburchaklardir, chunki AB ularning gipotenizasi bo‘lib, bir vaqtning o‘zida aylana diametri ham dir.*

Masala shartidan $CE = \frac{1}{2} \cdot OA = \frac{1}{2} \cdot R.$ $\quad \frac{S_{\Delta ACB}}{S_{\Delta ADB}} = \frac{\frac{1}{2} \cdot 2R \cdot \frac{1}{2} \cdot R}{\frac{1}{2} \cdot 2R \cdot R} = \frac{1}{2}.$

10) To‘g‘ri burchakli uchburchakning katetlari $3 - 2\sqrt{5}x + x^2 = 0$ tenglama ildizlariga teng bo‘lsa, shu uchburchakning yuzini toping.

Yechish: *Masala shartidan kelib chiqib, uchburchakning yuzi* $S = \frac{1}{2} \cdot x_1 \cdot x_2$ *ga teng bo‘ladi. Berilgan tenglamani yechib,* $x_{1,2} = \sqrt{5} \pm \sqrt{5 - 3} = \sqrt{5} \pm \sqrt{2},\ \ x_1 = \sqrt{5} - \sqrt{2}$ *va* $x_2 = \sqrt{5} + \sqrt{2}$ *ekanligini aniqlaymiz.*

Berilgan uchburchakning yuzi $S = \frac{1}{2} \cdot x_1 \cdot x_2 = \frac{1}{2} \cdot (\sqrt{5} - \sqrt{2}) \cdot (\sqrt{5} + \sqrt{2}) = 1{,}5.$

11) Agar $f(x+2)=2(2f(x)+1), f(2)=4$ bo‘lsa, $f(6)$ ni qiymatini toping.

Yechish: $x+2=4 \longrightarrow x=2;\ f(2+2)=f(4)=2(2\cdot 4+1)=18;$

$$f(6)=2(2f(4)+1)=2(2\cdot 18+1)=74.$$

12) Agar $f(x-3)=\frac{2x-1}{x+1}$ bo‘lsa, $f(f(3))$ ni qiymatini toping.

Yechish: $x-3=3 \longrightarrow x=6;\ f(3)=\frac{2\cdot 6-1}{6+1}=\frac{11}{7};\ x-3=\frac{11}{7} \longrightarrow x=\frac{32}{7};$

$$f(f(3))=\frac{2\frac{32}{7}-1}{\frac{32}{7}+1}=\frac{57}{39}=\frac{19}{13}.$$

13) $1; 5; 13; 29; \ldots$ ketma − ketlikning yigirmanchi hadini toping.

Yechish: $a_1=1=2^2-3;\ a_2=5=2^3-3;\ a_3=13=2^4-3;\ a_4=29=2^5-3;$

$$a_{20}=2^{21}-3.$$

14) $44+47+50+\cdots+(6n-1);\ n\in N.$ yig‘indini hisoblang.

Yechish: $(6n-1)=44+(k-1)3=44+3k-3 \Longrightarrow k=2(n-7).$ *Berilgan ifodaning yig‘indisi* $S_k=\frac{44+6n-1}{2}\cdot 2(n-7)=(43+6n)\cdot(n-7).$

15) a;3;b arifmetik, a;3;b geometrik progressiya bo‘lsa, a^2+b^2 ni toping.

Yechish: *Birinchi ifoda arifmetik progressiya bo‘lgani uchun* $3-a=b-3$ *yoki* $a+b=6.$ *Ikkinchi ifoda geometrik progressiya bo‘lgani uchun* $\frac{3}{a}=\frac{b}{3}$ *bundan* $ab=9.$ $a^2+b^2=36-2ab=36-18=18.$

16) $A(-6;-1)$ nuqta $C(n;5)$ va $B(-2;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqda yotsa, *n* ning qiymati nechaga teng.

Yechish: *BC nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzamiz:*

$$\frac{y-1}{5-1}=\frac{x+2}{n+2}\ \textit{bundan}\ n=\frac{4x-2y+10}{y-1}=\frac{4\cdot(-6)-2\cdot(-1)+10}{-1-1}=6.$$

17) Agar arifmetik progressiya $a_n=-\frac{n-2}{5}$ ketma-ketlik bilan berilgan bo‘lsa, uni ayirmasini toping.

Yechish: $n=1$ *da* $a_1=-\frac{1-2}{5}=\frac{1}{5}$ *bo‘ladi.* $a_n=\frac{1}{5}+(n-1)d=-\frac{n-2}{5}$ *yoki* $1+5(n-1)d=2-n$ *bundan* $d=-\frac{1}{5}.$

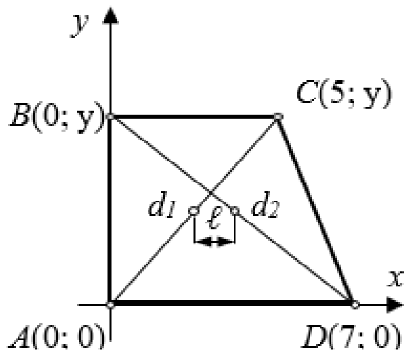
18) $f(x)=cos\left(\frac{p}{4}\cdot tg\frac{px}{4}\right)$ bo‘lsa, $f'(1)$ ni toping.

Yechish: $f'(1)=-\left(\frac{p}{4}\cdot tg\frac{px}{4}\right)'\cdot sin\left(\frac{p}{4}\cdot tg\frac{px}{4}\right)=-\frac{p^2}{16}\cdot\frac{1}{cos^2\frac{px}{4}}\cdot sin\left(\frac{p}{4}\cdot tg\frac{px}{4}\right)=$

$$=-\frac{p^2}{16}\cdot\frac{1}{cos^2\frac{p\cdot 1}{4}}\cdot sin\left(\frac{p}{4}\cdot tg\frac{p\cdot 1}{4}\right)=-\frac{p^2\sqrt{2}}{16}.$$

19) Uchlari $A(0;0)$, $B(0;y)$, $C(5;y)$ va $D(7;0)$ nuqta-larda bo‘lgan to‘rtburchakning diagonallari o‘rtasidagi masofani toping (rasmga qarang).

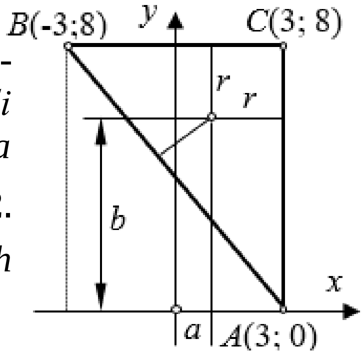
Yechish: *Berilgan nuqtalarni koordinata teksligiga joylashtiramiz.* d_1 *diagonali o'rtasining koordinatasini topamiz* $d_{1o'r}(2{,}5;0{,}5y)$, d_2 *diagonalni o'rtasining koordinatasini topamiz* $d_{2o'r}(3{,}5;0{,}5y)$.



Ikki nuqta masofasini topish formulasidan

$\ell=\sqrt{(3{,}5-2{,}5)^2+(0{,}5y-0{,}5y)^2}=1.$

20) Uchlari $A(3;0)$, $B(-3;8)$, $C(3;8)$ nuqtalarda bo'lgan uchburchakka ichki chizilgan aylana tenglamasini tuzing.



Yechish: *Berilgan nuqtalarni koordinata teksligiga joylashtiramiz. Chizmadan ko'rinadiki uchburchak to'g'ri burchakli bo'lib, uning katetlari 8 va 6 ga, gipotenuzasi 10 ga teng. Unga ichki chizilgan aylana radiusi* $r=\frac{2S}{a+b+c}=\frac{6\cdot 8}{6+8+10}=2.$ *Chizmadan aylana markazining koordinatasini aniqlash mumkin:*

$a=3-r=3-2=1;\quad b=8-r=8-2=6.$

Aylana tenglamasi $(x-a)^2+(y-b)^2=r^2$. *Demak* $(x-1)^2+(y-6)^2=4.$

21) $\sqrt{1+log_2x}+\sqrt{4log_4x}-2=4$ tenglamani yeching.

Yechish: *Berilgan tenglamadagi logarifmni 4 asosdan 2 asosga o'tib, quyidagi ko'rinishga keltiramiz:* $\sqrt{2(log_2x-1)}=4-\sqrt{1+log_2x}$ *bu tenglikni ikkala tomonini kvadratga oshirib, so'ldalashtirgandan so'ng, kuyidagi kvadrat tenglamani hosil qilamiz:*

$$log_2^2x-102log_2x+33\cdot 9=0.$$

Bu tenglamani yechib, $x=8$ *va* $x\neq 2^{99}$*ekanligini aniqlash mumkin.*

22) Agar $tg11^0=a$ bo'lsa, $\frac{sin22^0\cdot ctg191^0}{sin79^0\cdot cos349^0}$ ni hisoblang.

Yechish: *Berilgan* $\frac{sin22^0\cdot ctg191^0}{sin79^0\cdot cos349^0}$ *ifodani quyidagicha yozamiz:*

$$\frac{sin22^0\cdot ctg191^0}{sin79^0\cdot cos349^0}=\frac{2\cdot sin11^0\cdot cos11^0\cdot ctg(180^0+11^0)}{sin(90^0-11^0)\cdot cos(360^0-11^0)}=\frac{2\cdot cos^2 11^0}{cos11^0\cdot cos11^0}=2.$$

23) Agar $xy+yz+zx=16$ bo'lsa, $(x+y+z)^2$ ning eng kichik qiymatini toping.

Yechish: *Bunda* $x=y=z$ *deb olib,* $x^2+x^2+x^2=16$ *yoki* $3x^2=16$ *bundan* $x^2=\frac{16}{3}$. $(x+y+z)^2=(x+x+x)^2=(3x)^2=9x^2=9\cdot\frac{16}{3}=48.$

24) $y=e^{2x-3x^2}\cdot log_2(3x^2-2x)$ funksiya hosilasini $x=1, f'(1)$ da hisoblang.

Yechish:

$$y'=f'(1)=(2-6x)e^{2x-3x^2}\cdot log_2(3x^2-2x)+\frac{6x-2}{(3x^2-2x)ln5}\cdot e^{2x-3x^2}=$$

$$=(2-6\cdot 1)e^{2\cdot 1-31^2}\cdot log_2(31^2-2\cdot 1)+\frac{6\cdot 1-2}{(31^2-2\cdot 1)ln5}\cdot e^{2\cdot 1-31^2}=\frac{4}{e\cdot ln5}.$$

25) $\int_1^2\sqrt[3]{x\sqrt[5]{x}}\,dx$ ni hisoblang.

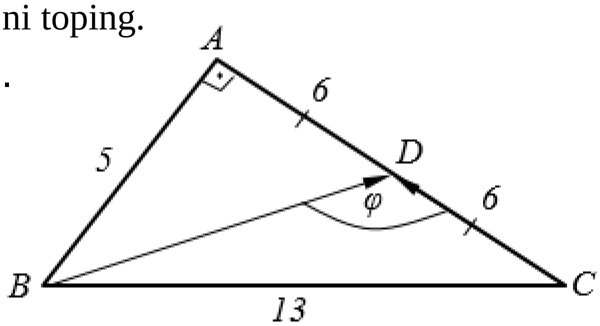
Yechish: *Berilgan integralni quyidagi ko'rinishda yozamiz:*
$\int_1^2 \sqrt[3]{x\sqrt[5]{x}}\,dx = \int_1^2 x^{\frac{2}{5}}\,dx = x^{\frac{2}{5}+1} = \frac{5}{7}x^{\frac{7}{5}} = \frac{5}{7}(2^{\frac{7}{5}} - 1^{\frac{7}{5}}) = \frac{5}{7}\left(2\sqrt[5]{4} - 1\right).$

26) $1\frac{5}{16} + z^2 - \frac{\sqrt{85}}{4}z = 0$ tenglamaning katta va kichik ildizlari kublarining ayirmasini toping.

Yechish: *Berilgan tenglamani quyidagi ko'rinishda yozamiz:* $z^2 - \frac{\sqrt{85}}{4}z + \frac{21}{16} = 0.$ *Bu tenglamada* $x_1 + x_2 = \frac{\sqrt{85}}{4}$; $x_1 \cdot x_2 = \frac{21}{16}$. $x_1^2 + x_2^2 = \frac{85}{16} - 2x_1x_2 = \frac{85}{16} - 2\cdot\frac{21}{16} = \frac{43}{16}$. *Tenglamaning ildizlari* $x_{1,2} = \frac{\sqrt{85}}{8} \pm \sqrt{\frac{85}{64} - \frac{21}{16}} = \frac{\sqrt{85}\pm 1}{8}$. *Masala shartiga asosan* $x_2^3 - x_1^3 = (x_2 - x_1)(x_1^2 + x_1x_2 + x_2^2) =$

$$= \left(\frac{\sqrt{85}+1}{8} - \frac{\sqrt{85}-1}{8}\right)\left(\frac{43}{16} + \frac{21}{16}\right) = \frac{1}{4}\cdot 4 = 1.$$

27) Chizmada berilganlar bo'yicha $\overrightarrow{BD}\cdot\overrightarrow{CD}$ ni toping.



Yechish: *Chizmadan:* $AC = \sqrt{13^2 - 5^2} = 12.$

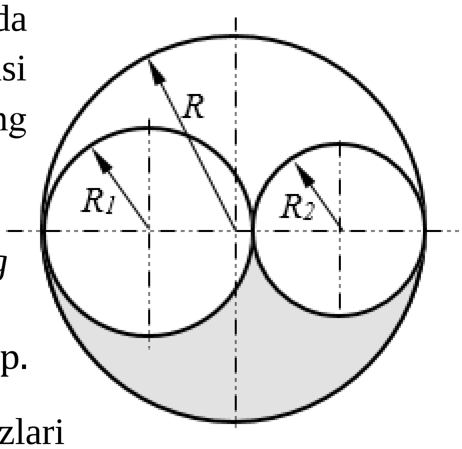
D nuqta AC ning o'rtasi bo'lgani uchun $AD = CD = 6.$ $\left|\overrightarrow{BD}\right| = \sqrt{6^2 + 5^2} = \sqrt{61}.$

ΔBDC *dan kosinuslar teoremasidan foydalanib,*

$169 = 61 + 36 - 2\cdot 6\cdot\sqrt{61}\cdot cos$ц.

Bundan cosц $= -\frac{6}{\sqrt{61}}$. $\overrightarrow{BD}\cdot\overrightarrow{CD} = \left|\overrightarrow{BD}\right|\cdot\left|\overrightarrow{CD}\right|\cdot cos$ц $= -\sqrt{61}\cdot 6\cdot\frac{6}{\sqrt{61}} = -36.$

28) Rasmda aylana ichida markazlari bir chiziqda yotuvchi uchta aylana berilgan. Bunda katta aylana radiusi R, $R_1 = 6$ va $R_2 = 5$ bo'lsa, rasmdagi bo'yalgan shaklning yuzini toping.



Yechish: Berilgan katta aylananing radiusi

$R = R_1 + R_2 = 11.$ *Rasmdagi bo'yalgan shaklning yuzi:*

$S_b = \frac{\text{p}}{2}\cdot(R^2 - R_1^2 - R_2^2) = \frac{\text{p}}{2}\cdot(11^2 - 6^2 - 5^2) = 30\text{p}.$

29) $1 - log_{2x-1}\frac{x^4-2}{2x+1} = 0.$ tenglamaning barcha ildizlari yig'indisini toping.

Yechish: *Bu tenglamani ildizi* $x \neq -\frac{1}{2}$; *uning yechimi esa* $0 < x < \frac{1}{2}$ *va* $x > 1$ *oraliqda bo'lishi kerak. Berilgan tenglamani quyidagi ko'rinishda yozamiz:*

$log_{2x-1}\frac{x^4+2}{2x+1} = log_{2x-1}(2x-1)$. *Bundan* $\frac{x^4+2}{2x+1} = 2x - 1$ *yoki*

$x^4 - 4x^2 + 3 = 0.$*Bu tenglamaning ildizlari* $x_1 \neq -\sqrt{3}$; $x_2 \neq -1$; $x_2 \neq 1$; $x_1 = \sqrt{3}$.

30) $\sqrt{2^{\sqrt{6}}\cdot 2^{\sqrt{x+1}}} = 4^{\sqrt{x+1}}$ tenglama ildiziga teskari bo'lgan sonni toping.

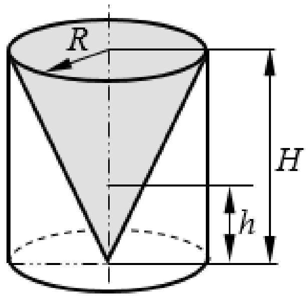
Yechish: *Berilgan tenglamani ikkala tomonini kvadratga oshirib, quiydagi ko'ri-nishga keltiramiz:* $2^{4\sqrt{x+1}} - 2^{2\sqrt{x+1}} \cdot 2^{\sqrt{6}} = 0$ *yoki* $2^{2\sqrt{x+1}}\left(2^{2\sqrt{x+1}} - 2^{\sqrt{6}}\right) = 0$ *bundan* $2^{2\sqrt{x+1}} \neq 0$ *va* $2^{2\sqrt{x+1}} - 2^{\sqrt{6}} = 0$. *Bundan* $2\sqrt{x+1} = \sqrt{6} \Rightarrow x = \frac{1}{2}$.

31) To'g'ri burchakli uchburchakning perimetri 40 *dm* ga, yuzasi esa 60 dm^2 ga teng bo'lsa, uni katetlarini toping.

Yechish: *Masala shartiga asosan uchburchakning perimetri* $a + b + c = 40$. $ab = 120$. *Bundan* $b = \frac{120}{a}$. *Birinchi tenglikdan* $a + b = 40 - c$. *Bularni ikkala tomo-nini kvadratga oshirsak* $(a + b)^2 = 1600 - 80c + c^2$.

Hamda $(a + b)^2 = c^2 + 2ab$ *ekanligini hisobga olib, bularni tenglashtirsak, quyi-dagi tenglik* $1600 - 80c = 2ab = 2 \cdot 120$ *hosil bo'ladi. Bundan* $c = 17$. *ekanligini topamiz.* $a + b = 40 - 17 = 23$. *Yoki* $a + \frac{120}{a} = 23$.

Bundan $a = 15$. $b = \frac{120}{a} = \frac{120}{15} = 8$.



32) Rasmda silindr ichida suv bilan to'ldirilgan konus joylash-gan. Silindrning balandligi H = 12 dm ga teng. Agar konusning uchi teshib qo'yilsa, silindrdagi suv qanday balandlikka ko'tari-ladi.

Yechish: *Konus ichidagi suvning hajmi, silindrga o'tgan suv hajmiga teng bo'ladi* $V = \frac{\text{p}}{3} \cdot R^2 \cdot H = \text{p} \cdot R^2 \cdot h$.

Bundan $h = \frac{1}{3} \cdot H = 4dm$.

33) $\sqrt[n]{81^2 + 9^4 + 3^8}$ ifodaning qiymati natural bo'ladigan *n* ning eng katta qiymatini toping.

Yechish: *Berilgan ifodani quyidagi ko'rinishda yozamiz:* $\sqrt[n]{3^8 + 3^8 + 3^8} = \sqrt[n]{3^9}$ *bunda* $n = 9$.

34) $(2^{10} + 2^8)(2^5 - 2^3)$ ifoda qiymatining natural bo'luvchilari sonini toping.

Yechish: *Berilgan ifodani quyidagi ko'rinishda yozamiz:*

$2^8 \cdot (2^2 + 1) \cdot 2^3 \cdot (2^2 - 1) = 2^{11} \cdot 3 \cdot 5$. $NBS = (11 + 1) \cdot (1 + 1) \cdot (1 + 1) = 48$.

35) $lg8 \cdot lpg_2 10 + log_5 9 \cdot log_3 5$ ni hisoblang.

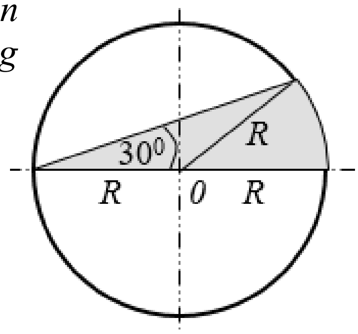
Yechish: *Berilgan ifodani quyidagi ko'rinishda yozamiz:*

$$\frac{3log_2 2}{log_2 10} \cdot lpg_2 10 + \frac{2log_3 3}{log_3 5} \cdot log_3 5 = 5.$$

36) $\sqrt{8^x \cdot \sqrt[3]{64^{-x} \cdot \sqrt[3]{128^{-x}}}} = \sqrt[18]{4}$ tenglamani yeching.

Yechish: *Berilgan tenglamani ikkala tomonini kvadratga oshirib, quyidagi ko'rinishda yozamiz:* $2^{3x} \cdot \sqrt[3]{2^{-6x} \cdot \sqrt[3]{2^{-7x}}} = 2^{\frac{2}{9}}$ *bundan* $\sqrt[3]{2^{-6x} \cdot \sqrt[3]{2^{-7x}}} = 2^{\frac{2}{9}-3x}$ *buni ikkala tomo-nini kubga oshiramiz:* $2^{-6x} \cdot \sqrt[3]{2^{-7x}} = 2^{\frac{2}{3}-9x}$*bundan* $\sqrt[3]{2^{-7x}} = 2^{\frac{2}{3}-3x}$ *buni ikkala tomo-nini kubga oshiramiz:* $2^{-7x} = 2^{\,2-9x}$ *bundan* $x = 1$.

37) Rasmda ko‘rsatilgan aylananing radiusi $R = 6\ dm$ ga teng. Rasmdagi bo‘yalgan shaklni yuzini toping.



Yechish: *Chizmada markaziy burchak* $в = 120^0$.*Bo‘yalgan shakilni yuzi, uchburchak va yoy yuzalarining yig‘ndisiga teng bo‘ladi.* $S_b = S_\Delta + S_{yoy} = \frac{1}{2} \cdot R^2 \cdot sinв + \frac{р}{180^0} \cdot б \cdot R^2$.

$S_b == \frac{1}{2} \cdot 6^2 \cdot sin120^0 + \frac{р}{180^0} \cdot 30^0 \cdot 6^2 = 6р + 9\sqrt{3}$.

38) Agar $tg4б = -\frac{2}{5}$ bo‘lsa, $ctgб - tgб - 2tg2б$ ni qiymatini toping.

Yechish: $tgб - tgб - 2tg2б = \frac{1}{tgб} - tgб - 2tg2б = \frac{1-tg^2б}{tgб} - 2tg2б =$

$= \frac{2(1-tg^2б)}{2tgб} - 2tg2б = 2\left(\frac{2tgб}{1-tg^2б}\right)^{-1} - 2tg2б = 2(tg2б)^{-1} - 2tg2б =$

$= 2\left(\frac{2(1-tg^2 2б)}{2tg2б}\right) = 4\left(\frac{2tg2б}{1-tg^2 2б}\right)^{-1} = 4tg4б = 4 \cdot \left(-\frac{2}{5}\right)^{-1} = -10.$

39) Qandaydir a, b va c lar uchun $cos4x = acos^4x + bcos^2x + c$ ayniyat bajarilsa, $a + 2b + c$ toping.

Yechish: *To‘rtlangan burchak kosinusi formulasidan foydalanib, quyidagi tenglikni tuzamiz:* $8cos^4б - 8cos^2б + 1 = acos^4x + bcos^2x + c$. *Ayniyatning xossasidan foydalanib* $a = 8$, $b = -8$, $c = 1$ *ekanligini aniqlaymiz.*

Demak $a + 2b + c = 8 + 2(-8) + 1 = -7$.

40) $log_{(\sqrt{6}-\sqrt{5})}(241 + 44\sqrt{30})$ ni hisoblang.

Yechish: *Berilgan ifodani a ga teng deb olamiz:* $log_{(\sqrt{6}-\sqrt{5})}(241 + 44\sqrt{30}) = a$.

Buda $241 + 44\sqrt{30} = (\sqrt{6} + \sqrt{5})^4$ *ga teng. Shunda* $log_{(\sqrt{6}-\sqrt{5})}(\sqrt{6} + \sqrt{5})^4 = a$ *bo‘ladi. Logarifm formulasidan foydalanib,* $(\sqrt{6} + \sqrt{5})^4 = (\sqrt{6} - \sqrt{5})^a$*ekanligini aniqlaymiz. Bu tenglikni ikkala tomonini* $(\sqrt{6} + \sqrt{5})^a$*ga ko‘paytirib, quyidagi tenglikni hosil qilamiz:* $(\sqrt{6} + \sqrt{5})^a \cdot (\sqrt{6} + \sqrt{5})^4 = (\sqrt{6} - \sqrt{5})^a \cdot (\sqrt{6} + \sqrt{5})^a$

$$(\sqrt{6} + \sqrt{5})^{a+4} = [(\sqrt{6} - \sqrt{5}) \cdot (\sqrt{6} + \sqrt{5})]^a = 1^a = 1 = (\sqrt{6} + \sqrt{5})^0$$

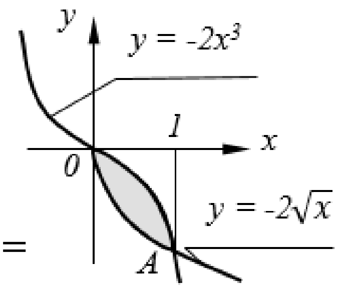
Daraja formulasidan foydalanib, $a + 4 = 0$ *ekanligini aniqlaymiz. Bundan* $a = -4$.

41) Agar $f(x) = \ln(2x + \sqrt{x^2 + 1})$ bo‘lsa, $f'(0)$ ni hisoblang.

Yechish: $f'(0) = \frac{(2x+\sqrt{x^2+1})'}{(2x+\sqrt{x^2+1})} = \frac{2+\frac{(\sqrt{x^2+1})'}{2\sqrt{x^2+1}}}{(2x+\sqrt{x^2+1})} = \frac{2+\frac{(x^2+1)'}{2\sqrt{x^2+1}}}{(2x+\sqrt{x^2+1})} =$

$$= \frac{2+\frac{2x}{2\sqrt{x^2+1}}}{\left(2x+\sqrt{x^2+1}\right)} = \frac{2+\frac{2\cdot 0}{2\sqrt{0^2+1}}}{(2\cdot 0+\sqrt{0^2+1})} = 2.$$

42) $y = -2\sqrt{x}$ va $y = -2x^3$ chiziqlar bilan chegaralangan soha yuzasini toping.



Yechish: *Bu funksiyalarni grafiklarini chizib, ularning kesishishidan hosil bo'ldan yuzani Nyuton-Leybnis formulasi orqani aniqlaymiz:*

$$S = \left|-2\int_0^1 \sqrt{x}\,dx - 2\int_0^1 x^3\,dx\right) = 2\left(\int_0^1 x^3\,dx - \int_0^1 \sqrt{x}\,dx\right) =$$

$$= 2\left|\left(\frac{x^4}{4} - \frac{2}{3}\sqrt{x^3}\right)\right| = 2\left|\left(\frac{1^4}{4} - \frac{2}{3}\sqrt{1^3}\right) - \left(\frac{0^4}{4} - \frac{2}{3}\sqrt{0^3}\right)\right| = \frac{5}{6}.$$

43) $(x+6)\cdot(x+4)\cdot(x+2)\cdot x$ ning eng kichik qiymatini toping.

Yechish: *Berilgan ifodani $f(x)$ deb qarab, uni quyidagi ko'rinishga keltiramiz:* $f(x) = (x^2+6x)\cdot(x^2+6x+8)$ *Bu funksiyani hosilasini olib, uni 0 ga tenglaymiz:* $f'(x) = (2x+6)(x^2+6x+8) + (2x+6)(x^2+6x) =$
$= 2(2x+6)(x^2+6x+4) = 0.$ *Bundan* $x_1 = -3-\sqrt{5}, x_2 = -3, x_3 = -3+\sqrt{5}$, *bu qiymatlarni berilgan ifodaga qo'yib, ularning ichida eng kichigi -16 ekanligini aniqlash mumkin.*

44) $\int \frac{dx}{3+x^2}$ integralni hisoblang.

Yechish: *Berilgan ifodani* $\int \frac{dx}{\left(\sqrt{3}\right)^2+x^2}$ *ko'rinishda yozib, uni iltegralning 13 formulasidan foydalanib hisoblaymiz:* $\int \frac{dx}{\left(\sqrt{3}\right)^2+x^2} = \frac{1}{\sqrt{3}} arctg\frac{x}{\sqrt{3}} + C.$

45) Agar $\frac{4^x+8^x+12^x}{5^x+10^x+15^x} = \frac{250}{128}$ bo'lsa, *x* ni toping.

Yechish: *Berilgan tenglikni quyidagich yozamiz:*

$$\frac{4^x+2^x\cdot 4^x+3^x\cdot 4^x}{5^x+2^x\cdot 5^x+3^x\cdot 5^x} = \frac{4^x(1+2^x+3^x)}{5^x(1+2^x+3^x)} = \frac{2\cdot 5^3}{2\cdot 4^3} = \frac{5^3}{4^3} = \left(\frac{5}{4}\right)^3 \text{ yoki } \left(\frac{4}{5}\right)^x = \left(\frac{4}{5}\right)^{-3}$$

Bundan $x = -3.$

46) $(x^2+x-2)^2 + (x^2+x-2) - 2 = x$ tenglamaning nechta natural yechimlari mavjud.

Yechish: *Berilgan tenglikni quyidagich yozamiz:*

$(x^2+x-2)^2 + x^2 + x - 2 - 2 - x = 0$ *yoki*

$(x+2)^2\cdot(x-1)^2 + (x-2)\cdot(x+2) = 0$

$(x+2)\cdot\left((x+2)\cdot(x-1)^2 + x - 2\right) = 0$ $x+2=0 \to x=-2.$

Bu natural son emas. $(x+2)\cdot(x-1)^2 + x - 2 = 0$

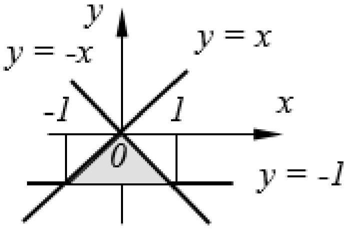
$(x+2)\cdot(x^2-2x+1) + x - 2 = 0$

$x^3 - 2x^2 + x + 2x^2 - 4x + 2 + x - 2 = 0$ $\quad x^3 - 2x = 0,\ x\cdot(x^2-2) = 0.$

Bundan $x = 0$; $x = -\sqrt{2}$; $x = \sqrt{2}$ *bu qiymatlar ham natural son emas.*

Demak, berilgan tenglamaning natural yechimlari mavjud emas.

47) $y = x$, $y = -x$ va $y = -1$ chiziqlar hosil qilgan uchburchakning yuzini toping.



Yechish: *Bu funksiyalarni grafiklarini chizib, ularning kesishishidan hosil bo'ldan uchburchakning yuzani topamiz. Chizmadan uchburchakning asosi 2 ga, balandligi 1 ga teng-ligini ko'rish mumkin.*

$S_\Delta = \frac{1}{2} \cdot 2 \cdot 1 = 1$ *kv.birlik.*

48) $f(x) = -3x^2 + 9x + t - 3$ funksiyaning maksimumi 4 ga teng bo'lsa, t ning qiymatini toping.

Yechish: *Funksiya maksimumining argumentini aniqlash uchun, berilgan funksiyadan birinchi tartibli hosila olib, uni nolga tenglaymiz:* $f'(x) = -6x + 9 = 0$ *bundan* $x = 1{,}5$. $f(x)_{max} = 4 = -3 \cdot (1{,}5)^2 + 9 \cdot 1{,}5 + t - 3$. *Bundan* $t = 0{,}25$.

49) $log_3 log_{\frac{1}{2}}(x - 4) > 0$, tengsizlikni yeching.

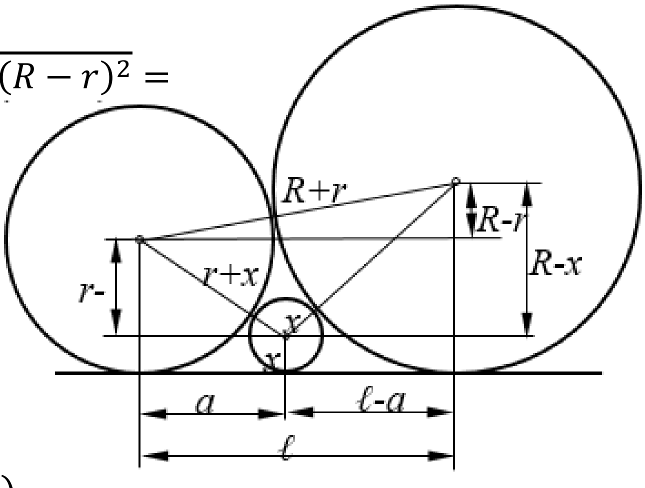
Yechish: *Logarifmning aniqlanish sohasi* $x - 4 > 0 \rightarrow x > 4$.

Berilgan tengsizlikni quyidagi ko'rinishda yozamiz: $log_3 log_{\frac{1}{2}}(x - 4) > log_3 1$, *yoki*

$log_{\frac{1}{2}}(x - 4) > 1$ *bundan* $(x - 4) < \left(\frac{1}{2}\right)^1$ *yoki* $x < 4{,}5$.

Tengsizlikni yechimi $4 < x < 4{,}5$.

50) Agar R va r radiusli aylanalar tashqi urunsa, ularga tashqi urungan aylana radiusi x ni toping (chizmaga qarang).

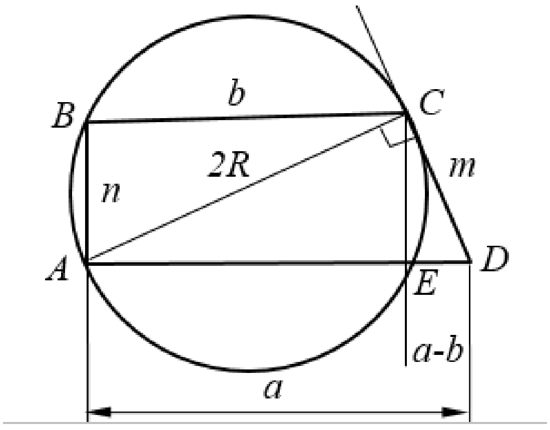


Yechish: *Chizmadan* $\ell = \sqrt{(R + r)^2 - (R - r)^2} =$
$= \sqrt{(R + r - R + r)(R + r + R - r)} =$
$= 2\sqrt{Rr}$. $a = \sqrt{(r + x)^2 - (r - x)^2} =$
$= \sqrt{(r + x - r + x)(r + x + r - x)} =$
$= 2\sqrt{rx}$.
$\ell - a = \sqrt{(R + x)^2 - (R - x)^2} =$
$= \sqrt{(R + x - R + x)(R + x + R - x)} =$
$= 2\sqrt{Rx}$.

$2\sqrt{Rr} = 2\sqrt{rx} + 2\sqrt{Rx} = 2\sqrt{x}\left(\sqrt{R} + \sqrt{r}\right)$.

Bundan $x = \frac{Rr}{\left(\sqrt{R}+\sqrt{r}\right)^2}$.

51) $ABCD$ trapetsiyaga aylana tashqi chizilgan bo'lib, trapetsiyaning ABC uchari aylanada yotadi. $AB = n$ va $CD = m$ bo'lib, trapetsiyaning C uchidagi aylanaga DC urunma o'tkazilgan hamda aylana tra-petsiyaning E nuqtasida kesishadi. Trapetsiyaning yuzini toping.



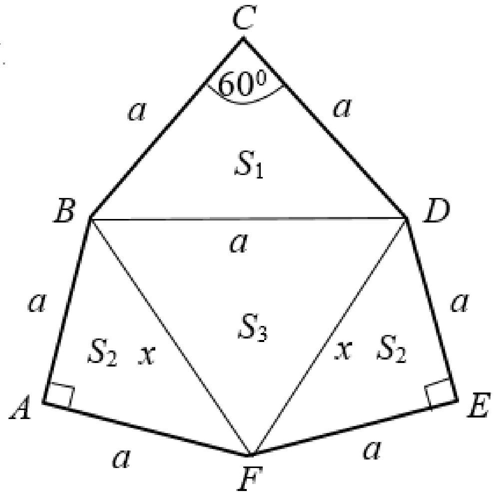
Yechish: *Trapetsiyani to‘go‘ri burchakli deb olib, masala shrti bo‘yich chizma chizamiz:*

Chizmadan $(2R)^2 = b^2 + n^2 = a^2 - m^2$ *bundan* $a^2 - b^2 = n^2 + m^2$.

$(a-b)(a+b) = n^2 + m^2$. $a - b = \sqrt{m^2 - n^2}$ *bo‘lgani uchun,* $a + b = \frac{n^2+m^2}{\sqrt{m^2-n^2}}$

Trapetsiyaning yuzi $S = \frac{a+b}{2} \cdot n = \frac{n^2+m^2}{2\sqrt{m^2-n^2}} \cdot n.$

52) Tomonlari $a = \frac{1}{\sqrt{\sqrt{7}+\sqrt{5}+4}}$ ga teng bo‘lgan *ABCDEF* oltiburchakning $\angle A = \angle E = 90^0$, $\angle C = 60^0$ga teng bo‘lsa, shu oltiburchakning yuzini toping.



Yechish: *Masala shartiga asoslanib chizma chizamiz.*

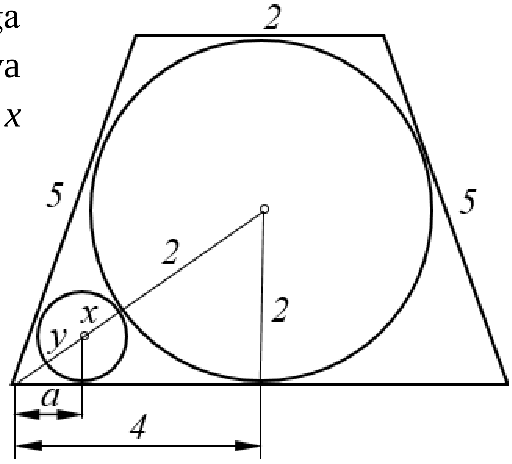
Trapetsiyaning yuzi $S = S_1 + 2S_2 + S_3$. (1)

ΔABC *teng tomonli uchburchak bo‘lgani uchun unung yuzi* $S_1 = \frac{\sqrt{3}}{4}a^2$ *ga teng.*

ΔFAB *to‘go‘ri burchakli uchburchak ning yuzi* $S_2 = \frac{1}{2}a^2$ *ga teng.* ΔBDF *dan* $x = \sqrt{2}a$. *Bu uchburchakning yuzini Geron formulasidan aniqlaymiz:*

Uchburchakning yarim perimetri $p = \frac{2\sqrt{2}a+a}{2}$ *ga teng bo‘ladi. Buni Geron formulasiga qo‘yib,* ΔBDF *ni yuzini topamiz:* $S_3 = \frac{\sqrt{7}}{4}a^2$. *Aniqlangan yuzalarni (1) formulaga qo‘ysak* $S = S_1 + 2S_2 + S_3 = \frac{\sqrt{3}}{4}a^2 + 2\frac{1}{2}a^2 + \frac{\sqrt{7}}{4}a^2 = 0{,}25.$

53) Asoslari 8 va 2 bo’lgan teng yonli trapetsiyaga aylana ichki chizilgan. Katta asosga, yon tomonga va ichki chizilgan aylanaga urinuvchi aylana radiusini *x* nitoping (chizmaga qarang).



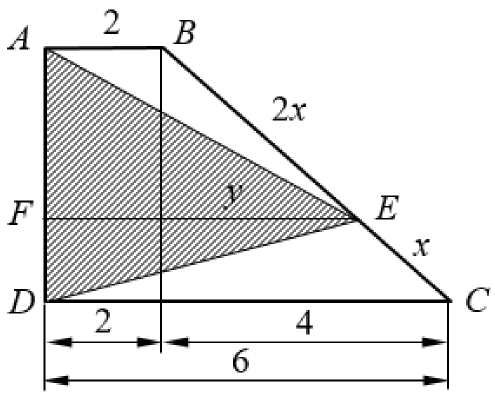
Yechish: *Chizmadan* $\frac{2}{4} = \frac{x}{a}$ *bundan* $a = 2x$.

$y = \sqrt{x^2 + a^2} = \sqrt{x^2 + 4x^2} = \sqrt{5}x$.

$\frac{2}{x+y+2} = \frac{x}{y}$ $\frac{2}{x+\sqrt{5}x+2} = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$ *bundan*

$x = \frac{2\sqrt{5}-2}{\sqrt{5}+1} = 3 - \sqrt{5}$.

54) *ABCD* trapetsiyaning yuzi 48 ga teng, asoslari *DC* = 6, *AB* = 2. *BC* tomondan *E* nuqta olingan bo‘lib, *BE* = 2*EC* bo‘lsa, *ADE* uchburchak yuzini toping (chizmada shtrixlangan yuza).



Yechish: *Chizmadan ABCD trapetsiyaning yuzi* $48 = \frac{2+6}{2}AD$ *bundan* $AD = 48:4 = 12.$

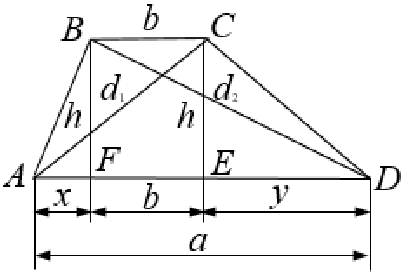
$\frac{4}{3x} = \frac{y}{2x}$ *bundan*

$y = \frac{8}{3}$ $EF = 2 + y = 2 + \frac{8}{3} = \frac{14}{3}$

ADE uchburchak yuzini

$S_{ADE} = \frac{1}{2} \cdot AD \cdot EF = \frac{1}{2} \cdot 12 \cdot \frac{14}{3} = 28.$

55) *ABCD t*rapetsiyaning diogonallari $d_1 = 26,$ $d_2 = 30$ va balandligi $h = 24$ ga teng. Uni yuzini toping.



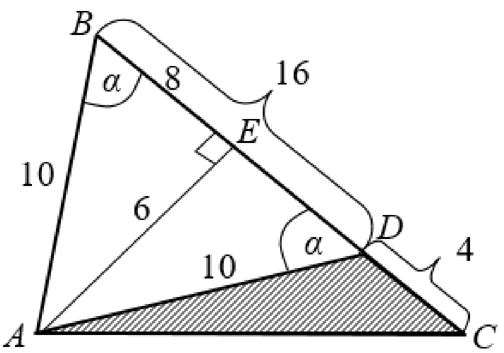
Yechish: *Masala shartiga asosan uning chizmasini chizamiz. Chizmadan* $x + y = a - b$ *hamda togri burchakli uchburchak ACE dan* $b + x = \sqrt{d_1^2 - h^2}$ *va burchakli uchburchak FBD dan* $b + y = \sqrt{d_2^2 - h^2}$ *larni aniqlash mumkin.Bularni bir biriga qo'shib, quyidagini hosil qilamiz:*

$$2b + x + y = \sqrt{d_1^2 - h^2} + \sqrt{d_1^2 - h^2} \text{ yoki } a + b = \sqrt{d_1^2 - h^2} + \sqrt{d_1^2 - h^2}.$$

Trapetsiyaning yuzi $S = \frac{a+b}{2} \cdot h = \frac{1}{2} \cdot h \cdot \left(\sqrt{d_1^2 - h^2} + \sqrt{d_1^2 - h^2}\right)$ *bu formulada* d_1, d_2 *va h larning qiymatlarini qo'ysak*

$$S = \frac{1}{2} \cdot 24 \cdot \left(\sqrt{26^2 - 24^2} + \sqrt{30^2 - 24^2}\right) = 336.$$

56) *ABC* uchburchakning *BC* tomonida *D* nuqta olingan. Agar *BD* = 16, *DC* = 4 va *AB* =*AD* = 10 bo'lsa, *ADC* uchburchakning yuzini toping.



Yechish: *Chizmadan ABC uchburchak*ning yuzi

$S_{ABC} = \frac{1}{2} \cdot AE \cdot (BE + EC) = \frac{1}{2} \cdot 6 \cdot 20 = 60.$

*ABD uchburchak*ning yuzi

$S_{ABD} = \frac{1}{2} \cdot BD \cdot AE = \frac{1}{2} \cdot 16 \cdot 6 = 60 = 48.$

*Izlanayotgan ABD uchburchak*ning yuzi

$S_{ABD} = S_{ABC} - S_{ABD} = 60 - 48 = 12.$

57) Agar $x\sqrt{x} - 8\sqrt{x} = 7$ bo'lsa, $x - \sqrt{x}$ ning qiymatini toping.

Yechish: *Berilgan tenglikni quyidagi ko'rinishda yozamiz:*

$$x\sqrt{x} - \sqrt{x} - 7\sqrt{x} - 7 = 0.$$

$\sqrt{x}(x-1) - 7(\sqrt{x}+1) = 0$ $\sqrt{x}(\sqrt{x}-1) \cdot (\sqrt{x}+1) - 7(\sqrt{x}+1) = 0$

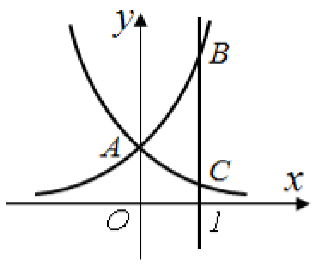
$(\sqrt{x}+1) \cdot (x - \sqrt{x} - 7) = 0$ *bundan* $\sqrt{x} \neq -1$ *va* $x - \sqrt{x} = 7.$

58) Ikki xonali sonning ko'paytmasi to'rt xonali sonning eng kichigiga teng bo'lsa, berilgan ikki xonali sonlar yig'indisini eng kichigini toping.

Yechish: *To'rt xonali sonning eng kichigi* 1000 *soni bo'lgani uchun,*

$$25 \times 40 = 1000 \text{ demak, } 25 + 40 = 65.$$

59) $x = 1,\ y = e^x$ va $y = e^x$ chiziqlar bilan chegaralangan soha yuzini toping.



Yechish: *Masala shartida berilgan funksiyalarni grafiklarini chizamiz.Masala shartiga asosan ABC yuzani toppish talab qilinadi. Bu yuzani toppish uchun OAB1 yuzadan OAC1 yuzani ayirish kerak.*

$$S_{ABC} = S_{OAB1} - S_{OAC1} = \int_0^1 e^x dx - \int_0^1 e^{-x} dx =$$

$$= e^x /_0^1 + \int_0^1 e^{-x} d(-x) = (e^x + e^{-x}) /_0^1 = e^1 + e^{-1} - (e^0 + e^{-0}) =$$

$$= e + \frac{1}{e} - 2 = \frac{(e-1)^2}{e}$$

60) Agar $f(2x-3) = 3x + 5$ bo‘lsa, $f\big(f(1)\big)$ ni toping.

Yechish: $2x - 3 = 1$ *dan* $x = 2$ $f(2 \cdot 2 - 3) = f(1) = 3 \cdot 2 + 5 = 11.$

$$f\big(f(1)\big) = f(11) = 3 \cdot 11 + 5 = 38.$$

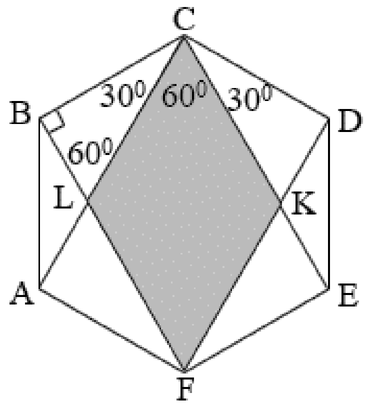
61) 3; 5; 9; 17; 33; 65; ... ketma-ketliknini yig‘indisini toping.

Yechish: *Berilgan sonlarni quyidagi ko‘rinishda yozamiz:* $3 = 2^1 + 1, 5 = 2^2 + 1,$ $9 = 2^3 + 1, 17 = 2^4 + 1, 33 = 2^5 + 1,\ 65 = 2^6 + 1$ *va hokozo.*

Bu yerda darajali ifoda geometric progressiyani ifodalaydi:

$$S_{ng} = \frac{2(2^n-1)}{2-1} = 2^{n+1} - 2, \qquad S_n = 2^{n+1} + n - 2.$$

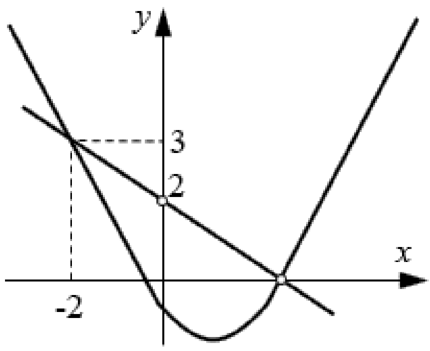
62) *ABCDEF* muntazam oltiburchakda *AC, CE, BF, FD* diagonallar o‘tkazilgan. *AC* va *BF* diagonallar *L* nuqtada, *CE* va AD diagonallar *K* nuqtada kesishadi. Agar oltiburchak tomoni $2\sqrt{3}$ ga teng bo‘lsa, *LCKF* to‘rtburchak yuzini toping.



Yechish: *Chizmadan* $LC = CE = KF = FL = b$ *bo‘lgani uchun LCKF to‘rtburchak romb ekanligini ko‘rish mum-kin.*ΔLBC *dan* $LC = b = 4$ *bo‘lgani uchun*

$$S_{LCKF} = b^2 \cdot sin60^0 = 4^2 \cdot \sqrt{3}/2 = 8\sqrt{3}.$$

63) Agar $f(x) = ax^2 + bx + c$ bo‘lca, berilgan grafikdan foydalanib, $2a + b$ ni toping.



Yechish: *To‘g‘ri chiziq tenglamasi* $y = kx + d$ *dagi k ni chizmadan foydalanib, aniqlaymiz:*

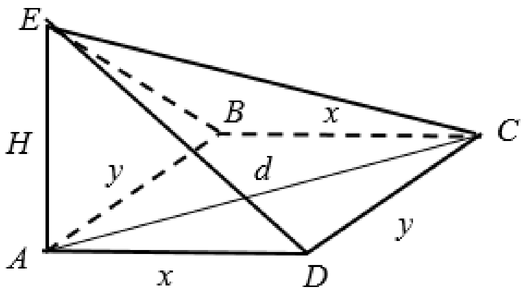
$3 = k(-2) + 2$ *bundan* $k = -\frac{1}{2}$. *Demak to‘g‘ri chiziq tenglamasi quyidagi ko‘rinishga ega bo‘ladi*

$$y = -\frac{1}{2}x + 2, \quad 0 = -\frac{1}{2}x + 2 \ \text{bundan} \ x = 4.$$

$-\frac{1}{2}x + 2 = ax^2 + bx + c$ *bu tenglikka* $x = -2$ *va* $x = 4$ *larni qoyib, ularni birga-likda sistema qilib yechamiz:*

$\begin{cases} 4a-2b+c=3 \\ 16a+4b+c=0 \end{cases}$ *bundan* $12a+6b=-3$ *bu tenglikni 6 ga bo'lsak,* $2a+b=-0{,}5$.

64) *ABCD* to'g'ri to'rtburchakning *A* uchidan shu to'g'ri to'rtburchak yotgan tekslikka perpendi-kulyar qilib *AE* to'g'ri chiziq o'tkazildi. Agar $BE=10\ sm$, $CE=13\ sm$ va $DE=11\ sm$ bo'l-sa, *AE* va *DC* to'g'ri chiziqlar orasidagi masofani toping.



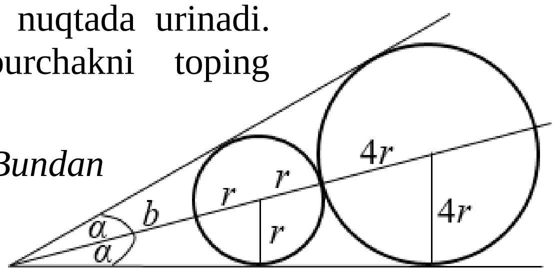
Yechish: *Masala shartiga asoslanib uning chizmasini chiamiz. To'g'ri to'rtburchak-ning tomonlarini x va y deb belgilaymiz. Chizmadan:* $d^2=x^2+y^2$

$H^2=13^2-d^2=13^2-(x^2+y^2)$ (1) $\quad H^2=10^2-y^2$ (2)

$H^2=11^2-x^2$ (3) *bu tengliklarni (2) ni (3) ga qo'shib, natijani (1) dan ayiramiz.*

Shunda $H^2=10^2+11^2-13^2=52$. *AE va DC to'g'ri chiziqlar orasidagi masofa x ga teng bo'lgani uchun uni aniqlaymiz:* $x=\sqrt{11^2-H^2}=\sqrt{121-52}=\sqrt{69}$.

65) Raduslari r va $4r$ bo'lgan aylanalar bir nuqtada urinadi. Ularning umumiy urunmalari orasidagi burchakni toping (chizmaga qarang).

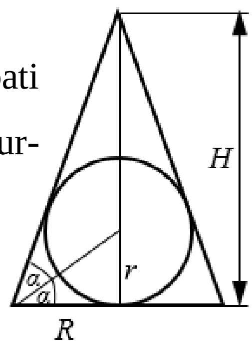


Yechish: *Chizmadan* $tgб=\frac{r}{b+r}=\frac{4r}{b+6r}$ *Bundan*

$b=\frac{2}{3}\cdot r.\quad tgб=\frac{r}{b+r}=\frac{r}{\frac{2}{3}\cdot r+r}=\frac{3r}{5r}=\frac{3}{5}$

$$tg2б=\frac{2tgб}{1-tg^2б}=\frac{2\cdot\frac{3}{5}}{1-\left(\frac{3}{5}\right)^2}=\frac{\frac{6}{5}}{\frac{16}{25}}=\frac{15}{8}\quad б=arctg(15/8).$$

66) Konusga shar ichki chizigan bo'lib, ularning hajimlarni nisbati $\frac{V_k}{V_{sh}}=2\frac{1}{4}$ kabi bo'lsa, konus asosi bilan yasovchish orasidagi bur-chakni toping.



Yechish: *Masala shartia asosan* $\frac{V_k}{V_{sh}}=\frac{\frac{1}{3}p\cdot R^2\cdot H}{\frac{4}{3}p\cdot r^3}=\frac{9}{4}$.

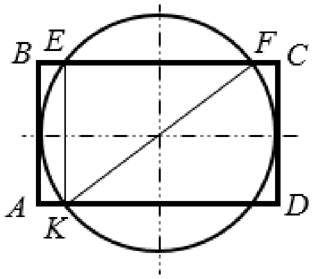
Bundn $\quad R^2\cdot H=9\cdot r^3$ (1).

Chizmadan $tgб=\frac{r}{R}$ *va* $tg2б=\frac{H}{R}$ *ekanligini hamda* $tg2б=\frac{2tgб}{1-tg^2б}$ *bilgan holda* $tg2б=\frac{2\cdot\frac{r}{R}}{1-\left(\frac{r}{R}\right)^2}=\frac{2\cdot r\cdot R}{R^2-r^2}=\frac{H}{R}$ *aniqlaymiz. Bundan* $H=\frac{2\cdot r\cdot R^2}{R^2-r^2}$ *buni* (1) *ga qoyamiz* $R^2\cdot\frac{2\cdot r\cdot R^2}{R^2-r^2}=9\cdot r^3$ *va quyidagi* $2R^4-9r^2\cdot R^2+9r^4=0$ *tenglamani hosil qilamiz, bundan* $R=\sqrt{3}\cdot r$. *Shunda* $tgб=\frac{r}{R}=\frac{r}{\sqrt{3}\cdot r}=\frac{1}{\sqrt{3}}$ *bundan* $б=30^0$ *yoki* $2б=60^0$.

67) $A = 99^2 - 77^2 - 22^2$ soni quyidagi 9, 7, 4, 2 sonlarning qaysi biriga bo‘linmaydi.

Yechish: *Berilgan sonni quyidagi ko‘rinishda yozamiz:*
$9^2 \cdot 11^2 - 7^2 \cdot 11^2 - 2^2 \cdot 11^2 = (81 - 49 - 4) \cdot 11^2 = 4 \cdot 7 \cdot 11^2$. *Bundan ko‘rinadili berilgan son* 9 *ga bo‘linmaydi.*

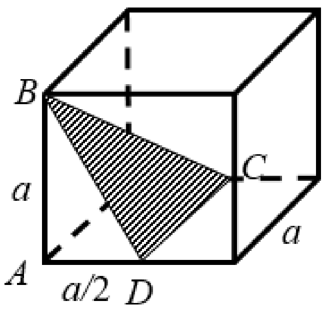
68) To‘g‘ri to‘rtburchakning tomonlari 8 va 6 teng. Markazi to‘rtburchakning diodanallari kesish nuqtasida bo‘lgan aylana bu to‘rtburchakning ikki kichik tomoniga urinadi. Shu aylana to‘rtburchakning katta tomonidan ajratgan kesma uzunligini toping.



Yechish: *Masala shartiga asosan* $AB = EK = 6$, $AD = BC = 8$.*Bu esa aylana diametri FK ga teng.*

To‘g‘ri burchakli uchburchak EFK dan so‘ra-layotgan EF kesma uzunligini Pifagar teoremasidan foydalanib toppish mumkin: $EF = \sqrt{FK^2 - EK^2} = \sqrt{8^2 - 6^2} = 2\sqrt{7}$.

69) D va C nuqtalar kub tomonining o‘rtalari bo‘lib, BCD uchburchakning yuzi $\sqrt{5}$ ga teng bo‘lsa, kub-ning hajmini toping.



Yechish: *Chizmadan* $BD = \sqrt{a^2 + (a/2)^2} = \frac{\sqrt{5}}{2} \cdot a$.
BCD uchburchakning yuzi
$S_{BCD} = \frac{1}{2} \cdot a \cdot BD = \frac{1}{2} \cdot a \cdot \frac{\sqrt{5}}{2} \cdot a = \sqrt{5}$.
Bundan $a = 2$. *Shunda kubning hajmi* $V = a^3 = 2^3 = 8$.

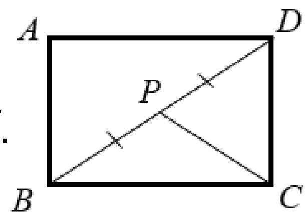
70) $f(x) = 15cos3x - 8sin3x + 4$ funksiyaning qiymatlar sohasini toping.

Yechish: *Bunda* $15cos3x - 8sin3x$ *funksiyaning qiymatlar sohasini topamiz:*
$y = \sqrt{15^2 + (-8)^2} = \pm 17$. *Demak, berilgan funksiyaning aniqlanish sohasi*
$\pm 17 + 4$. *Bundan* [-13;21].

71) $\left(\frac{1}{3}\right)^{\frac{log_5 log_{1/4} x}{log_{1/25} 3}} = log_{1/4}^2 5$ tenglamani yeching.

Yechish: *Bunda tenglamalar kamayuvchko‘rsatkichli va logarifmik funksiyalardan tashkil topgan uchun yechim (0;1) oraliqda bo‘lishi kerak. Berilgan tenglikni quyidagi korinishda yozamiz:* $3^{\frac{-log_5 log_{1/4} x}{-1/2 \cdot log_5 3}} = log_{1/4}^2 5$ *buni soddalashtirib,quyidagini hosil qilamiz:* $lpg_{1/4} x = \pm log_{1/4} 5 = log_{1/4} 5^{\pm 1}$*bundan* $x \neq 5$ *va* $x = 0{,}2$.

72) $ABCD$ to‘g‘ri to‘rtburchakda $|BP| = |PD|$, $|DC| = 4\ sm$, $|BC| = 6\ sm$, bolsa (rasmga qarang), $\overrightarrow{CB} \cdot \left(\overrightarrow{AB} + \overrightarrow{CP}\right) =$?

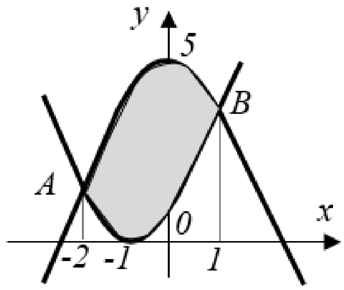


Yechish: *Chizmadan* $BD = \sqrt{BC^2 + DC^2} = \sqrt{6^2 + 4^2} = 2\sqrt{13}$.
$\cos(\angle DBC) = \frac{BC}{BD} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}$. $\overrightarrow{CB} \cdot \left(\overrightarrow{AB} + \overrightarrow{CP}\right) =$

$= \overrightarrow{CB}\cdot\overrightarrow{AB} + \overrightarrow{CB}\cdot\overrightarrow{CP} = |\overrightarrow{CB}|\cdot|\overrightarrow{AB}|\cdot cos90^0 + |\overrightarrow{CB}|\cdot|\overrightarrow{CP}|\cdot\cos(\angle DBC) =$

$= 6\cdot 4\cdot 0 + 6\cdot\sqrt{13}\cdot\frac{3}{\sqrt{13}} = 18.$

73) $y = x^2 + 2x + 1$ va $y = -x^2 + 5$ chiziqlar bilan chegaralangan soha yuzasini toping.



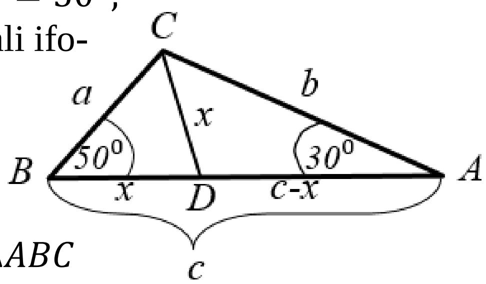
Yechish: *Ikkita parabolaning kesishish nuqtasining absissalarini toppish uchun ularning chap tomonlarini tenglashtiramiz:* $x^2 + 2x + 1 = -x^2 + 5$ *demak, ularning kesishish nuqtasining absissalari* $x_1 = -2$; $x_2 = 1$ *bo'ladi. Chiziqlar bilan chegaralangan sohaning yuzasini quyida-gicha aniqlanadi.*

$$S_{-1A5B-1} = S_{-2A5B1} - S_{-2A-1B1} = \int_{-2}^{1}(5 - x^2)dx - \int_{-2}^{1}(x^2 + 2x + 1)dx =$$

$$= \left(5x - \frac{x^3}{3} - \left(\frac{x^3}{3} + x^2 + x\right)\right)\Big|_{-2}^{1} = \left(4x - x^2 - \frac{2x^3}{3}\right)\Big|_{-2}^{1} =$$

$$= \left(4 - 1 - \frac{2}{3} - (-8 - 4 - \frac{-16}{3})\right) = 15 - 6 = 9.$$

74) *ABC* uchburchakning burchaklari $\angle A = 30^0$, $\angle B = 50^0$, $\angle C = 100^0$ga teng bo'lsa, uning *a* tomonini *b* va *c* orqali ifo-dalanig (rasmga qarang).



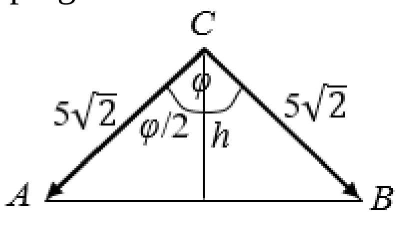
Yechish: *Uchburchakning C uchidan AB tomo-niga CD bissiktrissa tushiramiz (burchak C ikkiga bo'-linib, har biri* 50^0 *dan bo'ladi).*

Shunda $\angle ACB = \angle ADC = 100^0$ *bo'ladi. Bunda* ΔABC *va* ΔABC *lar o'xshash bo'ladi.*

Shundan foydalanib, $\frac{x}{c-x} = \frac{a}{b}$, (1) $\quad \frac{c-x}{b} = \frac{b}{c}$, *bundan* $c - x = \frac{b^2}{c}$, $\; x = \frac{c^2-b^2}{c}$

bularni (1) *ga qo'yamiz;* $a = \frac{c^2-b^2}{b}$.

75) *A, B, C* nuqta uchun $\overrightarrow{CB}(-3;5;-4)$ va $\overrightarrow{CA}(4;3;-5)$. *ABC* uchburchakning *C* uchidan *AB* tomoniga tushirilgan balandlik uzunligini toping.



Yechish: *Berilgan vektorlarning uzunliklarini;* $|\overrightarrow{CD}| = 5\sqrt{2}$, $|\overrightarrow{CA}| = 5\sqrt{2}$, *g teng. Bu esa berilgan uchburchak teng yonli ekanligini ko'rsatadi. Uni chizmasini chizamiz va C uchidagi burchakni* ц *deb olamiz.*

Vektorlar orasidagi burchak $cos ц = \frac{\overrightarrow{CB}\cdot\overrightarrow{CA}}{|\overrightarrow{CB}|\cdot|\overrightarrow{CA}|} = \frac{-12+15+20}{5\sqrt{2}\cdot 5\sqrt{2}} = \frac{23}{50}$,

$cos\frac{ц}{2} = \sqrt{\frac{1+cos ц}{2}} = \sqrt{\frac{1+\frac{23}{50}}{2}} = \frac{\sqrt{73}}{10}$. $\quad h = 5\sqrt{2}\cdot cos\frac{ц}{2} = 5\sqrt{2}\cdot\frac{\sqrt{73}}{10} = \sqrt{36{,}5}.$

76) Tekslikda $A(1;1)$, $B(3;4)$, $C(x;y)$ niqtalar berilgan.

Agar $\overrightarrow{AB} = \overrightarrow{AC}$ bo‘lsa, $\frac{x}{y} + \frac{y}{x}$ ni toping.

<u>Yechish:</u> *Berilgan nuqtalar bo‘yicha vector va uning koordinatalarini tuzamiz;* $\overrightarrow{AB}(3-1; 4-1) = \overrightarrow{AB}(2;3)$ $\overrightarrow{AC}(x-1; y-1)$. *Bu vektorlar teng bo‘lgani uchun*

$$\begin{cases} x-1=2 \\ y-1=3 \end{cases} = \begin{cases} x=3 \\ y=4 \end{cases} \quad \frac{x}{y}+\frac{y}{x} = \frac{3}{4}+\frac{4}{3} = \frac{9+16}{12} = \frac{25}{12} = 2\frac{1}{12}$$

77) Ikki natural sonning EKUKi 168 ga teng va ularning nisbati 3:4 kabi bo‘lsa, kichik sonni toping.

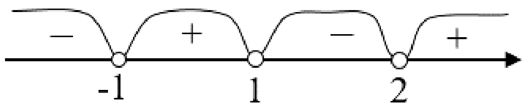
<u>Yechish:</u> $168:4 = 42$.

78) Ikki natural sonning EKUKi va EKUBi mos ravishda 420 va 35 ga teng bo‘lsa, ularning ko‘paytmasi 35^2 dan necha marta kata?

<u>Yechish</u>: *Natural sonlarni n va m deb belgilaymiz;* (1.1) *formuladan foydalanib, ularni ko‘paytmasini topamiz.* $n \cdot m = 420 \cdot 35$. *Shartga asosan* $\frac{n \cdot m}{35^2} = \frac{420 \cdot 35}{35 \cdot 35} = 12$.

79) $x^4 \cdot 5^x + 25 \geq 25x^4 + 5^x$ tengsizlikni yeching.

<u>Yechish</u>: *Berilgan tengsizlikni quyidagicha yozamiz;* $x^4 \cdot 5^x + 25 - 25x^4 - 5^x \geq 0$, *bundan* $5^x(x^4-1) - 25(x^4-1) \geq 0$ yoki $(x^4-1)(5^x-25) \geq 0$ *buni shartli ravishda nolga tenglab,* $x=-1, x=1, x=2$ *larni aniqlaymiz.*



Bularni sonlar o‘qiga qo‘yib, intervallar tuzib, ular orasidagi ishoralarni aniqlaymiz.

Shartga asosan tengsizlikning techimi; $[-1;1] \cup [2;\infty)$.

80) $\left(\sqrt[4]{2}\right)^{4x+5} = \left(\sqrt{2}\right)^{\frac{-4x}{3}}$ tenglamani yeching.

<u>Yechish</u>: *Berilgan tenglamani quyidagi ko‘rinishda yozamiz;*

$\left(\sqrt[4]{2}\right)^{4x+5} = \left(\sqrt[4]{4}\right)^{\frac{-4x}{3}}$ *bu tenglikni ikkala tomonini* $\left(\sqrt[4]{4}\right)^{\frac{-4x}{3}}$ *ga bo‘lsak,*

$$\left(\sqrt[4]{\frac{2}{4}}\right)^{4x+5+\frac{4x}{3}} = 1 = \left(\sqrt[4]{\frac{2}{4}}\right)^{0}$$ *bundan* $4x+5+\frac{4x}{3} = 0 \to x = -\frac{15}{16}$.

81) $|x-7| \cdot log_2(x-2) = 3(x-7)$ tenglamaning ildizlari yig‘indisini toping.

<u>Yechish</u>: *Berilgan tenglamaning bir ildizi* $x=7$ *ga teng. Berilgan tenglamani ikkala tomonini* $|x-7|$ *bo‘lib, quyidagi tenglikni hosil qilamiz;* $log_2(x-2) = \frac{3(x-7)}{|x-7|} = \pm 3$. *Logarifmlar formulasidan foydalanib,* $x-2 = 2^{\pm 3}, x = 2+2^{\pm 3}$*bundan* $x_1 = 2+\frac{1}{8}$, $x_2 = 2+10$. $x + x_1 + x_2 = 7+2+\frac{1}{8}+10 = 19\frac{1}{8}$.

82) Tenglamalar sistemasini yeching. $\begin{cases} 2\cdot 3^x + 3y = 4 \\ 3^{x+1} - 2y = 6 \end{cases}$

Yechish: *Sstemaning birinchi tenglamasini 2 ga, ikkinchi tenglamasini 3 ga ko'pay-tiramiz:* $\begin{cases} 4\cdot 3^x+6y=8 \\ 3\cdot 3^{x+1}-6y=18 \end{cases} = \begin{cases} 4\cdot 3^x+6y=8 & (1) \\ 9\cdot 3^x-6y=18 & (2) \end{cases}$ (1) *ni* (2) *ga qoshib, quyidagi tenglikni hosil qilamiz:* $13\cdot 3^x=26$, *bundan* $3^x=2$, $log_3 3^x=log_3 2\rightarrow x=log_3 2$. *Buni sistemaning birinchi tenglamasiga qo'yib, y ni topamiz:* $2\cdot 3^{log_3 2}+3y=4$ *yoki*
$2\cdot 2+3y=4$ *bundan* $y=0$.

14. TEST SAVOLLARI

V-1

1. Hisoblang $\frac{0,4\cdot 0,15\cdot 1,6}{6,4\cdot 2,5\cdot 0,03}$ A) 2 B) -2 C) 0,4 D) 0,2

2. Soddalashtiring $\frac{(32-16a^{1/4})(2a^{1/4}+a^{1/2})}{8a^{1/4}-2a^{3/4}}$ A) 8 B) 6 C) -6 D) 4

3. Hisoblang $\sqrt{27+10\sqrt{2}}+\sqrt{27-10\sqrt{2}}$ A) 12 B)11 C)10 D)13

4. Go‘sht qaynaganda 40 foiz vaznini yo‘qotadi.25 kg go‘sht qaynatilganda qancha go‘sht qoladi ? A) 15 B) 14 C) 8 D) 10

5. *a;* 4,2*;* 3,1*;* 1,1 sonlari o‘rta arifmetigi 2,95 ga teng. *a* ni toping.
A) 3,4 B) 2,4 C) 4 D) 3

6. Sistemani yeching $\begin{cases} 2x+5y=15 \\ x-2y=3 \end{cases}$ A) (1;5) B) (3;4) C) (5;1) D) (2;3)

7. Arifmetik progressiyaning uchinchi va to‘qqizinchi hadining yig‘indisi 8 ga teng bo‘lsa shu progressiyaning dastlabki 11 ta hadining yig‘ indisini toping.
A) 43 B) 45 C) 46 D) 44

8. Tenglamani yeching $2x-3+4(x-1)=5$ A) 3 B) 2 C) 4 D) 5 |

9. Agar x_1 va x_2 $3x^2-2x-6=0$ tenglamani ildizlari bo‘lsa, $\frac{1}{x_1}+\frac{1}{x_2}$ ni toping.
A) 1/3 B) 2/3 C) -1/3 D) 3/4

10. Tenglamani yeching $\frac{1}{2x-3}-\frac{3}{x(2x-3)}=\frac{5}{x}$ A) $1\frac{2}{3}$ B) $1\frac{1}{3}$ C) $2\frac{1}{3}$ D) $-1\frac{1}{3}$

11. $|x-5|=3$ tenglama ildizlari ayirmasini toping. A) 3 B) 4 C) 6 D) 5

12. A(2;-3;7) nuqtaning *oxz* o‘qiga nisbatan simmetrik bo‘lgan nuqtaning koordinatalarini toping. A) (2;3;7) B) (-2;3;4) C) (2;3;5) D) (2;3;6)

13. Tekislikka og‘ma va perpendikulyar tushirilgan. Og‘maning tekislikdagi proeksiyasi 7 ga teng bo‘lib, uning uzunligi 25 ga teng. Og‘ma va tekislik orasidagi burchakni toping. A) *arcsin*24/25 B) *arcsin*7/24 C) *arcsin*7/25 D) *arccos*7/50

14. $y = 5 - 2x - x^2$ funksiyaning qiytmatlar sohasini toping.
A) (-∞; 0) B) (-∞; ∞) C) (0; ∞) D) (-∞;6]

15. $y = x^2 - 12x + 65$ parabola uchining koordinatalari ko`paytmasini toping.
A)174 B)164 C)184 D) 182

16. Tengsizliklarning eng katta butun yechimini toping. $5^{x-1}<2$
A) 3 B) 4 C) 1 D) 2.

17. Agar $a=\log_{\frac{1}{2}}3;$ $b=\log_{\frac{1}{4}}3;$ $c=\log_{\frac{1}{2}}5$ bo`lsa, *a,b,c* sonlari uchun quyidagi munosabatlardan qaysi biri o`rinli? A) $a<c<b$ B) $c<a<b$ C) $b<a<c$ D) $c<b<a$

18. Hisoblang $ctg\frac{31}{6}\pi$ A) $\sqrt{2}$ B) 1 C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{3}$

19. Tenglamani yeching $2\sin 6x-\sqrt{2}=0$
A) $(-1)^k \pi/6+\pi k$ B) $(-1)^k \pi/12+\pi k$ C) $(-1)^k \pi/24+\pi k/6$ D) $(-1)^k \pi/24+\pi k/3$

20. Rombning tomoni 4 ga teng, unga ichki chizilgan aylananing radiusi $\sqrt{3}/4$ ga teng bo`lsa, rombning o`tkir burchagi sinusini toping? A) 3 B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{8}$ D) 2.

21. To`g`ri burchakli uchburchakning balandligi gipotenuzani 5 va 20 ga teng kesmalarga ajratadi. H=? A) 10 B) 12 C) 14 D) 8

22. Agar $|\vec{a}| = 7$, $|\vec{b}| = 17$, va $|\vec{a} - \vec{b}| = \sqrt{192}$ bo`lsa, $|\vec{a} + \vec{b}| =$ ni toping.
A) 12 B) 14 C) 20 D) 22

23. 1:2500000 masshtabli xaritadagi 50 mm masofa necha (km) ni tashkil qiladi.
A) 200 B) 125 C) 150 D) 100

24. Hisoblang. $173 \cdot 3,6 + 2,7 \cdot 64 + 2,7 \cdot 36 + 17,3 \cdot 64$
A) 2000 B) 1000 C) 1500 D) 2500

25. *a* ning qanday qiymatlarida $4x^2-ax+a-3=0$ tenglama bitta ildizga ega bo‘ladi.
A) 10;4 B) 8;12 C) 4;8 D) 4;12

26. *a* ning qanday qiymatida $\begin{cases} x + ay = 15 \\ x - 3y = 2a + 3 \end{cases}$ yechimga ega emas.
A) 1 B)2 C)-3 D) 3

27. Tenglamani yeching $\frac{3}{x+2} - \frac{2x-1}{x+1} = \frac{2x+1}{x^2+3x+2}$ A)1 B)2 C)3 D)-3.

28. $y = \sqrt{\frac{(x-4)(2-x)}{x(x+1)}}$ funksiyaning aniqlanish sohasini toping.
A) [2;4] B)(-1;0)U[2;4] C) (-1;0) D) (2;4)

29. Hisoblang $\frac{log_3 12 - 4log_3^2 2 + log_3^2 + 4log_3 2}{3log_3 12 + 6log_3 2}$ A) 3 B)4 C)1 D) 2

30. Sexda 120 ta somovar, 25 ta patnis yasalgan. Sarf qilingan hamma materiallarning 0,96 qismi somovarga ketgan. Agar har bir samovar og‘irligi 3,6 kg bo‘lsa, har bir patnis necha grammdan yasalgan ? A) 960 B)250 C)720 D) 360

31. Birinchi kun ish normasining 2/5 qismi bajarildi. Ikkinchi kun birinchi kunda bajarilgan ishning 1/6 qismicha ko‘p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi? A) 12/15 B)13/15 C) 11/15 D) 10/15

32. $(\alpha x + 2y)(2x + \beta y) = \gamma x^2 + 6xy + 2y^2$ ayniyatdagi noma’lum koeffitsient γ ni toping.
A) 5/3 B)3 C) 4 D) 7/3

33. 3/2 va 24 sonlar orasiga uchta musbat son shunday joylashtirilganki, natijada u sonlar berilgan sonlar bilian birgalikda geometrik progressiyani tashkil qiladi.qo‘yilgan sonlar yig‘indisini toping. A) 17 B)23 C) 18 D) 21

34. Uchburchakli piramidaning uchlari A(3;0;1), B(-1;4;;1), c(5;2;3) va D(0;-5;4) nuqtalarda joylashgan. O-nuqta ACD uchburchak medianalarining kesishgan nuqtasi. AO vektorning uzunligini toping. A) $5\sqrt{2}$ B)$\sqrt{35}/3$ C) $\sqrt{53}/3$ 7/3 D)

35. $y = \sqrt{x}$, $y=0$ va $x=2,25$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.
A) 6,25 B)6,75 C) 2,25 D) 2,75

36. Konusning o‘q kesimi teng tomonli uchburchakdan, silindrniki esa kvadratdan iborat. Agar ularning to‘la sirtlari tengdosh bo‘lsa, xajimlarining nisbatini toping.
A)2:3 B)$\sqrt{2}$:$\sqrt{3}$ C) 2: $\sqrt{3}$ D)1:3

V-2

1. $\left(4\frac{2}{5}\cdot 3\frac{6}{10}-1\frac{2}{5}\cdot 4{,}4\right)\cdot\frac{5}{22}$ ni qiymatini toping. A) 2 B) 2,2 C) 2,4 D) 4

2. Soddalashtirig. $\frac{x^2(x-2)}{3x+3}\cdot\frac{3x^2+9x+6}{x^2-4}$ A) x B) x-2 C) 2 D) x^2

3. Hisoblang $\sqrt{29-12\sqrt{5}}-\sqrt{29+12\sqrt{5}}$ A) 6 B) - 5 C) 5 D)-6

4. 32 dan 62,4 necha foiz ortiq. A) 75% B)85% C)60% D)95%

5. Uch son o‘rta arifmetigi 8,1 ga teng. Birinchi son 7,35 ga teng, agar keyingi har bir son avvalgisidan ayni bir songa farq qilsa keyingisini oldingisidan ayirmasini toping.

A)0,75 B)0,65 C)0,85 D) 0,95

6. Sistemani yeching $\begin{cases}3x+5y=21\\2x-y=1\end{cases}$ A) (2;-3) B)(-2;3) C)(2;3) D) (-4;3)

7. Arifmetik progressiyaning birinchi hadi 1,2 ga va to‘rtinchi hadi esa 1,8 ga teng shu progressiyaning dastlabki oltita hadining yig‘indisini toping.

A)12,2 B)10 C) 12 D)10,2

8. Tenglamani yeching $2x-3+2(x-1)=4(x-1)-7$ A) 1 B)2 C) ∅ D)3

9. Agar x_1 va x_2 $x^2+x-5=0$ tenglamani ildizlari bo‘lsa, $x_1^2+x_2^2$ ni toping.

A) 12 B)11 C)13 D)10

10. Tenglamani yeching $\frac{x}{2}+\frac{x}{6}+\frac{x}{12}+\frac{x}{20}+\frac{x}{30}+\frac{x}{42}=-6$

A)7 B)-7 C) 8 D) 6

11. Tenglamani yeching $|x+4|=2x$ A) 4 B) 5 C) 3 D) 2

12. A(2;-3;5) nuqta to‘g‘ri chiziqning boshini va C(-1;2;-7) nuqta shu to‘g‘ri chiziqni o‘rtasining koordinatalari bo‘lsa, to‘g‘ri chiziqning oxiri bo‘lgan B nuqtaning koordinatalarini toping. A)(4;-7;19) B)(7;4;-19) C)(-4;7;-19) D)(19;4;7)

13. $y=\frac{2+x}{x-1}$ funksiyaning aniqlanish sohasini toping.

A) (-∞;1) B) (1; ∞) C) (-∞;1)U(1; ∞) D)[1;∞)

14. $y=\sqrt{x^2-2x+3}$ funksiyaning qiytmatlar sohasini toping.

A) (-3;2) B) $[\sqrt{2};\ \infty)$ *C) [2;3] D)[-3;2)*

15. b va c ning qanday qiymatida M(-6; -23) nuqta $y=x^2+bx+c$ parabolaning uchi bo`ladi. A) $b=-12,\ c=-13$ B) $b=12,\ c=13$ C) $b=12,\ c=-13$ D) $b=-12,\ c=12$

16. Tengsizlikni yeching: $9^{\log_9(x-4)}>3$ A) $x>7$ B) $x<7$ C) $x>3$ D) $x<3$

17. Agar $a=\log_{1/5}4;\ \ b=\log_{1/5}6\ \ c=\log_{1/6}4$ bo`lsa, $a,b,c$ sonlari uchun quyidagi munosabatlardan qaysi biri o`rinli? A) $a<b<c$ B) $b<a<c$ C) $c<a<b$ D) $b<c<a$

18. $\sin75^0\cdot\sin15^0$ ni hisoblang A) 0,45 B) 0,75 C) 0,50 D)0,25

19. Tnglamani yeching 2cos3x-1=0

A) ±π/6+2πk/3 B)±π/9+2πk/3 C) ±π/3+2πk/3 D) π/9+2πk/3

20. Ikki parallel to`g`ri chiziqni 3- to`g`ri chiziq kesib o`tganda hosil bo`lgan ichki bir tomonli burchaklardan biri ikkinchisidan 100^0 kichik bo`lsa, shu burchaklardan kattasini toping. A)145 B)150 C)140 D)155

21. To`g`ri burchakli uchburchakning balandligi gipotenuzani 3 va 12 ga teng kesmalarga ajratadi. H=? A) 8 B)6 C)10 D)12

22. Agar $|\vec{a}| = 6$, $|\vec{a}+\vec{b}| = 11$ *va* $|\vec{a}-\vec{b}| = \sqrt{79}$ bo`lsa, $|\vec{b}|$ ni toping.

A) 4 B)6 C) 8 D)10

23. Ikki shahar orasidagi masofa 400 km yuo'lsa, 1:5000000 masshtabli xaritada bu masofa necha *mm* ga teng bo'ladi? A) 60 B)50 C)70 D) 80

24. Hisoblang $4{,}2\cdot13{,}5-8{,}7\cdot4{,}2-5{,}8\cdot8{,}7+13{,}5\cdot5{,}8$

A) 46 B)48 C)50 D) 58

25. *a* ning qanday qiymatlarida $x^2+ax+\frac{1}{4}=0$ tenglama bitta ildizga ega bo'ladi.

A) -2;2 B)-1;1 C) 2;3 D) 2;6

26. *a* ning qanday qiymatida $\begin{cases}-4x+ay=1+a\\(6+a)x+2y=a+3\end{cases}$ sistema yechimga ega emas.

A)-4 B)4 C) 6 D)-6

27. Mis, qo'rg'oshindan iborat qotishmaning 60% i mis. Mis qo'rg'oshindan 6 kg ko'p bo'lsa mis necha kilogramm ? A)16 B)18 C)14 D)12

28. Tekislikka og'ma va perpendikulyar tushirilgan. Og'maning tekislikdagi proek-siyasi 24 ga teng. Og'ma va tekislik orasidagi burchak *arccos*0,96 bo'lsa, perpendikul-yarning uzunligini toping. A)8 B)6 C)7 D)9

29. Hisoblang $\dfrac{log_2^2 14+log_2 14\cdot log_2 7-2log_2^2 7}{log_2 14+2log_2 7}$ A) 2 B)7 C)1 D)14

30. Birinchi kun ish normasining 2/5 qismi bajarildi. Ikkinchi kun birinchi kunda bajarilgan ishning 1/8 qismicha ko'p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi ? A)16/21 B)19/20 C)13/20 D)17/20

31. $(\alpha x+3y)(2x+\beta y)=\gamma x^2+8xy+6y^2$ ayniyatdagi nomaъlum koeffitsiyent γ ni toping

32. Matematikadan o'tkazilgan imtihonda o'quvchilarning 10% i birorta ham masalani yecha olmadi. 325 ta o'quvchi masalalarni yechishda xatolikka yo'l qo'ydi. Agar barcha masalalarni to'liq yechgan o'quvchilarning masalalarni umuman yecha olmagan o'quv-chilarga nisbati 5:2 kabi bo'lsa, qancha o'quvchi imtihon topshirgan.

A)520 B)325 C)500 D) 425

33. Geometrik progressiya $b_1+b_5=51$ va $b_2+b_6=102$ bo'lsa S_7 ni toping.

A)381 B)380 C)370 D)260

34. A(1;-2;2) , B(1;4;0), C(-4;1;1) va D(-5;-5;3) nuqtalar berilgan. AC va BD vektor-lar orasidagi burchakni toping. A) 45^0 B) 90^0 C) 30^0 D) 60^0

35. $y=x^2$, y=0 va x=-2 chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

A) -8/3 B) 5/3 C) 8/3 D) -5/3

36. Teng tomonli silindr va konusning balandliklari o'zaro teng. Ularning to'la sirtla-ri nisbatini toping. A) 3:8 B)6:5 C)3:4 D)3:2

V-3

1. Hisoblang $\dfrac{0{,}26}{0{,}00026}+\dfrac{0{,}24}{0{,}0015}-\dfrac{0{,}7}{0{,}0014}$ A)600 B)680 C) 660 D)650

2. Soddalashtiring. $a(b-c)-b(c-a)-c(a-b)$
A) *ab-ac* B) *ab+ac* C) *0* D) *2ab-2ac*

3. $\frac{\sqrt{(b+2)^2-8b}}{\sqrt{b}-\frac{2}{\sqrt{b}}}$ ni $b = 0{,}0025$ da hisoblang.
A) 0,05 B)-0,05 C) 0,5 D)-0,5.

4. Uch sonning o'rta arifmetigi 13,9 ga teng. Ikkitasi 20,2 va 21,7. Uchinchisini toping. A)-0,2 B)0,2 C) 0,4 D)0,-4

5. 40 dan 29,2 necha foiz kam ? A) 27% B)37% C) 40% D) 45%

6. Sistemani yeching $\begin{cases} \frac{1}{2}x-\frac{1}{3}y=1 \\ 3x-5y=-3 \end{cases}$ A) (4;3) B)(4;-3) C) (4;4) D) (3;3)

7. 7,5+9,8+12,1+…+53,5 yig'indini hisoblang.
A) 630,5 B)650,5 C) 660,5 D)640,5.

8. Tenglamani yeching $\frac{17}{5x}=2-\frac{7}{x}$ A)-5,2 B)5,2 C) 6,2 D) -6,2

9. Agar x_1 va x_2 $x^2-2x-9=0$ tenglamani ildizlari bo'lsa, $x_1^3+x_2^3$ ni toping.
A)62 B)64 C) 66 D) 68

10. Tenglamani yeching $\frac{3}{2-\frac{3}{2}}=\frac{21}{8}$ A) 6 B)4 C)8 D) 2

11. Tenglamani yeching $|x+5|=-3$ A) 3 B) 5 C) ø D) 2

12. *A(3;-2;5) va B(-1;3;2)* nuqtaoar orasidagi masofani toping.
A)2 B)2√5 C) √5 D) 5√2

13. Tekislikka og'ma va perpendikulyar tushirilgan. Og'maning tekislikdagi proeksiyasi 14 ga teng. Og'ma va tekislik orasidagi burchak *arccos0,28* bo'lsa, perpendikulyarning uzun-ligini toping. A)58 B)68 C)48 D)49

14. Agar $k>0,\ b<0$ bo'lsa, $y=kx+b$ to'g'ri chiziq qaysi choraklardan o'tadi.
A) III,IV B) I,II,III C) I,III,IV D) II,IV

15. *a* ning qanday qiymatlarida *6x+3y=7* va *2ax+3y=3* to'g'ri chiziqlar kesishish nuqtasining abssassasi manfiy bo'ladi. A) *a*<2 B) *a*>3 C) *a*>1 D) *a*>2

16. Tengsizlikni yeching $5^{\log_5(x-7)}>4$ A) *x*>11 B) *x*<11 C) *x*>7 D) *x*<7

17. Agar $a=\log_{\frac{1}{5}}4$; $b=\log_{\frac{1}{5}}6$; $c=\log_{\frac{1}{6}}4$ bo`lsa, *a ,b, c* sonlari uchun quyidagi munosabatlardan qaysi biri o`rinli? A) *a<c<b* B) *c<b<a* C) *b<c<a* D) *b<a<c*

18. $2\sin\frac{\pi}{6}+tg\frac{\pi}{4}$ ni hisoblang A)2 B)3 C) 1 D) 4

19. Tenglamani yeching $2\cos 4x-\sqrt{3}=0$
A) π/24+πk/2 B)±π/24+πk/2 C) π/12+πk/2 D) π/6+πk/2

20. Rombning tomoni 10 ga, unga ichga aylananing radiusi 3 ga teng. Rombning o`tkir burhagi kosinusini toping. A) 3/5 B)2/5 C)4/5 D)1/5 $\frac{4}{5}$

21. Uchburchak burchklari o`zaro 1:1:2 nisbatda. uchburchak katta tomoni 12 ga tang. Katta tomonga tushirilgan balandlikni toping. A) 2 B)4 C)6 D)8

22. Agar $|\vec{a}| = \sqrt{85}$, $|\vec{a} + \vec{b}| = 20$ va $|\vec{a} - \vec{b}| = 9\sqrt{2}$ bo`lsa, $|\vec{b}|$ ni toping.

A)14 B)12 C) 13 D) 11

23. Turist butun yo'lning 0,85 qismini bosib o'tganda, ko'zlangan manzilga 9,66 km qolganligi ma'lum bo'ldi. Butun yo'lning uzunligini toping.

A)64 B)62,4 C) 64,4 D)60,4

24. Hisoblang $\dfrac{0{,}2^2 + 2\cdot 0{,}2\cdot 0{,}3 + 0{,}3^2}{0{,}5\cdot 0{,}4 - 0{,}5\cdot 0{,}8}$ A) 2,5 B) 1,25 C) 1,75 D) -1,25

25. a ning qanday qiymatlarida ax^2-6x+9=0 tenglama bitta ildizga ega bo'ladi.

A) -1 B)1 C)2 D)-2

26. a ning qanday qiymatida $\begin{cases} 16x + ay = 4 \\ ax + 9y - 3 = 0 \end{cases}$ sistema yechimga ega emas.

A)12 B)-12 C)13 D)-13

27. $\dfrac{6}{x^2-1} - \dfrac{2}{x-1} = 2 - \dfrac{x+4}{x+1}$ tenglamani yeching. A) 3 B)5 C) 7 D)2

28. $y = \sqrt{\dfrac{(x-3)(x-1)}{x(4-x)}}$ funksiyaning aniqlanish sohasini toping.

29. Hisoblang $\dfrac{2\log_3^2 - \log_3^2 18 - \log_3 2\cdot \log_3 18}{2\log_3 2 + \log_3 18}$ A) 2 B)4 C)-4 D)-2

30. Massasi 400 g , konsentrasiyasi 24% bo'lgan, massasi 600 g konsentrasiyasi 16% bo'lgan eritma aralashtirildi. Aralashma foizini toping.

A)19,2 B)20 C)19 D)18,2

31. Velosipedchi bir soatda butun yo'lning 0,65 qismini bosib o'tdi, bu esa yo'lning yarimidan 9,75 km ko'p. Butun yo'lning uzunligiri toping.

A) 60 B)55 C) 65 D) 70

32. Matematikadan o'tkazilgan imtihonda o'quvchilarning 12% i birorta ham masalani yecha olmadi. 210 ta o'quvchi masalalarni yechishda xatolikka yo'l qo'ydi. Agar barcha masalalarni to'liq yechgan o'quvchilarning masalalarni umuman yecha olmagan o'quv-chilarga nisbati 7:3 kabi bo'lsa, qancha o'quvchi imtihon topshirgan.

A) 360 B)350 C) 340 D) 330

33. Geometrik progressiyaning birinchi hadi 486 ga, maxraji 1/3 ,bo'lsa uning dastlabki to'rtta hadining yig'indisini toping.

A)720 B)660 C) 740 D) 710

34. $x^2 + y^2 - 4x - 6y - 3=0$ tenglama bilan berilgan aylananing radiusini toping.

A) 6 B)3 C)4 D) 5

35. Sferaga balandligi asosining diametriga teng bo‘lgan konus ichki chizilgan. Agar sfera sirtining yuzi 125 ga teng bo‘lsa, konus asosining yuzini toping.

A) 15π B)10π C)10 D)20

36. $y=x^3$, $y=0$ va $x=2\sqrt{2}$ chiziqlar bilan chegaralangan figuraning yuzini taping.

A)8 B)16 C)4 D)9

V-4

1. Aylanaga tashqi chizilgan oltiburchakning beshta tomoni mos ravishda 5,6,7,8,9 ga teng bo‘lsa, uning oltinchi tomonini toping. A) 6 B) 5 C) 7 D) 8

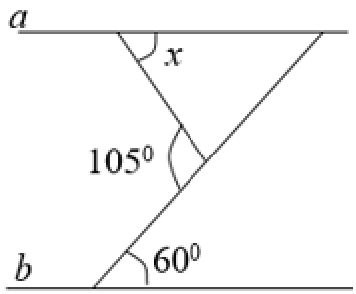
2. $f(x) = cosx + x$ funksiya uchun $f'(x) = 0$ bo‘lsa, x ning qiymatini toping.

$A)\ x = -\frac{\pi}{2} + 2\pi n, n\epsilon Z$ $B)\ x = -\frac{\pi}{2} + \pi n, n\epsilon Z$ $C)\ x = \pi + \pi n, n\epsilon Z$ $D)\ x = \frac{\pi}{2} + 2\pi n, n\epsilon Z$

3. $(1 + 3x + x^2)(3 - 3x - x^2) \leq -5$ tengsizlikni eching.

$A)\ [-0; -1]$ $B)\ [-4; -2] \cup [1; \infty)$ $C)\ [-2; -1] \cup [1; \infty)$ $D)\ (-\infty; -4] \cup [-2; -1] \cup [1; \infty)$

4. Agar $a \parallel b$ bo‘lsa, $\angle x$ ni toping.



$A)\ 60^0$ $B)\ 30^0$ $C)\ 45^0$ $D)\ 35^0$

5. Uchlari C (4;3), B (3;-1), A (2;4) nuqtalarda bo‘lgan uchburchakning medianalari kesishgan nuqta O bo‘lsa, $\overrightarrow{OA}$ bektorning uzunligini toping.

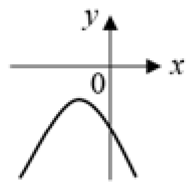
$A)\ 3$ $B)\ \sqrt{5}$ $C)\ 5$ $D)\ 2\sqrt{5}$

6. $\begin{cases} log_3\left(log_{\frac{1}{9}}\frac{x}{y}\right) = 0 \\ log_{\sqrt{x}}xy = 8 \end{cases}$ tenglamalar sistemasini yeching.

7. ABC uchburchakning A uchidan BC tomoniga AD mediana tushurilgan, bunda $AD=BD=AC$ bo‘lsa, BC tomonning AB tomoniga bo‘lgan nisbatini toping.

$A)\ \frac{2\sqrt{3}}{3}$ $B)\ \sqrt{3}$ $C)\ \frac{1}{2}$ $D)\ \frac{3\sqrt{3}}{2}$

8. $y = ax^2 + bx + c$ $(D = b^2 - 4ac)$ funksiyaning berilgan grafigidagi to‘g‘ri tasdiqni ko‘rsating.



$A)\ bc < 0$ $B)\ bD < 0$
$C)\ aD < 0$ $D)\ ab > 0$

9. $\left(sinx - \frac{1}{2}\right)(sinx + 1) = 0$ tenglamani yeching.

$A)\ (-1)^n\frac{\pi}{6} + \pi n;\ -\frac{\pi}{2} + 2\pi n.$ $B)\ (-1)^n\frac{\pi}{6} + \pi n.$
$C)\ \frac{\pi}{2} + \pi n.$ $D)\ -\frac{\pi}{2} + 2\pi n$

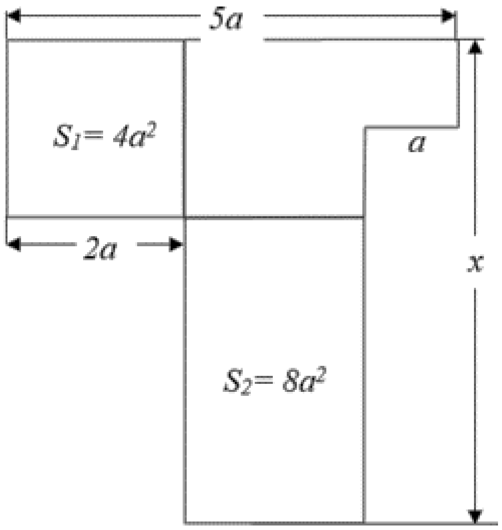
10. $y = 5 + e^{4x} + 4x^3$ funksiyaning hosilasini toping.

$A)\ 4e^{4x} + 12x^2$ $B)\ e^{2x} - x$ $C)\ \frac{1}{2}e^{4x} + x$ $D)\ e^{4x} + 7x$

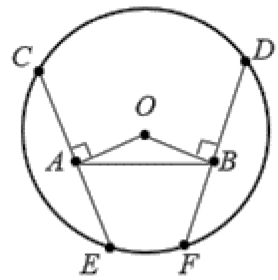
11. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro perpendikulyar birlik vektorlar bo‘lsa, $|4\vec{a} - 3\vec{b}|$ ni toping.

A) 5 B) $\frac{7}{\sqrt{2}}$ C) $5\sqrt{2}$ D) 4

12. Rasmda berilganiga ko‘ra x ni toping. (bunda a =1).



A) 6 B) 4 C) 8 D) 10



13. $CE=DF$, $AO\perp CE$, $\angle AOB=100^0$ bo‘lsa, $\angle OAB$ ni toping.

A) 40^0 B) 30^0 C) 50^0 D) 60^0

14. $3\sqrt{x^2+1} = x^2+3$ tenglama ildizlarining o‘rta arifmetigini toping.

$A) -\frac{1}{2}$ $B)\ 3$ $C)\ 0$ $D) -1$

15. Arifmetik progressiyada [illegible] = 6 ga teng, [illegible] − 2 ni toping

16. $\int_0^1 (1+x^2)\,dx$ ni hisoblang. $A)\ 1\frac{1}{3}$ $B)\ \frac{1}{3}$ $C)\ 1$ $D)\ \frac{2}{3}$

17. $7^{lgx} - 3\cdot 5^{lgx-1} = 5^{lgx+1} - 13\cdot 7^{lgx-1}$ tenglamani yeching.

A) 400 B) 80 C) 1000 D) 100

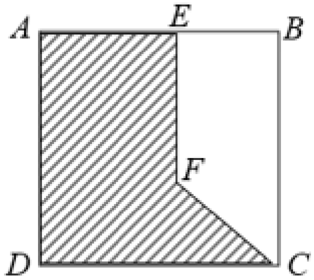
18. $9^{\frac{1}{2}+log_9 sinx} = 27^{\frac{1}{6}+log_3\sqrt[3]{cosx}} + \sqrt{6}$ tenglamaninig yechimini toping.

$A)\ \frac{2\pi}{12} + 2\pi n.$ $B)\ \frac{5\pi}{12} + 2\pi n.$ $C)\ \frac{5\pi}{12} + \pi n.$ $D)\ \frac{\pi}{4} + 2\pi n.$

19. Ifodani so‘ldalashtiring.

$$\sqrt[3]{\frac{1+x}{x^2-2x+1}} - \sqrt[3]{\frac{x-1}{x^2+2x+1}} - \sqrt[3]{\frac{1}{x^4-2x^2+1}} - (x^2-1)^{-\frac{2}{3}}$$

$A)\ x^{-1}$ $B)\ 1$ $C)\ 0$ $D)\ \frac{1}{x^2-1}$



20. Tomoni 10 ga teng bo‘lgan kvadratda rasmdagi kabi ko‘pburchak chizilgan. Shtrixlangan ko‘pburchakning yuzi 68 sm^2 ga teng bo‘lsa, EF kesmaning uzunligini toping. Bunda $\angle BCF = 45^0$ ga teng va $FE//BC$. A) 6 *sm* B) 8 *sm* B) 3 *sm* D) 5 *sm*.

21. $\frac{1+\frac{1+\frac{1\ldots}{5}}{5}}{5}+1=x$ bo‘lsa, x ni toping. $A)\ \frac{11}{6}$ $B)\ \frac{7}{5}$ $C)\ \frac{4}{5}$ $D)\ \frac{5}{4}$ |

22. $\frac{x-7}{\sqrt{12+4x^2-19x}}<0$ tengsizlikni yeching.

$A)\ x>\frac{3}{4}$ $B)\ x<\frac{3}{4};4<x<7$ $C)\ -4<x<7$ $D)\ -7<x<-4$

23. $x^2+x-2=\frac{x^2+x-2}{x^2-1}$ tenglama ildizlarining ko‘paytmasini toping.

A) 4 B) -2 C) 6 D) 1.

24. $|8^{14}-7^{22}|$ ayirmaning oxirgi raqamini toping. A) 2 B) 3 C) 5 D) 1.

25. Agar konus o‘q kesimining yuzi N ga, asosining yuzi M gat eng bo‘lsa, konus yon sirtining yuzini toping. $A)\ \sqrt{MN}$ $B)\ \sqrt{\pi MN}$ $C)\ 2\sqrt{MN}$ $D)\ \sqrt{M^2+N^2}$

26. $3,1,\frac{1}{3}\cdots$ geometrik progressiyaning oltinchi va yettinchi hadini toping.

$A)\ 1{,}3$ $B)\ \frac{1}{27},\frac{1}{81}$ $C)\ \frac{1}{81},\frac{1}{243}$ $D)\ \frac{1}{9},\frac{1}{27}$

27. $sin^4\alpha+2sin^2\alpha\cdot cos^2\alpha+cos^4\alpha$ ni so‘ldalashtiring.

A) $sin\alpha$ B) -1 C) $1+cos^2\alpha$ D) 1

28. $\left(\frac{2}{3}\right)^{-1}-\frac{3}{4}+\left(6-4\cdot\left(\frac{5}{16}\right)^0\right)^{-2}$ ni hisoblang.

29. Ratsional ifodani kanonik ko‘rinishga keltiring.

$$\frac{1-\frac{1-\frac{1-x}{1+2x}}{1+2\frac{1-x}{1+2x}}}{1+2\frac{1-\frac{1-x}{1+2x}}{1+2\frac{1-x}{1+2x}}}$$

$A)\ \frac{1-x}{1+2x}$ $B)\ \frac{1+2x}{1-x}$ $C)\ \frac{1+x}{1+2x}$ $D)\ \frac{1}{2}$

30. 13478+1347+134+13 yig‘indi quyidagi qaysi songa qoldiqsiz bo‘linadi.

A) 5 B) 9 C) 3 D) 4

V-5

1. $\frac{(a-b)^2+ab}{(a+b)^2-ab}:\frac{a^5+b^5+a^2b^3+a^3b^2}{a^3+b^3+ab^2+a^2b}\cdot(a^3-b^3)$ ni so‘lalashtiring.

$A)\ ab$ $B)\ \frac{1}{a-b}$ $C)\ a+b$ $D)\ a-b$

2. r radiusli aylanaga tashqi chizilgan teng yonli trapetsiyaning o‘tkir burchagi α bo‘lsa, uning yuzini toping.

$A)\ \frac{4r^2}{sin\alpha}$ $B)\ 2r^2sin\alpha$ $C)\ \frac{2r^2}{cos\alpha}$ $D)\ r^2tg\alpha$

3. $|x+4|+|x-3|+|x-2|=7$ tenglamaning ildizlari yig‘indisini toping.

A) -2 B) 0 C) ildizi yo‘q D) 2

4. Usta ishning 0,75 qismini $\frac{17}{5}$ soatda bajaradi. U shu ishni $\frac{4}{5}$ qismini qancha soatda bajaradi. $A)\ 3\frac{47}{75}$ $B)\ 4\frac{7}{15}$ $C)\ 4\frac{8}{15}$ $D)\ 2\frac{1}{5}$

5. Romning diaganallari 6 va 4 ga teng bo‘lsa, uning yuzini toping.

A) 24 B) 10 C) 12 D) 16

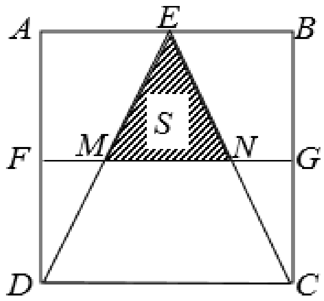
6. Arifmetik progressiyada $a_1 + a_3 + a_5 + \cdots + a_9 = 50$ va $a_2 + a_4 + a_6 + \cdots + a_{10} = 75$ bo‘lsa, uning ayirmasini toping.

A) 6 B) 5 C) 3 D) 15

7. $arccos(x+1) = \frac{2p}{3}$ tenglamani yeching. $A) -\frac{3}{2}$ $B) -2$ $C) -\frac{1}{2}$ $D) -1$

8. Teng yonli trapetsiyaning diagonallari o‘zaro perpendikulyar hamda yuzi 32 ga teng bo‘lsa, uning diagonali uzunligini toping. $A)\ 8$ $B)\ 6$ $C)\ 4$ $D)\ 4\sqrt{2}$

9. Rasmda $AE = EB = BG = GC = \frac{a}{2}$ bo‘lib, $a = 8$ bo‘lsa, shtrixlangan yuza (S) ni toping.



$A)\ 2$ $B)\ \frac{1}{8}$ $C)\ 1$ $D)\ 8$

10. Agar $y = e^{cosx} + 2^{sinx}$ bo‘lsa, y'(p) ni hisoblang.

$A) -ln3$ $B)\ ln2$ $C)\ ln3$ $D) -ln2$.

11. Aylana tashqarisidagi nuqtadan aylanaga ikkita urinma o‘tkazilgan. Agar urunmalar orasidagi burchak 84^0 bo‘lsa, aylananing urinish nuqtalari orasidagi katta yoyni toping.

A) 240^0 B) 248^0 C) 252^0 D) 264^0

12. $7^{lgx} - 3 \cdot 5^{lgx-1} = 5^{lgx+1} - 13 \cdot 7^{lgx-1}$ tenglamani yeching.

A) 100 B) 80 C) 1000 D) 400

13. Quyidagi funksiyalarning qaysi biri davriy.

$A)\ y = \{x\}$ $B)\ y = cosx^2$ $C)\ y = x^2$ $D)\ y = cos\sqrt{x}$

14. Agar $log_2 10 = b,\ log_7 2 = a$ bo‘lsa, $log_4 78{,}4$ ni a va b orqali ifodalang.

$A)\ 2 - \frac{1}{a} + \frac{b}{2}$ $B)\ 2 - \frac{1}{a} - \frac{b}{2}$ $C)\ 2 + \frac{1}{a} - \frac{b}{2}$ $D)\ 2 + \frac{1}{a} + \frac{b}{2}$

15. $f(x) = sin\left(\frac{x}{4} + 5\right)$ funksiyaning boshlang‘ichini toping.

$A)\ F(x) = 4cos\left(\frac{x}{4} + 5\right) + C$ $B)\ F(x) = -4cos\left(\frac{x}{4} + 5\right) + C$

$C)\ F(x) = 5cos\left(\frac{x}{4} + 5\right) + C$ $D)\ F(x) = \frac{1}{4}cos\left(\frac{x}{4} + 5\right) + C$

16. $1 - sin3x = cos^2 2x - cos\left(\frac{p}{2} + x\right)$ tenglamani yeching.

$A)\ arcsin\frac{1}{\sqrt{3}} + 2pk$ $B)\ \pm\frac{p}{6} + 2pk$ $C) 2pk$ $D)\ \frac{pk}{2}$

17. Ikkita qo‘shni yoqlarining markazlari orasidagi masofa $\sqrt{18}$ ga teng bo‘lgan kubga tashqi chizilgan shar sirtining yuzini toping.

A) 108p B) 125p C) 144p D) 120p

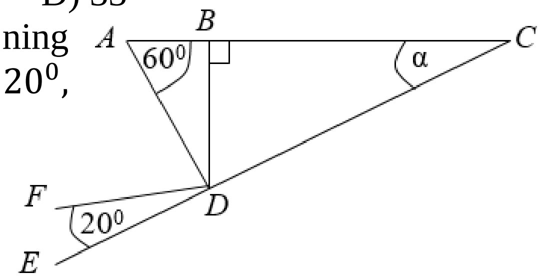
18. $f(x) = \frac{2}{x}$ funksitaning M(e;4) nuqtadan o‘tuvchi boshlang‘ich funksiyasini toping.

$A)\ lnx + 5 \quad B)\ lnx^2 \quad C)\ lnx + 3 \quad D)\ 2lnx + 2$

19. 144 va 128 sonlarining umumiy bo‘luvchilari yig‘indisini toping.

A) 35 B) 32 C) 31 D) 33

20. Rasmda berilgan ma’lumotlarga ko‘ra, $б = \angle C$ ning qiymatini toping. Bu yerda $\angle A = 60^0, \angle FDE = 20^0$, $DB \perp AC$ va $\angle ADF = \angle BDC$.



A) 15^0 B) 20^0 C) 25^0 D) 30^0

21. $\vec{b}(8;-4)$ va $\vec{a}(-5;0)$ vektorlar uchun $|\vec{a} + \vec{b}|$ ni hisoblang.

A) 5 B) 6 C) 10 D) 4

22. Nechta btun a va b sonlar jufti $a^2 - b^2 = -17$ tenglikni qanoatlantiradi.

A) 1 B) 4 C) ∅ D) 2

23. Tengsizlikni yeching. $\frac{5}{x-1} + \frac{3}{2x+7} > \frac{7}{3x-3}$

$A) \left(-\frac{7}{2};1\right)\ B) \left(-\frac{47}{25};1\right)\ C) \left(-\frac{7}{2};-\frac{47}{25}\right)\ D) \left(-\frac{7}{2};-\frac{47}{25}\right) \cup (1;\infty)$.

24. $ax^2 + bx + c = 0$ tenglamaning ildizlari x_1 va x_2 bo‘lsa, ildizlari $\frac{1}{x_1}$ va $\frac{1}{x_2}$ bo‘lgan kvadrat tenglama tuzing.

$A)\ cx^2 - bx + a \quad B) - cx^2 + bx - a \quad C)\ cx^2 - bx - a \quad D)\ cx^2 + bx + a$

25. $y = 3sinx + 4cosx$ funksiyaning qiymatlar to‘plamini toping.

A) [-4;4] B) [-5;5] C) [-1;1] D) [-7;7]

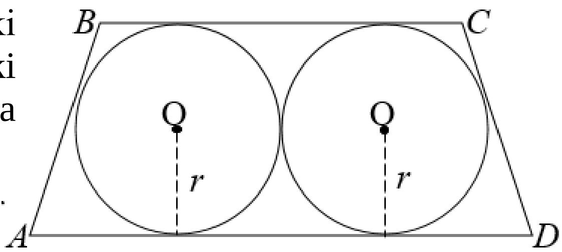
26. $\frac{\sqrt{24}}{\sqrt{5}+\sqrt{2}-\sqrt{3}}$ kasrni irratsionallikdan qutqaring.

$A)\ \sqrt{5} + \sqrt{2} - \sqrt{3} \quad B)\ 2 \quad C)\ \sqrt{5} - \sqrt{2} + \sqrt{3} \quad D)\ \sqrt{5} + \sqrt{2} + \sqrt{3}$

27. $0{,}3 \cdot 10^{-4} + 0{,}2 \cdot 10^{-5} + 0{,}001 \cdot 10^{-2}$ ni hisoblang.

A) $42\cdot 10^{-5}$ B) $42\cdot 10^{-6}$ C) $0{,}42\cdot 10^{-5}$ D) $4{,}2\cdot 10^{-6}$

28. Radiuslari bir xil 3 *sm* gat eng bo‘lgan ikki aylana rasmdagidek *ABCD* trapetsiyaga ichki chizilgan. Agar uning *AB* va *BC* asoslari moa ravishda 15 *sm* va 10 *sm* gat eng. rapetsiyaning yon tomonlari *AB* va *CD* lar uzunliklari yig‘indisini toping.



A) 19 *sm* B) 12 *sm* C) 13 *sm* D) 25 *sm*

29. $\frac{\frac{1}{\frac{1}{10}-\frac{1}{12}}}{\frac{1}{\frac{1}{8}-\frac{1}{6}} + \frac{1}{\frac{1}{5}-\frac{1}{6}}}$ ni hisoblang. $A)\ 1 \quad B)\ \frac{1}{2} \quad C)\ 12 \quad D)\ 10$

30. $\sqrt{5-8x+2x^2}=x-2$ tenglamani yeching.
$A)\ 4-\sqrt{3}$ $B)\ \sqrt{14}$ $C)\ 2+\sqrt{3}$ $D)\ 2-\sqrt{3}$

V-6

1. Teng yonli uchburchakning perimetri 52 *sm*. Agar uning asosi yon tomonining 0,6 qismini tashkil qilsa, o‘rta perpendikulyarlari kesishgan nuqtasidan uchlarigacha bo‘l-gan masofani toping. A) $100\sqrt{91}$ B) $\frac{6\sqrt{91}}{13}$ C) $\frac{13\sqrt{91}}{6}$ D) $\frac{100\sqrt{91}}{91}$

2. $arcsin(sin(cosx))$ ni so‘ldalashtiring. $A)\ x$ $B)\ 1$ $C)\ cosx$ $D)\ sinx$

3. Sharga konus ichki chizilgan. Konusning yasovchisi asosining diametriga teng. Shar hajmining konus hajmiga nisbatini toping. A) 8:3 B) 32:9 C) 16:9 D) 27:4

4. Muntazam parallelepipedning balandligi asosining tomonidan $\sqrt{6}$ marta katta. Parallelepipedning diagonali asos tekisligi bilan qanday burchak tashkil qiladi.
$A)\ 30^0$ $B)\ arccos\frac{1}{2\sqrt{3}}$ $C)\ arccos\frac{1}{\sqrt{6}}$ $D)\ 60^0$

5. Kamayuvchi geometrik progressiyani tashkil etuvchi uchta sondan uchinchisi 24 ga teng. Agar uchunchi son o‘rniga 18 soni olinsa, shu uchta son arifmetik progressiyani tashkil etadi. Arifmetik progressiyaning birinchi hadini toping.
A) 63 B) 54 C) 50 D) 36

6. $\begin{cases} sin^2x+cos^2y=1 \\ x-y=\frac{4p}{3} \end{cases}$ sistemani yeching.

$A)\ \left(\frac{7p}{6}+pk;\ -\frac{p}{6}+pk\right),$ $B)\ \left(\frac{7p}{6}+\frac{pk}{6};\ -\frac{p}{6}+pk\right),$
$C)\ \left(\frac{7p}{6}+2pk;\ -\frac{p}{6}+2pk\right),$ $D)\ \left(\frac{7p}{6}+\frac{pk}{2};\ -\frac{p}{6}+\frac{pk}{2}\right)$

7. $\frac{x^{33}-1}{x^{11}+x^{22}+x^{33}}$ kasirni qisqartiring.
$A)\ \frac{x^{11}-1}{x^{11}}$ $B)\ x^{11}+1$ $C)\ x^{11}+1$ $D)\ 1+\frac{1}{x^{11}}$

8. $y=\frac{8}{3}\sqrt{x},\ y=-x^3$ va $y=8$ chiziqlar bilan chegaralangan yuzani aniqlang.
$A)\ 44$ $B)\ 32$ $C)\ 36$ $D)\ 48$

9. $\int_0^{\frac{p}{4}} cos2xsin2xdx$ hisoblang. A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$

10. a ning $\sqrt{24}x+ax^2+a+2>0$ tengsizlik yechimga ega bo‘lmaydigan butun qiy-matlari orasidan eng kattasini toping. A) -5 B) -3 C) -4 D) -2

11.(-3;4) nuqtaning abssissa, ordinata o‘qlariga va koordinata boshiga simmetrik bo‘lgan nuqtalarni tutashtirishdan hosil bo‘lgan uchburchakning eng katta tomonini toping.
A) 10 B) 12 C) 14 D) 24

12. $x^2\cdot 5^x+2x\cdot 5^x=x^2\cdot 5^{\sqrt{x+6}}+2x\cdot 5^{\sqrt{x+6}}$tenglama nechta ildizga ega.
A) 2 ta B) 3 ta C) 4 ta D) 1 ta

13. $y=\sqrt{x^3}+ln3x+3^x$ funksiyaning $x=1$ nuqtadagi hosilasini toping.
$A)\ 3+ln3$ $B)\ 8$ $C)\ 3ln6+4$ $D)\ 2{,}5+ln27$

14. $7x-2x^2+x^3-1$ ko‘phadni x^2-x+1 ko‘phadga bo‘lgandagi qoldiqni toping.

A) x-2 B) -5x C) 2-x D) 5x

15. $2014 \cdot 2011 - 2012 \cdot 2013$ ni hisoblang.

A) 2 B) -2 C) -4 D) 2010

16. $log_x 9 log_9(12-x) = \frac{1+2log_9 2}{log_9 x} - 1$ tenglamani yeching.

A) 7 B) 12 C) 6 D) 9

17. $\overline{1234ab}$ (olti xonali) son 18 va 24 ga qoldiqsiz bo‘linsa, a va b lar ayirmasining mo‘lulini toping. A) 8 B) 9 C) 2 D) 4

18. $x-4 < \sqrt{x^2-6x}$ tengsizlikni yeching.

$A)$ $(-\infty;\infty)$ $B)$ $(8;\infty)$ $C)$ $(-\infty;0] \cup (8;\infty)$ $D)$ $(-\infty;0] \cup (6;\infty)$

19. $a = \frac{15}{32}$, $b = \frac{21}{24}$ va $c = \frac{33}{38}$ sonlarini o‘sish tartibida joylashtiring.

$A)$ $c < b < a$ $B)$ $b < a < c$ $C)$ $a < c < b$ $D)$ $b < c < a$

20. $\frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} + \cdots + \frac{1}{\sqrt{2011}+\sqrt{2014}}$ ni hisoblang.

A) $\frac{\sqrt{2014}-1}{3}$ B) 3 C) $\frac{1}{3}$ D) $\frac{1-\sqrt{2014}}{3}$

21. x va y sonlar ayirmasining uchlanganligini yozing va uni $x=-0{,}37, y=-0{,}42$ bo‘lgandagi qiymatini toping.

A) -0,79 B) 0,15 C) 0,12 D) -0,15

22. $\overline{abc} + \overline{cba} = 1453$ tenglamada $\overline{abc}$ va $\overline{cba}$ uch xonali sonlar bo‘lsa, $(a+c) \cdot b$ ning qiymatini toping. A) 80 B) 64 C) 91 D) 45

23. To‘g‘ri burchakli uchburchakning yuzasi 6 ga teng, katetlari esa 3:4 nisbatda bo‘l-sa, uning tomonlarini toping.

A) 3;4;6 B) 3;4;5 C) 4;5;6 D) 5;6;8

24. Teng yonli trapetsiyaning asosidagi burchakning sinusi 0,6 ga, asoslarining ayirma-si 4 ga teng bo‘lsa, uning yo tomonini toping.

A) 2 B) 4 C) 3 D) 2,5

25. $y = \sqrt{x^2-|x|-2}$funksiyaning aniqlanish sohasini toping.

$A)$ $(-\infty;-1] \cup [-1;\infty)$ $B)$ $(-\infty;-2] \cup [1;\infty)$

$C)$ $(-\infty;-2] \cup [2;\infty)$ $D)$ $(-\infty;-1] \cup [2;\infty)$

26. $\left|\frac{4-5x+x^2}{x^2-4}\right| \le 1$ tengsizlikni yeching.

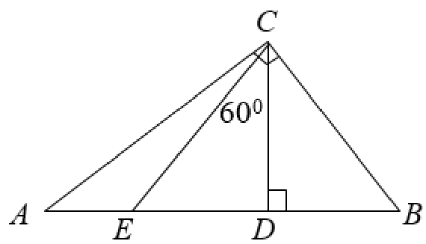
A) [0;1,6] U [2,5;∞) B) [0;1,6] U [2,5] C) [0;1,6] D) (0;1,6)

27. $\vec{x} + \vec{y} = (-2;-3)$ va $|\vec{x}| = \sqrt{13}$, $|\vec{y}| = \sqrt{26}$ bo‘lsa, $\vec{x} \cdot \vec{y}$ ni toping.

A) -26 B) -32 C) -8 D) -13

28. Rasmda berilgan ma’lumotlarga ko‘ra, S_{ABC} ni toping. Bu yerda $AC \perp BC$, $\angle ECD = 60^0$, $CD \perp AB$ va $CE = EB = 14$

A) 98 B) 88 94 D) 90



29. OX o‘qqa parallel, M(-3;1) nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

$A)\ x+y=1\quad B)\ x=-3\quad C)\ x=1\quad D)\ y-1=0$

30. Agar б $\in\left(\frac{p}{4};\frac{p}{2}\right)$ bo‘lsa, quyidagi shartlarning qaysi biri to‘g‘ri.

$A)\ (cos\text{б})^{sin\text{б}}<(cos\text{б})^{cos\text{б}}<(sin\text{б})^{cos\text{б}}$ $B)\ (sin\text{б})^{cos\text{б}}<(cos\text{б})^{sin\text{б}}<(cos\text{б})^{cos\text{б}}$
$C)\ (cos\text{б})^{cos\text{б}}<(sin\text{б})^{cos\text{б}}<(cos\text{б})^{csin}$ $D)\ (cos\text{б})^{cos\text{б}}<(cos\text{б})^{csin}<(sin\text{б})^{cos\text{б}}$

V-7

1. $5:(-8)\cdot 288-(-76+13):(-9)+(-13)$ ifodaning 3/4 qismini aniqlang.

A) -139,5 B) 139.5 C) 140,5 D) -150

2. $\sqrt[3]{5^{(\sqrt{5}+1)^2}\cdot 25^{-\sqrt{5}}}\cdot\left(\sqrt{5^3}-\sqrt{\frac{1}{125}}\right):\left(\sqrt{5}-5^{-\frac{1}{2}}\right)$ ni hisoblang.

A) 155 B) 1 C) 50 D) 25

3. Natural bo‘luvchilari soni eng ko‘p bo‘ladigan uch xonali sonni toping.

A) 908 B) 480 C) 804 D) 840

4. Aylananing ikkita vatari O nuqtada kesishadi va kesishish nuqtasida birinchi vatar 12 va 18 ga, ikkinchi vatar esa 3:4 nisbatda bo‘linadi. Ikkinchi vatarni uzunligini toping.

A) $21\sqrt{2}$ B) $16\sqrt{2}$ C) $14\sqrt{2}$ D) $12\sqrt{2}$

5. Agar konus o‘q kesimining yuzi N ga, asosining yuzi M gat eng bo‘lsa, konus yon sirtini yuzini toping. A) $\sqrt{\pi MN}$ B) $\sqrt{M^2+N^2\cdot\pi}$ C) $\sqrt{MN}$ D) $2\sqrt{MN}$

6. $f(x)=\frac{1}{x-4}+\frac{x}{4}+\frac{1}{4}$ bo’lsa, fungsiya uchun $f(a)=0$ bo’lsa a ni toping.

A) 0;3 B) 2; 3 C) 3 D) 0

7. $A(2;-5)$ nuqtadan o‘tuvchi va $y=x^2-6x+5$ parabolaga urinuvchi to‘g‘ri chiziqning urinish nuqtasi abssissalari ko‘paytmasini toping.

A) 2 B) 4 C) -2 D) -4

8. $tg(2+3x)=-1$ tenglamani yeching.

A) $\frac{\pi}{12}-\frac{2}{3}+\frac{\pi k}{3}$ B) $-\frac{\pi}{12}-\frac{2}{3}+\frac{\pi k}{3}$ C) $-\frac{\pi}{12}+\frac{2}{3}-\frac{\pi k}{3}$ D) $\frac{\pi}{12}+\frac{2}{3}+\frac{\pi k}{3}$

9. $\sqrt{1-x^2}-\sqrt{x^2+6x+9}+\sqrt{2x^2+10x+4}=0$ tenglamani yeching.

A) $1;2$ B) $-1\frac{2}{7};-1;5$ C) $-1;5$ D) $1;0$

10. Qo‘shni burchaklardan biri ikkinchisidan 11 marta katta bo‘lsa, shu burchaklar-dan kichigini toping. A) 15^0 B) 12^0 C) 30^0 D) 20^0

11. Sayyoh 70 km yo‘l bosishi kerak edi. Birinchi kuni u yo‘lning $\frac{1}{2}$ qismini, ikkinchi kuni qolgan masofaning $\frac{1}{2}$ qismini o‘tdi. U necha km yo‘l yurgan.

A) 17,5 B) 85 C) 52,5 D) 62,5

12. 639 sonini 2:3:4 kabi nisbatda bo‘ling.

A) 142;215;282 B) 142;213;284 C) 171;239;259 D) 171;142;326

13. 1,16(6) + 0,12(3) ni hisoblang.

A) $\frac{7}{30}$ B) $1\frac{29}{100}$ C) $1\frac{7}{30}$ D) 19

14. Uchlari $A(-3;2), B(3;4)$ va $C(5;-2)$ nuqtalarda bo‘lgan uchburchakning AB va BC tomonlarining o‘rtalarini tutashtiririshdan hosil bo‘lgan chiziqning uzunligini toping.

A) $5\sqrt{2}$ B) $2\sqrt{5}$ C) $3\sqrt{5}$ D) 5

15. $\frac{3\sqrt{2}-\sqrt{12}}{2-\sqrt{6}} \cdot x > 1$ tengsizlikni yeching.

$A)\ x > -\frac{\sqrt{3}}{3}$ $B)\ x < 1$ $C)\ x > 1$ $D)\ x < -\frac{\sqrt{3}}{3}$

16. $f(x) = -3x^2 + 5x^4$ funksiya boshlang‘ichini toping.

$A)\ x^3 + \frac{x^5}{5} + C$ $B)\ \frac{x^2}{4} + \frac{x^4}{5} + C$ $C)\ x^3 + x^5 + C$ $D) -x^3 + x^5 + C$

17. $\frac{\left(0,(6)+\frac{1}{3}\right):0,25}{(0,12(3):0,0925)} + 12,5 \cdot 0,64$ ni hisoblang. A) 8 B) 11 C) 3 D) 7

18. $\frac{2x+5}{1+3x} + \frac{x-3}{3x+2} \geq 1$ tengsizlikni yeching.

A) $(-\frac{2}{3};-\frac{1}{3})$ B) $[-\frac{5}{2};-\frac{2}{3})$ C) $[-\frac{5}{2};-\frac{1}{3})$ D) $[-\frac{5}{2};-\frac{1}{3}) \cup (-\frac{1}{3};\infty)$

19. $|x^2 - x - 3| + 1 + x = 0$ tenglamani yeching.

A) $-1; 3$ B) $-\sqrt{2}; 1-\sqrt{5}$ C) $-\sqrt{2}; \sqrt{5}-1$ D) $\sqrt{2}; \sqrt{5}-1$

20. Balandligi 4 sm, asosi 16 sm bo‘lgan teng yonli uchburchakka tashqi chizilgan aylana radiusini toping. A) 8 B) 10 C) 11 D) 9

21. Agar x_n ketma-ketlik uchun $x_n = \frac{1-n}{n} - (-1)^n$ bo‘lsa, $x_4 - x_3$ ni toping.

A) $\frac{1}{6}$ B) $2\frac{1}{12}$ C) $-2\frac{1}{12}$ D) 5

22. Teng yonli uchburchakning yon tomoniga tushirilgan medianasi 5 sm, asosi $4\sqrt{2}$ sm bo‘lsa, uchburchakning yon tomonini toping.

A) 7 B) 5 C) 6 D) $4\sqrt{2}$

23. $A(7;-6)$ nuqtadan abssissa o‘qiga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini toping. $A)\ x = -5$ $B)\ x = 5$ $C)\ x = 7$ $D)\ x = -7$

24. Piramida asosinig yuzi 512 ga, balandligi 16 ga teng. Asosiga parallel bo‘lgan kesimning yuzi 50 ga teng bo‘lsa, u asosidan qanday masofada joylashgan.

A) 8 B) 9 C) 10 D) 11

25. $\begin{cases} cosx \cdot siny = \frac{2}{3} \\ sinx \cdot cosy = -\frac{1}{3} \end{cases}$ bo‘lsa, $ctg(x-y)$ ni toping.

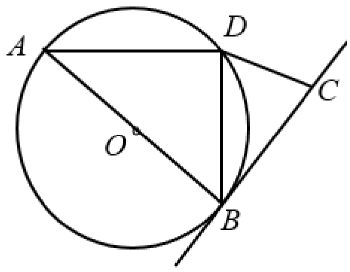
A) -1 B) 0 C) 1 D) 2

26. Agar $y = ln(x-1)^2 + x^5 + e^{-\frac{x^2}{2}}$ bo‘lsa, $y'(0)$ ni toping.

A) -2 B) -3 C) 2 D) 3

27. Rasmga ko‘ra $\angle BAD = 40^0, BD = BC$ bo‘lsa, $\angle DCB$ ni toping.

A) 80^0 B) 70^0 C) 40^0 D) 50^0



28. m ning qanday butun qiymatida quyidagi ifodani qisqartirish mumkin.

$\frac{36+mx+x^2}{7+8x+x^2}$ A) 35 B) -37 C) -35 D) 37

29. $sin\left(\frac{x}{2}+\frac{p}{5}\right)=\frac{\sqrt{3}}{2}$ tenglamani yeching.

$A) -\frac{2p}{5}+(-1)^k\frac{2p}{3}+2pk$ $B) -\frac{2p}{5}+(-1)^k\frac{2p}{3}+pk$

$C) \frac{2p}{5}+(-1)^k\frac{2p}{3}+2pk$ $D) \frac{2p}{5}+2pk$

30. $log_x 3\cdot log_3 5\cdot log_{3x}3=log_{9x}3\cdot log_9 25$ tenglama yechimlarining ko‘paytmasini toping. A) 1 B) $\sqrt{3}$ C) 3 D) $\frac{2}{\sqrt{3}}$

V-8

1. $\frac{58,4\cdot 31,2-27,2}{31,2+58,4\cdot 30,2}$ ni hisoblang. A) 1 B) $\frac{1}{2}$ C) 2 D) $\frac{1}{4}$

2. x ning qanday qiymatlarida $|44+2x^2-26x|=-(26x-44-2x^2)$ tenglik o‘rinli bo‘ladi.

A) $(-\infty;2]\cup[11;\infty)$ B) $(-\infty;2)\cup(11;\infty)$C) $(2;11)$ D) $(-2;0)\cup(0;11)$

3. $5+\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)=0$ tenglamaning haqiqiy yechimlari ko‘paytmasini toping.

A) $\frac{3+\sqrt{5}}{2}$ B) 1 C) 0 D) $\frac{3+\sqrt{5}}{2}$

4. To‘g‘ri burchakli uchburchak katetlari 4 va 3 ga teng. Katetlarning gipotenuzadgi pro-yeksiyasiyalarining uzunliklari farqini toping. A) 1,8 B) 1,4 C) 1,6 D) 0,4

5. Barcha ikki xonali sonlar ichida 3 ga ham, 2 ga ham bo‘linmaydiganlari nechta.

A) 30 B) 32 C) 33 D) 28

6. $\sqrt{(\sqrt{5}-\sqrt{3})\sqrt{\sqrt{8+\sqrt{60}}\,(\sqrt{5}+\sqrt{3})}}$ ni hisoblang. A) 4 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) 2

7. $y=\sqrt{x^2-|x|-2}$ funksiyaning aniqlanish sohasini toping.

A) $(-\infty;2]\cup[1;\infty)$ B) $(-\infty;-1]\cup[2;\infty)$

C) $(-\infty;-1]\cup[1;\infty)$ D) $(-\infty;-2]\cup[2;\infty)$

8. $cos50^0-sin40^0+ctg135^0$ ni hisoblang.

A) -1 B) 1 C) -2 D) 2

9. $\vec{x}+\vec{y}=(-2;-3)\ va\ |\vec{x}|=\sqrt{13}, |\vec{y}|=\sqrt{26}$ bo‘lsa, $\vec{x}\cdot\vec{y}$ ni toping.

A) -8 B) -32 C) -13 D) -26

10. $2-\sqrt{x-\sqrt{x+8}}=0$, tenglamaning haqiqiy ildizlari yig‘ndisini toping.

A) 9 B) 8 C) 1 D) 12

11. $\left(\frac{1}{8}\right)^{\sqrt{x}}\geq\frac{x^2}{2^{3\sqrt{x}}}$ tengsizlikning butun yechmlari nechta.

A) 3 ta B) ∅ C) 1 ta D) 2 ta

12. Teng yonli uchburchakning uchidagi burchagi α gat eng. Uchburchakka ichki va tashqi chizilgan doiralar radiuslari nisbatini toping.

$A)\ ctg\frac{p+б}{4}$ $B)\ cosбtg\frac{p+б}{4}cosб$ $C)\ tg\frac{p-б}{4}sinб$ $D)\ tg\frac{p-б}{4}cosб$

13. $f(x)=\frac{1}{x}+\sqrt{x}$ funksiyaning hosilasini toping.

A) $\frac{1}{x^3}-\frac{1}{2\sqrt{x}}$ B) $\frac{1}{2\sqrt{x}}-\frac{1}{x^2}$ C) $\frac{1}{x^2}-\frac{1}{2}\sqrt{x}$ D) $\frac{1}{x^2}+\frac{1}{2\sqrt{x}}$

14. $cos^2 2x+2sin^3 x=sinx$ tenglamani yeching.

A) $\frac{\pi}{4}+\frac{\pi k}{2},\ \frac{\pi}{6}+\frac{2\pi k}{3}$ B) $\frac{\pi}{4}+\frac{\pi k}{2},\ \frac{\pi}{6}+\frac{\pi k}{3}$ C) $\frac{\pi}{4}+\pi k,\ \frac{\pi}{6}+\frac{2\pi k}{3}$ D) $\frac{\pi}{4}+\pi k,\ \frac{\pi}{6}+\frac{\pi k}{3}$

15. Ordinatasi $y_0=-2$ bo‘lgan nuqtadan $y=1-4x+x^2$ egri chiziqqa uriuvchi to‘g‘ri chiziq tenglamasini ko‘rsating.

$A)\ y=2x\ va\ y=8-2x$ $B)\ y=8-2x$ $C)\ y=-2x\ va\ \ 2x-8$ $D)\ y=2x$

16. Silindr asosining radiusi 10 sm, balandligi 16 sm. Silindrning o‘qiga parallel kesim o‘tlazilgan va u o‘qdan 60 mm uzoqlikda yotadi. Shu kesmaning yuzini (sm^2) toping. A) 256 B) 196 C) 216 D) 208

17. Kvadrat yuzini $\frac{4}{9}$ barobar ortirish uchun uning tomonini nech marta oshirish kerak.

A) $\frac{2}{3}$ B) $\sqrt{\frac{2}{3}}$ C) $\sqrt{\frac{3}{2}}$ D) $\frac{3}{2}$

18. Sakkizburchakning tashqi burchaklari 159^0, 14^0, 17^0, 42^0, 33^0, 45^0, 32^0, 18^0 bo‘lsa, ichki burchaklari yig‘indisini toping.

A) 1440^0 B) 1020^0 C) 1080^0 D) 1000^0

19. $log_c(3-x^2+2x)>log_c(x^2-x-2)$ tengsizlikni $x=2{,}25$ soni qanoatlantirsa, tengsizlikni yeching.

A) (1,5;3,5) B) (2;2,5) C) (2,5;3) D) (2;3)

20. Yig‘indisi 6 ga teng, birinchi 5 ta hadining yig‘indisi esa $5\frac{13}{16}$ ga teng bo‘lgan cheksiz kamayuvchi geometrik progresssiyaning uchinchi hadini toping.

A) 0,75 B) 3 C) 0,5 D) 1,5

21. Quyidagilardan qaysi biri $n(n\in N)$ ning istalgan qiymatida natural son bo‘ladi.

$A)\ \frac{3^n+3^{n+1}+3^{n+2}}{12}$ $B)\ \frac{2^n+2^{n+1}+2^{n+2}}{21}$ $C)\ \frac{4^n+4^{n+1}+4^{n+2}}{22}$ $D)\ \frac{n^3}{6}+\frac{n^2}{2}+\frac{n}{3}$

22. $\frac{x^{33}-1}{x^{11}+x^{22}+x^{33}}$ ni qisqartiring. $A)\ x^{11}-1$ $B)\ \frac{x^{11}-1}{x^{11}}$ $C)\ x^{11}+1$ $D)\ 1+\frac{1}{x^{11}}$

23. $\frac{x^2-x}{x^2-x-1}-1=\frac{x^2-x+2}{x^2-x-2}$ tenglama ildizlarining o‘rta arifmetigini toping.

A) 0,5 B) 1 C) 2 D) 1,5

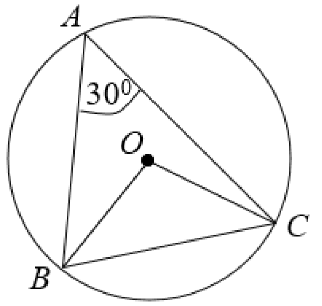
24. $\left(sinб+\frac{1}{sinб}\right)^2+\left(cosб+\frac{1}{cosб}\right)^2-\left(tgб+\frac{1}{tgб}\right)^2$ ifodani so‘ldalashtiring.

$A)\ 3$ $B)\ 1$ $C)\ 5$ $D)\ sinб\cdot cos^2б$

25. ABC to‘g‘ri burchakli uchburchakning *C* to‘g‘ri burchagidan *CD* balandlik tushirilgan. *D* nuqtadan *AC* va *BC* tomonlarigacha bo‘lgan masofa mos ravishda *m* va *n* ga eng bo‘lsa, uchburchakning yuzini toping.

$A)\ \frac{(m^2+n^2)^2}{2nm}$ $B)\ \frac{m^2+n^2}{2nm}$ $C)\ \frac{(m+n)^2}{2nm}$ $D)\ \frac{m+n}{2nm}$

26. Rasimda berilishiga ko‘ra *R* ni toping. *BC* = 4.



A) $2\sqrt{3}$ B) 2 C) 4 D) $2\sqrt{2}$

27. $33\cdot 18^2\cdot 24^3$ sonni kanonik ko‘rinishda yozing.

A) $3^4\cdot 5^2\cdot 11$ B) $2^8\cdot 3^5\cdot 11$ C) $2^5\cdot 3^{10}\cdot 11$ D) $2^{11}\cdot 3^8\cdot 11$

28. To‘g‘ri burchakli uchburchakning perimetri 24 sm ga, yuzi 24 sm^2 ga teng bo‘lsa, unga tashqi chizilgan doiraning yuzini toping.

A) 25π B) 20π C) 30π D) 35π

29. $f(x)=e^{\frac{2}{5}-\frac{3x}{5}}+sinx-1$ funksuyaning boshlang‘ichini toping.

$A)\ F(x)=\frac{5}{3}e^{\frac{2}{5}-\frac{3x}{5}}+cosx+x+C$ $B)\ F(x)=-\frac{5}{3}e^{\frac{2}{5}-\frac{3x}{5}}-cosx-x+C$

$C)\ F(x)=\frac{5}{3}e^{\frac{2}{5}-\frac{3x}{5}}+cosx+C$ $D)\ F(x)=-\frac{3}{5}e^{\frac{2}{5}-\frac{3x}{5}}-cosx-x+C$

30. $sin^2x=3sinx\cdot cosx$ tenglamani yeching.

$A)\ pk$ $B)\ arctg3+2pk$ $C)\ pk, arctg3+pk$ $D)\ arctg3+pk$

V-9

1. $2,(3)+5\frac{1}{3}:\left(6\frac{3}{4}\cdot 1\frac{1}{3}-8\frac{1}{9}\right)$ ni hisoblang. $A)\ \frac{26}{3}$ $B)\ 8\frac{1}{3}$ $C)\ 7\frac{1}{3}$ $D)\ 9\frac{1}{3}$

2. Katetlari 4 va 3 bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi asosidan katta katetigacha bo‘lgan masofani toping.

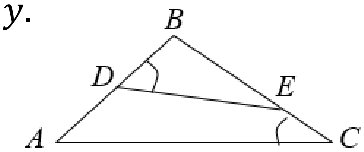
A) 1,92 B) 2,4 C) 5,76 D) 1,44

3. $x^3-32-4x\sqrt{x}=0$ tenglama ildizlarining ko‘paytmasini toping.

A) 2 B) 3 C) 1 D) 4.

4. Rasimda berilishiga ko‘ra $x+y$ ning qiymatini toping. Bunda $\angle BDE=\angle BCA$, $BD=5,\ BE=6,\ AC=18,\ DE=9, AD=x,\ EC=y$.

A)31 B) 11 C) 30 D) 18



5. $\vec{a}(-5;0)$ va $\vec{b}(m;n)$ vektorlarning skalyar ko‘paytmasi $m+12$ ga teng, *m* ni toping.

$A)-2$ $B)-1$ $C)-3$ $D)-4$

6. $\frac{3tgx-tg^3x}{1-3tg^2x}$ funksiyaning eng kichik musbat davrini toping.

$A)$ p $B)\frac{p}{3}$ $C)\frac{p}{2}$ $D)\frac{p}{4}$

7. Ishchi har kuni kunlik rejada ko'rsatilganidan 25 ta detal ortiq yasab, uch kunda rejadagi 7 kunga mo'ljallanganidan 15 ta ortiq detal yasadi. Ishchi har kuni nechtadan detal yasagan? A) 47 B) 15 C) 40 D) 44

8. $\frac{a^3-b^3}{a^2+ab+b^2}\cdot\left(\frac{1}{a-\sqrt{ab}}+\frac{1}{a+\sqrt{ab}}\right)$ ni so'ldalashtiring. A) $\frac{1}{a+b}$ B) 1 C) 2 D) a

9. $2^{\frac{2-x}{2x+1}}=2^{\frac{3x-1}{2x+1}}-1$ tenglamani yeching. A) 5 B) 1 C) 3 D) 2

10. $\overline{2014xy}$ ko'rinishidagi 6 xonali sonlar orasida 36 ga qoldiqsiz bo'linadiganlari nechta? A) 1 ta B) 2 ta C) 3 ta D) 4 ta

11. $xy + yz + zx = 16$ bo'lsa, $x^2 + y^2 + z^2$ ning eng kichik qiymatini toping.
A) 32 B) 16 C) 24 D) 18

12. *sin(arcsin4/5 + arccos3/5)* ni hisoblang.
A) 24/25 B) 7/25 C) 12/25 D) -24/25

13. Boshi A(2;4) nuqtada bo'lgan $\overline{AB}$ (3;-2) vektorning B nuqtasi koordinatalarini aniqlang. A) (-5;-2) B) (1;-6) C) (5;6) D) (5;2)

14. $\frac{1+\frac{1+\frac{\cdots}{5}}{5}}{5}+1=x$ tenglamadan x ni toping. A) 5/4 B) 7/5 C) 11/6 D) 4/5

15. $f(x) = (2x – 4)^7$ funksiyaning hosilasini toping.
A) $14(4 – 2x)^6$ B) $14(4 + 2x)^6$ C) $-14(4 + 2x)^6$ D) $-14(4 - 2x)^6$

16. $|x + 4| + |x – 3| + |x – 2| = 7$ tenglamaning ildizlari yig'indisini toping.
A) 2 B) ildizi yo'q C) -2 D) 0

17. $|8-x^2|^{x^2-x-2}=1$ tenglama nechta yechimga ega? A) 6 B) 2 C) 7 D) 4

18. $y = 3sinx – 2cosx - x$ funksiyaning hosilasini toping.
A) 2cosx – 1 B) 2sinx – x C) 2sinx + 3cosx -1 D) 2cosx + 3sinx -1

19. 144 va 128 sonlarining umumiy bo'luvchilari yig'indisini toping.
A) 35 B) 32 C) 31 D) 33

20. Uchlari A(-3;0), B(-1;4) va C(3;2) nuqtalarda bo'lgan uchburchak qanday uch-burchak?
A) teng yonli to'g'ri burchakli B) to'g'ri burchakli C) teng tomonli D teng yonli

21. Ikkita shar berilgan bo'lib, ularning radiuslari 3 dm va 25 sm. Sharlar kesishish chizi-g'ining uzunligi 48π sm bo'lsa, ularning markazlari orasidagi masofani (sm) toping.
A) 11 yoki 25 B) 11 yoki 24 C) 18 yoki 24 D) 12 yoki 20

22. Agar $x + y + x^2y + xy^2 = 24$ va $x + y = 5$ bo'lsa, $x^3 + y^3$ ni toping.
A) 68 B) 60 C) 42 D) 30

23. $f(x) = e^{\frac{1}{2}x}$ - sinx, $F(x)$ - ?
A) cosx + $2e^{\frac{1}{2}x}$ + C B) –cosx + $e^{\frac{1}{2}x}$ + C C) cosx - $2e^{\frac{1}{2}x}$ + C D) –cosx + $e^x + C$

24. $(1-sin2x)(1+tgx) = 1 – tgx$ tenglamasini yeching.
A) π/4 + πn, πn B) π/2 + 2πn C) πn D) π/4 + πn

25. Qo'shni burchaklardan biri ikkinchisidan 46^o ga katta bo'lsa, shu burchaklardan kattasini toping. A) 113^o B) 115^o C) 103^o D) 117^o

26. Geometrik progressiyada 5 + 15 + 45 + … + 1215 yig'indini hisoblang.

A) 1820 B) 1840 C) 1720 D) 1920

27. $\frac{32-6x^2+x^3}{x^2-8x+16}$ ni so‘ldalashtiring.

A) $1-x$ *B)* $x+2$ *C)* $3x-2$ *D)* $(x-1)/(x+1)$

28. $\lg(x-2)+\lg(x-3)=1-\lg5$ tenglamani yeching

A) {2;6} B) {1;4} C) {2;3} D) {4}

29. $\int(7cosx-6sinx)dx$ ni hisoblang.

A) -7sinx – 6cosx + C B) 7sinx – 6cosx + C
C) 6cosx – 7sinx + C D) 6cosx + 7sinx + C

30. Ketma-ket toq sonlar yig’indisi quyidagilardan qaysi biriga albatta bo’linadi?

A) 3 B) 5 C) 4 D) 6

V-10

1. $\int\frac{1}{x+2}dx$ ni hisoblang.

A) lnx + C B) ln|x + 2| + C C) 0,5lg(x + 2) + C D) $-\frac{1}{(x+2)^2}+C$

2. $\frac{3tgx-tg^3x}{1-3tg^2x}$ funksiyaning eng kichik musbat davrini toping

A) 2π B) π/3 C) π D) 3π/2

3. $x^2+(\sqrt{x})^2-2=0$ tenglama ildizlari ko’paytmasini toping. A) -1 B) 1 C) -2 D) 4

4. $\begin{cases}x^2+y^2=2(xy+2)\\x+y=6\end{cases}$ tenglamalar sistemasidan $|x-y|$ ni toping.

A) 0 B) 3 C) 1 D) 2

5. $(\pi-e)^{\ln(1-2x)}\cdot cos^2x^{)}\geq1$ tengsizlikning [0;π] oraliqqa tegishli barcha yechimlarini aniqlang.

A) [π/4; π/2) B) [0; π/2] C) (π/4; 3π/4) D) [0; π/2)

6. 1; 5; 13; 29; … ketma ketlikning yigirmanchi hadini toping.

A) 3^7-1 B) $4^{10}-7$ C) $2^{21}-3$ D) $2^{22}-3$

7. Sharga konus ichki chizilgan. Konusning yasovchisi asosining diametriga teng. Shar hajmining konus hajmiga nisbatini toping. A) 32; 9 B) 16; 9 C) 27; 4 D) 8;3

8. M(2;3) va N(-3;-1) nuqtalar berilgan. MN kesmaning ordinata o’qi bilan kesishgan nuqta koordinatalari toping. A) (0; -7/5) B) (0; 7/5) C) (7/5; 0) D) (-7/4; 0)

9. $\begin{cases}\sqrt{x}-\sqrt{y}=3\\x-y=21\end{cases}$ sistemadan $x+y$ ni toping. A) 30 B) 38 C) 25 D) 29

10. $\sqrt[4]{\frac{2\sqrt{6}+5}{6x}}\cdot(3\sqrt{2x}-2\sqrt{3x})^{0,5}$ ni so‘ldalashtiring $x>0$. A) 2 B) $\sqrt{3}$ C) x D) 1

11. ABC muntazam uchburchakda AD mediana $10\sqrt{3}$ ga teng bo’lsa, D nuqtadan AB tomonning o’rtasigacha bo’lgan masofani toping. A) 5 B) $5\sqrt{3}$ C) $10\sqrt{3}$ D) 10

12. To’rtburchakli muntazam piramidaning balandligi 12 m, asosining tomoni 10 m bo’lsa, piramida to’la sirtining yuzini (m^2) toping.

A) 360   B) 540   C) 345   D) 480

13. $y = 3ctgx$ funksiyaning $F(\pi/2) = 9$ shartni qanoatlantiruvchi boshlang'ichini topib, $F(5\pi/2)$ ning qiymatini toping. A) 10   B) 18   C) 12   D) 9

14. Tenglamani yeching. $2^{x-1} + 2^{x-2} + 2^{x-3} = 448$. A) 7   B) 11   C) 10   D) 9

15. Markazi ABCD (BC||AD) trapetsiyaning AC diagonalida yotuvchi aylana A va B nuqtalardan o'tib CD ga C nuqtada urinadi hamda AD ni E nuqtada kesib o'tadi. Agar $CD = 5\sqrt{52}$ va $AB = 5\sqrt{2}$ bo'lsa, trapetsiyaning yuzini hisoblang.

A) 130   B) 136   C) 135   D) 134

16. Arifmetik progressiyada $a_{11} + a_7 = 6$ ga teng, $a_9^2 - 2$ ni toping

A) 5   B) 3   C) 2   D) 7

17. x musbat butun son bo'lib, 487 sonini x ga bo'lganda bo'linma $4x$ ga teng bo'lsa, qoldiqni toping. A) 3   B) 4   C) 2   D) 5

18. D(-1;0), C(0;-1), B(1;0), A(0;1) nuqtalardan hosil bo'lgan ABCD to'rtburchak simmetriya nuqtasi koordinatalarini toping. A) (0;-1)   B) (0;0)   C) (-1;-1)   D) (1;1)

19. Uchburchakning tomonlari 15, 14, 13 bo'lsa, uchburchakka ichki chizilgan aylana radiusini toping. A) 3   B) 2   C) 7   D) 4

20. Agar $2ctgx + 1/sinxcosx = 4$ va x € $(0; \pi/2)$ bo'lsa, $1/sinxcosx - 2ctg2x$ ni toping.

A) 8   B) 10   C) 2   D) 0

21. Rombning diagonallari 6 va 4 ga teng bo'lsa, uning yuzini toping.

A) 12   B) 24   C) 10   D) 16

22. Agar $2 \le x \le y \le z \le t \le 128$ bo'lsa, $x/y + z/t$ ifodaning eng kichik qiymatini toping.

A) 0,25   B) 0,5   C) 1,6   D) 0,75

23. $4log_{cos2x}sinx - 4 + 3log_{sin^3x}cos2x = 0$ tenglamani yeching

A) $(-1)^k \arcsin(\pm\frac{1}{\sqrt{3}}) + \pi k$, k € Z   B) $\arcsin\frac{1}{\sqrt{3}} + 2\pi k$, k € Z

C) $(-1)^{k+1} \arcsin\frac{1}{\sqrt{3}} + \pi k$, k € Z   D) $(-1)^k \arcsin\frac{1}{\sqrt{3}} + \pi k$

24. To'g'ri parallelepiped ikkita yon yog'ining yuzi 24 sm^2 va 20 sm^2, ular orasidagi burchak 60^o ga teng. Agar parallelepipedning yon qirrasi 4 sm ga teng bo'lsa, uning hajmini (sm^3) toping. A) $80\sqrt{2}$ B) $60\sqrt{6}$ C) $120\sqrt{3}$ D) $60\sqrt{3}$

25. Ishchi har kuni kunlik rejada ko'rsatilganidan 25 ta detal ortiq yasab, uch kunda rejadagi 7 kunga mo'ljallanganidan 15 ta ortiq detal yasadi. Ishchi har kuni nechtadan detal yasagan? A) 47   B) 44   C) 15   D) 40

26. $y = 5x^3 - 5x^4 + x^5 - 1$ funksiya minimumining abssissasi qaysi oraliqqa tegishli?

A) (0;2)   B) (-2;2)   C) (5;9)   D) (1;5)

27. $|6 + 5x + x^2| \le 6 + 5x + x^2$ tengsizlikni yeching.

A) (-∞;-3] U [-2;∞)   B) [-3;-2]   C) {-3;-2}   D) (-3;-2)

28. Agar $f(a,b,c) = a/b\text{-}c$ bo'lsa, $f(f(1,2,3), f(2,3,1)\ f(3,1,2)$ ni toping.

A) 0   B) -1/4   C) -1/2   D) 1

29. Ikkita natural sonni 3 ga bo'lganda qoldiqda 1 va 2 qoldi. Bu sonlar kvadratlarining musbat ayirmasini uchga bo'lganda qanday qoldiq qoladi?

A) 1   B) 1 yoki 2   C) 0   D) 2

30. $A = 15/32$, $b = 21/24$ va $c = 33/38$ sonlarini o'sish tartibida joylashtiring.

A) $a<c<b$ B) $c<b<a$ C) $b<a<c$ D) $b<c<a$

V-11

1. Agar $\overline{abc}$, $\overline{bca}$, $\overline{cab}$ uch xonali natural sonlar yigdndisi 777 ga teng bo‘lsa, $a+b+c$ ni toping. A) 7 B) 6 C) 8 D) 2

2. $4{,}8 = x + \frac{y}{5}$ tenglikda x va y sonlar 5 dan kichik natural sonlar bo‘lsa, y ning qiymatini toping. A) 1 B)3 C) 4 D) 0

3. $2 < a < 6$ va $2 < b < 10$ bo‘lsa, a va b butun sonlar uchun $\frac{1+\frac{a}{b}}{1+\frac{b}{a}}$ kasrning eng katta qiymatini toping. A) $\frac{7}{3}$ B) $\frac{5}{3}$ C) 7 D) 15

4. Hisoblang: $\left(1\frac{1}{7}\right)\cdot\left(1\frac{1}{8}\right)\cdot\left(1\frac{1}{9}\right)\cdot\ldots\cdot\left(1\frac{1}{62}\right)$ A) 9 B) $\frac{11}{7}$ C) 7 D) $\frac{10}{7}$

5. Besh xonali $\overline{x734y}$ sonini 55 ga bo‘lganda natural son hosil bo‘ladi. *x* ning barcha qiymatlari yig;indisini toping. A)11 B) 9 C)3 D) 14

6. Hisoblang: $\frac{1}{2}+\frac{2}{3}+\frac{3}{2}+\frac{4}{3}+\cdots+\frac{15}{2}+\frac{16}{3}$. A) 72 B) 24 C) 65 D) 56

7. $(x^2+x)+(x^2+2x)+\cdots+(x^2+19x)=1425$ tenglamani qanoatlantiruvchi *x* natural sonni toping. A) 6 B)10 C) 5 D) 8

8. So‘ldalashtiring: $tg б\cdot tg в + (tg б + tg в)\cdot ctg(б + в)$ A) -1 B) 1 C) 2 D) 0

9. Hisoblang: $sin1^0 + sin2^0 + sin3^0 + \cdots + sin359^0$.

A) 1 B) -1 C) $sin179^0$ D) 0

10. Agar $x < -2$ bo‘lsa, $\sqrt{x^2+6x+1+\sqrt{9-12x+4x^2}}$ ifodani so‘ldalashtiring.

A) $2-x$ B) $2+x$ C) $-2-x$ D) $-2x$

11. Agar $2^a = 81$, $3^b = 8$ bo‘lsa, $a\cdot b$ ning qiymatini toping.

A) 14 B) 12 C) 15 D) 18

12. Ifodani so‘ldalashtiring: $\frac{a^4-10a^2+169}{a^2+6a+13}$

A) $a^2-5a+13$ B) a^2+13 C) $a^2-6a+13$ D) $a^2-3a+13$

13. *a* ning qanday qiymatida $\frac{9x^2-6x+1}{9} = (x+a)^2$ tenglik ayniyat bo‘ladi?

A) $-\frac{1}{3}$ B) -1 C) $-\frac{1}{4}$ D) $-\frac{1}{2}$

14. $(a^2-2a+1)\cdot x = a^2+2a-3$ tenglama *a* ning qanday qiymatida cheksiz ko‘p yechimga ega? A) $a=-3$ B) $a=-3,\ a=1$ C) $a=1$ D) $a\neq 1$

15. k ning qanday eng kichik natural qiymatida $x^2+(k+2)^2\cdot x+2k-4=0$ teng-lamaning ildizlari 2 dan kichik bo‘ladi? A) 4 B) 3 C) 2 D) 1

16. $(3-x)\cdot(x+2) > 0$ tengsizlikning butun yechimlari yigdndisini toping.

A) -3 B) 2 C) 0 D) -5

17. Agar $f(x) = \begin{cases} -x+2, & x<2 \\ \frac{x-1}{2}, & x\geq 2 \end{cases}$ bo‘lsa, $f(f(-1))$ ni toping.

A) -1 B) 3 C) 1 D) -2

18. Agar $f(x) = (a + b - 4) \cdot x^3 + 2x^2 + (b - 1) \cdot x$ juft funksiya berilgan bo‘lsa, $f(a)$ ning qiymatini toping. A) 12 B) 14 C) 20 D) 18

19. Hisoblang: $\int_1^2 \left(e^x + \frac{1}{x}\right) dx$ A) $e^2 + e - ln2$ B) $e^2 - e + ln2$ C) $e^2 + e + ln2$ D) $e^2 - e - ln2$

20. $\int \frac{3}{x \cdot ln2} dx$ ni hisoblang. A) $3l2x + C$ B) $6lnln2x + C$ C) $1{,}5lnln2x + C$ D) $3lnln2x + C$

21. *ABC* uchburchakning *BC* tomonida *D* nuqta olingan. Agar *BD* = 16, *DC* = *4* va *B* = *AD* = 10 bo‘lsa, *ADC* uchburchakning yuzini toping.

A) 12 B) 14 C) 10 D) 16

22. To‘g‘ri burchakli *ABCD* trapetsiyaning *B* va *C* burchaklari to‘g‘ri, *AB* = 8, *BC*= 6 va *DC* = 4. Trapetsiyaning *D* uchidan *AC* diagonaligacha bo‘lgan masofani toping.

A) 3,6 B)3 C) 2,4 D) 2

23. *ABCD* trapetsiyaning yuzi 48 ga teng, asoslari *DC* = 6, *AB* = 2. *BC* tomondan *E* nuqta olingan bo‘lib, *BE* = 2*EC* bo‘lsa, *ADE* uchburchak yuzini toping.

A) 32 B) 18 C) 24 D) 28

24. *ABC* uchburchak lichlarining koordinatalari berilgan: A(8;12), *B{*-8; 0) va C(-2;8). Uchburchakning *CM* medianasi yotgan to‘g‘ri chiziq tenglamasini tuzing.

A) $x + 2y + 3 = 0$ B) $x + y + 6 = 0$ C) $x + y = 6$ D) $x - y - 6 = 0$

25. $A = \{x : |x - 2| < 3, x € N\}$ to‘plamning elementlari sonini toping.

A)3 B) 4 C) 6 D) 5

V-12

1. $a = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 40 \cdot 41$, $b = 5 \cdot 4 + 10 \cdot 6 + 15 \cdot 8 + \cdots + 200 \cdot 82$ bo‘lsa, $\frac{a}{b}$ nmg qiymatmi toping. A) $\frac{1}{12}$ B) $\frac{1}{6}$ C) $\frac{1}{10}$ D) $\frac{1}{8}$

2. $\frac{7}{1+\frac{2}{x-1}}$ kasr ma'noga ega bodmaydigan barcha *x* lar yigdndisini toping.

A) 0 B) -1 C) 1 D) -2

3. Agar $x < -1$, $y > 1$ bo‘lsa, quyidagi javoblardan qaysi biri har doim o‘rinli?

A) $x^4 > y$ B) $y^3 > x^3$ C) $x^2 > y^2$ D) $y^2 > x^6$

4. Hlsobtag: $\left(1\frac{1}{7}\right) \cdot \left(1\frac{1}{8}\right) \cdot \left(1\frac{1}{9}\right) \cdot ... \cdot \left(1\frac{1}{69}\right)$ A) 7 B) $\frac{10}{7}$ C) $\frac{69}{7}$ D) 10

5. 180 gramm suvga 70 gramm tuz aralashtirildi. Hosil bodgan aralashmaning necha foizi tuzdan iborat bo‘ladi? A) 28 B) 25 C)30 D) 22

6. Agar $\sqrt{a + \sqrt{a + \sqrt{a + \cdots}}} = 2$ bo‘lsa, $\sqrt{a - \sqrt{a - \sqrt{a - \cdots}}}$ ning qiymatini toping.

A) 1 B) 2 C) 4 D) 3

7. Hisoblang: $1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \cdots + 9 \cdot 28$

A) 900 B) 740 C) 1210 D) 960

8. Agar $ctg б = -\frac{1}{2}$ bo‘lsa, *tg3a* ning qiymatini toping.

A) $-\frac{1}{11}$ B) 5,5 C) $-\frac{2}{11}$ D) $\frac{1}{6}$

9. Hisoblang: $sin2^0 + sin3^0 + sin4^0 + \cdots + sin358^0$.

A) 1 B) sin 179° C) 0 D) -1

10. Agar $x < -2$ bo‘lsa, $\sqrt{x^2 + 5x + 2 + \sqrt{4 - 4x + x^2}}$ ifodani so‘ldalashtiring.

A) $2 + x$ B) $2 - x$ C) D) $-2 - x$

11. $x,\ y,\ z$ butun sonlar bo‘lib, $y < 0$ va $\frac{2}{3x} = -\frac{3}{4y} = \frac{4}{5z}$ bo‘lsa, $x,\ y,\ z$ sonlarni o‘sish tartibida joylashtiring.

A) $x < y < z$ B) $z < y < x$ C) $y < x < z$ D) $y < z < x$

12. Iifodani soddalashtiring: $\frac{\frac{1}{a} + \frac{1}{b+c}}{\frac{1}{a} - \frac{1}{b+c}} \cdot \left(1 + \frac{b^2+c^2-a^2}{2bc}\right) : \frac{(a+b+c)^2}{bc}$

A) 1 B) 0,5 C) $b + c - a$ D) $a + b + c$

13. Agar $x \neq 0$ bo‘lsa, $5 + 5^{2x+y} - 5^{x+1} - 5^{x+y} = 0$ tenglamadagi x ni y orqali ifodalang.

A) $x = -1 - y$ B) $x = 1 - y$ C) $x = y - 1$ D) $x = y + 1$

14. Agar $x\sqrt{x} - 7\sqrt{x} = 6$ bo‘lsa, $x - \sqrt{x}$ ning qiymatini toping.

A) 7 B) 6 C) 8 D) 3

15. Agar $x^2 + (k + 2)^2 \cdot x + 2k - 4 = 0$ tenglamaning ildizlari 2 dan kichik bo‘lsa, k ning eng katta butun manfiy qiymatini toping.

A) -2 B) -4 C) -1 D) -5

16. $\left|\frac{1-2x}{1+3x}\right| > 0$ tengsizlikni yeching.

A) $\left(-\infty; -\frac{1}{3}\right) \cup (2; \infty)$ B) $\left(-\infty; -\frac{1}{3}\right) \cup \left(-\frac{1}{3}; \infty\right)$

C) $\left(-\infty; -\frac{1}{3}\right) \cup \left(-\frac{1}{3}; 2\right) \cup (2; \infty)$ D) $(-\infty; \infty)$

17. Agar $f(2x - 3) = 3x + 5$ bo‘lsa, $f\big(f(1)\big)$ ni toping.

A) 11 B) 38 C) 26 D) 16

18. $y = cos^2\left(\frac{x}{3} - \frac{\text{p}}{4}\right) + 2sinx$ funksiyaning eng kichik musbat davrini toping.

A) 2p B) 6p C) 3p D) *davriy emas.*

19. $x = 1,\ \ y = e^x$ va $y = e^{-x}$ chiziqlar bilan chegaralangan soha yuzini toping.

A) $\frac{(e-1)^2}{e}$ B) $e - 1$ C) $\frac{e-1}{e}$ D) $\frac{(e-2)^2}{e}$

20. Muntazam ko‘pburchak tomoni unga tashqi chizilgan aylananing 36°li yoyini tortib turadi. Muntazam ko‘pburchakning tomonlari sonini toping.

A) 12 B) 10 C) 6 D) 8

21. *ABC* uchburchakda *D* va *E* nuqtalar *BC* tomonni uchta teng qismlarga bodadi (*BD* = *DE* = *EC*), *F* va *G* nuqtalar esa *AD* kesmani uchta teng qismlarga bo‘ladi (*AF*= *FG* = *GD*). *AFE* uchburchak yuzining *ABC* uchburchak yuziga nisbatini toping.

A) $\frac{1}{12}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{9}$

22. *ABCDEF* muntazam oltiburchakda *AC, CE, BF, FD* diagonallar o‘tkazilgan. *AC* va *BF* diagonallar *L* nuqtada, *CE* va AD diagonallar *K* nuqtada kesishadi. Agar oltiburchak tomoni $2\sqrt{3}$ ga teng bo‘lsa, *LCKF* to‘rtburchak yuzini toping.

A) $5\sqrt{3}$ B) $8\sqrt{3}$ C) $9\sqrt{3}$ D) $6\sqrt{3}$

23. *ABCD* trapetsiyaning yuzi 24 ga teng, asoslari $DC = 6$, $AB = 2$. *BC* tomondan *E* nuqta olingan bo‘lib, $BE = 2EC$ bo‘lsa, *ADE* uchburchak yuzini toping.

A) 12 B) 21 C) 14 D) 16

24. A(3;0) va B(- 1; 2) nuqtalardan o‘tuvchi hamda markazi $y = x + 2$ to'g‘ri chiziqda yotgan aylana tenglamasini toping.

A) $(x-3)^2 + (y-5)^2 = 25$ B) $(x-4)^2 + (y-5)^2 = 25$
C) $(x-3)^2 + (y-4)^2 = 25$ D) $(x-5)^2 + (y-3)^2 = 25$

25. A = {1; 3: 5; 6; 8; 10} va {5; 6; 7; 8; 10} tocplamlar berilgan. A U B to‘plam elementlari sonini toping. A) 8 B) 11 C) 7 D) 6

V-13

1. *a* va *b* raqamlar yig‘indisi 13 ga qoldiqsiz bo‘linadi. Agar $\overline{abc}$ ko‘rinishdagi uch xonali sonlarni 13 ga bo‘lganda bir xil qoldiq qolsa, shu qoldiqni toping.

A) 6 B) 4 C) 0 D) 2

2. Agar *n, m* va *k* natural sonar uchun $nm = 25$ va $mk = 4$ bo‘lsa, $n + m + k$ ifodaning qiymatini toping.

A) 30 B) 42 C) 35 D) 19

3. Hisoblang:
$(-9)^3:(-9)^2 + (-10)^3:(-10) -$
$-(-2)^8:(-2)^7$

A) -89 B) 93 C) 89 D) -197

4. Hisoblang: $2019\frac{5}{26} - 2017\frac{2}{13}$

A) $\frac{27}{13}$ B) $\frac{51}{26}$ C) $\frac{24}{13}$ D) $\frac{53}{26}$

5. Ota o‘g‘lidan 3 marta katta, qizidan esa 25 yoshga katta. Agar ota 51 yoshda bo‘lsa, o‘g‘li qizidan necha yoshga kichik.

A) 8 B) 7 C) 9 D) 10

6. Piyoda 3 soatda 7,8 km yo‘l yurdi. Agar piyoda shu tezlik bilan yursa 4 soatda necha km yo‘l yuradi

A) 10,6 B) 10,4 C) 10,8 D) 10,5

7. Hisoblang:
$125^{\frac{2}{3}} \cdot 4 \cdot \sqrt[3]{(0{,}027)^2} + 3$

A) 12 B) 13 C) 6 D) 3

8. Hisoblang:

$$\sqrt{(3^{-2})^{-1} + (2^{-2})^{-2}} + 1$$

A) 6 B) 5 C) 8 D) 4

9. (b_n) geometrik progressiyada $b_6 - b_3 = 84$ va $b_5 - b_2 = 42$ bo‘lsa, $b_2 + b_4$ ni toping.

A) 27 B) 30 C) 36 D) 51

10. Sakkista haddan iborat arifmetik prodressiyaning toq o‘rindagi hadlari yig‘indisi 168 ga, juft o‘rindagi hadlari yig‘indisi 200 ga teng. Shu progressiyaning oltinchi hadini toping.

A) 58 B) 42 C) 66 D) 50

11. Soddalashtiring:
$3(4x - 3y) - 2(-3y + 4x)$

A) $-4x + 3y$ B) $4x - 3y$
C) $4x + 3y$ D) $-4x - 3y$

12. Soddalashtiring:
$(-1{,}5a^2b^3)^2 \cdot (-100a^3b^4)$

A) $225a^7b^9$ B) $-225a^7b^9$
C) $-225a^7b^{10}$ D) $225a^7b^{10}$

13. m ning qanday qiymatlarida $\frac{m^2-9}{m^2-1}$ va $\frac{2m+6}{m-1}$ ifodalar bir-biriga teng bo‘ladi?

A) -5;6 B) -5;3 C) -5;-3 D) -3;5

14. $\frac{(x^2-9):(3+x)-2}{(x^2-16):(x-4)+2} = (x-1):(x+1)$ tenglamaning ildizini toping

A) 4/5 B) 3,5 C) 1/9 D) x ϵ Ø

15. Tengsizlikni yeching:

$\frac{(2x-x^2-4)(x+3)}{x^2-9} > 0$

A) $(-\infty;2)\cup(2;3)$ B) $(-\infty;3)$ C) $(-\infty;-3)\cup(-3;3)$ D) $(2;3)$

16. Nechta butun son $\sqrt[4]{2x^2+7} - \sqrt{x-2} < 0$ tengsizlikni yechimi bo‘ladi?

A) 0 B) 2 C) 1 D) 3

17. Agar $f(x)$ chiziqli funksiya grafigi koordinatalar boshidan o‘tsa va $f(2)=5$ tenglik o‘rinli bo‘lsa, $f(1)$ ni toping.

A) 2 B) 4 C) 2,5 D) 3,5

18. $f(x) = kx + 3$ funksiya k ning qanday qiymatlarida toq funksiya bo‘ladi?

A) k ning hech bir qiymatida B) $k > 0$ C) $k > 0$ D) $k \in R$

19. $\frac{4^{x^2-5x+6}-1}{2^{x^2-4x+4}-1} = 1$ tenglamaning ildizlari yig‘indisi (yoki ildizi, agar u bitta bo‘lsa) 12 dan qanchaga kam?

A) 4 B) 8 C) 6 D) 10

20. Agar $2^{x-3}\cdot 3^{x+1} = 15$ tenglamaning ildizi x_0 bo‘lsa, $x_o - \frac{1}{lg6}$ ni toping.

A) $\log_6 12$ B) $3\log_6 2$ C) $2\log_6 2$ D) $\log_6 2$

21. Agar $f(x) = log_2 x^3 + 1$ bo‘lsa, $f(2) + f\left(\frac{1}{x}\right) = f(x)$ tenglamani yeching.

A) $\sqrt[4]{8}$ B) 2 C) $\sqrt[3]{4}$ D) $2\sqrt{2}$

22. Agar $f(x+2) = log_3(x^2 - 6x + 27) + 6$ bo‘lsa, $f(2)$ ning qiymatini toping.

A) $6 + log_3 7$ B) $6 + log_3 19$ C) 9 D) 8

23. Ikkita to‘g‘ri chiziq kesishidan hoail bo‘lgan burchaklardan uchtasining yig‘indisi 215^0 ga teng bo‘lsa, shu burchaklardan kattasini toping.

A) 145^0 B) 135^0 C) 115^0 D) 125^0

24. Ixtiyoriy uchtasi bir to‘g‘ri chiziqda yotmagan 10 ta nuqtani o‘zaro tutashtirib, ko‘pi bilan nechta bir xil kesma hosil qilish mumkin.

A) 10 B) 90 C) 55 D) 45

25. Agar prizmaning qirralari soni yoqlari sonidan 24 taga ko‘p bo‘lsa, prizmaning diagonallari soni uning uchlari sonidan nechtaga ko‘p.

A) 91 B) 130 C) 104 D) 117

26. Agar piramidaning qirralari soni bilan uchlari soni yig‘indisi 34 bo‘lsa, uning yoqlari sonini toping.

A) 11 B) 12 C) 10 D) 13

27. Uchlari $A(2;2), B(0;3)$ va $C(0;0)$ nuqtalarda bo‘lgan uchburchakning CM bissektrisasi bo‘lsa, M nuqtaning koordinatalarini toping.

A) $\left(\frac{4}{3};\frac{4}{3}\right)$ B) $\left(\frac{6}{5};\frac{6}{5}\right)$ C) $\left(\frac{5}{6};\frac{5}{6}\right)$ D) $\left(\frac{3}{4};\frac{3}{4}\right)$

28. $A(-9;12;-16)$ nuqtadan Oxy tekslikkacha bo‘lgan masofani toping.

A) 16 B) 20 C) 12 D) 9

29. $f(x) = (x^2+x)\cdot\sqrt{x^2+1}$ funksiyaning $x_0 = 0$ nuqtadagi hosilasini toping.

A) 2 B) 0 C) 01 D) -1

30. $f(x) = (x-1)^{20}\cdot(\cos x + \sin x)$ funksiyaning $x_0 = 0$ nuqtadagi hosilasini toping.

A) –19 B) 19 C) 20 D) –20

31. $\int \frac{x^3}{\sqrt{5+x^4}}dx$ integralni hisoblang.

A) $2\sqrt{5+x^4}+C$ B) $\sqrt{5+x^4}+C$
C) $\frac{1}{2}\sqrt{5+x^4}+C$ D) $4\sqrt{5+x^4}+C$

32. $\int(2x+1)\cos(x^2+x)dx$ integralni hisoblang.

A) $-\cos(x^2+x)+C$
B) $\cos(x^2+x)+C$
C) $\sin(x^2+x)+C$
D) $-\sin(x^2+x)+C$

33.
$$2\left(1-\sin 2\alpha+\text{ctg}\left(\frac{3\pi}{4}-\alpha\right)\cdot\cos 2\alpha\right)+1$$
ifodaning $\alpha = 15^\circ$ dagi qiymatini toping. A) 1 B) $\sqrt{2}$ C) 0 D) –1

34. Ifodani soddalashtirting:
$$\frac{\cos\left(\frac{5\pi}{2}-6\alpha\right)+\sin(\pi+4\alpha)+\sin(3\pi+\alpha)}{\sin\left(\frac{5\pi}{2}+6\alpha\right)+\cos(4\alpha-2\pi)+\cos(\alpha+\pi)}$$
A) cosα B) tgα C) sinα D) ctgα

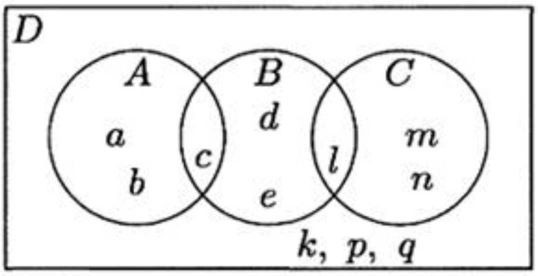
35. [1; 200] sonlar to‘plamida nechta natural son 6 ga (qoldiqsiz) bo‘linib, 9 ga (qoldiqsiz) bo‘linmaydi?

A) 22 B) 33 C) 44 D) 11

36. Tekislikda o‘zaro kesishmaydigan a va b to‘g‘ri chiziqlar berilgan. a to‘g‘ri chiziqda 3 ta, b to‘g‘ri chiziqda 4 ta nuqta belgilangan. Uchlari bu nuqtalarda bo‘lgan jami nechta uchburchak mavjud?

A) 32 B) 30 C) 36 D) 24

37. Rasmdan foydalanib, $((A\cap B)\cup C)\cap D$ to‘plamning elementlari sonini toping.



A) 4 B) 1 C) 3 D) 0

38. Tekislikda o‘zaro kesishmaydigan a va b to‘g‘ri chiziqlar berilgan. a to‘g‘ri chiziqda 2 ta, b to‘g‘ri chiziqda 4 ta nuqta berilgan. Uchlari bu nuqtalarda bo‘lgan jami nechta to‘rtburchak mavjud?

A) 8 B) 5 C) 6 D) 12

39. Quyidagilarning qaysi biri ayniyat?
1) $(x+a)\cdot(x-b)=x^2-(a-b)x-ab$;
2) $(x-c)\cdot(x-d)=x^2-(c+d)x+cd$;
3) $(x-c)\cdot(x+d)=x^2-(c-d)x-cd$;
4) $5a^2-3b^2-((a^2-2ab-b^2)-(5a^2-2ab-b^2))=9a^2-3b^2$;
5) $3a-(2c-(6a-(c-b)+c+(a+8b)-6c))=2a+7b-8c$;

A) 1; 4; 5 B) 2; 3; 4 C) 1; 3; 5
D) 2; 4; 5 E) 1; 2; 3

40. Tengsizliklar uchun quyida keltirilgan xossalardan qaysilari to‘g‘ri?

1) agar $a > b$ bo‘lsa, u holda $b - a > 0$ bo‘ladi;
2) agar $a > b$ va $c > 0$ bo‘lsa, u holda $ac - bc < 0$ bo‘ladi;
3) agar $a > b$ va $c < 0$ bo‘lsa, u holda $ac - bc < 0$ bo‘ladi;
4) agar $a > b$ bo‘lsa, u holda $c - a < c - b$ bo‘ladi;
5) agar $a > b > 0$ va $m > 0$ bo‘lsa, u holda $\frac{m}{a} - \frac{m}{ab} < 0$ bo‘ladi.

A) 3; 4; 5 B) 2; 3; 4 C) 1; 3; 5
D) 1; 2; 4 E) 1; 2; 5

41. Quyida keltirilgan tasdiqlardan qaysilari to‘g‘ri?

1) arifmetik progressiyaning ayirmasi uchun $d = \frac{a_n - a_1}{n-1}$ $(n \neq 1)$ munosabat orinli;

2) $\sin(б + в)$, $sinб \cdot cosв$ va $\sin(б - в)$ sonlar arifmetik progressiyaning ketma-ket keladigan hadlari bo‘ladi;

3) arifmetik progressiyaning dastlabki n ta hadining yig‘indisi uchun $S_n = \frac{2a_1 + (n-1) \cdot d}{2} \cdot n$ formula o‘rinli;

4) cheksiz kamayuvchi geometrik progresssiyaning yig‘indisi $S = \frac{b_1}{q-1}$ formula bilan aniqlanadi;

5) geometrik progresssiyaning progressiyaning dastlabki n ta hadining yig‘indisi uchun $S_n = \frac{b_1(1-q^n)}{q-1}$ $(q \neq 1)$ formula o‘rinli.

A) 2; 4; 5 B) 1; 2; 4 C) 3; 4; 5
D) 1; 3; 5 E) 1; 2; 3

42. Korsatlichli va logarifmik funksiyalar uchun quyida keltirilgan xossalardan qaysilari to‘g‘ri?

1) $y = a^x$ $(a > 0, a \neq 1)$ funksiyaning aniqlanish shasi - barcha musbat sonlar to‘plamidan iborat;

2) $y = a^x$ $(a > 0, a \neq 1)$ funksiyaning qiymatlar to‘plami - barcha musbat haqiyqiy sonlar to‘plamidan iborat;

3) $y = a^x$ $(a > 0, a \neq 1)$ funksiya $0 < a < 1$ bo‘lganda barcha haqiyqiy sonlar to‘plamida o‘suvchi, $a > 1$ bo‘lganda esa kamayivchi bo‘ladi;

4) logarifmik funksiyaning aniqlanish sohasi – barch musbat sonlar to‘plamidan iborat;

5) $y = log_a x$ logarifmik funksiya $x > 0$ oraliqda, agar $a > 1$ bo‘lsa, o‘suvchi, $0 < a < 1$ bo‘lsa, kamayuvchi bo‘ladi.

A) 2; 4; 5 B) 1; 3; 4 C) 2; 3; 5
D) 1; 2; 4 E) 1; 3; 5

43. Quyida keltirilgan tasdiqlardan qaysilari noto‘g‘ri?

1) yuzasi S va tomonlari a, b, c bo‘lgan uchburchakka tashqi chizilgan aylananing radiusi $R = \frac{a \cdot b \cdot c}{4S}$ formula bilan aniqlanadi;

2) tomonlari a, b va c bo‘lgan uchburchakka ichki chizilgan aylananing radiusi $r = \frac{4S}{a+b+c}$ formula bilan aniqlanadi;

3) tomonlari a va b ga, ular orasidagi burchak б ga teng bo‘lgan parallelogramm-ning yuzi $S = \frac{1}{2} a \cdot b \cdot sinб$ formula bilan aniqlanadi;

4) tomonlari a va b ga, ular orasidagi burchak б ga teng bo‘lgan uchburchakning yuzi $S = \frac{1}{2} a \cdot b \cdot sinб$ formula bilan aniqlanadi;

5) oxshash shakillar yuzalarining nisbati ularning mos chiziqli olchovlarining nisbatiga teng.

A) 1; 3; 5 B) 2; 4; 5 C) 1; 2; 4
D) 2; 3; 5 E) 1; 3; 4

44. To‘g‘ri to‘rtburchakning bo‘yi 25% ga oshirildi. Uning yuzi o‘zgarmay qolishi uchun eni necha foizga kamaytirish kerak.

A) 25 B) 20 C) 30 D) 15

45. Quyida keltirilgan tasdiqlardan qaysilari to‘g‘ri?

1) piramidaning hajmi asosining yuzi bilan balandligi ko‘paytmasining uchdan bir qismiga teng;

2) silndrning hajmi asosining yuzi bilan balandligi ko‘paytmasining uchdan bir qismiga teng;

3) sharning hajmi $\frac{4}{3}$ р $\cdot R^3$ ga teng;

4) asosining radiusi R ga, balandligi h ga teng bo‘lgan silindr yon sirtining yuzi р $\cdot R \cdot h$ ga teng;

5) radiusi R ga, segmentining balandligi H ga teng bo‘lgan shar sektorining hajmi $\frac{2}{3}$ p $\cdot R^2 \cdot H$ ga teng.

A) 1;3;4 B) 2;3;4 C) 3;4;5
D) 2;3;8 E) 3;4;5

V-14

1. a va b raqamlar yig‘indisi 7 ga qoldiqsiz bo‘linadi. Agar $\overline{abc}$ ko‘rinishdagi uch xonali sonlarni 7 ga bo‘lganda bir xil qoldiq qolsa, shu qoldiqni toping.

A) 0 B) 4 C) 2 D) 6

2. a va b raqamlar yig‘indisi 7 ga qoldiqsiz bo‘linsa, $37a + 9b$ ni 7 ga bo‘lgandagi qoldiqni topin.

A) 2 B) 1 C) 0 D) 6

3. Hisoblang:

$$\underbrace{-2019 + 2019 - 2019 + \cdots + 2019 - 2019}_{2019\ ta}$$

A) 0 B) 2019 C) -2019 D) 2018

4. Hisoblang: $8{,}6(7) - 3{,}(8)$

A) $\frac{451}{99}$ B) $\frac{431}{99}$ C) $\frac{431}{90}$ D) $\frac{451}{90}$

5. Biri ikkinchisidan 3marta kata bo‘lgan ikki sonning yig‘indisi 9,64 ga teng. Shu sonlarning kichigini toping.

A) 2,31 B) 2,41 C) 2,16 D) 2,21

6. Oltita sonning yig‘indisi 70 ga teng. Ulardan birini 2 marta oshirib, qolganlarini o‘zgartirmagan holda yig‘indisi 78 ga teng bo‘lsa, o‘zgartirilmagan beshta sonning yig‘indisini toping.

A) 62 B) 67 C) 63 D) 66

7. $19^{123} - 27 \cdot 26 \cdot 23 \cdot 22$ ayirma qanday raqam bilan tugaydi?

A) 4 B) 5 C) 9 D) 7

8. $((a^{-2})^{-3} \cdot (a^3)^{-1} \cdot a^{-2})^3 : a$ ifodaning $a = -1/6$ dagi qiymatini toping.

A) $1/36$ B) -6 C) $-1/6$ D) 36

9. x ning qanday qiymatida $\frac{4}{x-3}$; $\frac{2}{3}$ va $\frac{8-x}{x-3}$ lar berilgan tartibda arifmetik progressiyaning ketma-ket hadlari bo‘ladi?

A) $4\frac{6}{7}$ B) $4\frac{3}{7}$ C) $6\frac{6}{7}$ D) $6\frac{3}{7}$

10. Hadlari $x_n = 4n^2 + cn + 2$ formula bilan berilgan ketma-ketlikda $x_4 - x_2 = \ = 52$ bo‘lsa, bu ketma-ketlikning uchinchi hadini toping.

A) 44 B) 52 C) 56 D) 50

11. Agar $P = 3a^2 + 4b,$ $Q = -2a^2 - 3b$ bo‘lsa, $P + Q + 4b$ ni toping.

A) $a^2 + 5b$ B) $-a^2 - 5b$
C) $-a^2 + 5b$ D) $a^2 - 5b$

12. Soddalashtiring:
$(5a - 3b) - (-4b + 7a)$

A) $-2a - b$ B) $2a + b$
C) $-2a + b$ D) $2a + b$

13. $(x + 2)^2 + 15 = 8|x + 2|$ tenglamaning barcha ildizlari yig‘indisini toping.

A) -14 B) -8 C) 6 D) 16

14. $\frac{(x^4-16)\sqrt{12-x-x^2}}{\sqrt{-x}} = 0$ tenglamaning haqiqiy ildizlari ko‘paytmasini toping.

A) 16 B) 8 C) 6 D) -8

15. Quyidagi tengsizliklardan qaysi biri a ning har qanday haqiqiy qiymatida o'rinli bo'ladi?
A) $4a^2+7a>(a+3)(a+4)-12$
B) $(a-6)(a+2)+a^2\le(a-5)(a+1)-21$
C) $a^2+(a-2)(a-4)<(a+2)(a+4)$
D) $(a-1)(a+3)+3a^2>(a+4)(a-2)-4$

16. Tengsizlikni yeching:
$$\frac{(x^2+2x+1)(x-3)(x+4)}{x^2-4x+4}<0$$
A) $(-4;-1)\cup(2;3)$
B) $(-4;-1)\cup(-1;2)\cup(2;3)$
C) $(-\infty;-4)\cup(3;\infty)$
D) $(-\infty;-4)\cup(-1;2)\cup(3;\infty)$

17. Agar $k<0$, $b>0$ bo'lsa, $y=kx+b$ chiziqli funksiyaning grafigi qaysi chorak-larda yotadi?
A) I, II va III B) I, II va IV
C) II, III va IV D) I, III va IV

18. Agar $f(x)$ o'zgarmas funksiya uchun $f(2)=3$ bo'lsa, $f(1)$ ni toping.
A) 1 B) 3 C) 2 D) aniqlab bo'lmaydi

19. $\left(\frac{1}{4}\right)^x-\left(\frac{1}{2}\right)^x\le 12$ tengsizlikning (-4; 4) oralig'idagi butun yechimlar sonini toping. A) 3 B) 2 C) 5 D) 6

20. $x^2\cdot 3^{\sqrt{x+2}}-9x^2=6\cdot 3^{\sqrt{x+2}}-54$ tenglamaning ildizlari kvadratlarining yig'indisini toping.
A) 16 B) 4 C) 10 D) 12

21. Hisoblang:
$$\sqrt{(log_2 3+4log_3 2-4)\cdot log_2 3}+log_2 12$$
A) 4 B) $log_2 9$ C) 2 D) 8

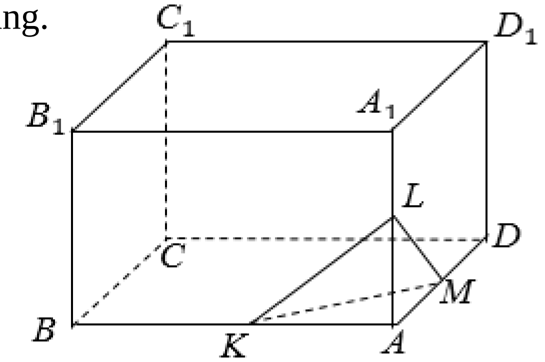
22. $y=\sqrt{\frac{2}{lg(x-2)}-3}-1$ funksiya grafigi abssissalar o'qini qaysi nuqtasida kesib o'tadi?
A) (102; 0) B) kesib o'tmaydi
C) (0; 102) D) $(\sqrt{10}+2;0)$

23. Ikkita to'g'ri chiziq kesishidan hoail bo'lgan burchaklardan biri ikkinchsidan 36^0 ga katta bo'lsa, ularning nisbatini toping.
A) 6:5 B) 4:3 C) 5:4 D) 3:2

24. *ABC* uchburchakning burchaklari 2:3:1 kabi nisbatda. Agar eng kichik tomoni 5 sm bo'lsa, uning eng katta tomoni uzun-ligini (sm) toping.
A) 8 B) 10 C) $5\sqrt{3}$ D) $5\sqrt{2}$

25. Agar prizmaning diagonallari soni 54 bo'lsa, uning qirralari soni yoqlari sonidan nechtaga ko'p.
A) 14 B) 16 C) 20 D) 98

26. To'g'ri burchakli parallelepipedning *A* uchidan chiquvchi 8; 9 va 12 dm qirra-laridan mos ravishda *A* nuqtadan boshlab hisoblaganda qirralari 3; 5 va 6 dm bo'lgan piramida qirqib olindi (rasmga qarang). Uning qolgan qismining hajmini (dm^3) hi-soblang.



A) 774 B) 834 C) 819 D) 849

27. $A(a;-1)$, $B(1-a;2a+1)$ va $C(a+1;-3)$ nuqtalar bitta to'g'ri chiziq-da yotsa, shu to'g'ri chiziq tenglamasini tuzing.
A) $y=-2x+3$ B) $y=2x+3$
C) $y=-3x+2$ D) $y=3x+2$

28. Uchlari $A(2;2)$ va $B(6;6)$ nuqtalar-da bo'lgan AB kesmaning C o'rtasidagi nuqta. AB kesmaga perpendikulyar va C nuqtadan otuvchi to‘g‘ri chiziq tenglamasini tuzing.

A) $-x-y+8=0$ B) $2x-y+12=0$
C) $x+y-8=0$ D) $-x+y+8=0$

29. $f(x)=ln\sqrt{x^2-4x+5}+3x$ funksiyaning $x_0=0$ nuqtadagi hosilasini toping.

A) 13/5 B) 22/5 C) 17/5 D) 12/5

30. $f(x)=(2x+1)^5\cdot\sqrt{x^6+16}$ funksiya-ning $x_0=0$ nuqtadagi hosilasini toping.

A) 20 B) 10 C) 40 D) 5

31. $\int\frac{(x-2)dx}{x^2-4x+17}$ integralni hisoblang.

A) $\ln(x^2-4x+17)^{-2}+C$
B) $\ln\sqrt{x^2-4x+17}+C$
C) $\ln(x^2-4x+17)^2+C$
D) $\ln(x^2-4x+17)+C$

32. $\int\frac{(x-2)dx}{\sqrt{x^2-4x+8}}$ integralni hisoblang.

A) $\sqrt{x^2-4x+8}+C$
B) $-\sqrt{x^2-4x+8}+C$
C) $-\frac{1}{2}\sqrt{x^2-4x+8}+C$
D) $\frac{1}{2}\sqrt{x^2-4x+8}+C$

33. Agar $tg\alpha=\sqrt{7}$ bo‘lsa, $\frac{4\sin^4\alpha}{5\sin^2\alpha+15\cos^2\alpha}$ ifodaning qiymatini toping.

A) 0,5 B) 0,49 C) 0,47 D) 0,48

34. $\frac{1}{2}(1+\cos^{-1}2\alpha+tg2\alpha)\cdot(1-\cos^{-1}2\alpha+tg2\alpha)$ ifodaning $\alpha=15°$ dagi qiymatini toping.

A) $\frac{2}{\sqrt{3}}$ B) $2\sqrt{3}$ C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{3}$

35. A to‘plam 48 sonining butun bo‘luvchilaridan tashkil topgan bo‘lsa, A to‘plamning elementlari sonini aniqlang.

A) 20 B) 18 C) 10 D) 16

36. A va B to‘plamlarning elementlari mos ravishda 24 va 36 sonlarining natural bo‘luvchilaridan iborat bo‘lsa, $A\cap B$ to‘plamning elementlari sonini aniqlang.

A) 6 B) 4 C) 8 D) 12

37. Markazlari har xil nuqtalarda bo‘lgan 3 ta aylana ko‘pi bilan nechta nuqtada kesishadi?

A) 3 B) 6 C) 4 D) 7

38. Tog‘ning cho‘qqisiga 8 ta yo‘l olib boradi. Borgan yo‘lidan qaytmaslik sharti bilan tog‘ning cho‘qqisiga jami necha xil usulda borib kelish mumkin?

A) 42 B) 56 C) 28 D) 21

39. Quyidagilarning qaysi biri ayniyat emas?

1) $(x+a)\cdot(x-b)=x^2-(a-b)x-ab$;
2) $(x-e)\cdot(x+d)=x^2-(e-d)x-ed$;
3) $12x^2+y^2-(8x^2-5y^2-(-10x^2+(5x^2-6y^2)))=-x^2+12y^2$;
4) $6ab+(2a^3+b^3-(3ab^2-(a^3+2ab^2-b^3)))=3a^3-ab^2+6ab$;
5) $3a-(2c-(6a-(c-b)+c+(a+8b)-6c))=2a+7b-8c$;

A) 2; 4; 5 B) 1; 2; 4 C) 2; 3; 5
D) 1; 3; 4 E) 1; 3; 5

40. Tengsizliklar uchun quyida keltirilgan xossalardan qaysilari to‘g‘ri?

1) agar $a > b$ va $b > c$ bo‘lsa, u holda $a - c < 0$ bo‘ladi;

2) agar $a > b$ va $c > 0$ bo‘lsa, u holda $ac - bc > 0$ bo‘ladi;

3) agar $a > b$ bo‘lsa, u holda $b - c < a - c$ bo‘ladi;

4) agar $a > b$ bo‘lsa, u holda $c - a < c - b$ bo‘ladi;

5) agar $a > b > 0$ va $m > 0$ bo‘lsa, u holda $\frac{m}{a} - \frac{m}{b} > 0$ bo‘ladi.

A) 1; 2; 4 B) 2; 3; 4 C) 3; 4; 5
D) 1; 2; 5 E) 1; 3; 5

41. $a_1, a_2, \dots, a_n$ $(d \neq 0)$ arifmetik progressiya berilgan. Quyidagi sonlardan qaysilari arifmetik progressiyani tashkil qiladi?

1) $a_1, a_3, a_5, \dots, a_{2n-1}$
2) $\sqrt{a_2}, \sqrt{a_4}, \sqrt{a_6}, \dots, \sqrt{a_{2n}}$
3) $a_1 + a_2, a_2 + a_3, a_3 + a_4, \dots, a_{2n-1} + a_{2n}$
4) $\sqrt{a_1 + a_3}, \sqrt{a_3 + a_5}, \sqrt{a_5 + a_7}, \dots, \sqrt{a_{2n-3} + a_{2n-1}}$
5) $a_2 + a_3, a_3 + a_4, a_4 + a_5, \dots, a_{2n-2} + a_{2n-1}$

A) 1; 2; 4 B) 1; 3; 4 C) 2; 4; 5
D) 2; 3; 5 E) 1; 3; 5

42. Korsatlichli va logarifmik funksiyalar uchun quyida keltirilgan xossalardan qaysilari noto‘g‘ri?

1) $y = a^x$ $(a > 0, a \neq 1)$ funksiyaning qiymatlar to‘plami - barcha haqiyqiy sonlar to‘plamidan iborat;

2) $y = a^x$ $(a > 0, a \neq 1)$ funksiya $0 < a < 1$ bo‘lganda barcha haqiyqiy sonlar to‘plamida o‘suvchi, $a > 1$ bo‘lganda esa kamayivchi bo‘ladi;

3) logarifmik funksiyaning aniqlanish sohasi – barch musbat sonlar to‘plamidan iborat;

4)) $y = log_a x$ logarifmik funksiya $x > 0$ oraliqda, agar $a > 1$ bo‘lsa, o‘suvchi, $0 < a < 1$ bo‘lganda esa, kamayuvchi bo‘ladi;

5) agar $a > 1$ bo‘lsa, u holda $y = log_a x$ funksiya $x > 1$ da manfiy qiymatlar, $0 < x < 1$ da musbat qiymatlarni qabul qiladi. A) 1; 2; 5 B) 2; 3; 4 C) 1; 3; 5
D) 2; 4; 5 E) 1; 3; 4

43. Quyida keltirilgan tasdiqlardan qaysilari to‘g‘ri?

1) tomonlari a, b va c bo‘lgan uchburchakka ichki chizilgan aylananing radiusi $r = \frac{2S}{a+b+c}$ formula bilan aniqlanadi;

2) tomoni a ga, burchaklaridan biri б ga teng bo‘lgan rombning yuzi $S = \frac{1}{2} a^2 sin$б formula bilan aniqlanadi;

3) tomonlari a va b ga, ular orasidagi burchak б ga teng bo‘lgan parallel-gramm-ning yuzi $S = a \cdot b \cdot sin$б formula bilan aniqlanadi;

4) tomonlari a va b ga, ular orasidagi burchak б ga teng bo‘lgan uchburchakning yuzi $S = a \cdot b \cdot sin$б formula bilan aniqlanadi;

5) dioganallari d_1 va d_2 ga, ular orasidagi burchak б ga teng ixtiyoriy qavariq tortburchakning yuzi $S = \frac{1}{2} d_1 \cdot d_2 \cdot sin$б formula bilan aniqlanadi; teng bo‘lgan

A) 2; 3; 4 B) 1; 3; 5 C) 2; 4; 5
D) 1; 3; 4 E) 1; 2; 5

44. Trapetsiyaning o‘rta chizigi va unga ichki chizilgan aylana radiusi 2 marta oshirilsa, uning yuzi necha marta oshadi?

A) 4 B) 6 C) 8 D) 2

45. Shar segmentining balandligi 4 marta oshirilib, uning radiusi 2 marta kamaytirilsa, shar sektorininr hajmi qanday o‘zga-radi?

A) 8 marta oshadi B) 8 marta kamayadi
C) o‘zgarmaydi D) 2 marta oshadi

I L O V A

1.Raqamlar.

Hozirda biz foydalaniladigan 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 raqamlari arab raqamlaridir, bundan tashqari rim raqmlari ham mavjid, ular I-bir, II-ikki, III-uch, IV-to‘rt, V-besh, VI-olti, VII-yetti, VIII-sakkiz, IX-to‘qqiz, X-o‘n. L-ellik, G-yuz, D-beshyuz, M-ming.

Agar katta raqamdan keyin kichik raqam yozilsa, u son bu raqamlarining qiymatlari yig‘indisiga teng bo‘ladi.

XV = 10 + 5 = 15 ; LXX = 50 + 10 + 10 = 70.

MDGLXVIII = 1000 + 500 + 100 +50 + 10 + 8 = 1668 sonini ifodalaydi.

Agar katta raqam oldiga kichik raqam yozilsa, u bu raqamlar ayirmasiga teng sonni ifodalaydi: IX = 10 – 1 = 9; XG = 100 - 10 = 90; GGGXGVII = 397.

$10^6 -$ million; $10^9 -$ milliard; $10^{12} -$ trillion; $10^{15} -$ kvadrillion;
$10^{18} -$ kvintillion.

2. Ayrim birliklar.

$1\ gaz = 71{,}12\ sm$; $1\ duym = 2{,}54\ sm$; $1\ arshin = 71\ sm$; $1\ mil = 1609\ m$.
$1\ fut = 30{,}48\ sm$; $1\ tanob = 1821\ m^2$; $1\ barrel = 159\ l$; $1\ arpa = 0{,}045\ g$.
$1\ misqol = 4\ g$.

3. Bo‘linish alomatlari.

1. Oxirgi raqami juft son yoki 0 bilan tugaydigan sonlar 2 ga qoldiqsiz bo‘linadi.
M: $12, 20, 32, 336$.

- Raqamlarining yig‘indisi 3 ga bo‘linadigan sonlar 3 ga qoldiqsiz bo‘linadi.
M: $384; 3+8+4=15.\ 687; 6+8+7=21.$
- Berilgan sonnig oxirgi ikki raqami 4 ga yoki 0 ga bo‘linsa, berilgan son 4 ga bo‘linadi. M: $1692.\ 92:4\ 23.\ \ 200:4=50.$
- Oxirgi raqami 0 yoki 5 bilan tugaydigan sonlar 5 ga qoldiqsiz bo‘linadi.
M: 70 yoki 135.
- 2 va 3 ga bo‘linadigan sonlar 6 ga qoldiqsiz bo‘linadi.
M: 24 yoki 42.
- Berilgan sondagi o‘nlar xonasidagi sondan birlar xonasidagi raqamning ikkilanganini ayirib, ayirma 7 ga bo‘linsa, berilgan son 7 ga bo‘linadi.
M: $91.\ 9-2\cdot 1=7.\ 1134.\ 113-2\cdot 4=105=7\cdot 15.$
- Oxirgi uchta raqami 0 yoki 8 ga bo‘linsa, berilgan son 8 ga bo‘linadi.
M: 2048 yoki 2000.
- Raqamlar yig‘indisi 9 ga bo‘linadigon sonlar 9 ga qoldiqsiz bo‘linadi.
M: 5058 yoki 78804.
- Oxirgi raqami 0 bo‘lgan sonlar 10 ga qoldiqsiz bo‘linadi.
- Berilgan sonning toq o‘rnidagi raqamlari yig‘indisi bilan juft o‘rnidagi raqamlari yig‘indisining ayirmasi 0 bo‘lsa yoki 11 ga bo‘linsa, berilgan son 11 ga bo‘linadi.
M: $50457.\ (5+4+7)-(0+5)=11.$
- Oxirgi ikkita raqami 0 yoki 25 ga bo‘linsa berilgan son 25 ga bo‘linadi.
M: $500, 1625, 2875.$

2. Qolgan sonlarga bo‘linish belgilarni bevosita ularni tub ko‘paytuvchilarga ajratish yo‘li bilan topiladi.

3. Agar qo‘shiluvchilardan har biri ma‘lum bir songa bo‘linsa, yig‘indi ham shu son-ga bo‘linadi

4. Agar kamayuvchi va ayriluvchi sonlar ma‘lum bir songa bo‘linsa, ayirma ham shu songa bo‘linadi.

5. Agar ko‘paytiriluvchilardan biri ma‘lum bir songa bo‘linsa, ko‘paytma ham shu songa bo‘linadi.

Ikki xonali sonni quyidagi ko‘rinishda yozish mumkin:

$$\overline{xy} = 10\overline{x} + \overline{y} \qquad (1)$$

bu yerda $\overline{x}$*-ikki xonali sonning birinchi raqamii,* $\overline{y}$*-ikki xonali sonning ikkinchi raqami.*

Ikki xonali son o‘z raqamlarining yig‘ndisidan necha marta kattaligini aniqlash:

$$n = \frac{\overline{xy}}{\overline{x}+\overline{y}} \qquad (2)$$

Natural sonlarni kvadratini aniqlash :

Ikki xonali sonlar uchun;

$(\overline{xy})^2 = 100 \cdot (\overline{x})^2 + 20 \cdot (\overline{x}) \cdot (\overline{y}) + (\overline{y})^2$ ketma-ket kelgan natural son bo‘lib, ular 10…99 oraliqda bo‘ladi.

Bu formulani isbotini (1) formulani ikkala tomonini kvadratga oshirib, isbotlash mumkin.

M: $(87)^2 = 100 \cdot (8)^2 + 20 \cdot (8) \cdot (7) + (7)^2 =$

$= 64 \cdot 100 + 20 \cdot 56 + 49 = 6400 + 1120 + 49 = 7529.$

Uch xonali sonlar uchun;

$(\overline{xyz})^2 = 100 \cdot (\overline{xy})^2 + 20 \cdot (\overline{xy}) \cdot (\overline{z}) + (\overline{z})^2$ ketma-ket kelgan natural son bo‘lib, ular 100…999 oraliqda bo‘ladi.

M: $(871)^2 = 100 \cdot (87)^2 + 20 \cdot (87) \cdot (1) + (1)^2 =$

$= 100 \cdot 7569 + 1740 + 1 = 756900 + 1740 + 1 = 758641.$

4. a^n sonnni *b* soniga bo‘lgandadi qoldiqni topish:

Uni quyidagi ko‘rinishda yozamiz; $a^n = (b \cdot d + c)^n$ va bundan c^n ni *b* ga bo‘lib, qoldiq topiladi.

M1: 3^{24}*sonini* 7 *soniga bo‘lgandagi qoldiqni toping:*

<u>Yechich:</u>

$3^{24}: 7 \Rightarrow (81)^6: 7 \Rightarrow (7 \cdot 11 + 4)^6: 7 \Rightarrow 64^2: 7 \Rightarrow (7 \cdot 9 + 1)^2: 7 \Rightarrow 1^2: 7 \Rightarrow 1: 7$. *Demak qolgiq* 1 *ga teng ekan.*

M2: 7^8*sonini 13 soniga bo‘lgandagi qoldiqni toping:*

<u>Yechich:</u>

$7^8: 13 \Rightarrow (49)^4: 13 \Rightarrow (3 \cdot 13 + 10)^4: 13 \Rightarrow (100)^2: 13 \Rightarrow (7 \cdot 13 + 9)^2: 13 \Rightarrow 9^2: 13 \Rightarrow 81: 13 \Rightarrow (6 \cdot 13 + 3): 13 \Rightarrow 3: 13$. *Demak qolgiq* 3 *ga teng ekan.*

5. Kvadrat ildizdan ildiz chiqarish.

1) $\sqrt{676} = 26$ 2) $\sqrt{1369} = 37$ 3) $\sqrt{12544} = 112$

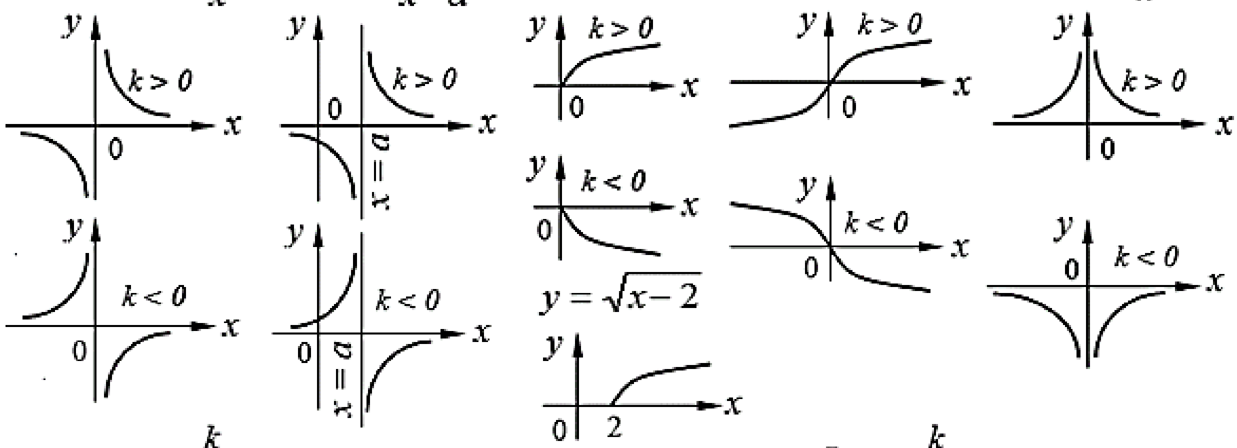
$\sqrt{6‘76} = 26$: -4; 46×6: $276 - 276 = 0$

$\sqrt{13‘69} = 37$: -9; 67×7: $469 - 469 = 0$

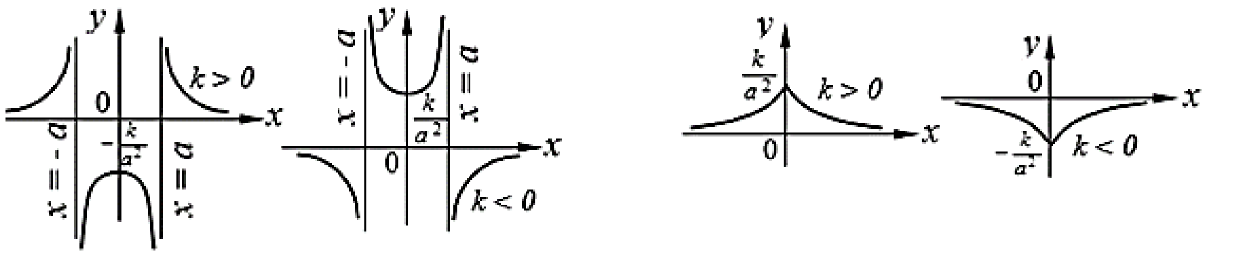
$\sqrt{1‘25‘44} = 112$: -1; 21×1: $25 - 21$; 222×2: $444 - 444 = 0$

6. Ayrim funksiyalarning grafiklari.

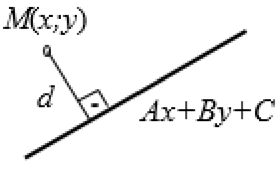
1. $y = \frac{k}{x}$ 2. $y = \frac{k}{x-a}$ 3. $y = k \cdot \sqrt{x}$ 4. $y = k \cdot \sqrt[3]{x}$ 5. $y = \frac{k}{x^2}$



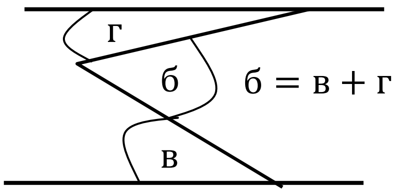
6. $y = \frac{k}{x^2 - a^2}$ 7. $y = \frac{k}{x^2 + a^2}$



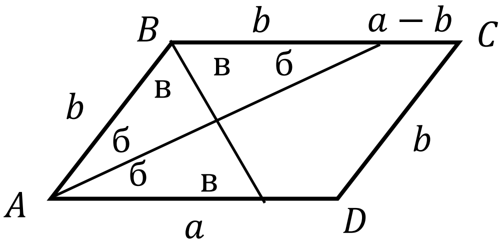
Qo‘shimcha ma’limolar.



$d = \frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$

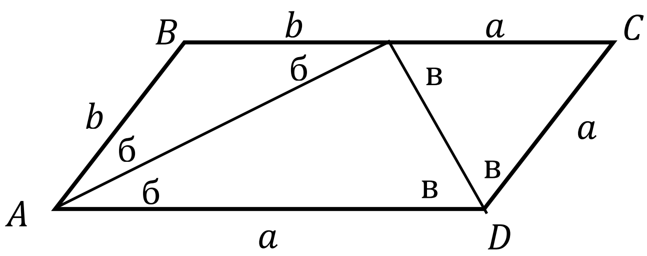


б = в + г



$b - (a - b) > 0$

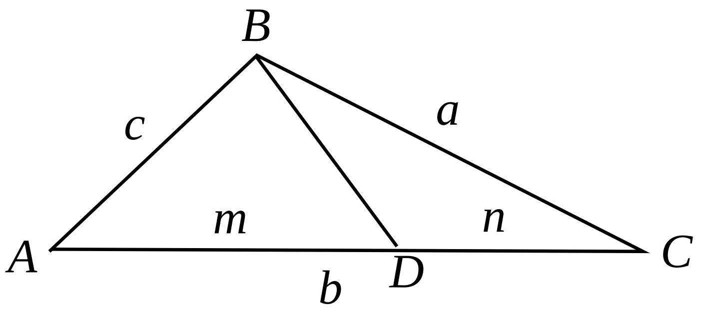
$2b > a \quad 2AB > AD$

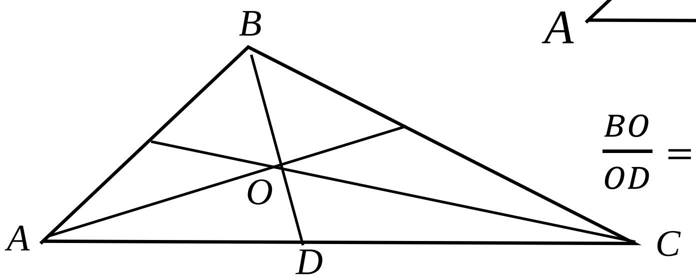


$b = (a - b) \quad 2b = a \quad 2AB = AD$

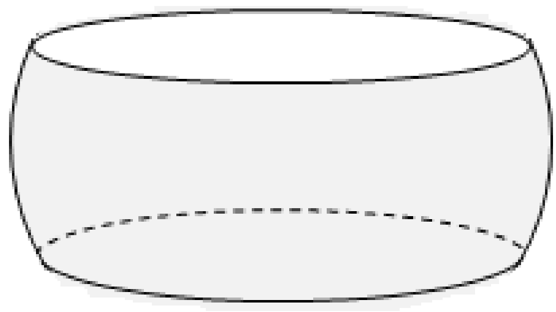
Styuvard teoremasi.

$$(BD)^2 = \frac{a^2 \cdot m + c^2 \cdot n}{m+n} - m \cdot n$$





$$\frac{BO}{OD} = \frac{AB+BC}{AC}$$



Shar kamarining hajmi:

$$V_{kam} = \frac{2}{3} \mathrm{p} \cdot (2 \cdot R^3 - 3 \cdot H^2 \cdot R + H^3)$$

Shar kamarining yuzasi:

$$S_{kam} = 4\mathrm{p}R(R-H)$$

Istalgan qavariq ko‘pyoqli uchun Eyler teoremasi.

$$N + F - L = 2$$

bu yerda *N*- ko‘pyoqli uchining soni; *F*- ko‘pyoqli yoqlariining soni; *L*- ko‘pyoqli qirralarining soni.

1-jadval

Ko‘pyoqlining ko‘rsatkichlari.

Ko‘pyoqlining nomi	*N*- uchining soni	*L*- qirralarining soni	*F*- yoqlariining soni
Uchburchakli piramida	4	6	4
To‘rtburchakli piramida	5	8	5
Uchburchakli prizma	6	9	5
To‘rtburchakli prizma	8	12	6
n-burchakli piramida	*n* +1	2*n*	*n* +1
n-burchakli prizma	2*n*	3*n*	*n* +2
n-burchakli kesik piramida	2*n*	3*n*	*n* +2

Test savollarining javoblari.

V	Savol javoblari																	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1.	D	A	C	A	A	C	D	B	C	B	C	A	A	D	A	C	B	D
2.	B	D	D	D	A	C	D	C	B	B	A	C	C	B	B	A	B	D
3.	C	D	B	A	A	A	D	B	A	B	C	D	C	C	B	A	D	A
4.	C	D	D	C	B	A	A	D	A	A	A	A	A	C	D	A	D	B
5.	D	A	D	A	C	B	A	A	D	D	D	A	B	C	B	D	A	D
6.	D	C	B	D	B	A	A	A	D	C	A	B	D	D	B	C	A	C
7.	D	A	D	A	B	A	A	B	D	A	C	B	B	B	D	D	B	D
8.	A	A	B	B	A	B	D	A	C	B	D	C	B	A	C	A	B	C

9.	B	A	D	B	A	B	B	C	D	D	B	A	D	A	D	A	A	C
10.	B	B	B	D	D	C	A	B	D	D	C	A	D	D	C	B	A	B
11.	A	C	B	A	A	D	C	B	D	B	B	C	A	C	D	B	A	D
12.	C	A	A	D	A	A	A	C	C	A	A	B	B	B	C	C	B	C
13.	C	A	B	D	C	B	A	A	B	A	B	A	C	C	C	C	A	A
14.	A	C	C	C	B	A	C	A	C	A	A	C	B	B	A	B	B	B
V	Savol javoblari																	
	19	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1.	C	C	A	D	B	A	D	C	A	B	C	C	B	C	D	B	C	B
2.	B	C	B	C	D	B	B	A	B	C	C	D	D	C	A	B	C	D
3.	B	C	C	A	C	D	B	B	D	D	D	A	C	B	A	C	D	B
4.	C	A	D	B	A	C	D	C	D	D	A	D						
5.	C	C	A	B	D	D	C	C	B	C	D	C						
6.	C	A	B	C	B	D	C	A	D	A	D	A						
7.	B	B	C	C	C	D	B	A	B	D	A	A						
8.	B	A	D	B	A	C	A	C	D	A	B	C						
9.	C	A	A	A	A	A	A	A	B	D	D	C						
10.	D	C	A	A	D	D	C	D	A	B	C	A						
11.	B	C	A	C	D	C	B											
12.	A	A	D	B	C	A	C											
13.	B	C	C	C	B	A	B	B	B	A	C	A	C	C	A	B	B	A
	37	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**									
	B	C	B	A	E	A	D	B	A									
14.	D	C	B	D	D	B	B	B	A	C	A	C	A	A	B	C	C	D
	37	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**									
	B	B	E	B	E	A	B	D	C									

Foydalanilgan adabiyotlar.

1. Aduhamidov A.U. va boshqalar. Algebra va matematik analiz asoslari (Akademik litseylar uchun darslik). I, II-qismlar. Toshkent. «O'qituvchi» NMIU, 2013.
2. Антонов Н.П., и др. Сборник эадач по элементарной математике. Москва. «Наука»,1973.
3. Егерев В.К., и др (под ред. М.И. Сканави). Сборник эадач по математике для поступающих во ВУЗы. Минск. «Высшая школа»,1990.
4. Гусев В.А., Мордкович А.Г. Математика (Ўқувчилар учун ўқув қўлланма). Тошкент. «Ўқитувчи»,1988.
5. Haydarov B.Q., Sariqov E.S., Qo'chqorov A.Sh. Geometriya (9-sinf uchun darslik). Toshkent. «O'zbekiston» NMIU, 2010.
6. Husanov O', Po'latov M. G'olipov N. Matematika (Oliy o'qiv yurtlariga kiruvchilar uchun ma'lumotnoma). 1,2-kitob, 2-nashr. Toshkent. «O'qituvchi» NMIU, 2012.
7. Лидский В.Б., и др. Задачи по элементарной математике. Москва. «Наука»,1973.
8. Mahmudov A. Abituriyent gazetasidan matematika testlar to'plami-3). Toshkent. «Tafakkur qanoti», 2014.
9. Mirzaahmedov M.A. va boshqalar «Matematikani» (Umumiy o'rta ta'lim maktablarning 11-sinflari va o'rta maxsus, kasb-hunar ta'limi muassasalari uchun darslik I va II bo'lim). Toshkent. «O'ZBEKISTON», 2018.
10. Sharifboyev Yo.S. Matematika (Oliy o'qiv yurtlariga kiruvchilar uchun ma'lumotnoma). Toshkent. «Turon-iqbol», 2010.
11. Сайдаматов Э.М и др. «Алгебра и основы математического анализа». Ташкент. «ILM ZIYO», 2007.
12. Turdiyev N.Sh. Matematika (Oliy o'qiv yurtlariga kiruvchilar uchun ma'lumotnoma). Toshkent. «Vektor-press», 2008.
13. Умирбеков А.У., Шаабзалов Ш.Ш. Математикани такрорланг (Олий ўқув юртларига кирувчилар учун қўлланма). Тошкент. «Ўқитувчи»,1989.
14. Вафоев С.Т. Олий ўқув юртларига кирувчилар учун ўтказиладиган тестлар ва уларнинг саволлари ҳақида. «Ta'lim texnologiyalari» журналининг № 3-сони, 25-29 бетлар. Тошкент, 2006 й.
15. Vafoyev S.T. va boshqalar «Matematikani takrorlang» (Oliy o'qiv yurtlariga kiruvchilar uchun o'quv qo'llanma). Toshkent. «Extremum-press», 2017.
16. Погорелов А.В. Геометрия (7-11 синфлар учун ўқув қўлланма). Тошкент, «Ўқитувчи», 1990.

M U N D A R I J A

VAFOYEV SAFO TO‘RAYEVICH
VAFOYEVA OZODA SAFOYEVNA
VAFOYEV RUSTAM SAFO O‘G‘LI
VAFOYEV O‘TKIRBEK SAFO O‘G‘LI

MATEMATIKA

Oliy o‘quv yurtlari kiruvchilar uchun o‘quv qo‘llanma

O‘quv qo‘llanma O‘z.R. FA ning akademiki, t.f.d., profeccor M.M.Mirsaidov tahriri ostida tayyorlandi

Bosishga ruxsat etildi 2023 yil -------. Qog‘oz o‘lchami 60x84 - 1/16,
Shartnoma № Hajmi: 11 b.t. Adadi 50 nusxa