Topic 9.

Derive application Private derivative

Derivatives different issues solution in doing is used . An example by doing in other words , a function to the graph given at the point don't try equation in making and of the thigh approx value in the calculation , of the function the most small and the most big values in determining is used . Below this issues derivative using solve formulas given .

y function to the graph x_0 at the point don't try equation :

$$y = f(x_0) + f'(x_0)\cdot(x - x_0)$$

Number approx value count for the following from the formula used :

$$f(x+\Delta x) = f(x) + \Delta f(x) \approx f(x) + dy = f(x) + f'(x)\Delta x$$

➢ **Private derivative**

Definition 9.1. If $\lim\limits_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x} = \frac{\partial z}{\partial x} = z'_x = f'_x(x,y)$ and $\lim\limits_{\Delta y\to 0}\frac{\Delta_y z}{\Delta x} = \frac{\partial z}{\partial y} = z'_y = f'_y(x,y)$ finite limits there is if they are suitable respectively $z = f(x,y)$ of the function x and y to variables relatively **private** are called **derivatives** .

$z = f(x,y)$ of the function second in order private first that derivatives in order private from derivatives received private to derivatives it is said . Second in order private derivatives as follows is :

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = z''_{xx} = f''_{xx}(x,y);$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y\partial x} = z''_{yx} = f''_{yx}(x,y);$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x\partial y} = z''_{xy} = f''_{xy}(x,y);$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y^2}=z''_{yy}=f''_{yy}(x,y);$$

Solved from examples samples

Example 9.1. Given of the function the first in order private derivatives define :

$$z=x^2e^{y^2}$$

Solution :

First in order derivative to determine for initially , y the unchanging x according to private derivative is taken :

$$z'_x=2xe^{y^2}$$

Then x the unchanging y according to private derivative is taken :

$$z'_y=x^2e^{y^2}\cdot 2y=2x^2ye^{y^2}$$

Example 9.2. Given two variable of the function second in order private derivatives define :

$$z=\ln(1+x+2y).$$

Solution :

Initially , the function the first in order private derivatives defined as :

$$z'_x=\frac{1}{1+x+2y};\ z'_y=\frac{2}{1+x+2y}.$$

First in order private from derivatives used , the second in order private derivatives defined as :

$$z''_{xx}=\frac{1}{(1+x+2y)^2};$$

$$z''_{xy} = z''_{yx} = -\frac{2}{(1+x+2y)^2};$$

$$z''_{yy} = -\frac{4}{(1+x+2y)^2};$$

Example 9.3. $u = x + y^2 - z^3$ function and $M_0(1;2;-1)$ point given $\overrightarrow{M_0M_1}$ vector direction according to M_0 at the point of the function derivative find it on the ground $M_1 = (3;-4;2)$.

Solution :

M_0 at the point of the function the first order private derivative is :

$$u'_x = 1;\ u'_y = 2y;\ u'_z = -3z^2,$$

$$u'_x(1,2,-1) = 1;\ u'_y(1,2,-1) = 4;\ u'_z(1,2,-1) = -3.$$

$\overrightarrow{M_0M_1}$ vector coordinates as follows will be :

$$\overrightarrow{M_0M_1} = (3-1;-4-2;2+1) = (2;-6;3).$$

This $\overrightarrow{M_0M_1}$ of the vector referrer cosines is :

$$\left|\overrightarrow{M_0M_1}\right| = \sqrt{4+36+9} = 7, \quad \cos\alpha = \frac{2}{7},\ \cos\beta = -\frac{6}{7},\ \cos\gamma = \frac{3}{7}.$$

From this looking for derivative as follows will be :

$$\frac{\partial u(M_0)}{\partial M_0M_1} = 1\cdot\frac{2}{7} + 4\cdot\left(-\frac{6}{7}\right) - 3\cdot\frac{3}{7} = \frac{2-24-9}{7} = -\frac{31}{7};$$

Derivative negative that it was for , given direction given at the point function decreased is going

Example 9.4. y function to the graph x_0 at the point don't try equation write :

$$y = \frac{10x-1}{x+2}, x_0 = 1$$

Solution :

y function to the graph x_0 at the point don't try equation

$$y = f(x_0) + f'(x_0)\cdot(x - x_0)$$

formula through is made . of the function given at the point value is :

$$f(x_0) = f(1) = \frac{10\cdot 1 - 1}{1+2} = \frac{9}{3} = 3$$

y from the function derivative is taken :

$$f'(x) = \left(\frac{10x-1}{x+2}\right)' = \frac{(10x-1)'\cdot(x+2) - (10x-1)\cdot(x+2)'}{(x+2)^2} =$$

$$= \frac{10\cdot(x+2) - (10x-1)\cdot 1}{(x+2)^2} = \frac{10x + 20 - 10x + 10}{(x+2)^2} = \frac{30}{(x+2)^2};$$

Derivative in the function at the point value is :

$$f'(x_0) = f'(1) = \frac{30}{(1+2)^2} = \frac{30}{9} = \frac{10}{3};$$

y function to the graph x_0 at the point don't try equation as follows will be :

$$y = f(x_0) + f'(x_0)\cdot(x - x_0) = 3 + \frac{10}{3}(x-1) = 3 + \frac{10}{3}x - \frac{10}{3} = \frac{10}{3}x - \frac{1}{3}$$

$$y = \frac{10}{3}x - \frac{1}{3}.$$

Example 9.5.

Side a has been square shaped of the sheet from the corners side h has been squares cut From this then , sheet in the picture as shown , crossed out field according to when folded , high part open box harvest will h be of how in value of the box volume the most big will be

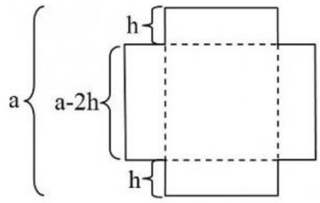

Figure 9.1

Solution :

From the picture of the box height h , square of the basis side while $a-2h$ to be to see can of the volume of the parallelepiped from the formula box using volume is written :

$$V=(a-h)^2\cdot h$$

Now this of the function the most big value is studied . In this h height negative value acceptance to do and $\frac{a}{2}$ from big to be possible it's not :

$$0<h<\frac{a}{2}$$

h to the variable relatively from the function derivative get through of the box the most big to volume have to be check can :

$$V'=2\cdot(a-2h)\cdot(-2)\cdot h+(a-2h)^2=$$

$$=(a-2h)(-4h+a-2h)=(a-2h)(a-6h);$$

This found derivative to 0 is equal to :

$$(a-2h)(a-6h)=0;$$

$$h_1=\frac{a}{2}; \qquad h_2=\frac{a}{6};$$

Determined $h_1=\frac{a}{2}$ and $h_2=\frac{a}{6}$ from the values $0<h<\frac{a}{2}$ condition satisfying only $h_2=\frac{a}{6}$ will be That is , $h=\frac{a}{6}$ will be

This has been determined $h = \frac{a}{6}$ in value box maximum or to a minimum size have that to determine for second derivative is :

$$V'' = -8a + 24h;$$

$$V''\left(\frac{a}{6}\right) = -8a + 24 \cdot \frac{a}{6} = -4a$$

Second derivative value negative because it is a box volume $h = \frac{a}{6}$ in value maximum will be

Example 9.6. Function from the differential using the following of the thigh approx value calculate : $\sqrt[3]{8,03}$

Solution :

Number approx value count for the following from the formula used :

$$f(x + \Delta x) = f(x) + \Delta f(x) \approx f(x) + dy = f(x) + f'(x)\Delta x$$

In this $f(x) = \sqrt[3]{x}$

$$\sqrt[3]{x + \Delta x} \approx \sqrt[3]{x} + \frac{1}{3\sqrt[3]{x^2}}\Delta x$$

$x = 8$ and $\Delta x = 0,03$ that it was for

$$\sqrt[3]{8,03} \approx \sqrt[3]{8} + \frac{1}{3\sqrt[3]{8^2}} \cdot 0,03 = 2 + \frac{0,03}{12} = 2,0025$$

will be

Independent work for issues

Given the 5th-order derivative of the function calculate :

9.1 x^3;

9.2 $2x^5 + 4x^4$;

9.3 $\frac{1}{ax+b}$;

9.4 $x^3 \ln x$;

$f(x)$ function to the graph x_0 at the point don't try equation write :

9.5 $f(x) = e^{\sqrt{5-x^2}}, \quad x_0 = 2;$

9.6 $f(x) = \frac{3x+5}{x+4}, \quad x_0 = -3;$

9.7 $f(x) = \frac{x^2+5x-3}{x^2+4}, \quad x_0 = -1;$

9.8 $f(x) = \sin x - \cos x, \quad x_0 = \frac{\pi}{4};$

of the function second in order private derivatives find :

9.9 $z = e^{x-y}(2x-1)$;

9.10 $z = \sin\left(x+\sqrt{y}\right)$;

9.11 $z = xe^y + x^y$;

9.12 $z = \ln\sqrt{x+y^2}$;

9.13 $z = \ln\left(\sqrt{x}+\sqrt{y}\right)$;

9.14 $z = x^{\sqrt{y}}$;

9.15 $z = arctg\left(\frac{y}{x}+1\right)$;

9.16 $z = xye^{xy}$;

9.17 $z = e^{xy}(x+y)$;

9.18 $z = \frac{\cos y^2}{x}$;

9.19 $z = \ln\left(1+e^x+y^2\right)$;

9.20 $z = \frac{\sqrt{x}-\sqrt{y}}{x+y}$;

9.21 $z = \sin\left(\frac{x}{y}\right)$;

9.22 $z=\frac{x\arcsin y}{y}$;

9.23 $z=x^y+y^x$;

9.24 $u=x^3-2x^2y+xy^2+1$ function and $M_0(1;2)$ point given $\vec{a}=(3;-4)$ vector direction according to M_0 at the point of the function derivative be found

$\overrightarrow{M_0M_1}$ vector direction according to M_0 at the point u of the function derivative be found

9.25 $u=\ln\left(x+\frac{y}{2z}\right), M_0(1,2,1), M_1(-2,3,5)$;

9.26 $u=\frac{y}{x}+\frac{z}{y}-\frac{x}{z}, M_0(1,1,2), M_1(8,-1,-4)$;

9.27 $u=\frac{\sin(x-y)}{z}, M_0\left(\frac{\pi}{2},\frac{\pi}{3},\sqrt{3}\right), M_1\left(\pi,\frac{\pi}{6},2\sqrt{3}\right)$;

9.28 $u=8\cdot\sqrt[5]{x^3+y^2+z}, M_0(3,2,1), M_1(5,8,4)$;

9.29 Depth $a=3$ was m conical ora digging demand will be done . How in depth this wrap up volume the most big will be

9.30 Radius R has been ball into drawn cylinder volume the most big to be of the cylinder find the height .

9.31 $\frac{x^2}{128}+\frac{y^2}{32}=1$ to the ellipse area the most big has been rectangle drawn If a rectangle sides ellipse if it is parallel to the axes , this of the rectangle find the sides .

9.32 Maximum to capacity have open cylindrical bakni preparation demand will be done . If it work release for $S=18,84\ \text{dm}^2$ If there is material , please dimensions (radius R and height H) how to be do you need

9.33 High part open reservoir square justified to the shape of a rectangular parallelepiped have If the tank is 256 liters water capacity fit possible if , his dimensions how when , the reservoir work in release the most less how much material is used ?

F function from the differential using the following of the thigh approx value calculate :

9.34 $\sqrt[4]{16,5}$;

9.35 $\sqrt[3]{26}$;

9.36 $\sin 31°$;

9.37 $\cos 89°$;

9.38 $e^{0,03}$;

9.39 $\ln 0,95$;

9.40 $y = e^{1-x^2}$, $x = 1,05$.

Answers

9.1. 0

9.2. 240

9.3. $-\dfrac{120a^5}{(ax+b)^6}$

9.4. $-\dfrac{6}{x^2}$

9.5. $-2e^x+e$

9.6. $7x+17$

9.7. $0.04x-1.36$

9.8. $\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$

9.9. $\dfrac{\partial z^2}{\partial x^2}=(2x-1)e^{x-y}+4e^{x-y};\ \dfrac{\partial z^2}{\partial y^2}=(1-2x)e^{x-y}$

9.10. $\dfrac{\partial z^2}{\partial x^2}=-\sin\left(x+\sqrt{y}\right);\ \dfrac{\partial z^2}{\partial y^2}=-\dfrac{\sin\left(x+\sqrt{y}\right)}{4y}-\dfrac{\cos\left(x+\sqrt{y}\right)}{4y\sqrt{y}}$

9.11. $\dfrac{\partial z^2}{\partial x^2}=x^y\cdot\dfrac{y^2}{x^2}-x^y\cdot\dfrac{y}{x^2};\ \dfrac{\partial z^2}{\partial y^2}=xe^y+x^y\ln^2 x$

9.12. $\dfrac{\partial z^2}{\partial x^2}=-\dfrac{1}{2\left(x+y^2\right)^2};\ \dfrac{\partial z^2}{\partial y^2}=-\dfrac{2y^2}{2\left(x+y^2\right)^2}+\dfrac{1}{x+y^2}$

9.13. $\dfrac{\partial z^2}{\partial x^2}=-\dfrac{1}{4x\left(\sqrt{x}+\sqrt{y}\right)^2}-\dfrac{1}{4x\sqrt{x}\left(\sqrt{x}+\sqrt{y}\right)};$

$$\frac{\partial z^2}{\partial y^2}=-\frac{1}{4y\left(\sqrt{x}+\sqrt{y}\right)^2}-\frac{1}{4y\sqrt{y}\left(\sqrt{x}+\sqrt{y}\right)}$$

9.14. $\dfrac{\partial z^2}{\partial x^2}=-\dfrac{x^{\sqrt{y}}\sqrt{y}}{x^2}+\dfrac{x^{\sqrt{y}}y}{x^2};\ \dfrac{\partial z^2}{\partial y^2}=\dfrac{x^{\sqrt{y}}\ln^2 x}{4y}-\dfrac{x^{\sqrt{y}}\ln(x)}{4y\sqrt{y}}$

9.15. $\dfrac{\partial z^2}{\partial x^2}=\dfrac{2y}{x^3\left(\left(1+\dfrac{y}{x}\right)^2+1\right)^2}-2y^2\dfrac{1+\dfrac{y}{x}}{x^4\left(\left(1+\dfrac{y}{x}\right)^2+1\right)^2};\ \dfrac{\partial z^2}{\partial y^2}=-2\dfrac{1+\dfrac{y}{x}}{x^2\left(\left(1+\dfrac{y}{x}\right)^2+1\right)^2}$

9.16. $\dfrac{\partial z^2}{\partial x^2}=xy^3e^{xy}+2y^2e^{xy};\ \dfrac{\partial z^2}{\partial y^2}=x^3ye^{xy}+2x^2e^{xy}$

9.17. $\frac{\partial z^2}{\partial x^2} = y^2(x+y)e^{xy} + 2ye^{xy}; \frac{\partial z^2}{\partial y^2} = x^2(x+y)e^{xy} + 2xe^{xy}$

9.18. $\frac{\partial z^2}{\partial x^2} = \frac{2\cos y^2}{x^3}; \frac{\partial z^2}{\partial y^2} = -\frac{4y^2\cos y^2}{x} - \frac{2\sin y^2}{x}$

9.19. $\frac{\partial z^2}{\partial x^2} = \frac{e^x}{y^2+e^x+1} - \frac{e^{2x}}{\left(y^2+e^x+1\right)^2}; \frac{\partial z^2}{\partial y^2} = -\frac{4y^2}{\left(y^2+e^x+1\right)^2} + \frac{2}{y^2+e^x+1}$

9.20. $\frac{\partial z^2}{\partial x^2} = \frac{2\left(\sqrt{x}-\sqrt{y}\right)}{(x+y)^3} - \frac{1}{\sqrt{x}(x+y)^2} - \frac{1}{4x\sqrt{x}(x+y)};$

$\frac{\partial z^2}{\partial y^2} = \frac{2\left(\sqrt{x}-\sqrt{y}\right)}{(x+y)^3} + \frac{1}{\sqrt{y}(x+y)^2} + \frac{1}{4y\sqrt{y}(x+y)}$

9.21. $\frac{\partial z^2}{\partial x^2} = -\frac{\sin\left(\frac{x}{y}\right)}{y^2}; \frac{\partial z^2}{\partial y^2} = -\frac{x^2\sin\left(\frac{x}{y}\right)}{y^4} + \frac{2x\cos\left(\frac{x}{y}\right)}{y^3}$

9.22. $\frac{\partial z^2}{\partial x^2} = 0; \frac{\partial z^2}{\partial y^2} = \frac{x}{\left(1-y^2\right)^{\frac{3}{2}}} - \frac{2x}{y^2\sqrt{1-y^2}} + \frac{2x\arcsin y}{y^3}$

9.23. $\frac{\partial z^2}{\partial x^2} = y^x\ln^2 y + \frac{x^y y^2}{x^2} - \frac{x^y y}{x^2}; \frac{\partial z^2}{\partial y^2} = \frac{x^2y^x}{y^2} - \frac{xy^x}{y^2} + x^y\ln^2 x$

9.24. $-\frac{11}{5}$

9.25. $-\frac{37}{4\sqrt{26}}$

9.26. $-\frac{16}{\sqrt{89}}$

9.29. $R = H = 3dm$

9.30. $H = \frac{2R}{\sqrt{3}}, V_{\max} = \frac{4\sqrt{3}\pi R^3}{9}$

9.31. $a = 16; b = 8$

9.32. $H = 2; R = 1; V_{\max} = 2\pi$

9.33. $a = 8; a = 8; c = 6$

9.34. $\frac{129}{64}$

9.35. $\frac{80}{27}$

9.36. 5.15

9.38. 0.9

9.39. -0.05