Topic 7.

Exponent and logarithm

$f(x) = 2^x$ function is called exponential (exponential) function because unknown x variable is an indicator . Him $g(x) = x^2$ such as level functions with mixed up do not send should , then variable basis will be

In general is indicative function

$$f(x) = a^x$$

in appearance is a function . In this a - positive constant number.

➢ **Exponent properties**

If a and b are positive numbers and x and y are any real numbers, the following holds:

1) $a^{x+y} = a^x a^y$; 2) $a^{x-y} = \frac{a^x}{a^y}$; 3) $\left(a^x\right)^y = a^{xy}$ 4) $(ab)^x = a^x b^x$

If $a > 0$ and $a \neq 1$ if , exponential function $f(x) = a^x$ grow up is going or is decreasing and continuously that it was for , u f^{-1} reverse to the function have it is a justified **logarithmic** is called a **function** and $\log_a$ with is determined . If the opposite function from the formula if used

$$f^{-1}(x) = y \Leftrightarrow f(y) = x$$

then , the following expression come output :

$$\log_a x = y \Leftrightarrow a^y = x$$

Har one $x \in R$ for $\log_a\left(a^x\right) = x$; $a^{\log_a x} = x$ will be .;

➢ Logarithm properties

If x and y are positive numbers, the following holds:

1) $\log_a(xy) = \log_a x + \log_a y$;2) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$; 3) $\log_a\left(x^r\right) = r\log_a x$

Solved from examples samples

Example 7.1. Given the equation solve : $\log_x \frac{1}{32} = -5$.

Solution:

Given logarithmic in Eq x only

$$\begin{cases} x > 0 \\ x \neq 1 \end{cases}$$

condition according to $x \in (0;1) \cup (1;+\infty)$ in the interval values acceptance do it takes

The solution of the equation is defined as:

$$\log_x \frac{1}{32} = -5 \Leftrightarrow$$

$$x^{-5} = \frac{1}{32} \Leftrightarrow$$

$$x^{-5} = 2^{-5} \Leftrightarrow$$

$$x = 2$$

This determined $x = 2$ solution $x \in (0;1) \cup (1;+\infty)$ in the interval there is . So , it is given logarithmic equation the solution $x = 2$ will be

Example 7.2. Given the equation solve : $5e^{-3x} = 42$.

Solution:

$$e^{-3x} = \frac{42}{5};$$

$$\ln e^{-3x} = \ln\left(\frac{42}{5}\right);$$

$$-3x = \ln\left(\frac{42}{5}\right);$$

$$x = \frac{\ln\left(\frac{42}{5}\right)}{-3} = -\frac{1}{3}\ln\left(\frac{42}{5}\right).$$

Example 7.3. The following inequality solve : $\log_{0,5}(x-1) > -1$

Solution:

x of identification field will be:

$$x-1>0 \Rightarrow$$

$$x>1 \Rightarrow x\in(1;+\infty)$$

Now the given inequality is solved:

$$x-1<0,5^{-1};$$

$$x-1<2 \Rightarrow x<3;$$

So , inequality the solution $x\in(1;+\infty)$ with intersection as a result $x\in(1;3)$ will be

Example 7.4. The following inequality solve : $4^x - 29\cdot 2^x + 168 \le 0$.

Solution:

To solve the given inequality, first, the exponent bases are equated:

$$\left(2^2\right)^x - 29\cdot 2^x + 168 \le 0;$$

$$2^{2x} - 29\cdot 2^x + 168 \le 0;$$

$$2^{2x} = \left(2^x\right)^2;$$

$$\left(2^x\right)^2 - 29\cdot 2^x + 168 \le 0;$$

Now this harvest has been in inequality $t = 2^x$ replacement done is increased . This leads to the following inequality:

$$t^2 - 29t + 168 \le 0$$

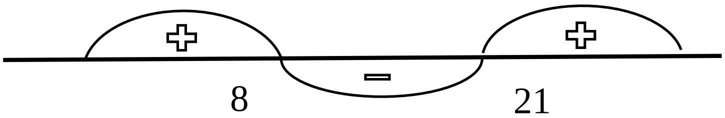
This inequality is solved as follows:

$$D = 29^2 - 4 \cdot 168 = 169;$$

$$t_1 = \frac{29 + 13}{2} = 21;$$

$$t_2 = \frac{29 - 13}{2} = 8;$$

$$(t - 8)(t - 21) \le 0$$



So ,t variable according to inequality the solution $t \in [8; 21]$ will be Now initial x variable according to the solution get for $2^{2x} = t$ used :

$$8 \le 2^x \le 21 \Leftrightarrow \begin{cases} 2^x \ge 8, \\ 2^x \le 21 \end{cases};$$

$$\begin{cases} 2^x \ge 8, \\ 2^x \le 21 \end{cases} \Rightarrow \begin{cases} 2^x \ge 2^3, \\ 2^x \le 2^{\log_2 21} \end{cases} \Rightarrow \begin{cases} x \ge 3, \\ x \le \log_2 21 \end{cases}$$

Hence , inequality the solution $x \in \left[3; \log_2 (21)\right]$ the fact that known will be

Example 7.5 .Given $y = \ln \frac{x+6}{x}$ function complete learn and graph it.

<u>Solution:</u>

a) Initially given of the function identification field found :

Logarithm ik function argument only is considered positive .

$$\frac{x+6}{x} > 0 \Rightarrow \frac{(x+6) \cdot x}{x^2} > 0 \text{ .}$$

In this fraction in the photo $x_1 = -6, \quad x_2 = 0$ roots will be Logarithmic function argument positive the interval $(-\infty; -6) \cup (0, +\infty)$ in which will be That is , given of the function identification field $(-\infty; -6) \cup (0, +\infty)$ will be

b) Vertical asymptotes to find for $x_1 = -6$ at the point - on the left , $x_2 = 0$ at the point - right on the side limits is :

$$\lim_{x \to -6-0} \ln \frac{x+6}{x} = \left\{ \ln \frac{-6-0+6}{-6} \right\} = \left\{ \ln \frac{0}{6} \right\} = -\infty \Rightarrow x = -6$$ - vertical asymptote ;

$$\lim_{x \to +0} \ln \frac{x+6}{x} = \left\{ \ln \frac{0+6}{0} \right\} = \left\{ \ln \frac{6}{0} \right\} = \infty \Rightarrow x = 0$$ - vertical asymptote ;

c) Function even-odd checked :

$$f(-x) = \ln \frac{-x+6}{-x} \neq f(x) = \ln \frac{x+6}{x};$$

$$f(-x) = \ln \frac{-x+6}{-x} \neq -f(x) = -\ln \frac{x+6}{x}.$$

From this given function it turns out that it is neither even nor odd.

d) Given function logarithmic function is not periodic.

e) Now function of the graph coordinate arrows the crossing points are determined:

Function graph Oy arrow with does not intersect because $x = 0$ point of the function identification to the field belongs to it's not .

Function of the graph Ox axis cut passing the point to determine the given function for is equal to 0:

$$f(x) = 0 \Rightarrow \ln \frac{x+6}{x} = 0 \Rightarrow \frac{x+6}{x} = 1 \Rightarrow x + 6 = x$$

The resulting equation has no solutions. Therefore , the function of the graph Ox arrow with intersected points are not available.

f) of the function slope asymptotes found :

$$k = \lim_{x \to \pm\infty} \frac{\ln \frac{x+6}{x}}{x} = \left\{ \ln \frac{1}{\infty} \right\} = 0;$$

$$b = \lim_{x \to \pm\infty} \ln \frac{x+6}{x} = \ln 1 = 0.$$

So , is a function of the graph horizontal asymptote $y=0$, that is Ox arrow will be

g) Given of the function the first in order the derivative is found:

$$f'(x)=\left(\ln\frac{x+6}{x}\right)'=\left(\ln(x+6)-\ln x\right)'=\frac{1}{x+6}-\frac{1}{x}.$$

The notation of the first order derivative is studied:

$$f'(x)=\frac{1}{x+6}-\frac{1}{x}=\frac{-6}{x(x+6)}.$$

x	$x<-6$	$x=-6$	$-6<x<0$	$x=0$	$x>0$
$f'(x)$	$-$	Not available	Not available	Not available	$+$
$f(x)$	Decreasing	$-\infty$	Not defined	∞	Growing

So , it is being studied of the function there are no extremum points.

h) Given of the function second in order the derivative is found:

$$f''(x)=\left(\frac{1}{x+6}-\frac{1}{x}\right)'=-\frac{1}{(x+6)^2}+\frac{1}{x^2}=\frac{-x^2+x^2+12x+36}{x^2(x+6)^2}=\frac{12(x+6)}{x^2(x+6)^2}=\frac{12}{x^2(x+6)}.$$

The notation of the second order derivative is studied:

x	$x<-6$	$x=-6$	$-6<x<0$	$x=0$	$x>0$
$f''(x)$	$-$	Not available	Not available	Not available	$+$
$f(x)$	Convex graph	$-\infty$	Not defined	∞	Concave graph

i) Given function graph while as follows :

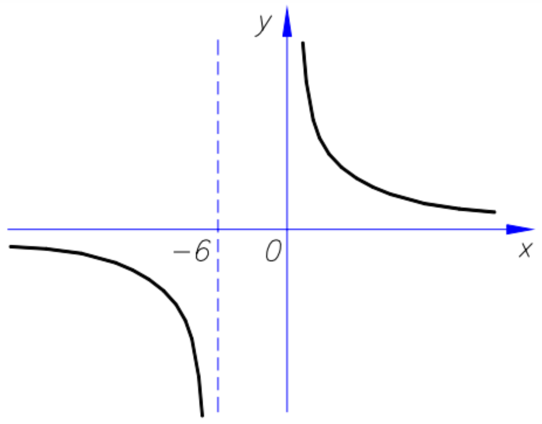


Figure 7.1

Independent work for issues

The following given equations solve (7.1-7.15):

7.1 $\ln x = -3$;

7.2 $2\lg x = \lg 2 + \lg(3x-4)$;

7.3 $\log_3(x+25) - \log_3(x-1) = 3$;

7.4 $\lg x + \lg(x-3) = 1$;

7.5 $\lg(3x-2) = 2$;

7.6 $\lg x + \lg(x-1) = \lg(4x)$;

7.7 $\log_9(x-5) + \log_9(x+3) = 1$;

7.8 $\log_2(x-2) + \log_2(x+1) = 2$;

7.9 $3^x - 2 = 12$;

7.10 $3^{1-x} = 2$;

7.11 $4^x = 5^{x+1}$;

7.12 $6^{1-x} = 10^x$;

7.13 $\dfrac{10}{1+e^{-x}} = 2$;

7.14 $e^{2x} - 2e^x = 15$;

7.15 $5^{2x} - 5^x - 12 = 0$;

The following given inequalities solve (7.16-7.23):

7.16 $\log_{\frac{1}{5}}(5x-1) \geq 0$;

7.17 $\log_{\frac{1}{3}} x \geq \log_x 3 - \frac{5}{2}$;

7.18 $\log_7 \frac{2x-6}{2x-1} > 0$;

7.19 $2\log_5 x - \log_x 125 < 1$;

7.20 $25^{-x} - 5^{-x+1} \geq 50$;

7.21 $\frac{4^x + 2x - 4}{x-1} \leq 2$;

7.22 $\sqrt{9^x + 3^x - 2} \geq 9 - 3^x$;

7.23 $\frac{6 - 3^{x+1}}{x} > \frac{10}{2x-1}$;

Given function learn and graph make (7.24-7.40):

7.24 $f(x) = xe^x$;

7.25 $f(x) = (x^2 - 1)e^x$;

7.26 $f(x) = (x+1)e^{-x}$;

7.27 $f(x) = x^2 e^{-x}$;

7.28 $f(x) = x\ln x$;

7.29 $f(x) = (x+2)\ln x$;

7.30 $f(x) = (x-2)\ln x$;

7.31 $f(x) = (x^2 - 4)\ln(x+1)$;

7.32 $f(x) = (x^2 - 7x + 12)\ln x$;

7.33 $f(x) = \frac{e^x}{x}$;

7.34 $f(x) = \frac{e^x}{x^2 - 1}$;

7.35 $f(x) = \frac{\ln x}{x}$;

7.36 $f(x)=\dfrac{\ln x}{x^2-7x+12}$;

7.37 $f(x)=(x+2)\log_2 x$;

7.38 $f(x)=x\log_2 x$;

7.39 $f(x)=x\cdot 2^x$;

7.40 $f(x)=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$;

Answers

7.1. $x = e^{-3}$

7.2. $x_1 = 2, x_2 = 4$

7.3. $x = 2$

7.4. $x_1 = -2, x_2 = 5$

7.5. $x = 34$

7.6. $x_1 = 0, x_2 = 5$

7.7. $x_1 = -4, x_2 = 6$

7.8. $x_1 = -2, x_2 = 3$

7.9. $x = \log_3 14$

7.10. $x = 1 - \log_3 2$

7.11. $x = \dfrac{\log_4 5}{1 - \log_4 5}$

7.12. $x = \dfrac{\lg 6}{1 + \lg 6}$

7.13. $x = -\ln 4$

7.14. $x = \ln 5$

7.15. $x = \log_5 4$

7.16. $x \in \left(\dfrac{1}{5}; \dfrac{2}{5}\right]$

7.17. $x \in (0;1) \cup \left(\sqrt{3};9\right)$

7.18. $x \in \left(-\infty; \dfrac{1}{2}\right)$

7.19. $x \in \left(0; \dfrac{1}{5}\right) \cup \left(1; 5\sqrt{5}\right)$

7.20. $x \in \left(-\infty; -\log_5 10\right]$

7.21. $x \in \left[\dfrac{1}{2}; 1\right)$

7.22. $x \in \left[\log_3 \dfrac{83}{19}; \infty\right)$

7.23. $x \in \left(0; \frac{1}{2}\right)$