“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

Kompleks sonlar va ular ustida amallar

Fan nomi: Hisob (Calculus)

REJA:

1. **Kompleks sonlar haqida dastlabki ta'riflar**
2. **Kompleks sonlar ustida asosiy amallar**
3. **Kompleks sonni darajaga ko'tarish va kompleks sondan ildiz chiqarish**
4. **Mustaqil yechish uchun misollar**

KOMPLEKS SONLAR TA'RIFI

Kompleks son deb

$$z = a + ib \quad (1)$$

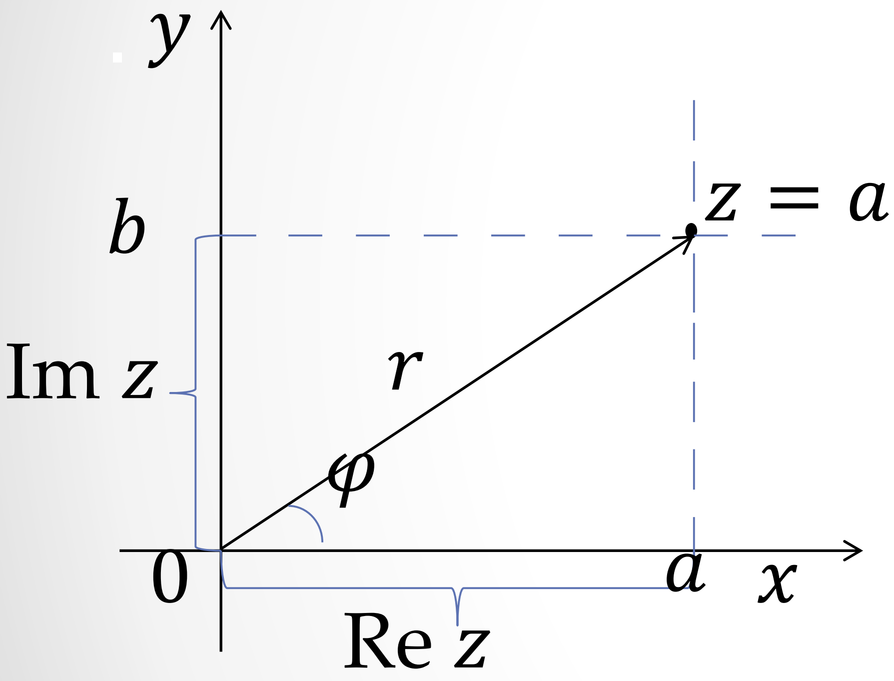
ifodaga aytiladi, bu erda *a* va *b* haqiqiy sonlar, *i* - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \quad \text{yoki} \quad i^2 = -1 \quad (2)$$

a- kompleks son z ning haqiqiy qismi, *ib* - mavhum qismi deyiladi. Ular bunday belgilanadi: *a=Re z, b=Imz*. Agar *a=0* bo'lsa, *0+ib=ib* sof mavhum son deyiladi; *b=0* agar bo'lsa, haqiqiy son hosil bo'ladi: *a+I*0=a*. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: *z=a+ib* va *z=a-ib* bir-biriga qo'shma deyiladi.

KOMPLEKS SONNING GEOMETRIK TASVIRI

$z = a + bi$



$z = a + bi$ kompleks son *moduli* $r = |z|$ kabi belgilanadi, $r = \sqrt{a^2 + b^2}$

$z = a + bi$ kompleks sonning *argumenti* $\varphi = arg\, z$ belgilanadi,

$cos\varphi = \frac{a}{r}$, $sin\varphi = \frac{a}{r}, 0 \leq \varphi < 2\pi$

$z = a + bi$ $\bar{z} = a - bi$ ko‘rinishidagi kompleks sonlar o‘zaro qo‘shma kompleks sonlar deyiladi.Masalan, $z = 2 + 3i$ $\bar{z} = 2 - 3i$

KOMPLEKS SONNING KO‘RINISHLARI

1. Vektor ko‘rinishi: $z = \overrightarrow{(x; y)}$

2. Algebraik ko‘rinishi : $z = x + iy$

3. Trigonometrik ko‘rinishi : $z = r \cdot (\cos\varphi + i \cdot \sin\varphi)$

4. Ko‘rsatkichli ko‘rinishi : $z = r \cdot e^{i\varphi}$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

1. $(a+bi)+(c+di)=(a+c)+(b+d)i$

$z_1=-0{,}13+2i$ va $z_2=7+3{,}6i$ sonlarini qo‘shing.

$$z_1+z_2=(-0{,}13+2i)+(7+3{,}6i)=(-0{,}13+7)+(2+3{,}6)i=$$
$$=6{,}87+5{,}6i$$

2. $(a+bi)-(c+di)=(a-c)+(b-d)i$

$z_1=13-7i$ va $z_2=-5+4i$ sonlarini ayiring.

$$z_1+z_2=(13-7i)-(-5+4i)=\big(13-(-5)\big)+(-7-4)i=$$
$$=18-11i$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

3. $(a+bi)\cdot(c+di)=(ac-bd)+(ad+bc)i$

$z_1=3+2i$ va $z_2=7+6i$ sonlarini ko‘paytiring.

$$z_1\cdot z_2=(3+2i)\cdot(7+6i)=3\cdot 7+3\cdot 6i+2i\cdot 7+2i\cdot 6i=$$
$$=21+18i+14i+12(i)^2=9+32i$$

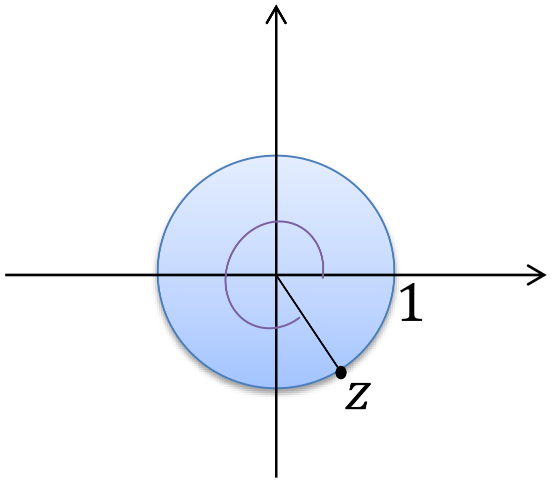
4. $\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i$

$z_1=2-i$ sonni $z_2=-3+2i$ songa bo‘ling.

$$\frac{2-i}{-3+2i}=\frac{(2-i)(-3+2i)}{(-3+2i)(-3-2i)}=\frac{-6+4i+3i-2}{(-3)^2-(2i)^2}=\frac{-8-i}{13}=$$
$$=\frac{-8}{13}-\frac{1}{13}i$$

Masalalar yechish

1-Masala Quyidagi sonni trigonometrik va ko‘rsatkichli ko‘rinishga keltiring: $z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$



Yechish: Dastlab r va φ ni topib olamiz:

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

$$\begin{cases} cos\varphi = \frac{1}{2} \\ sin\varphi = -\frac{\sqrt{3}}{2} \end{cases} => \ \varphi = \frac{5\pi}{3} = 300°$$

$$z = 1 \cdot \left(cos\frac{5\pi}{3} + i\, sin\frac{5\pi}{3}\right)$$

$$z = 1 \cdot e^{i \cdot \frac{5\pi}{3}}$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko‘rinishdagi kompleks sonlarni ko‘paytirishni qaraymiz.

$$z_1 = r_1(cos\varphi_1 + i\,sin\varphi_1), \quad z_2 = r_2(cos\varphi_2 + i\,sin\varphi_2)$$

$$z_1 \cdot z_2 = r_1(cos\varphi_1 + i\,sin\varphi_1) \cdot r_2(cos\varphi_2 + i\,sin\varphi_2) =$$
$$= r_1 \cdot r_2 \cdot \big((cos\varphi_1 \cdot cos\varphi_2 - sin\varphi_1 \cdot sin\varphi_2) +$$
$$+ i \cdot (cos\varphi_1 \cdot sin\varphi_2 + sin\varphi_1 \cdot cos\varphi_2)\big)$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 \cdot \big(cos(\varphi_1 + \varphi_2) + i\,sin(\varphi_1 + \varphi_2)\big)$$

$$z_1 = 6(cos70° + i\,sin70°), \quad z_2 = 4(cos25° + i\,sin25°)$$

$$z_1 \cdot z_2 = 6 \cdot 4\big(cos(70° + 25°) + i\,sin(70° + 25°)\big) =$$
$$= 24(cos95° + i\,sin95°)$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko‘rinishdagi kompleks sonlarni bo‘lishni qaraymiz.

$$z_1 = r_1(cos\varphi_1 + i\,sin\varphi_1), \qquad z_2 = r_2(cos\varphi_2 + i\,sin\varphi_2),$$

$$\frac{z_1}{z_2} = \frac{r_1(cos\varphi_1 + i\,sin\varphi_1)}{r_2(cos\varphi_2 + i\,sin\varphi_2)} = \frac{r_1}{r_2} \cdot \frac{(cos\varphi_1 + i\,sin\varphi_1)(cos\varphi_2 - i\,sin\varphi_2)}{(cos\varphi_2 + i\,sin\varphi_2)(cos\varphi_2 - i\,sin\varphi_2)} =$$

$$= \frac{r_1}{r_2} \cdot \frac{(cos\varphi_1 \cdot cos\varphi_2 + sin\varphi_1 \cdot sin\varphi_2) + i(sin\varphi_1 \cdot cos\varphi_2 - cos\varphi_1 \cdot sin\varphi_2)}{cos^2\varphi_1 + \ sin^2\varphi_2}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(cos(\varphi_1 - \varphi_2) + i\,sin(\varphi_1 - \varphi_2)\right)$$

$$z_1 = 6(cos70° + i\,sin70°), \qquad z_2 = 4(cos25° + i\,sin25°),$$

$$\frac{z_1}{z_2} = \frac{6}{4}\left(cos(70° - 25°) + i\,sin(70° - 25°)\right) = 1{,}5\,(cos45° - i\,sin45°)$$

Kompleks sonni natural darajaga ko‘tarish

$z = r(\cos\varphi + i\sin\varphi)$ kompleks son uchun $\forall n \in \mathbb{N}$ soni uchun Muavr formulasi:

$$z^n = r^n(\cos\varphi + i\sin\varphi)^n = r^n\big(\cos(n\varphi) + i\sin(n\varphi)\big)$$

$z = 3(\cos15° + i\sin15°)$ kompleks son 4-darajasini toping:

$$z^4 = 3^4\big(\cos(4\cdot15°) + i\sin(4\cdot15°)\big) = 81\cdot(cos60° + i\,sin60°) =$$

$$= 81\cdot\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{81}{2}\left(1 + \sqrt{3}i\right)$$

Javob: $\frac{81}{2}\left(1 + \sqrt{3}i\right)$

Masalalar yechish

2 masala $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$ kompleks sonning 10-darajasini toping.

Yechish: Dastlab trigonometrik ko‘rinishda yozib olamiz:

$$r=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{1}=1,\quad \varphi=\frac{5\pi}{3}=300°$$

$$z=1\cdot(cos300°+i\ sin300°)$$

$$z^{10}=1^{10}(cos3000°+i\ sin3000°)=1\cdot(cos120°+i\ sin120°)=$$

$$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

Javob: $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Masalalar yechish

3 masala

$(1+i)^{100}$ ifodaning qiymatini toping.

Yechish: $(1+i)^{100} = \left((1+i)^2\right)^{50} = (1+2i+$

Javob: -2^{50}

Kompleks sondan ildiz chiqarish

Kompleks sondan natural tartibli ildiz olishni ildiz tartibiga teskari qiymatli darajaga oshirish sifatida qabul qilish mumkin. Bunda ham Muavr formulasidan foydalanib quyidagicha formula xosil qilish mumkin

$$\sqrt[n]{z} = z^{\frac{1}{n}} = r^{\frac{1}{n}}(\cos\varphi + i\,\sin\varphi)^{\frac{1}{n}} =$$

$$= r^{\frac{1}{n}}\left(\cos\left(\frac{\varphi + 2\pi k}{n}\right) + i\,\sin\left(\frac{\varphi + 2\pi k}{n}\right)\right)$$

Bu formulada k o‘rniga $0,\ \pm 1,\ \pm 2, \ldots.$ qiymatlarni qo‘yib turli ildizlarni topamiz.

Masalalar yechish

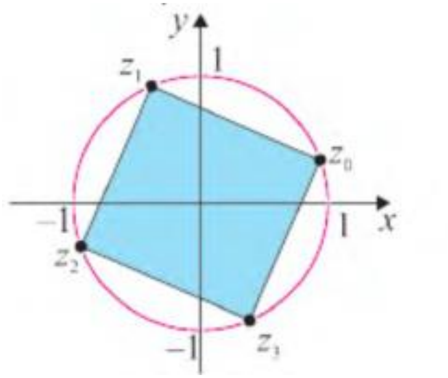
5 masala $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ kompleks sondan 4-darajali ildiz chiqaring.

Yechish: Dastlab trigonometrik ko‘rinishda yozib olamiz:

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1, \quad \varphi = \frac{\pi}{3} = 60°$$

$$z = 1 \cdot (cos60° + i\,sin60°)$$

$$\sqrt[4]{z} = z^{\frac{1}{4}} = 1^{\frac{1}{4}}\left(cos\frac{60°+2\pi k}{4} + i\,sin\frac{60°+2\pi k}{4}\right)$$



$k = 0 \;\; z_0 = cos15° + i\,sin15°$

$k = 1 \;\; z_1 = cos105° + i\,sin105°$

$k = 2 \;\; z_2 = cos195° + i\,sin195°$

$k = 3 \;\; z_3 = cos285° + i\,sin285°$

Uyga vazifa

MUSTAQIL YECHISH UCHUN MISOLLAR

1. $z_1 = 1 + i\sqrt{3} \qquad z_2 = 1 - i\sqrt{3}$

$z_1 \cdot z_2 = ? \quad z_1 + z_2 = ? \quad z_1 - z_2 = ? \quad \frac{z_1}{z_2} = ?$

2. $z = \frac{1}{(1 - i\sqrt{3})^6}$

3. $z = (1 + i\sqrt{3})^{15}$

4. $(-1)^{\sqrt{3}}$