Topic 5.

Limits

5. Definition 1. If $y = f(x)$ function $x = a$ of the point something around determined being desired $\varepsilon > 0$ for the thigh so $\delta > 0$ number is available if so , $|x-a| < \delta$ inequality satisfying all $x \neq a$ points for $|f(x) - A| < \varepsilon$ inequality if A done of a finite number $y = f(x)$ function $x = a$ at the point (or $x \to a$ in) is called the **limit** .

If $\lim\limits_{x \to a (x \to \infty)} f(x) = 0$ if $f(x)$ function **infinite small** , $\lim\limits_{x \to a (x \to 0)} f(x) = \infty$ infinite if , **infinite big** function will be

➢ Limit properties

1. $\lim c = c\,(c - const)$

2. $\lim (U_1 + U_2 + \ldots + U_k) = \lim U_1 + \lim U_2 + \ldots + \lim U_k$

3. $\lim (U_1 \cdot U_2 \cdot \ldots U_k) = \lim U_1 \cdot \lim U_2 \cdot \ldots \cdot \lim U_k$

4. $\lim \dfrac{U}{V} = \dfrac{\lim U}{\lim V}; \quad (\lim V \neq 0)$

➢ Great limits

First great limit: $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1.$

Second great limit: $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x = e.$

➢ Limit calculation formulas

$\lim\limits_{x \to 0} \dfrac{\sin kx}{kx} = \lim\limits_{x \to 0} \dfrac{tgkx}{kx} = \lim\limits_{x \to 0} \dfrac{shkx}{kx} = \lim\limits_{x \to 0} \dfrac{thkx}{kx} = 1,$ $k \in R$	$\lim\limits_{x \to \infty} \left(1 + \dfrac{k}{x}\right)^x = e^k$
$\lim\limits_{x \to 0} \dfrac{(1+kx)^m - 1}{kx} = m\,(m > 0).$	$\lim\limits_{x \to 0} (1 + x)^{\frac{k}{x}} = e^k$

$\lim\limits_{x\to 0}\dfrac{\ln(1+kx)}{kx}=1$	$\lim\limits_{x\to\infty}\left(1+\dfrac{1}{f(x)}\right)^{f(x)}=e.$ this on the ground $x\to\infty$ at $f(x)\to\infty$
$\lim\limits_{x\to 0}\dfrac{a^{kx}-1}{kx}=\ln a\,(a>0)$	$\lim\limits_{x\to 0}x^{\alpha}\ln x=\lim\limits_{x\to+\infty}x^{-a}\ln x=\lim\limits_{x\to+\infty}x^{a}e^{-x}=0\,(a>0)$

Solved from examples samples

Example 5.1. The following function limit calculate : $\lim\limits_{x\to 3}\dfrac{x^2+7}{2}$.

Solution :

In function x value when replaced by 3 , the following result is taken :

$$\lim_{x\to 3}\frac{x^2+7}{2}=\lim_{x\to 3}\frac{3^2+7}{2}=\lim_{x\to 3}8=8.$$

Example 5.2. The following function limit calculate : $\lim\limits_{x\to\infty}\dfrac{7x-2x^4}{4x^4+3x^2+1}$

Solution :

Given of the function fraction from both x the denominator and the numerator of the most big level bracket out is released , that is in this as follows to write can :

$$\lim_{x\to\infty}\frac{7x-2x^4}{4x^4+3x^2+1}=\lim_{x\to\infty}\frac{x^4\left(\frac{7}{x^3}-2\right)}{x^4\left(4+\frac{3}{x^2}+\frac{1}{x^4}\right)}=\frac{0-2}{4+0+0}=-\frac{2}{4}=-\frac{1}{2}$$

In this $\frac{7}{x^3}$, $\frac{3}{x^2}$, $\frac{1}{x^4}$ are $x\to\infty$ infinite in small functions the fact that in consideration received

Example 5.3. The following function limit calculate : $\lim\limits_{x\to\infty}\left(\sqrt{x^2-2}-\sqrt{x(x-2)}\right)$.

Solution :

To the border in transition infinity minus infinity of form uncertainty there is .

$$\lim_{x\to\infty}\left(\sqrt{x^2-2}-\sqrt{x(x-2)}\right)=\{\infty-\infty\}$$

Function of the roots the difference with is expressed . From uncertainty get rid of to be for this difference roots in total is multiplied .

$$\lim_{x\to\infty}\left(\sqrt{x^2-2}-\sqrt{x(x-2)}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-2}-\sqrt{x(x-2)}\right)\left(\sqrt{x^2-2}+\sqrt{x(x-2)}\right)}{\left(\sqrt{x^2-2}+\sqrt{x(x-2)}\right)}=$$

$$=\lim_{x\to\infty}\frac{2x-2}{\left(\sqrt{x^2-2}+\sqrt{x(x-2)}\right)}=\left\{\frac{\infty}{\infty}\right\}$$

As a result , infinity divided infinity uncertainty will come Uncertainty to open for fraction from the denominator and from the photo x bracket out is released and is shortened . As a result given function limit as follows will be :

$$\lim_{x\to\infty}\frac{2x-2}{\left(\sqrt{x^2-2}+\sqrt{x(x-2)}\right)}=\lim_{x\to\infty}\frac{x\left(2-\frac{2}{x}\right)}{x\left(\sqrt{1-\frac{2}{x^2}}+\sqrt{1-\frac{2}{x}}\right)}=\frac{2-0}{1+1}=1\,.$$

Example 5.4. The following function limit calculate : $\lim_{x\to-3}\frac{x^2-9}{x^2+7x+12}$

Solution :

Given in the function fraction photo and the denominator to multipliers is separated . In this fraction photo - reduced increase to the formula according to $a^2-b^2=(a+b)(a-b)$ and the denominator according to the following formula to multipliers is separated :

$$ax^2+bx+c=(x-x_1)(x-x_2)\text{, in which } x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

As a result function limit as follows is :

$$\lim_{x\to-3}\frac{x^2-9}{x^2+7x+12}=\lim_{x\to-3}\frac{(x-3)(x+3)}{(x+3)(x+4)}=\lim_{x\to-3}\frac{x-3}{x+4}=\lim_{x\to-3}\frac{-3-3}{-3+4}=-\frac{6}{1}=-6.$$

Example 5.5. The following function limit calculate : $\lim_{x\to0}\frac{\sin^2 3x}{tg^2 2x}$.

Solution :

Given function limit count for great from the limit used :

$$\lim_{x\to0}\frac{\sin^2 3x}{tg^2 2x}=\lim_{x\to0}\frac{9\cdot\sin^2 3x\cdot4x^2}{9x^2\cdot tg^2 2x\cdot4}=$$

$$=\lim_{x\to0}\frac{\sin^2 3x}{9x^2}\cdot\lim_{x\to0}\frac{4x^2}{tg^2 2x}\cdot\frac{9}{4}=1\cdot1\cdot\frac{9}{4}=\frac{9}{4}=2.25\,.$$

Example 5.6. The following function limit calculate : $\lim_{x\to\infty}\left(\frac{2x^2-3}{2x^2+1}\right)^{-4x^2}$.

Solution :

The limit count for originally , parenthesis inside of the fraction whole part separated is written :

$$\frac{2x^2-3}{2x^2+1}=\frac{2x^2+1-4}{2x^2+1}=\frac{2x^2+1}{2x^2+1}+\frac{-4}{2x^2+1}=1+\frac{-4}{2x^2+1}$$

Now the limit in the calculation from the excellent limit formula used ;

$$\lim_{x\to\infty}\left(\frac{2x^2-3}{2x^2+1}\right)^{-4x^2}=\lim_{x\to\infty}\left(1+\frac{-4}{2x^2+1}\right)^{\frac{2x^2+1}{-4}\cdot\frac{-4}{2x^2+1}\cdot\left(-4x^2\right)}=$$

$$=\lim_{x\to\infty}\left(\left(1+\frac{-4}{2x^2+1}\right)^{\frac{2x^2+1}{-4}}\right)^{\frac{16x^2}{2x^2+1}}=e^{\lim\limits_{x\to\infty}\frac{16x^2}{2x^2+1}},$$

In this $\lim_{\alpha(x)\to\infty}\left(1+\frac{1}{\alpha(x)}\right)^{\alpha(x)}=e$ from being was used . Now $a=\lim_{x\to\infty}\frac{16x^2}{2x^2+1}$ the limit count need will be :

$a=\lim_{x\to\infty}\frac{16x^2}{2x^2+1}=\lim_{x\to\infty}\frac{16x^2}{x^2\left(2+\frac{1}{x^2}\right)}=\lim_{x\to\infty}\frac{16}{2+\frac{1}{x^2}}=\frac{16}{2}=8$, in which $\lim_{x\to\infty}\frac{1}{x^2}=0$ from being was used . As a result given function limit as follows will be :

$$e^a=e^8 .$$

Independent work for issues

The following funk sia limits calculate (5.1-5.40).

5.1 $\lim_{x\to 8}\left(\log_2 x\right)$;

5.2 $\lim_{x\to 0}\frac{1}{x+5}$;

5.3 $\lim_{x\to 1}\frac{2}{3+x^3}$;

5.4 $\lim_{x\to -\infty}\left(1+4^x\right)$;

5.5 $\lim\limits_{x\to\infty}\frac{3x+1}{x}$;

5.6 $\lim\limits_{x\to\infty} e^{x}$;

5.7 $\lim\limits_{x\to\infty}\frac{5}{17+x^{3}}$;

5.8 $\lim\limits_{x\to\infty}\frac{1}{3x+9}$;

5.9 $\lim\limits_{x\to\infty}\frac{3^{x}+2}{3^{x+1}-1}$;

5.10 $\lim\limits_{x\to\infty}\frac{\sqrt{x-3}}{\sqrt{x}-\sqrt{3}}$;

5.11 $\lim\limits_{x\to\infty}\frac{7x^{4}+2x^{3}-1}{3x^{2}-2x^{4}+x}$;

5.12 $\lim\limits_{x\to 3}\frac{2x^{2}-9x+9}{x^{2}-5x+6}$;

5.13 $\lim\limits_{x\to 0}\frac{\sin 3x}{tg\,5x}$;

5.14 $\lim\limits_{x\to -2}\frac{x^{2}-4}{\sqrt{1-x}-3}$;

5.15 $\lim\limits_{x\to 2}\frac{3x^{2}-5x-2}{2x^{2}-x-6}$;

5.16 $\lim\limits_{x\to 1}\frac{x^{2}-1}{\sqrt{x}-1}$;

5.17 $\lim\limits_{x\to 2}\left(\frac{1}{x-2}-\frac{12}{x^{3}-8}\right)$;

5.18 $\lim\limits_{x\to 0}\frac{\sin 2x}{x}$;

5.19 $\lim\limits_{x\to 0}\frac{\sin^{5} 2x}{\sin^{4} 3x}$;

5.20 $\lim\limits_{x\to\infty}\frac{2x^{3}-2x+1}{3x^{2}+4x+2}$;

5.21 $\lim\limits_{x\to\infty}\left(\frac{5x-1}{5x+4}\right)^{2x+1}$;

5.22 $\lim\limits_{x\to 1}\frac{x^{3}-1}{5x^{2}-4x-1}$;

5.23 $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{x\sin 3x}$;

5.24 $\lim\limits_{x\to 0}\sin 3x ctg 5x$;

5.25 $\lim\limits_{x\to 1}\dfrac{x^3-2x^2-5x+6}{x^3-7x+6}$;

5.26 $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{(x-1)^3}$;

5.27 $\lim\limits_{x\to 2}\dfrac{x^2+3x-10}{x^2+2x-8}$;

5.28 $\lim\limits_{x\to -3}\dfrac{x^3+27}{x^2-9}$;

5.29 $\lim\limits_{x\to\infty}\left(x-\dfrac{3x^3}{3x^2+7}\right)$;

5.30 $\lim\limits_{x\to+\infty}\left(\sqrt{x^3+1}-\sqrt{x^3-1}\right)$;

5.31 $\lim\limits_{x\to+\infty}\left(\dfrac{x+1}{x+2}\right)^{2x}$;

5.32 $\lim\limits_{x\to+\infty}\left(1+\dfrac{1}{\sqrt{x}}\right)^{7x}$;

5.33 $\lim\limits_{x\to+\infty}\left(1+\dfrac{1}{x^3}\right)^{3x}$;

5.34 $\lim\limits_{x\to 0}\left(\dfrac{1+3x}{1+x}\right)^{\frac{5}{x}}$;

5.35 $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x-1}{x^2-2x+5}\right)^{-2x}$;

5.36 $\lim\limits_{x\to 0}\dfrac{\cos x-1}{3x^2}$;

5.37 $\lim\limits_{x\to 0}(8x\cdot ctgx)$;

5.38 $\lim\limits_{x\to 0}\dfrac{\sin^2 5x}{\sin^3 x}$;

5.39 $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{4x}$;

5.40 $\lim\limits_{x\to 0}\dfrac{\sin 2x+\sin 8x}{4x}$.

Answers

5.1. 3

5.2. $\frac{1}{5}$

5.3. $\frac{1}{2}$

5.4. 1

5.5. 3

5.6. ∞

5.7. 0

5.8. 0

5.9. $\frac{1}{3}$

5.10. 1

5.11. $-\frac{7}{2}$

5.12. 3

5.13. $\frac{3}{5}$

5.14. 0

5.15. 1

5.16. 4

5.17. $\frac{1}{2}$

5.18. $\sin 2$

5.19. 0

5.20. ∞

5.21. $\frac{1}{e^2}$

5.22. $\frac{1}{2}$

5.23. 6

5.24. $\frac{3}{5}$

5.25. $\frac{3}{2}$

5.26. ∞

5.27. $\frac{7}{6}$

5.28. $-\frac{9}{2}$

5.29. 0

5.30. 0

5.31. $\frac{1}{e^2}$

5.32. ∞

5.33. 1

5.34. e^{10}

5.35. $\frac{1}{e^6}$

5.36. $-\frac{1}{6}$

5.37. 8

5.38. $-\infty$

5.39. 0

5.40. $\frac{5}{2}$