

Quantum methods of image analysis and processing in the frequency domain

D. T. Mukhamedieva*^a, N. S. Mamatov^a, N. A. Niyozmatova^a, R. A. Sobirov^a, A. N. Samijonov^a

^aTashkent Institute of Irrigation and Agricultural Mechanization Engineers National Research University, 39, Kari Niyazi street, Tashkent, 100000, Uzbekistan

ABSTRACT

Quantum methods for image analysis and processing in the frequency domain represent an innovative approach to image processing using quantum computing. One of the key tools in this field is the quantum Fourier transform (QFT), which can be applied to quantum images to analyze and process data in the frequency domain. The process involves several steps, including quantum image preparation, QFT application, frequency domain data processing, and inverse Fourier transform to reconstruct the spatial domain image. Quantum image processing techniques offer new opportunities for solving complex problems in the field of image processing, especially in the context of the use of quantum computing. However, this area is still under active research and requires further research and technology development for practical application.

Keywords: Quantum computing, image processing, Qiskit, qubits, quantum circuit simulation, image conversion, quantum circuit, visualization of results, Fourier transform

1. INTRODUCTION

With the development of quantum computing and quantum information science in recent decades, the possibility of using quantum methods for image analysis and processing has been actively explored. Quantum image processing techniques represent a new and promising field that could lead to significant improvements in information processing and data analysis. One of the key tools in quantum image processing is the quantum Fourier transform (QFT), which is analogous to the classical Fourier transform, but can be performed on a quantum computer using quantum bits (qubits). The use of QFT makes it possible to analyze images in the frequency domain, which opens up new possibilities for data processing and analysis [1].

Let's consider current research and development in the field of quantum methods of analysis and image processing in the frequency domain. We will look at methods for representing images in quantum form, the use of the quantum Fourier transform for image analysis, and various algorithms and techniques for image processing in the quantum environment. Finally, we discuss the prospects and challenges in this field and possible directions for future research. The relevance of research in the field of quantum methods of analysis and image processing in the frequency domain is due to the growing interest in the application of quantum computing in various fields of science and technology. Quantum computing promises to revolutionize information processing with its ability to efficiently process large amounts of data using quantum principles of superposition and quantum entanglement. In the context of image processing, classical methods such as Fourier transform are widely used for image analysis and processing in the frequency domain. However, quantum imaging techniques represent a new approach that could provide more efficient and powerful tools for image analysis and processing. Specifically, the QFT is a version of the classical Fourier Transform that can be performed on a quantum computer. The application of QFT to images allows them to be analyzed in the frequency domain, which can be useful for identifying the main components of the image, filtering noise, data compression and other processing tasks [3].

*dilnoz134@rambler.ru

The application of quantum methods of image analysis and processing is of interest from both theoretical and practical points of view. On the one hand, this contributes to the development of the basic principles of quantum computing and its application in the field of image processing. On the other hand, this opens up new possibilities for solving complex image processing problems that can be applied in various fields such as medicine, biology, astronomy, and many others. However, it is worth noting that quantum imaging methods are still at an early stage of research, and require further research and technology development for their practical application. Currently, the main challenges are the development of efficient algorithms and software, as well as the creation of quantum devices capable of processing images with high accuracy and speed [4-5].

Quantum computers are revolutionizing the field of computing. Instead of the traditional use of bits, quantum computers operate on qubits, which, thanks to the quantum phenomena of superposition and entanglement, can exist in several states at the same time. Qubits are the basic building blocks of quantum computing. Unlike a classical bit, which can be either 0 or 1, a qubit can be in the 0, 1, or both states at the same time. This unique ability of quantum computers to perform many calculations simultaneously makes them very effective at solving certain types of problems. New opportunities that open up with the advent of quantum computers make it possible to solve complex problems faster and more efficiently than is possible using classical computers [6-8].

The difficulty of achieving a quantum advantage is great, especially given the susceptibility of quantum computers to errors. Quantum computers have specific characteristics, such as quantum entanglement and superposition, that make them sensitive to external influences. Even the slightest interference or errors in quantum operations can lead to distorted results or loss of information. The development and maintenance of quantum systems requires a high degree of precision and reliability. This includes the creation of stable quantum elements, as well as the development of error correction algorithms and methods for controlling quantum states. Despite these challenges, research and development in the field of quantum computing continues vigorously. Scientists and engineers around the world are working to improve quantum systems, reduce errors, and develop new techniques to overcome these obstacles. Hybrid approaches combining classical and quantum methods can help reduce the impact of errors on the final results, providing more reliable and efficient solutions to problems [9].

The difficulty of achieving a quantum advantage is great, especially given the susceptibility of quantum computers to errors. Quantum computers have specific characteristics, such as quantum entanglement and superposition, that make them sensitive to external influences. Even the slightest interference or errors in quantum operations can lead to distorted results or loss of information. The development and maintenance of quantum systems requires a high degree of precision and reliability. This includes the creation of stable quantum elements, as well as the development of error correction algorithms and methods for controlling quantum states. Despite these challenges, research and development in the field of quantum computing continues vigorously. Scientists and engineers around the world are working to improve quantum systems, reduce errors, and develop new techniques to overcome these obstacles. Hybrid approaches combining classical and quantum methods can help reduce the impact of errors on the final results, providing more reliable and efficient solutions to problems [9].

Using methods adapted from statistical inference to optimize tensor networks can significantly improve their efficiency. These methods can include various approaches, such as stochastic gradient descent or optimization methods based on machine learning algorithms, which can find optimal network parameters taking into account the statistical properties of the data. This has direct application to working with quantum systems, where tensor networks can be used to model and analyze the interactions of qubits and other quantum objects. Effective optimization of such networks can improve the accuracy and speed of calculations, which is important for the development of quantum computing and its applications [13-14].

2. MATERIALS AND METHODS

The Quantum Fourier Transform (QFT) is a version of the classical Fourier transform that can be performed on a quantum computer. In quantum computing, QFT can be used to analyze and process quantum images. Quantum images are quantum states that can contain information about the probability distribution of pixel values or other characteristics of the image. The image is encoded into quantum states using quantum bits (qubits). This can be done, for example, using quantum coupling circuits. The quantum Fourier transform is applied to the states representing the image. This allows you to analyze the image in the frequency domain, highlighting its main components. Once QFT

is applied to an image, various processing operations can be performed, such as filtering, smoothing, or highlighting specific frequency components. After processing an image in the frequency domain, an inverse quantum Fourier transform can be performed to return the image to the spatial domain [15-16].

This process could be particularly useful for image processing using quantum computing, especially in cases where classical image processing methods are limited or ineffective. However, it is worth noting that quantum computers are currently at an early stage of development, and the practical application of quantum imaging techniques may require further research and technology development. Just as image compression reduces file size with minimal loss of quality, choosing different structures for tensor networks provides different forms of computational “compression,” optimizing the way information is stored and processed [17].

Thus, just as JPEG provides efficient image compression, tensor networks allow us to efficiently store and process data, reducing size and computational costs with minimal information loss [18].

The Fourier transform of images (FFT - Fast Fourier Transform) is one of the main methods for analyzing and processing images in the frequency domain. It allows you to analyze images in terms of their frequency components, which can be useful for tasks such as noise filtering, sharpening, edge detection and data compression. The Fourier transform of an image works by breaking down an image into its frequency components. The result is a two-dimensional spectrum in which high frequencies are represented as bright spots and low frequencies as dark areas. The source image is subjected to preparation if necessary (for example, scaling, reduction to a power of two size for efficient FFT operation). Applies a two-dimensional Fourier transform to the image. For this purpose, the FFT algorithm is usually used, which efficiently calculates the Fourier transform by optimizing the operations. The outcome in the frequency domain is the image's spectrum. Here you can analyze the frequency components and their contribution to the original image. You can perform various frequency domain processing operations, such as filtering (such as removing low-frequency or high-frequency noise), sharpening, and others. After processing an image in the frequency domain, an inverse Fourier transform can be applied to return the image to the spatial domain [19-20].

The quantum circuit presented is a QFT for 7 qubits. In quantum computing, QFT is an important tool and is used to transform the states of qubits from the base (input) representation to the Fourier representation. Here is a step-by-step description of the quantum circuit: On the first qubit (q_3), Hadamard rotation (H) is applied. This rotation takes the qubit from the state $|0\rangle$ to the superposition $(|0\rangle + |1\rangle) / \sqrt{2}$. Then a controlled rotation (cr) occurs between qubits q_4 and q_3 with an angle of $\pi / 2(P(\pi / 2))$. This rotation depends on the state of q_3 and is applied to q_4 provided that q_3 is in state $|1\rangle$. On qubits q_5 , q_6 and q_7 , Hadamard rotations (H) are used, as well as controlled rotations with angles $2\pi / 2$, $4\pi / 4$, $8\pi / 8$ and $16\pi / 16$. These rotations form a quantum Fourier transform on these three qubits. Finally, on qubits q_5 , q_6 and q_7 , controlled rotations (cp) are applied between each pair of qubits with angles π , $\pi / 2$ and $\pi / 4$ respectively (Figure 1).

This circuit implements QFT for 7 qubits and can be used in various quantum computing problems such as quantum algebra, Shor's algorithm and others. In quantum computing, "cr" stands for controlled rotation, where the master qubit controls the rotation of the target qubit depending on its state. Formally, this can be represented as an operation:

$$CP(\lambda) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\lambda} \end{bmatrix} \quad (1)$$

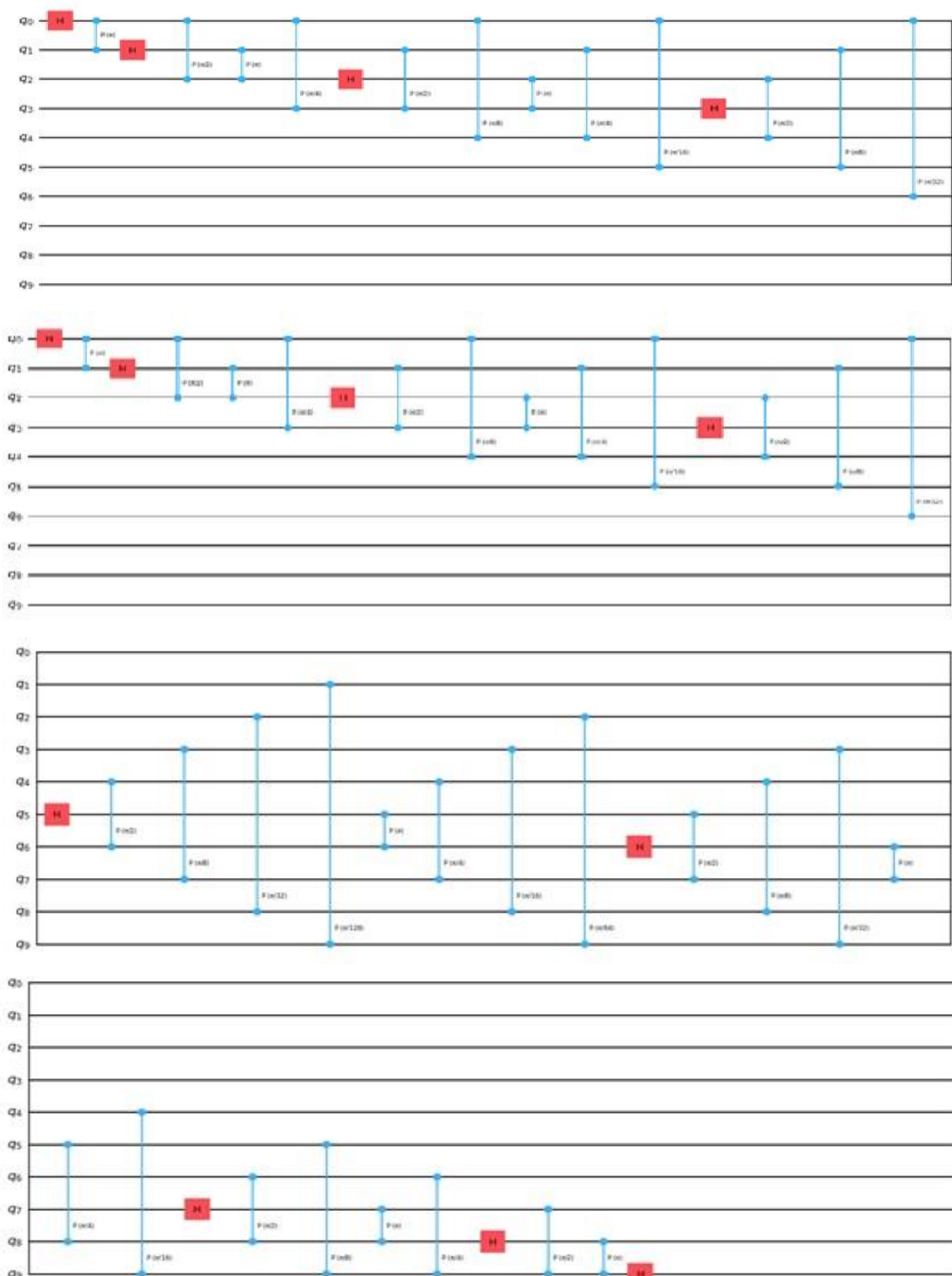


Figure 1. Quantum Fourier transform circuit.

Here λ is the rotation parameter, which depends on the state of the control qubit. If the control qubit is in the $\lambda|1\rangle|1\rangle$ state, then the target qubit undergoes rotation through an angle λ , otherwise no change occurs. In the circuit, the rotation angles, $2\pi/2$, $4\pi/4$, $8\pi/8$ and $16\pi/16$ are used in the (cr) operation to create a quantum Fourier transform. Usually $P(\theta)$ denotes a single-qubit rotation around an axis Z by an angle θ . Formally, this is the rotation matrix:

$$P(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \quad (2)$$

In the circuit $P(\pi/2)$ means a single-qubit rotation around the Z axis by an angle $2\pi/2$. Such rotations are important in quantum computing because they allow the phase of the qubit states to change. In the context of quantum computing $e^{i\theta}$ is often used to represent rotations on the complex plane, such as rotations of qubits around the Z axis.

This representation can be expressed using sine and cosine. The rotation matrix $P(\theta)$ around the Z axis can be written as:

$$P(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (3)$$

This matrix describes the rotation by an angle θ in the complex plane, where θ is measured in radians.

3. RESULTS

When displaying an image before the QFT, the Matplotlib (plt) library is used to display the image before the Fourier transform. The plt.imshow() function is used to display the image and plt.title() is used to set the title. To transform each pixel in the image, a number of qubits is used, which is calculated as the base 2 logarithm of the maximum image size value. This ensures that there are enough qubits to represent each pixel (Figure 2).

For each qubit in the quantum circuit, a Hadamard gadget (qc.h(j)) is applied, and then controlled rotation (qc.cp()) is applied between all qubits to implement the QFT. The circuit is drawn using circuit_drawer() from Qiskit so that the order of operations can be seen. Uses Aer's statevector_simulator backend to simulate a quantum circuit. The results obtained are presented in the form of accounts. Uses plot_histogram() from Qiskit to visualize the results as a histogram. Displaying the image after the quantum Fourier transform: Uses plt.imshow() to display the image after the Fourier transform. To transform each pixel in the image, a number of qubits is used, which is calculated as the base two logarithm of the maximum image size value. This ensures that there are enough qubits to represent each pixel. Create a quantum circuit object using Quantum Circuit from the Qiskit library. For each qubit in the quantum circuit, a Hadamard gadget (qc.h(j)) is applied, and then controlled rotation (qc.cp()) is applied between all qubits to implement the quantum Fourier transform. The circuit is drawn using circuit_drawer() from Qiskit so that the order of operations can be seen. Visualizing results: Uses plot_histogram() from Qiskit to visualize the results as a histogram.

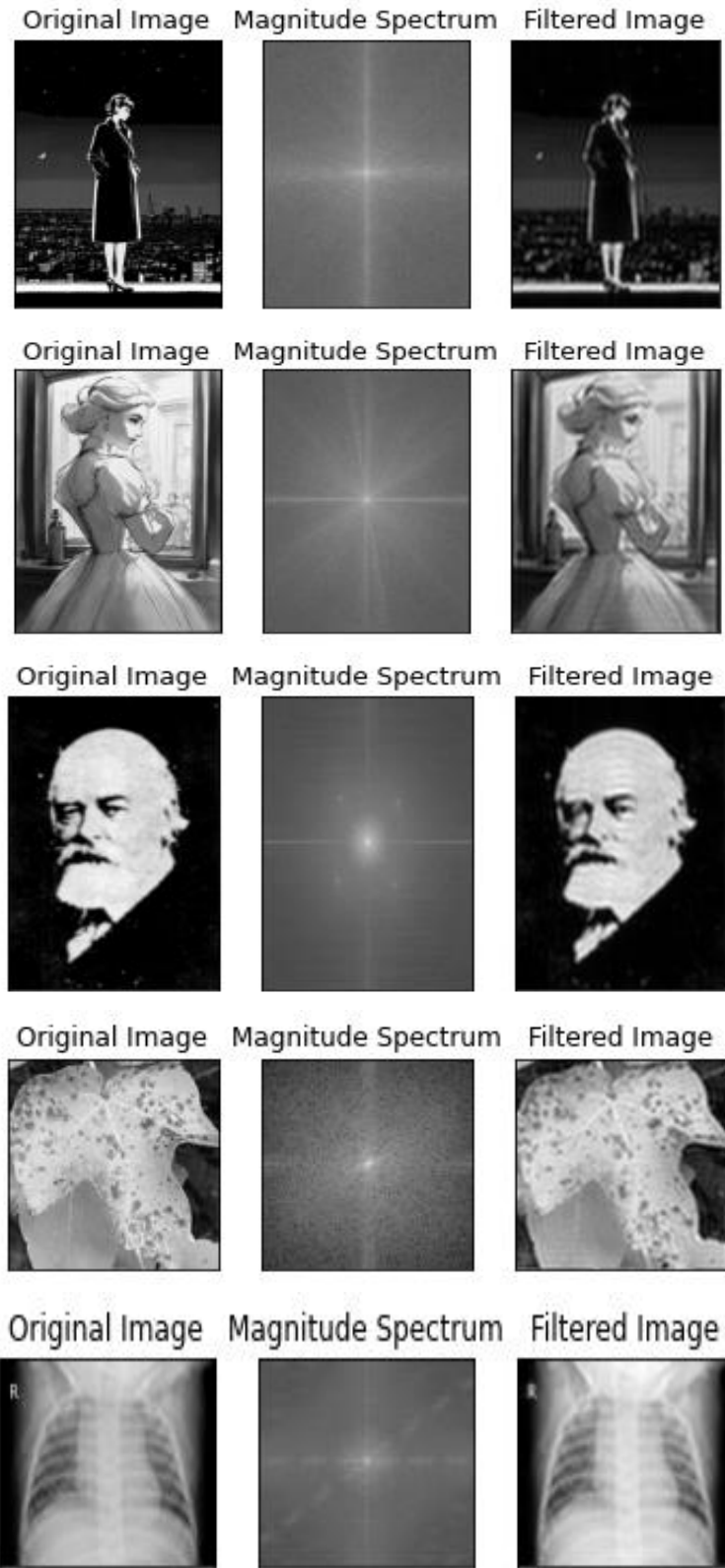


Figure 2. Result of the quantum Fourier transform.

The Fourier transform used in this program takes an image from pixel space to frequency space: The Fourier transform takes an image from pixel space to frequency space, where each pixel in the image is represented as the sum of different frequency components. This allows an image to be analyzed in terms of its constituent parts in the frequency domain. The Fourier transform isolates the main frequency components of an image, allowing you to determine the most significant elements in the image in terms of their frequency components. For example, sharp edges and contours of an image can be expressed as high-frequency components. The Fourier transform allows you to effectively compactly represent an image in the form of its frequency components. This can be useful for compressing images with minimal loss of quality. The Fourier transform is used for image processing and filtering, such as removing noise or applying blur effects. The use of frequency domain filters allows you to control various aspects of an image, such as sharpness and texture.

4. CONCLUSION

Thus, the Fourier transform performs important tasks related to the analysis, processing and representation of images in the frequency domain, making it a powerful tool in the field of image processing and computer vision. The programs are not used for image processing, such as removing noise or applying blur effects. Instead, they apply QFT to the image to examine it in the frequency domain using quantum circuits. The Fourier transform in the context of quantum computing helps analyze an image in quantum space, which can be useful in determining the fundamental frequency components that characterize the structure of the image. However, in this program, the use of QFT is not intended for image processing in the classical sense, since the result of the transformation is not used to modify or filter the image. The application of QFT in this program is used to analyze the image structure in the frequency domain using quantum circuits. This can be useful for a number of tasks, such as quantum signal processing: QFT allows the analysis of frequency components of a signal in a quantum system. This could be useful, for example, for analyzing signal spectra in quantum communications or signal processing in quantum sensor devices. The Fourier transform plays an important role in quantum information algorithms, such as Shor's algorithm for factorizing large numbers and algorithms for solving optimization problems. In quantum computing, QFT can be used to analyze and process data, including images. For example, it can help in identifying the main structural elements of an image or segmenting an image based on frequency characteristics.

In this research work, the use of QFT allows the analysis of image structure in a quantum context and can be useful for studying the main frequency components that characterize the image. The program you provided is used to apply QFT to an image and then analyze it. However, the use of the quantum Fourier transform itself is not a noise removal method. To remove noise from an image, various image processing methods such as filtering, smoothing or blurring are usually used. These techniques can be used to smooth pixel values, reduce high-frequency noise, or reduce differences between adjacent pixels. Although quantum techniques can also be applied to image processing, the QFT is not itself a noise removal technique. To remove noise from quantum images, you can use classical filtering methods or develop specialized quantum algorithms.

REFERENCES

- [1] Ozhigov, Yu. I., Quantum computing: educational method. allowance, Moscow State Publishing House. University, Moscow, 104 (2003).
- [2] Iris Cong, Soonwon Choi, and Lukin, M. D., Quantum convolutional neural networks, *Nature Physics*, 15(12):1273–1278 (2019).
- [3] Kaye, F., Laflamme, R. and Mosca, M.m Introduction to quantum computing, Regular and chaotic dynamics; Institute for Computer Research, Moscow–Izhevsk, 360 (2009).
- [4] Seunghyeok, Oh, Jaeho Choi and Joongheon Kim, A tutorial on quantum convolutional neural networks (qcn), In 2020 International Conference on Information and Communication Technology Convergence (ICTC), pages 236–239 (2020).
- [5] Russian Quantum Center, <https://www.youtube.com/channel/UCpOG8wlozPr6qXnO3kGoIxQ> (2024).
- [6] Yann LeCun and Corinna Cortes, MNIST handwritten digit database (2010).

- [7] Get started with IBM Quantum Experience, <https://quantum-computing.ibm.com/docs> (10 October 2020).
- [8] IBM Quantum Experience, <https://quantumcomputing.ibm.com/composer/new-experiment> (10 October 2020).
- [9] Han Xiao, Kashif Rasul and Roland Vollgraf, Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms. CoRR, abs/1708.07747 (2017).
- [10] Muhamediyeva, D. T., Fuzzy logical model for assessing soil salinity, IOP Conf. Series: Earth and Environmental Science, 1206, 012019, 1–7 (2023). doi:10.1088/1755-1315/1206/1/012019
- [11] Muhamediyeva, D. T., Application of artificial intelligence technologies to assess water salinity, IOP Conf. Series: Earth and Environmental Science 1206, 012020, 1–9 (2023).
- [12] Kaiming He, Xiangyu Zhang, Shaoqing Ren and Jian Sun, Deep residual learning for image recognition (2015).
- [13] Muhamediyeva, D. T. and Niyozmatova, N. A., Monitoring of biodiversity of water communities, E3S Web of Conferences, 411, 02042, 1-8 (2023).
- [14] Jeremy Howard and Sylvain Gugger, Fastai: A layered api for deep learning. Information, 11(2),108 (2020).
- [15] Xi-Wei Yao, Hengyan Wang, Zeyang Liao, Ming-Cheng Chen, Jian Pan, Jun Li, Kechao Zhang, Xingcheng Lin, Zhehui Wang and Zhihuang Luo, Quantum image processing and its application to edge detection: Theory and experiment. Physical Review X, 7(3), (2017).
- [16] Russian Quantum Center, <https://www.youtube.com/channel/UCpOG8wlozPr6qXnO3kGoIxQ> (2020).
- [17] Jae-Eun Park, Brian Quanz, Steve Wood, Heather Higgins and Ray Harishankar, Practical application improvement to quantum svm: theory to practice. arXiv preprint arXiv:2012.07725 (2020).
- [18] Quantum computing: textbook. allowance, Publishing house of the Kazan Federal University, Kazan, 100 (2010).
- [19] Kocczyk, D., Quantum machine learning for data scientists. arXiv preprint arXiv:1804.10068 (2018).