Topic 3.

Sequences

Definition 3.1. Har one n for a natural number determined in order x_n the number suitable placed if , then numbered $x_1, x_2, x_3, ..., x_n$ numbers collection $\{x_n\} = \{x_1, x_2, x_3, ..., x_n\}$ **numerous** is called a **sequence** . of the collection elements while of the sequence elements or are called limits .

Example :

1) $\{-1, 1, -1, 1, ...\} = \{(-1)^n\};$

2) $\left\{0, \frac{1}{2}, \frac{2}{3}, ...\right\} = \left\{\frac{n-1}{n}\right\};$

3) $\{1, 2, ...\} = \{n\};$

4) $x_{n+1} = \frac{x_n}{2}, x_1 = 1 \Rightarrow x_n = \frac{1}{2^{n-1}}, n = 1, 2, 3, ...;$

Numerical sequences arranged numerous collection that it was for , in them limited collections about theorems appropriate will be

➢ Numerical sequence limit

If optional positive ε for the thigh all $n > N$,$|x_n - a| < \varepsilon$ condition satisfactory so N for a natural number (ε ga bond) exists is a finite a number$\{x_n\}$ numerous **sequence limit** is called and as follows is defined as :

$$\lim_{n \to \infty} x_n = a$$

$$x_n \underset{n \to \infty}{\to} a$$

This is the following sentences can also be described through can :

-$\{x_n\}$ sequence a to approaches ;

-$\{x_n\}$ sequence a to equal to to the limit have

- x_n (of the sequence common let's go) a to strives

➢ Unlimited big and infinite small sequences

If optional M all positive numbers $n > N$ for s $\{x_n\} > M$ the satisfying N (M to connection) if a natural number is found , $\{x_n\}$ sequence **infinite big** is called

If $\lim\limits_{n\to\infty} x_n = 0$ if , to the limit definition according to $\{x_n\}$ sequence infinite small will be

➢ Monotonous sequences

If the sequence every one next Come on from before if it is not small (big) , that is all n digital limits for $x_n \le x_{n+1} (x_n \ge x_{n+1})$ inequality if $\{x_n\}$ done sequence decreasing non (growing is not) is called .

Decreasing and grower didn't happen sequences **monotonous** are called **sequences** .

If $x_n \ge x_{n+1}$ and $x_n \le x_{n+1}$ strictly didn't happen inequalities instead of $x_n < x_{n+1}$ or $x_n > x_{n+1}$ strictly inequalities appropriate if , the sequence suitable respectively grower and is called reducing .

Solved from examples samples

Example 3.1 . $a_n = \dfrac{4n-1}{n}$ of the sequence common expression given let it be This of the sequence a_{19} find the term .

Solution :

$\{a_n\}$ finding the 19th term of the sequence for n of instead of $n = 19$ put is :

$$a_{19} = \frac{4 \cdot 19 - 1}{19} = \frac{75}{19}.$$

3. Example 2. $1, 6, 11, 16, \ldots$ of the sequence general formula expression write and initial ten Come on the total count

Solution :

Above sequence elements to 5 increased going known , that is this sequence arithmetic progression organize is enough In this

$$a_n = a_1 + d(n-1);$$

$$d = \frac{a_n - a_1}{n-1} \text{ or } d = a_2 - a_1$$

the fact that known .

$$d = a_2 - a_1 \Rightarrow 6 - 1 = 5;$$

$$d = 5;$$

$$a_n = 1 + 5(n-1);$$

That's why $\{a_n\}$ sequence of the elements common expression as follows to write can :

$$a_n = 5n - 4 .$$

This of the sequence initial ten Come on the total as follows count can :

$$a_1 = 5 \cdot 1 - 4 = 1;$$
$$a_2 = 5 \cdot 2 - 4 = 6;$$
$$a_3 = 5 \cdot 3 - 4 = 11;$$
$$a_4 = 5 \cdot 4 - 4 = 16;$$
$$a_5 = 5 \cdot 5 - 4 = 21;$$
$$a_6 = 5 \cdot 6 - 4 = 26;$$
$$a_7 = 5 \cdot 7 - 4 = 31;$$
$$a_8 = 5 \cdot 8 - 4 = 36;$$
$$a_9 = 5 \cdot 9 - 4 = 41;$$
$$a_{10} = 5 \cdot 10 - 4 = 46;$$

$$S_{10} = \sum_{i=1}^{10} a_i = 1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 = 235$$

Or arithmetic in progression limits the total which determines s_n to the formula to put can :

$$S_n = \frac{a_1 + a_n}{2} n \Rightarrow S_{10} = \frac{a_1 + a_{10}}{2} n \Rightarrow S_{10} = \frac{1 + 46}{2} \cdot 10 = 235.$$

Example 3.3. Number 7 $a_n = \dfrac{15 + n}{n + 2}$ of the sequence Come on to be or that it won't be show me

Solution :

Number 7 above of the sequence Come on to be for this of the sequence which one one $n-$ element to be need That's why This issue is as follows to Eq is quoted :

$$7 = \frac{15+n}{n+2};$$

$$7 = \frac{15+n}{n+2} \Rightarrow 15+n = 7n+14;$$

$$6n = 1 \Rightarrow n = \frac{1}{6}$$

the fact that come comes out In this n that the number is not a natural number reason $\left(n = \frac{1}{6} \notin N\right)$ Number 7 $\{a_n\}$ of the sequence Come on be ca n't

Example 3.4. $\{a_n\}$ sequence $a_n = 2^n + n$ in the form of given let it be This of the sequence grower or decreasing that define

Solution :

This of the sequence grower or decreasing that to determine for last two come on the difference seeing will be :

$$a_{n+1} - a_n = 2^{n+1} + n + 1 - \left(2^n + n\right) = 2 \cdot 2^n + n + 1 - 2^n - n = 2^n + 1 > 0$$

that it was for , this sequence growing water that to say can

Example 3.5. $\{x_n\} = \left\{\sqrt{n}\right\}$ sequence to monotony check

Solution :

This of the sequence $n-$ Come on has been x_n and $(n+1)-$ Come on has been $x_{n+1} = \sqrt{n+1}$ limits the difference will be studied :

$$x_n - x_{n+1} = \sqrt{n} - \sqrt{n+1} = \frac{\left(\sqrt{n} - \sqrt{n+1}\right)\left(\sqrt{n} + \sqrt{n+1}\right)}{\left(\sqrt{n} + \sqrt{n+1}\right)} =$$

$$= \frac{n-n-1}{\sqrt{n} + \sqrt{n+1}} = -\frac{1}{\sqrt{n} + \sqrt{n+1}} < 0, \forall n \in N \Rightarrow x_n < x_{n+1}.$$

From this $\{x_n\}$ sequence growing water the fact that known will be

Example 3.6. $\{x_n\}$ sequence of terms common formula $x_n = \frac{2^n}{3^{n+1}}$ in the form of given let it be This of the sequence limited that prove it

Solution :

This sequence as follows to write can :

$$\frac{2^n}{3^{n+1}} = \frac{2^n}{3^n \cdot 3} = \frac{2^n}{3^n} \cdot \frac{1}{3} = \left(\frac{2}{3}\right)^n \cdot \frac{1}{3}.$$

From this $\frac{2}{3} < 1 \Rightarrow \left(\frac{2}{3}\right)^n < 1$ because it is , anyway n for a natural number $\frac{2^n}{3^{n+1}} < \frac{1}{3}$ the fact that known .

From this except , $x_n > 0$. So , whatever n for $|x_n| < \frac{1}{3}$ from the fact that $\{x_n\}$ sequence limited will be

Example 3.7. Mathematician induction method through the following equality prove :

$$\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\left(1-\frac{1}{16}\right)\ldots\cdot\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}, n \geq 2 .$$

Solution :

Initially $n = 2$ for checked :

$$\left(1-\frac{1}{4}\right) = \frac{2+1}{2\cdot 2} \Rightarrow \frac{3}{4} = \frac{3}{4}.$$

Now $k = n$ for this too equality right to $k = k+1$ be for checked :

$$\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\left(1-\frac{1}{16}\right)\ldots\cdot\left(1-\frac{1}{k^2}\right)\cdot\left(1-\frac{1}{(k+1)^2}\right) =$$

$$= \frac{k+1}{2k}\left(1-\frac{1}{(k+1)^2}\right) = \frac{k+1}{2k}\left(\frac{(k+1)^2-1}{(k+1)^2}\right) = \frac{1}{2k}\left(\frac{k^2+2k+1-1}{k+1}\right) = \frac{1}{2k}\left(\frac{k^2+2k}{k+1}\right) = \frac{k+2}{2(k+1)}$$

So , equality mathematician induction method through proved .

Example 3.8. Sequence limit count

$$\lim_{n\to\infty}\frac{3n^2+2n+4}{4n^2+n-3}.$$

Solution :

$n \to \infty$ when he aspires of the fraction photo and denominator also to infinity inclined will be That is, " $\frac{\infty}{\infty}$ " to uncertainty take will come This in the fraction photo and the denominator n^2 to to be through this sequence is changed to :

$$\lim_{n\to\infty}\frac{3n^2+2n+4}{4n^2+n-3}=\lim_{n\to\infty}\frac{3+\frac{2}{n}+\frac{4}{n^2}}{4+\frac{1}{n}-\frac{3}{n^2}}=\frac{\lim\limits_{n\to\infty}\left(3+\frac{2}{n}+\frac{4}{n^2}\right)}{\lim\limits_{n\to\infty}\left(4+\frac{1}{n}-\frac{3}{n^2}\right)}=$$

$$=\frac{\lim\limits_{n\to\infty}3+\lim\limits_{n\to\infty}\left(\frac{2}{n}\right)+\lim\limits_{n\to\infty}\left(\frac{4}{n^2}\right)}{\lim\limits_{n\to\infty}4+\lim\limits_{n\to\infty}\left(\frac{1}{n}\right)-\lim\limits_{n\to\infty}\left(\frac{3}{n^2}\right)}=\frac{3+0+0}{4+0-0}=\frac{3}{4}.$$

Independent work for issues

3.1 The following of sequences the first ten Come on find the sum of

a) $1,4,7,10,...;$

b) $a_n=\frac{100}{n+1};$

c) $a_n=\frac{n^2}{n+1};$

d) $a_n=\sin\frac{\pi n}{2};$

e) $a_n=n^{(-1)^n};$

3.2 $1,3,5,7,9,...$ from the elements of the sequence formula express and initial eight Come on the total count

3.3 The following sequences for general formula expression write

a) $\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},...$

b) $2,\ 12,\ 22,\ 32,\ 42,\ 52,...;$

c) $2,\ 5,\ 8,\ 11,\ 14,\ 17,\ ...;$

d) $1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ ...;$

e) $1,\ 3,\ 9,\ 27,\ 81,\ 243,\ 729,\ ...;$

f) $0,\ 1,\ 3,\ 7,\ 15,\ 31,\ 63,\ ...;$

g) 2, 4, 10, 28, 82, 244, 730, ...;

h) -1, 1, -1, 1, -1, 1, -1, 1, ...;

i) 0, 3, 3, 9, 15, 33, 63, ...;

j) 0, 1, 0, 2, 0, 4, 0, 8, 0, 16, ...

3.4 $a_n = 4 - n$ sequence given let it be $a_{10} + a_{20} - a_{30}$ the define

3.5 $a_n = (-1)^n n^2 + 2$ sequence given let it be $a_1 \cdot a_3 \cdot a_5$ the count

3.6 $a_n = 3n$ of the sequence find the first 5 terms and program make up

3.7 $a_n = 2n^2 + 1$ sequence given let it be This find the 38th term of the sequence .

3.8 $a_n = |n-3| + |2-n|$ sequence given let it be a_n of the sequence odd order in numbers standing the first five come on find the sum .

3.9 $a_n = n^2 - n - 2$ sequence given let it be This 5 of the sequence non-increasing limits find the sum of

3.10 $x_n = \frac{(-1)^n}{n}, n \in N$ find the 15th term of the sequence and program make up

3.11 c_n sequence for $c_1 = 4, c_{n+1} = c_n + 3$ known if , the first find the 5 words that appear .

3.12 Number 3 $a_n = \frac{51 + 2n}{n + 4}$ of the sequence Come on to be or that it won't be show me

3.13 $a_1 = 4, a_2 = 6$ and $a_{n+2} + a_n = 2a_{n+1}$ if a_{15} find the

3.14 Growing and decreasing to sequences examples bring

3.15 The following of sequences decreasing or grower that define

a) $a_n = n^2 - n$;

b) $a_n = \frac{n}{n+1}$;

c) $x_n = \left(1 + \frac{1}{n}\right)^n, (n = 1, 2, 3, \ldots)$

3.16 The following of the sequence limited or that it is not limited define :

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots;$$

3.17 $a_n = n^2 + 3n$ sequence given if so , the numbers 130;0;42 this of the sequence elements to be or that it won't be show me

3.18 Mathematician induction method using right that prove it

a) $1+3+5+...+(2n-1) = n^2, \forall n \in N$

b) $\frac{1}{1\cdot 5} + \frac{1}{5\cdot 9} + ... + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}, \forall n \in N$

3.19 The following of sequences limited prove it

a) $\left\{\frac{2n-1}{2n+1}\right\}, n \in N$;

b) $\left\{\frac{n^3+1}{n^3-1}\right\}, n \in N$.

3.20 Unlimited small of the sequence limited multiple about from the theorem or monotonous and limited of the sequence approach about from the theorem using the following of the sequence approach prove :

$$x_n = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{2^2}\right)...\left(1-\frac{1}{2^n}\right)$$

3.21 Sequence limit calculate :

a $\lim_{n\to\infty}\left\{\frac{3n^2+3}{4n^2-4}\right\}$)

b $\lim_{n\to\infty}\frac{(2-n)^3}{(n+1)^2-(n+1)^3}$)

c $\lim_{n\to\infty}\frac{n\sin n}{n^2+1}$)

d) $\lim_{n\to\infty}\left(\sqrt{n^2+1}-\sqrt{n^2}\right)$.

3.22 From the definition of limit using , of the sequence the limit calculate :

$$1; \frac{5}{7}; \frac{7}{11}; ...; \frac{(2n+1)}{(4n-1)}; ...$$

3.23 The following sequences to monotony check

a) $a_n = \frac{8n^2}{2n^3+n}$;

b) $x_n = \dfrac{n-1}{n^2}$

3.24 The following sequences to limitation check

a) $a_n = \dfrac{100}{n^2+1}$;

b) $a_n = n^2 - 8n - 20$.

3.25 $a_n = n^2 - 5n + 1$ of the sequence the most small find the term .

Answers

3.1. $a)S_{10}=145$; $b)S_{10}\approx 67$; $c)S_{10}\approx 85.45$; $d)S_{10}=0$; $e)S_{10}=-5$

3.2. $a_n=2n-1, S_8=64$

3.3. $a)a_n=\frac{1}{2n}$; $b)a_n=10n-8$; $c)a_n=3n-1$; $d)a_n=2^{n-1}$; $e)a_n=3^{n-1}$; $f)a_n=2^{n-1}-1$;

$g)a_n=3^{n-1}+1$; $h)a_n=(-1)^n$; $i)a_n=2^{n-1}+(-1)^n$; $j)a_n=\left((-1)^n+1\right)2^{(n-3)/2}$

3.4. 4

3.5. 161

3.6. $a_1=3, a_2=6, a_3=9, a_4=12, a_5=15$

3.7. 2889

3.8. 31

3.9. 2

3.10. $-\frac{1}{15}$

3.11. 50

3.12. yes $n=39$ Come on will be

3.13. $a_{15}=32$

3.14. Growing sequence : a) $1,2,3,4,5...$; b $\frac{1}{2},\frac{3}{4},\frac{5}{6},\frac{7}{8},...$) c $2,4,8,16,32,...$)

Decreasing sequence : a) $10,9,8,7,6,...$; b $\frac{7}{2},3,\frac{5}{2},2,...$) c)$1000,500,250,125,...$

3.16. Decreasing

3.17. Number 130 $n=10$ Come on will be Numbers 0 and 42 sequence Come on it won't be .

3.21. $a)\frac{3}{4}$; $b)1$; $c)0$; $d)0$

3.22. $\frac{1}{2}$

3.23. a) Decreasing

3.24. a) Bounded , $0 < a_n \leq 50$

3.25. -5