*“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI*

# Matritsa va ular ustida amallar

Fan nomi: Hisob (Calculus)

# Reja:

1. Matritsalar va ular ustida amallar.
2. Matritsalarni ko'paytirish, teskari matritsani topish.
3. Matritsaning rangi.

# Matritsalar va ular ustida amallar

$m \times n$ dona $a_{ij}$ $(i = \overline{1, m}, j = \overline{1, n})$ elementlardan tuzilgan to'g'ri burchakli jadval matritsa deyiladi va

$$A = \begin{array}{rl} & \begin{array}{ccccc} \phantom{a_{11}} & \phantom{a_{12}} & 3 & \phantom{\dots} & \phantom{a_{1n}} \end{array} \\ 2 \rightarrow & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \end{array} \quad \text{yoki} \quad A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix}$$

ko'rinishda yoziladi. Matritsaning elementlari ikkita indesklar bilan belgilanadi. Elementning birinchi $i$ indeksi satr nomini, ikkinchi $j$ indeks esa ustunning nomerini bildiradi. Matritsaning $a_{ij}$ elementi $i$ – satr va $j$ – ustun kesishgan joyda joylashgan. Matritsalar odatda katta lotin harflari bilan belgilanadi:

$A, B, C, \dots$

# Matritsalar va ular ustida amallar

Agar $A$ va $B$ matritsalarning mos $a_{ij}$ va $b_{ij}$ elementlari bir-biriga teng, ya`ni $a_{ij} = b_{ij}$ bo'lsa, bunday $A$ va $B$ matritsalar **teng matritsalar** deyiladi. Faqat bir xil o'lchovli matritsalargina bir-biriga teng bo'lishi mumkin. Har xil o'lchovli matritsalarning bir-biriga teng bo'lishi yoki teng emasligi tushunchalari kiritilmagan. Satrlarining soni ustunlarining soniga teng bo'lgan ($m = n$) matritsalar **kvadrat matritsalar** deyiladi. Agar $i = 1$ bo'lsa, u holda **satr-matritsa**ga ega bo'lamiz; agar $j = 1$ bo'lsa, biz **ustun-matritsa**ga ega bo'lamiz. Ular mos ravishda **satr-vektor** va **ustun-vektor** ham deb ataladi.

$A = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$ $A = B$ $A = (1, 2, 3)$

$A$ $m \times n$

$m = n$

# Matritsalar va ular ustida amallar

Masalan, ikkita

$$A = \begin{pmatrix} 1 & 6 \\ 2 & -4 \\ -3 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 \\ 3 & 7 \\ 8 & -11 \end{pmatrix}.$$

matritsalar berilgan bo'lsin. U holda

$$A + B = \begin{pmatrix} 1 + (-2) & 6 + 4 \\ 2 + 3 & -4 + 7 \\ -3 + 8 & 9 + (-11) \end{pmatrix} = \begin{pmatrix} -1 & 10 \\ 5 & 3 \\ 5 & -2 \end{pmatrix},$$

$$C = A - B = \begin{pmatrix} 1 - (-2) & 6 - 4 \\ 2 - 3 & -4 - 7 \\ -3 - 8 & 9 - (-11) \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -1 & -11 \\ -11 & 20 \end{pmatrix}.$$

# Matritsalar va ular ustida amallar

## Matritsani songa ko'paytirish.

$A$ matritsani $\lambda$ songa ko'paytmasi $\lambda A$ bilan belgilanadi.

$A$ matritsaning $\lambda$ songa $\lambda A$ ko'paytmasi deb shunday $B$ matritsaga aytiladiki, $B$ matritsaning elementlari $b_{ij} = \lambda a_{ij}$ dan iboratdir, bu yerda $a_{ij}$ – $A$ matritsaning elementlari. $A$ matritsani $\lambda$ songa ko'paytirganda hosil bo'ladigan $B$ matritsa $A$ matritsa bilan bir xil o'lchovli bo'ladi. Hullas, matritsani biror songa ko'paytirish uchun bu matritsaning har bir elementini shu songa ko'paytirib chiqish kerak.

# Matritsalar va ular ustida amallar

Masalan,

$$\lambda = -2,$$

$$A = \begin{pmatrix} 3 & 0 \\ 7 & -1 \end{pmatrix}$$

$2 \times 2$

bo'lsin. U holda

$$\lambda A = -2 \cdot \begin{pmatrix} 3 & 0 \\ 7 & -1 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ -14 & 2 \end{pmatrix}.$$

# Matritsalar va ular ustida amallar

**Misol 1.** $AB$ va $BA$ ko'paytmalarni toping.

$$A=\begin{pmatrix}4 & -5 & 8\\ 1 & 3 & -1\end{pmatrix},$$

$$B=\begin{pmatrix}-1 & 5\\ -2 & -3\\ 3 & 4\end{pmatrix}.$$

$AB \neq BA$

$AB = BA$

$AB$ ko'paytmani topamiz:

$$A\cdot B=\begin{pmatrix}4 & -5 & 8\\ 1 & 3 & -1\end{pmatrix}\begin{pmatrix}-1 & 5\\ -2 & -3\\ 3 & 4\end{pmatrix}=\begin{pmatrix}4\cdot(-1)+(-5\cdot(-2))+8\cdot3 & 4\cdot5+15+32\\ -1-6-3 & 5-9-4\end{pmatrix}=\begin{pmatrix}38 & 67\\ -10 & -8\end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} -1 & 5 \\ -2 & -3 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & -5 & 8 \\ 1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -4+5 & 5+15 & -8-5 \\ -8-3 & 10-9 & -16+3 \\ 12+4 & -15+12 & 24-4 \end{pmatrix} = \begin{pmatrix} 1 & 20 & -13 \\ -11 & 1 & -13 \\ 16 & -3 & 20 \end{pmatrix}$$

$$A \cdot B \neq B \cdot A$$

# Matritsalar va ular ustida amallar

**Misol 2.** $AB$ va $BA$ ko'paytmalarni toping.

$$A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -5 \\ -1 & 2 \end{pmatrix}.$$

Hisoblaymiz:

$$A \cdot B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3-5 & -15+10 \\ 1-2 & -5+4 \end{pmatrix} = \begin{pmatrix} -2 & -5 \\ -1 & -1 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 1 & -5 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 - 5 & 5-10 \\ -3+2 & -5+4 \end{pmatrix} = \begin{pmatrix} -2 & -5 \\ -1 & -1 \end{pmatrix}$$

$$A \cdot B = B \cdot A$$

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# Matritsa va ular ustida amallar (davomi)

Fan nomi: Hisob (Calculus)

# Reja:

1. ~~Matritsalar va ular ustida amallar.~~
2. ~~Matritsalarni ko’paytirish~~, teskari matritsani topish.
3. Matritsaning rangi.

# Teskari matritsa

$n-$ tartibli kvadrat matritsa berilgan bo'lsin:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

n x n

Agar $A$ matritsaning determinanti noldan farqli

det A ≠ 0

$$det\,A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \neq 0$$

bo'lsa, $A$ matritsa **aynimagan matritsa** deyiladi. Agar $det\,A = 0$ bo'lsa, $A$ matritsa **aynigan matritsa** deyiladi.

# Teskari matritsa

$A$ matritsaga teskari matritsa $A^{-1}$ ko'rinishda belgilanadi. Teskari matritsa tushunchasi faqat aynimagan kvadrat matritsalarga taalluqlidir.

1) 2)

Ushbu

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

kvadrat matritsa **birlik matritsa** deyiladi.

2x2

$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & -2 \\ 3 & 6 \end{pmatrix}$ $A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \\ 4 & 6 & 0 \end{pmatrix}$

Ushbu

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{pmatrix}$$

$A^T = \begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix}$ $\boxed{A^T = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -1 & 6 \\ 1 & 5 & 0 \end{pmatrix}}$

kvadrat matritsa $A$ matritsaga nisbatan **transponirlangan matritsa** deyiladi.

# Teskari matritsa

Aynimagan $A$ matritsa berilgan bo'lsin. Agar

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

bo'lsa, $A^{-1}$ matritsa $A$ matritsaga **teskari matritsa** deyiladi.

$A$ matritsaga teskari $A^{-1}$ matritsani topish formulasi:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix},$$

bu yerda $A_{ij}$ – berilgan $A$ matritsaga nisbatan transponirlangan $A^T$ matritsaning algebraik to'ldiruvchilari.

# Transpanerlangan matritsa

# Teskari matritsa

**Misol 1.** $A$ matritsa berilgan:

$$\overset{2\times 2}{\phantom{x}}$$

$$\text{a) } A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}; \quad \text{b) } A = \begin{pmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{pmatrix}.$$

$A$ matritsa aynimagan matritsa ekanligiga ishonch hosil qiling, $A$ matritsaga teskari $A^{-1}$ matritsani toping va $A \cdot A^{-1} = A^{-1} \cdot A = E$ tengliklarning bajarilishini tekshiring.

1) $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$ $\qquad A^{-1} - ?$

1) $\det A = \begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix} = -3 - 2 = -5 \neq 0$ aynimagan!

2) $A_{11} = (-1)^{1+1} \cdot 3 = 3$ $\qquad A_{12} = (-1)^{1+2} \cdot 1 = -1$

$A_{21} = (-1)^{2+1} \cdot 2 = -2$ $\qquad A_{22} = (-1)^{2+2} \cdot (-1) = -1$

$A_{to'ldiz} = \begin{pmatrix} 3 & -1 \\ -2 & -1 \end{pmatrix}$

$A^{T}_{to'ldiz} = \begin{pmatrix} 3 & -2 \\ -1 & -1 \end{pmatrix}$

$A^{-1} = \frac{1}{\det A} A^{T}_{to'ld} = -\frac{1}{5} \cdot \begin{pmatrix} 3 & -2 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

$$A^{-1} = \begin{pmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

$$A \cdot A^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} \frac{3}{5}+\frac{2}{5} & -\frac{2}{5}+\frac{2}{5} \\ -\frac{3}{5}+\frac{3}{5} & \frac{2}{5}+\frac{3}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

# Teskari matritsa

b) $A = \begin{pmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{pmatrix}$ matritsaning determinantini hisoblaymiz:

# Matrista rangi

$A$ matritsadan yaralgan determinantlar ichidan noldan farqlilarini ajratib olamiz. Ana shu noldan farqli determinantlar tartibining eng kattasi **$A$ matritsaning rangi** deyiladi ($rangA$ deb belgilanadi).

Agar $A$ matritsadan yaralgan $k$ –tartibli determinantlarning hammasi nolga teng bo'lsa, u holda $rangA < k$ bo'ladi.

*misol*. Quyidagi matritsaning rangini toping.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix}$$

rang $A = 2$

$\det M_1 = |2| \neq 0$

$\det M_2 = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 0$ $\det M_2 = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$

$\det M_2 = \begin{vmatrix} 2 & 3 \\ 10 & 15 \end{vmatrix} = 0$ $\det M_2 = \begin{vmatrix} 2 & 1 \\ 10 & 5 \end{vmatrix} = 0$

$\det M_2 = \begin{vmatrix} 4 & 2 \\ 10 & 5 \end{vmatrix} = 0$ $\det M_2 = \begin{pmatrix} 2 & 6 \\ 5 & 15 \end{pmatrix} = 0$

$\det M_3 = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 10 & 5 & 15 \end{vmatrix} =$

# Uyga vazifa

1. Amallarni bajaring:

$a)\ \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix};$ $\qquad b)\ \begin{pmatrix} -1 & 2 & 3 \\ -3 & 1 & 5 \end{pmatrix} + \begin{pmatrix} 2 & -2 & -3 \\ 3 & 0 & -5 \end{pmatrix}$

$c)\ 4\cdot\begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{pmatrix} + 3\begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 2 \end{pmatrix} - 2\cdot\begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \end{pmatrix};$

2. $A=\begin{pmatrix} 3 & 2 & 8 \\ 1 & -4 & 0 \end{pmatrix}$ , $B=\begin{pmatrix} 2 & -1 \\ 1 & -3 \\ 0 & 1 \end{pmatrix}$ matritsalar ko‘paytmasini toping.

3. Matritsalarni transponirlang:

$a)\ A=\begin{pmatrix} -1 & -3 & 2 \\ 0 & -4 & 0 \\ 2 & -5 & 3 \end{pmatrix};$ $\qquad b)\ B=\begin{pmatrix} 1 & 0 & 3 & -4 \\ 2 & 3 & 4 & 0 \\ 1 & 3 & 0 & 7 \\ -2 & 4 & 6 & 8 \end{pmatrix}$

4. Berilgan matritsaga teskari matritsani toping:

$$a)\ \begin{pmatrix} 2 & 5 & 0 \\ 3 & 3 & 1 \\ 0 & -2 & -1 \end{pmatrix} \quad b)\ \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$$

5. Matritsaning rangini toping:

$$a)\ \begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix} \quad b)\ \begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$$