Topic 6.

The opposite function . Function continuity . Unlimited small functions

Definition 6.1. If $y = f(x), x \in X, y \in Y$ function his own every one value one times acceptance if he does , that is every one $y \in Y$ for the only one $x \in X$ there is if , then $y = f(x), x \in X, y \in Y$ function reversible is called

Definition 6.2. Function reversible if , then X the collection Y to the collection reflective $y = f(x)$ to the function Y the collection X to the collection reflective $y = g(x)$ function suitable put can , in this $y = g(f(x))$. This is a function $f(x)$ **to the function reverse** is called and $f^{-1}(y)$ such as is determined .

➢ Unlimited small functions

okay $\alpha(x)$ and $\beta(x)$ $x \to x_0$ when striving $\lim\limits_{x \to x_0} \frac{\alpha(x)}{\beta(x)} = A$ let it be Unlimited small functions comparison for of the following used :

1) If $A \neq \infty$ and $A \neq 0$ if $\alpha(x)$ and $\beta(x)$ the first in order infinite small functions is called ;

2) If $A = 1$ if , then $\alpha(x)$ and $\beta(x)$ equivalent infinite small functions is called That is , $x \to x_0$ when striving $\alpha(x) \sim \beta(x)$;

3) If $A = 0$ if , then $\alpha(x)$ function $\beta(x)$ to relatively high in order infinite small is called a function . That is , $x \to x_0$ when striving $\alpha(x) = o(\beta(x))$;

4) If $A = \infty$ if , then $\alpha(x)$ function $\beta(x)$ to relatively lower in order infinite small is called a function . That is , $x \to x_0$ when striving $\beta(x) = o(\alpha(x))$;

5) If $\lim\limits_{x \to x_0} \frac{\alpha(x)}{\beta(x)}$ there is a limit if not , then $\alpha(x)$ and $\beta(x)$ functions incomparable functions will be

6) If $\lim\limits_{x \to x_0} \frac{\alpha(x)}{(\beta(x))^k} = A$, $0 < |A| < \infty$ if $\alpha(x)$ function $x \to x_0$ when striving $\beta(x)$ with in comparison k in order infinite small function will be

➢ Function continuity

$y = f(x)$ function x_0 **at the point continuously** is called , if

1) SHE IS x_0 point and his small around determined if
2) $y = f(x)$ of the function x_0 at the point limit there is if
3) of the function x_0 limit value at the point $\lim\limits_{x \to x_0} f(x) = f(x_0)$ to equal to if

If $x \to x_0$ when striving $x > x_0$ if $\lim\limits_{\substack{x \to x_0 \\ x > x_0}} f(x)$ right is called side limit and $\lim\limits_{x \to x_0 + 0} f(x)$ is written as If $x \to x_0$ when striving $x < x_0$ is called the left limit $\lim\limits_{\substack{x \to x_0 \\ x < x_0}} f(x)$ and $\lim\limits_{x \to x_0 - 0} f(x)$ is written as

Left and right bilaterally limits both of them **one bilaterally limits** is $\lim\limits_{x \to x_0} f(x) = A$ called to be for $\lim\limits_{x \to x_0 - 0} f(x) = \lim\limits_{x \to x_0 + 0} f(x) = A$ to be enough and necessary .

of the function at the point continuity meaning as follows is represented by :

$$f(x_0) = \lim_{x \to x_0} f(x) = \lim_{x \to x_0 - 0} f(x) = \lim_{x \to x_0 + 0} f(x)$$

If x_0 point of the function continuity disruptive if so , this x_0 point of the function **interruption point** is called

Okay x_0- of the function interruption point let it be If this point for one bilaterally limits there is if , then x_0 point **Type I interruption** is called a **point** .

If $f(x_0)$ there is if not , that is one bilaterally from limits at least one $x \to x_0$ when striving ∞ to equal to if , then x_0 point **Type II interruption** is called a **point.**

Solved from examples samples

Example 6.1. function to continuity check : $f(x) = \begin{cases} x + 2, & x < -2 \\ \dfrac{x^2 - 4}{2}, & -2 \le x < 0; \\ \sin x, & x \ge 0. \end{cases}$

Solution :

Function numbers read on all in value determined and $(-\infty;-2),(-2;0),(0;+\infty)$ in the interval continuously . Because elementary functions with expressed . Function $x=-2$ and $x=0$ at the points they are studied through of the function analytical specification will change .

If $x=-2$ if , then $f(-2)=\dfrac{(-2)^2-4}{2}=0$ will be

Now one sided limits defined as :

$$\lim_{x\to -2-0}(x+2)=0; \qquad \lim_{x\to -2+0}\left(\frac{x^2-4}{2}\right)=0;$$

In this , $\lim\limits_{x\to -2-0} f(x)=\lim\limits_{x\to -2+0} f(x)= f(-2)=0$. So , $x=-2$ – at the point function continuously .

If $x=0$ if , then $f(0)=\sin 0=0$ will be

Now one sided limits defined as :

$$\lim_{x\to 0-0}\left(\frac{x^2-4}{2}\right)=-2; \qquad \lim_{x\to 0+0}\sin x=0;$$

In this $\lim\limits_{x\to 0-0} f(x)\neq \lim\limits_{x\to 0+0} f(x)= f(0)$. So , $x=0$ – of the function $\omega=\left|\lim\limits_{x\to x_0-0} f(x)-\lim\limits_{x\to x_0+0} f(x)\right|=\left|-2-0\right|=2$ jump with the first kind of interruption point is considered

$\lim\limits_{x\to 0+0} f(x)=f(x)$ that it was for , $f(x)$ function $x=0$ at the point right by continuously will be

As a result , the function $R/\{0\}$ when continuously will be , in this $x=0$ – first type interruption point

6. Example 2. Given functions 1st order eng small functions to be prove it

$\alpha(x) = x^2 - 1$, $\beta(x) = x - 1$, $x \to 1$.

Solution :

$\alpha(x) = x^2 - 1, \beta(x) = x - 1$ functions $x \to 1$ when striving

$$\lim_{x \to 1} \alpha(x) = \lim_{x \to 1} \left(x^2 - 1\right) = 1^2 - 1 = 0\,;$$

$$\lim_{x \to 1} \beta(x) = \lim_{x \to 1} (x - 1) = 1 - 1 = 0$$

infinite small functions that to see can Now their $x \to 1$ when ratio will be studied :

$$\lim_{x \to 1} \frac{\alpha(x)}{\beta(x)} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x - 1} =$$

$$= \lim_{x \to 1} (x + 1) = 1 + 1 = 2\,.$$

So , the limit value is 0 and ∞ from different has been the value acceptance does From this ,$\alpha(x)$ function $x \to 1$ when $\beta(x)$ to the function relatively the first in order infinite small function the fact that known will be

Example 6.3. $x \to -1$ to when striving $y = x + 1$ of the function $\beta(x) = (x+1)^2$ to the function relatively smallness order define

Solution :

In this ,

$$\alpha(x) = x + 1 \text{ function } \left(\lim_{x \to -1} \alpha(x) = \lim_{x \to -1} (x + 1) = -1 + 1 = 0\right)$$

from the fact that $x = -1$ at the point infinite small function that to see can $\beta(x) = (x+1)^2$ function too

$$\left(\lim_{x \to -1} \beta(x) = \lim_{x \to -1} (x+1)^2 = (-1+1)^2 = 0\right)$$

from being his $x = -1$ at the point infinite small function the fact that known will be them to compare for the following calculations is executed :

$$\lim_{x\to -1}\frac{\alpha(x)}{\beta(x)}=\lim_{x\to -1}\frac{x+1}{(x+1)^2}=\lim_{x\to -1}\frac{1}{x+1}=\frac{1}{0}=\infty$$

From this , $\alpha(x)$ function $x\to -1$ when $\beta(x)$ to the function relatively lower in order infinite small function the fact that known will be

Example 6.4. Unlimited small functions compare :

$$\alpha(x)=\left(x^2-1\right)^2, \qquad \beta(x)=x-1, \qquad x\to 1\,.$$

Solution :

$\alpha(x)=\left(x^2-1\right)^2$ function

$$\left(\lim_{x\to 1}\alpha(x)=\lim_{x\to 1}\left(x^2-1\right)^2=0\right)$$

from the fact that $x=1$ at the point infinite small function that to see can $\beta(x)=x-1$ function too

$$\left(\lim_{x\to 1}\beta(x)=\lim_{x\to 1}(x-1)=0\right)$$

from being his $x=1$ at the point infinite small function the fact that known will be them to compare for the following calculations is executed :

$$\lim_{x\to 1}\frac{\alpha(x)}{\beta(x)}=\lim_{x\to 1}\frac{\left(x^2-1\right)^2}{x-1}=\lim_{x\to 1}\frac{(x-1)^2(x+1)^2}{x-1}=$$

$$\lim_{x\to 1}(x-1)(x+1)^2=0\cdot 2=0$$

From this , $\alpha(x)$ function $x\to 1$ at the point $\beta(x)$ to the function relatively high in order infinite small function the fact that known will be

6. Example 5. Given y to the function $f^{-1}(x)$ reverse function define : $y=-\frac{1}{2}x+7$, $2\le x\le 9$.

Solution :

Given function argument for and his value replaced by :

$$x = -\frac{1}{2}y + 7\,;$$

$$y = 2\cdot(-x+7) = -2x+14\,;$$

In this this function argument the following condition satisfaction need :

$$2 \le y \le 9 \Rightarrow$$

$$2 \le -2x+14 \le 9 \Rightarrow$$

$$-12 \le -2x \le -5 \Rightarrow$$

$$6 \ge x \ge 2{,}5$$

So , it is given of the function reverse function $f^{-1}(x) = -2x+14, \quad (2{,}5 \le x \le 6)$ will be

Independent work for issues

Given $f(x)$ function x_0 at the point to continuity check (6.1-6.4).

6.1 $\quad x_0 = 1,\ f(x) = \dfrac{1-x^2}{1+x^2};$

6.2 $\quad x_0 = 0,\ f(x) = \begin{cases} \dfrac{1-\cos x}{x^2}, & x \ne 0; \\ 3, & x = 0. \end{cases}$

6.3 $\quad x_0 = 1,\quad f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \ne 1; \\ 2, & x = 1. \end{cases}$

6.4 $\quad x_0 = 0,\ f(x) = \begin{cases} x\sin\dfrac{1}{x}, & x \ne 0; \\ 0, & x = 0. \end{cases}$

6.5 $\quad f(x)$ function $x_0 = 0$ at the point continuously has been A of find the value of

$$f(x)=\begin{cases}\dfrac{\sin 8x-x}{3x+\sin 4x}, & x>0;\\ A\cos x, & x\le 0.\end{cases}$$

6.6 $f(x)$ function $x_0=0$ at the point continuously has been A of find the value of

$$f(x)=\begin{cases}\dfrac{1-\cos x}{x^2}, & x\ne 0;\\ A, & x=0.\end{cases}$$

6.7 Given function continuity check : $y=\dfrac{x-4}{|x|}$;

6.8 Given function continuity check : $y=1+2^{\frac{1}{1-x}}$;

6.9 Given function continuity check : $y=\dfrac{1}{x^2-4}$;

Given y to the function reverse function define (6.10-6.24):

6.10 $y=3x+2$;

6.11 $y=-5x-7$;

6.12 $y=12x-3$;

6.13 $y=-8x+16$;

6.14 $y=\dfrac{2}{3}x-5$;

6.15 $y=-\dfrac{3}{4}x+5$;

6.16 $y=-\dfrac{5}{8}x+10$;

6.17 $y=\dfrac{1}{2}x+8$;

6.18 $y=x^2+5$;

6.19 $y=x^2-4$;

6.20 $y=(x+3)^2$;

6.21 $y=(x-6)^2$;

6.22 $y=\sqrt{x-2},\ y\ge 0$;

6.23 $y = \sqrt{x+5},\ y \ge 0$;

6.24 $y = \sqrt{x+8},\ y \ge 8$;

Given $f(x)$ to the function $g(x)$ function reverse to be check (6.25-6.30).

6.25 $f(x) = x + 6,\ g(x) = x - 6$;

6.26 $f(x) = 5x + 2,\ g(x) = \dfrac{x-2}{5}$;

6.27 $f(x) = 3x - 9,\ g(x) = \dfrac{1}{3}x - 3$;

6.28 $f(x) = 2x - 7,\ g(x) = \dfrac{x+7}{2}$;

6.29 $f(x) = -4x + 8,\ g(x) = -\dfrac{1}{4}x + 2$;

6.30 $f(x) = \dfrac{1}{2}x - 7,\ g(x) = 2x + 14$;

$x \to 0$ to when striving of functions $y = x$ to the function relatively smallness procedures define (6.31-6.36):

6.31 $\dfrac{2x}{x+1}$;

6.32 $\sqrt{x + \sqrt{x}}$;

6.33 $\sqrt[3]{x^2} - \sqrt{x^3}$;

6.34 $y = 1 - \cos x$;

6.35 $y = tgx - \sin x$;

6.36 $\dfrac{7x^8}{x^4 + 1}$;

Unlimited small functions Compare (6.37-6.43):

6.37 $\alpha(x) = \dfrac{3x^4 - 4}{x+1}, \qquad \beta(x) = x^3, \qquad x \to 0$

6.38 $\alpha(x) = \dfrac{1-x}{x+1}, \qquad \beta(x) = 1 - \sqrt[3]{x}, \qquad x \to 1$;

6.39 $\alpha(x) = \sqrt[3]{x^4 + 2x^3}, \qquad \beta(x) = \ln(1+x), \qquad x \to 0$;

6.40 $\alpha(x) = 1 - \cos^3 x, \qquad \beta(x) = \sin^2 x, \qquad x \to 0$;

6.41 $\alpha(x) = \dfrac{x+1}{x^2+1}, \qquad \beta(x) = \dfrac{1}{x}, \qquad x \to 0$;

6.42 $\alpha(x) = \dfrac{arctgx}{x^2+1}, \qquad \beta(x) = \dfrac{1}{x^2}, \qquad x \to \infty$;

6.43 $\alpha(x) = 1 + \sin^3 x, \qquad \beta(x) = \cos^2 x, \qquad x \to \dfrac{\pi}{2}$

Given functions 1st order eng small functions to be prove (6.44-6.45):

6.44 $f(x) = tgx \;\; va \;\; \varphi(x) = \arcsin x, \qquad x \to 0$;

6.45 $f(x) = 1 - \cos x \;\; va \;\; \varphi(x) = 3x^2, \qquad x \to 0$

Answers

6.1. At the point continuously

6.2. To interrupt have

6.3. At the point continuously

6.4. At the point continuously

6.5. $A = 1$

6.6. $A = \frac{1}{2}$

6.7. $x = 0$ at the point to disconnect have

6.8. $x = 1$ at the point to disconnect have

6.9. $x = -2$ and $x = 2$ at points to disconnect have

6.10. $f^{-1}(x) = \frac{x-2}{3}$

6.11. $f^{-1}(x) = -\frac{x+7}{3}$

6.12. $f^{-1}(x) = \frac{x+3}{12}$

6.13. $f^{-1}(x) = -\frac{x}{2} + 2$

6.14. $f^{-1}(x) = \frac{3x+15}{2}$

6.15. $f^{-1}(x) = \frac{20-4x}{3}$

6.16. $f^{-1}(x) = 16 - \frac{8x}{5}$

6.17. $f^{-1}(x) = 2x - 16$

6.18. $f^{-1}(x) = \pm\sqrt{x-5}$

6.19. $f^{-1}(x) = \pm\sqrt{x+4}$

6.20. $f^{-1}(x) = \pm\sqrt{x} - 3$

6.21. $f^{-1}(x) = \sqrt{x} \pm 6$

6.22. $f^{-1}(x) = x^2 + 2$

6.23. $f^{-1}(x)=x^2-5, x\in(-\infty;-\sqrt{5})\cup(\sqrt{5};\infty)$

6.24. $f^{-1}(x)=x^2-8, x\in(-\infty;-4)\cup(4;\infty)$

6.25. Functions to each other reverse will be

6.26. Functions to each other reverse will be

6.27. Functions to each other reverse it's not

6.28. Functions to each other reverse will be

6.29. Functions to each other reverse will be

6.30. Functions to each other reverse will be

6.31. First in order infinite small functions

6.32. Incomparable functions

6.33. $k=\frac{2}{3}$ good infinite small function

6.34. High in order

6.35. High in order

6.36. High in order

6.37. $\alpha(x)$ and $\beta(x)$ incomparable functions

6.38. $\alpha(x)$ and $\beta(x)$ the first in order infinite small functions

6.39. $\alpha(x)$ and $\beta(x)$ incomparable functions

6.40. $\alpha(x)$ and $\beta(x)$ the first in order infinite small functions

6.41. $\alpha(x)$ function $\beta(x)$ to relatively high in order infinite small function

6.42. $\alpha(x)$ and $\beta(x)$ the first in order infinite small functions

6.43. $\alpha(x)$ function $\beta(x)$ to relatively lower in order infinite k is internal function