“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

Determinantlar

Fan nomi: Hisob (Calculus)

REJA:

1. Ikkinchi tartibli determinant.
2. Uchinchi tartibli determinant.
3. Determinantning xossalari.

1. Ikkinchi tartibli determinant. To'rtta sondan iborat ushbu jadvalni qaraymiz va uni *matritsa,* aniqrog'i, ikkinchi tartibli kvadrat matritsa deb ataymiz:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \qquad (1)$$

$\Delta = a_{11}a_{22} - a_{21}a_{12}$ son (1) matritsaning *determinanti* deb ataladi. (1) matritsaning determinanti bunday belgilanadi.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. \qquad (2)$$

Shunday qilib, ta’rifga va belgilashga asosan quyidagiga egamiz:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}. \qquad (3)$$

Determinantni tashkil qiladigan sonlar uning elementlari deb ataladi. Ikkinchi tartibli determinant ikkita satrga va ikkita ustunga ega. Istalgan elementning belgilanishida birinchi indeks shu element turgan satr tartibini, ikkinchi indeks esa ustun tartibini ko’rsatadi. a_{11}, a_{12} elementlar birinchi satrni, a_{21}, a_{22} ikkinchi satrni tashkil etadi.

a_{11}, a_{21} elementlar birinchi ustunni, a_{12}, a_{22} elementlar ikkinchi ustunni tashkil etadi.

$a_{11}\,, a_{22}$ elementlar joylashgan diagonal determinantning *bosh diagonali*, a_{21}, a_{12} elementlar joylashgan diagonal esa *yordamchi diagonali* deb ataladi.

Shunday qilib,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

determinant mos ravishda bosh va yordamchi diagonallarda turgan elementlarning ko'paytmalari ayirmasiga, ya'ni $a_{11}a_{22} - a_{21}a_{12}$ ga teng.

1-misol. $\begin{vmatrix} 8 & -1 \\ 3 & 5 \end{vmatrix} = 8 \cdot 5 - 3 \cdot (-1) = 40 + 3 = 43$

2. Uchinchi tartibli determinant. Uchinchi tartibli kvadrat matritsani, ya'ni 3×3 ta sondan iborat ushbu jadvalni qaraymiz:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (4)$$

Bu matritsaning uchinchi tartibli determinant deb quyidagi

$$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - $$
$$-a_{31}a_{22}a_{13} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

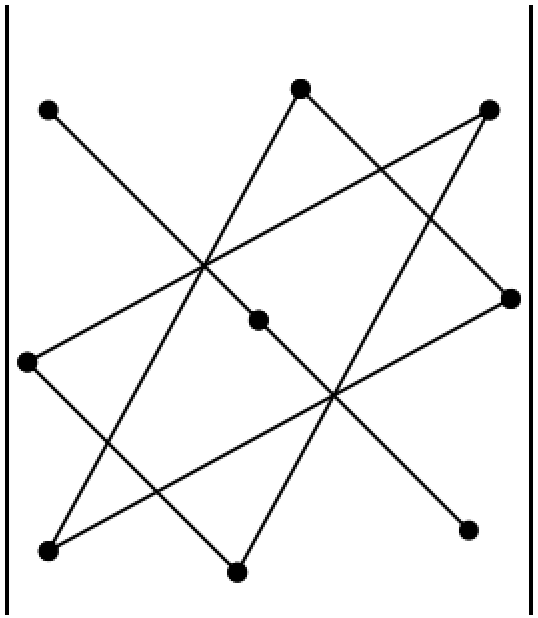
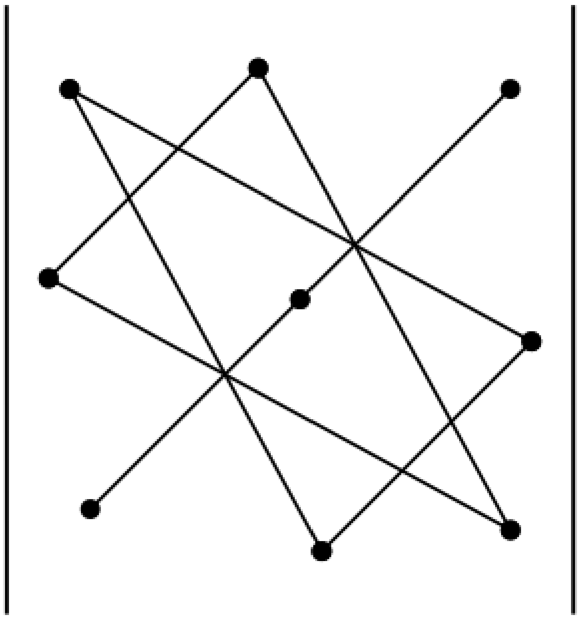
songa aytiladi. Uchinchi tartibli determinant bunday belgilanadi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Shunday qilib,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} +$$

$$+a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}. \quad (5)$$

	
+	–

(+) ишора билан, (-) ишора билан олинади.

3x3

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 4 & 6 \end{vmatrix} = \underbrace{6 + (-4) + 0}_{2} - \left(\underbrace{6 - 4 + 0}_{2} \right) = 2 - 2 = 0$$

3. Determinantning xossalari. Bu xossalarni uchinchi tartibli determinant uchun keltiramiz.

1-xossa. Determinantning satrlaridagi elementlari va ustunlaridagi elementlari o'rinlari alamshtirilganda uning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 3 & -2 \\ -3 & 0 & 4 \end{vmatrix} = 24 - 6 + 0 - (0 + 0 - 4) = 18 + 4 = 22$$

22 −22

$$\begin{vmatrix} 2 & 1 & -3 \\ -1 & 3 & 0 \\ 0 & -2 & 4 \end{vmatrix} = 24 + 0 - 6 - (0 + 0 - 4) = 18 + 4 = 22$$

$\Delta = \Delta$

2-xossa. Agar determinantning ikkita parallel satr (ustun) elementlarining o'rinlari almashtirilsa, uning ishorasi qarama-qarshi ishoraga almashadi. Masalan

$$\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = -\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} -3 & 0 & 4 \\ 1 & 3 & -2 \\ 2 & -1 & 0 \end{vmatrix} =$$

3-xossa. Agar determinant ikkita bir xil elementli satr (ustun)ga ega bo’lsa, u nolga teng. Haqiqatan, ikkita parallel bir xil elementli qatorlarning o’rinlarini almashtirish bilan determinant o’zgarmaydi, biroq 2- xossaga asosan uning ishorasi o’zgaradi. Demak, $\Delta = -\Delta$,

ya’ni, $2\Delta = 0$ yoki $\Delta = 0$. Masalan, $\begin{vmatrix} 2 & 3 & 7 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} = 0$

4-xossa. Determinant biror satr (ustun)ning barcha elementlarini istalgan λ songa ko’paytirish determinantni bu songa ko’paytirishga teng kuchlidir.

$$\begin{vmatrix} \lambda a_{11} & a_{12} & a_{13} \\ \lambda a_{21} & a_{22} & a_{23} \\ \lambda a_{31} & a_{32} & a_{33} \end{vmatrix} = \lambda \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

5-xossa. Agar determinant nollardan iborat bo'lgan satr (ustun)ga ega bo'lsa, u nolga teng. Bu xossa oldingi xossadan $\lambda = 0$ bo'lganda kelib chiqadi.

6-xossa. Agar determinant ikkita parallel proportsional satr (ustun)ga ega bo'lsa, u nolga teng.

Misol. $\begin{vmatrix} 3 & 4 & 2 \\ 6 & 8 & 4 \\ 7 & 3 & 5 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & 4 & 2 \\ 3 & 4 & 2 \\ 7 & 3 & 5 \end{vmatrix} = 0$

Minor va algebraik to'ldiruvchi

3. Минор ва алгебраик тўлдирувчилар

$$\Delta = \begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix}$$ детерминантда i- сатрни ва j- устунни ўчиришдан 2-тартибли детерминант ҳосил бўлади, бунга a_{ij} элементга мос ***минор*** дейилади ва M_{ij} билан белгиланади. Масалан,

$$M_{21} = \begin{vmatrix} a_{12} a_{13} \\ a_{32} a_{33} \end{vmatrix}, M_{22} = \begin{vmatrix} a_{11} a_{13} \\ a_{31} a_{33} \end{vmatrix}$$

ва бошқалар.

a_{ij} ***элементнинг алгебраик тўлдирувчиси*** деб унга мос минорнинг мусбат ёки манфий ишора билан олинган катталигига айтилади,бунда $i + j$ жуфт бўлса, мусбат ишора билан, $i + j$ тоқ бўлса манфий ишора олинади. a_{ij} элементнинг алгебраик тўлдирувчисини A_{ij} билан белгиланади. Демак,

$$A_{21} = -M_{21} = -\begin{vmatrix} a_{12} a_{13} \\ a_{32} a_{33} \end{vmatrix}, A_{22} = M_{22} = \begin{vmatrix} a_{11} a_{13} \\ a_{31} a_{33} \end{vmatrix}$$

бўлади ва бошқалар.

Uyga vazifa

1) $\begin{vmatrix} 1 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & -1 & 8 \end{vmatrix}$; **2)** $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$; **3)** $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$;

4) $\begin{vmatrix} 3 & -1 & -2 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{vmatrix}$; **5)** $\begin{vmatrix} -1 & 2 & 5 \\ 2 & 0 & 6 \\ 4 & 0 & 7 \end{vmatrix}$; **6)** $\begin{vmatrix} 1 & 7 & -1 \\ 2 & 6 & 2 \\ 1 & 1 & 4 \end{vmatrix}$.