Topic 4.

Functions . Function concept

A function is this two collection elements between compatibility to install applicable has been main mathematician from concepts one is considered

f the rule given let it be If X of the collection every one x to the element f the rule according to Y in the collection the only one y the item is suitable if put , then X in the collection $y = f(x), x \in X, y \in Y$ **function given** is called

X collection of the function **identification field** is called and $D(f)$ such as is determined .Y of the function changes collection of the function **values field** is called and $E(f)$ such as is determined .

Oxy of the plane $(x, f(x))$ points collection $y = f(x)$ of the function graph is called

Function : 1) analytical ; 2) graphic ; 3) schedule through to be given can

➢ Function main characteristics

If $f(x)$ function his own X identification in the field to zero relatively symmetrical is optional $x \in X$ for $f(x) = f(-x)$ equality if done couple is called

If $f(x)$ function X identification of the field optional $x \in X$ for $f(-x) = -f(x)$ equality if done odd is called

If so $T \neq 0$ number is available if and every how $x \in X$ for the following conditions : 1) $x + T \in X$; 2) $f(x+T) = f(x)$ if done , then $y = f(x)$ function periodic is $T - y = f(x)$ called of the function period is called

f function $E \subset D(f)$ in the collection limited is called if $\exists A : \forall x \in E \left| f(x) \right| \leq A$ if

Hypothesis let's do $y = f(x)$ function $D = (f)$ in the collection determined and $E \subset D = (f)$ let it be

If $\forall x_1, x_2 \in E$:

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ if (f) E at growing ;

$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ if (f) E decreasing it's not ;

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ if (f) E decreasing

$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ if (f) E growing water it's not .

Above given four get up E at monotonous is called growing and decreasing while E at strictly monotonous is called

Solved from examples samples

Example 4.1. $y = \dfrac{\sqrt{x^2 - 4}}{2^x (x-6)} + \ln(x+10)$ of fuchsia identification find the field .

<u>Solution :</u>

Couple level root under expression negative not to be , of the fraction the denominator to zero equal to absence , logarithm sign under expression while positive to be need because of , of the function identification field the following inequalities solve through found :

$$\begin{cases} x^2 - 4 \geq 0, \\ 2^x (x-6) \neq 0, \\ x + 10 > 0, \end{cases} \text{ or } \begin{cases} (x-2)(x+2) \geq 0, \\ x \neq 6, \\ x > -10, \end{cases}$$

From this

$$\begin{cases} x \in (-\infty; -2] \cup [2; +\infty), \\ x \neq 6, \\ x \in (-10; +\infty). \end{cases}$$

of the system all inequalities one of time in itself of the variable x that satisfies values $x \in (-10;-2] \cup [2;6) \cup (6;+\infty)$ will be So , y of the function identification field $D(y):(-10;-2] \cup [2;6) \cup (6;+\infty)$.

Example 4.2. $y = 3\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right) - 4$ of the function values field $E(y)$ find the

Solution :

$\arccos x$ of values field $E(y) = [0;\pi]$ or different $0 \le \arccos x \le \pi$ way to write can $\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right)$ function $\arccos x$ from x abscissa arrow across shift through get can Such changes to the range effect doesn't do it , that's it because of $0 \le \arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right) \le \pi$ will be

$3\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right)$ function $\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right)$ of Oy arrow across three times stretched out condition , that is $0 \le 3\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right) \le 3\pi$ will be

Now last stage ordinate arrow across four unity down shift done is increased . And this the following to inequality take comes :

$$0 - 4 \le 3\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right) - 4 \le 3\pi - 4 \Leftrightarrow$$

$$-4 \le 3\arccos\left(\frac{x}{3} + \frac{5\pi}{7}\right) - 4 \le 3\pi - 4$$

So , that's it $E(y) = [-4;3\pi - 4]$ the fact that come comes out

Example 4.3. $y = 2x^4 - 3x^2 + 6$ function even-odd check

Solution :

function even-odd check for $(-x)$ at the point function is considered .

$$f(-x) = 2\cdot(-x)^4 - 3\cdot(-x)^2 + 6 = 2x^4 - 3x^2 + 6 = f(x)$$

So , it is given function $f(-x) = f(x)$ condition satisfies . From this y of the function couple the fact that known will be

Example 4.4. $y = 2\sin 4x$ of the function the most small period define

Solution :

Periodic of the function definition according to every how x and $T \neq 0$ for $y(x+T) = y(x)$.

$f(x) = 2\sin 4x$ for :

$$2\sin\left(4\left(x+T\right)\right) = 2\sin 4x \text{ or } \sin\left(4x+4T\right) - \sin 4x = 0 \text{ ,}$$

From this

$$2\sin\frac{4x+4T-4x}{2}\cdot\cos\frac{4x+4T+4x}{2} = 0$$

will be That is $\sin 2T\cdot\cos\left(4x+2T\right) = 0$.

Received equality every how x for action does That is , if x the own into didn't get part to zero equal to if , that is $\sin 2T = 0$ that it was for the most small period $T = \frac{\pi}{2}$ will be

Example 4.5. $y = -2x^2 + 4x$ function graph draw

Solution :

Given function square function because it was , his graph to be a parabola clearly Now this of the parabola Ox axis cut passing points to determine need will be Of this for this square function to 0 equal to solve need :

$$-2x^2 + 4x = 0\ ;$$

$$x\left(-2x+4\right) = 0\ ;$$

In this every one bracket to 0 equating to the following the solution get can :

$$x = 0 \text{ and } -2x + 4 = 0;$$

$$x_1 = 0; \quad x_2 = 2$$

Now the tip of the parabola is determined . In this $y = ax^2 + bx + c$ square the tip of the parabola in the function to find from the formula

$$x_0 = -\frac{b}{2a} \Rightarrow x_0 = -\frac{4}{2\cdot(-2)};$$

$$x_0 = 1 \text{ and } y_0 = -2\cdot 1^2 + 4\cdot 1 = 2$$

the fact that come comes out This found Ox axis cut passing $(0,0);(2,0)$ points and the tip of the parabola calculated $(1,2)$ point through given function graph is made (Fig. 4.1):

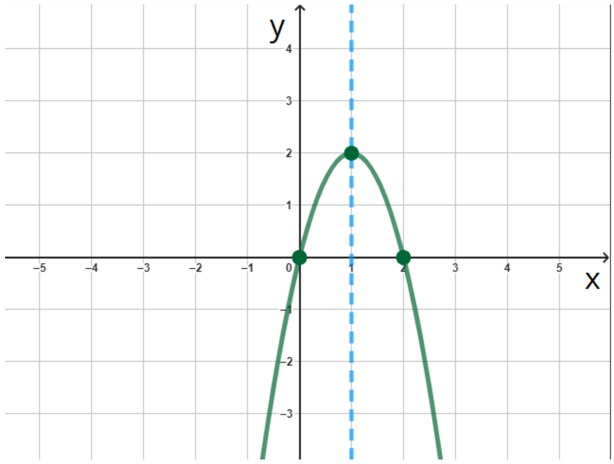


Figure 4.1

Independent work for issues

4.1 The following $f(x)$ function given let it be This function for $f(5),\ f(-1),\ f(1/2)$ s count

$$f(x) = \begin{cases} x^2 + 1, & agar \quad x > 0 \\ -4, & agar \quad x = 0 \\ 1 - 2x, & agar \quad x < 0 \end{cases}$$

Given y of the function identification field $D(y)$ find the (4.2-4.9).

4.2 $y = \frac{x-1}{(x+2)(x-3)}$;

4.3 $y = \sqrt{\frac{x}{x-2}}$;

4.4 $y = \frac{x}{\sqrt[4]{25-x^2}}$;

4.5 $y = \frac{3^{\sqrt{x}}}{\lg(3-x)}$;

4.6 $y = \sqrt{x+2} - \ln(4-x)$;

4.7 $y = \frac{\sqrt{1-x^2} \cdot \ln(x+1)}{(x^2+1)\sqrt{5^x}} - \frac{\sqrt[4]{x-1}}{x}$;

4.8 $y = \frac{\sqrt{4-x^2}}{arctgx} + \log_2(x-2)$;

4.9 $y = \frac{\arcsin x}{\sin 5x}$;

4.10 The following of functions identification field $D(y)$ and values field $E(y)$ find the

a $y = \frac{1}{x}$)

b) $y = x^2$

Given y of the function values field $E(y)$ find the (4.11-4.13).

4.11 $y = \frac{2\sqrt{2x}-1}{x^2+1}$;

4.12 $y = 6\sin x - 8\cos x$;

4.13 $y = 2 \cdot 5^{-2x^2}$;

Given y function even-odd check (4.14-4.20).

4.14 $y = (x-5)^2 + 3$;

4.15 $y = \frac{\sin x}{x^3}$;

4.16 $y = \left(\sin^2 x + \cos x\right) \cdot x^3$;

4.17 $y = x^2 \ln x$;

4.18 $y = 3^{4x} \cdot x^2 + \cos x$;

4.19 $y = \frac{tgx}{x^4 + x^2 + x}$;

4.20 $y = \frac{x^4}{\sin x} - x^3 \ln\left(1 + x^2\right)$;

4.21 $y = x - e^{2x}$ function to monotony check

Given y of the function the most small find the period (4.22-4.24).

4.22 $y = \sin^2 4x$;

4.23 $y = 2\sin\frac{x}{2}$;

4.24 $y = tg^2 x$;

4.25 $y = \ln x + 2$ and $y = x^2 + x - 6$ function draw

Answers

4.1. $f(5)=26,\ f(-1)=3,\ f\left(\frac{1}{2}\right)=1\frac{1}{4}$

4.2. $D(y)=\{x\in R: x\neq -2\ \text{ va } x\neq 3\}$

4.3. $D(y)=\{x\in R: x>2\}$

4.4. $D(y)=\{x\in R: -5<x<5\}$

4.5. $D(y)=\{x\in R: 0\le x<2\ \text{ va }\ 2<x<3\}$

4.6. $D(y)=\{x\in R: -2\le x<4\}$

4.7. $D(y)=\{x\in R: x=1\}$

4.8. $\varnothing$

4.9. $D(y)=\left\{x\in R: -1\le x<-\frac{\pi}{5}\ \text{ va }\ -\frac{\pi}{5}<x<0\ \text{ va }\ 0<x<\frac{\pi}{5}\ \text{ va }\ \frac{\pi}{5}<x\le 1\right\}$

4.10. $a)D(y)=\{x\in R: x\neq 0\},E(y)=\{y\in R: y\neq 0\},b)D(y)=\{x\in R\},E(y)=\{y\in R: y\ge 0\}$

4.11. $E(y)=\{y\in R: -1\le y\le 0.933638\}$

4.12. $E(y)=\{y\in R: -10\le y\le 10\}$

4.13. $E(y)=\{y\in R: 0<y\le 2\}$

4.14. Function neither even nor odd

4.15. Couple function

4.16. Odd function

4.17. Function neither even nor odd

4.18. Function neither even nor odd

4.19. Function neither even nor odd

4.20. Odd function

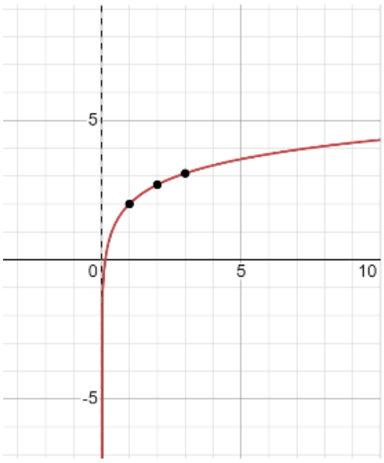
4.21. $\left(-\infty;\frac{\ln\left(\frac{1}{2}\right)}{2}\right)$ in between growing $\left(\frac{\ln\left(\frac{1}{2}\right)}{2};\infty\right)$ in between decreasing

4.22. $T=\frac{\pi}{4}$

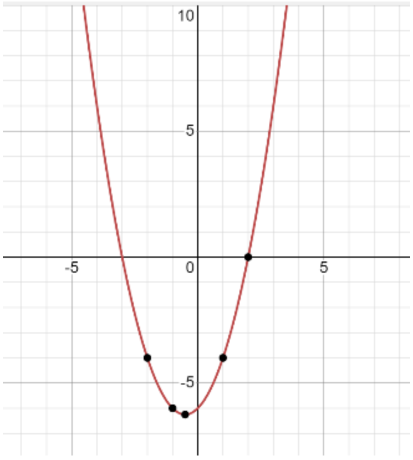
4.23. $T=4\pi$

4.24. $T = \pi$

4.25. Functions graph as follows will be :



$y = \ln x + 2$



$y = x^2 + x - 6$