



*“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH
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Kompleks sonlar va ular ustida amallar

Fan nomi: Hisob (Calculus)

REJA:

1. Kompleks sonlar haqida dastlabki ta'riflar

2. Kompleks sonlar ustida asosiy amallar

3. Kompleks sonni darajaga ko'tarish va kompleks sondan ildiz chiqarish

4. Mustaqil yechish uchun misollar

KOMPLEKS SONLAR TA'RIFI

Kompleks son deb

$$z = a + ib \quad (1)$$

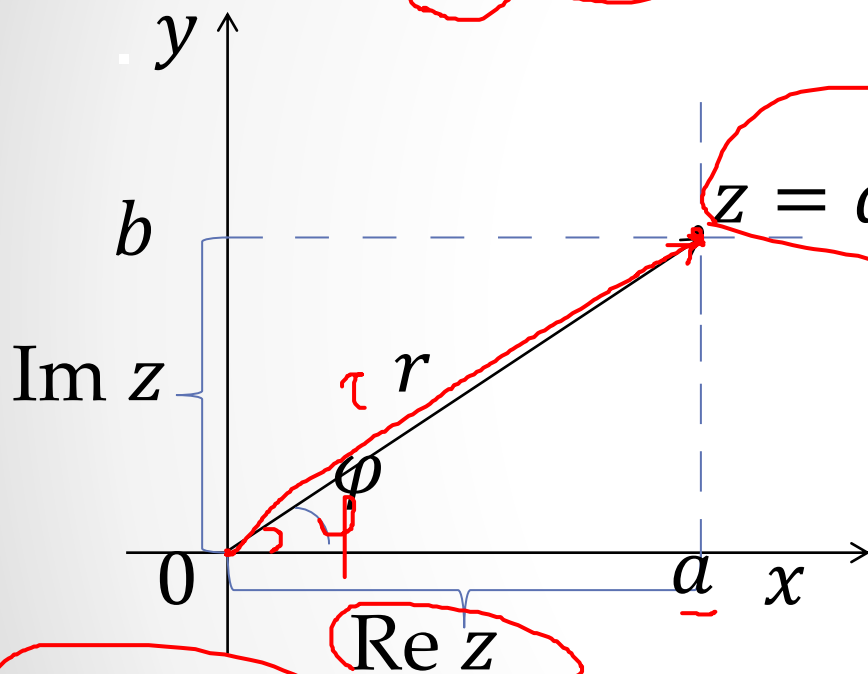
ifodaga aytiladi, bu erda a va b haqiqiy sonlar, i - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \quad \text{yoki} \quad i^2 = -1 \quad (2)$$

a - kompleks son z ning haqiqiy qismi, ib - mavhum qismi deyiladi. Ular bunday belgilanadi: $a = \operatorname{Re} z$, $b = \operatorname{Im} z$. Agar $a=0$ bo'lsa, $0+ib=ib$ sof mavhum son deyiladi; $b=0$ agar bo'lsa, haqiqiy son hosil bo'ladi: $a+i*0=a$. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: $z=a+ib$ va $z=a-ib$ bir-biriga qo'shma deyiladi.

KOMPLEKS SONNING GEOMETRIK TASVIRI

$$z = a + bi$$



$z = a + bi$ kompleks son moduli $r = |z|$ kabi belgilanadi, $r = \sqrt{a^2 + b^2}$

$z = a + bi$ kompleks sonning *argumenti* $\varphi = \arg z$ belgilanadi,
 $\cos \varphi = \frac{a}{r}$, $\sin \varphi = \frac{b}{r}$, $0 \leq \varphi < 2\pi$

$z = a + bi$ $\bar{z} = a - bi$ ko'rinishidagi kompleks sonlar o'zaro qo'shma kompleks sonlar deyiladi. Masalan, $z = 2 + 3i$ $\bar{z} = 2 - 3i$

KOMPLEKS SONNING KO'RINISHLARI

1. Vektor ko'rinishi: $z = \overrightarrow{(x; y)}$

2. Algebraik ko'rinishi : $z = x + iy$

3. Trigonometrik ko'rinishi : $z = r \cdot (\cos\varphi + i \cdot \sin\varphi)$

4. Ko'rsatkichli ko'rinishi : $z = r \cdot e^{i\varphi}$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

$$1. (a + bi) + (c + di) = (a + c) + (b + d)i$$

$z_1 = -0,13 + 2i$ va $z_2 = 7 + 3,6i$ sonlarini qo'shing.

$$z_1 + z_2 = (-0,13 + 2i) + (7 + 3,6i) = (-0,13 + 7) + (2 + 3,6)i = 6,87 + 5,6i$$

$$2. (a + bi) - (c + di) = (a - c) + (b - d)i$$

$z_1 = 13 - 7i$ va $z_2 = -5 + 4i$ sonlarini ayiring.

$$z_1 - z_2 = (13 - 7i) - (-5 + 4i) = (13 - (-5)) + (-7 - 4)i = 18 - 11i$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

$$3. (a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$$

$$ac + bci + bdi + bdi^2$$

$$i^2 = -1$$

$z_1 = 3 + 2i$ va $z_2 = 7 + 6i$ sonlarini ko'paytiring.

$$\begin{aligned} z_1 \cdot z_2 &= (3 + 2i) \cdot (7 + 6i) = 3 \cdot 7 + 3 \cdot 6i + 2i \cdot 7 + 2i \cdot 6i = \\ &= 21 + 18i + 14i + 12(i)^2 = 9 + 32i \end{aligned}$$

$$4. \frac{a+bi}{c+di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2} i$$

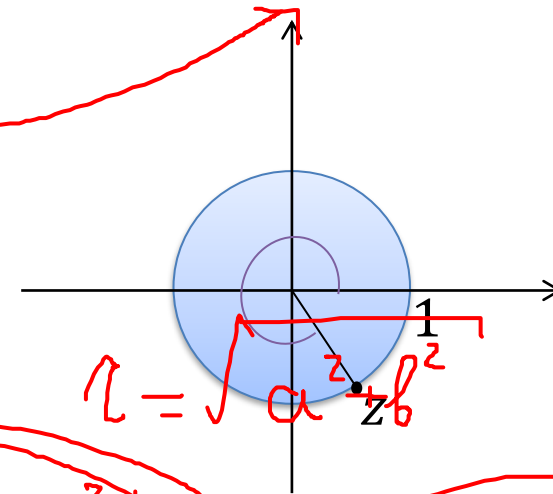
$z_1 = 2 - i$ sonni $z_2 = -3 + 2i$ songa bo'ling.

$$\begin{aligned} \frac{2-i}{-3+2i} &= \frac{(2-i)(-3-2i)}{(-3+2i)(-3-2i)} = \frac{-6+4i+3i-2}{(-3)^2 - (2i)^2} = \frac{-8-i}{13} = \\ &= \frac{-8}{13} - \frac{1}{13}i \end{aligned}$$

Masalalar yechish

1-Masala Quyidagi sonni trigonometrik va ko'rsatkichli ko'rinishga keltiring: $z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

Yechish: Dastlab r va φ ni topib olamiz:



$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

$$\begin{cases} \cos \varphi = \frac{1}{2} \\ \sin \varphi = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi = \frac{5\pi}{3} = 300^\circ$$

$$z = 1 \cdot \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$$

$$z = 1 \cdot e^{i \cdot \frac{5\pi}{3}}$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko'inishdagi kompleks sonlarni ko'paytirishni qaraymiz.

$$z_1 = r_1(\cos\varphi_1 + i \sin\varphi_1), \quad z_2 = r_2(\cos\varphi_2 + i \sin\varphi_2)$$

$$\begin{aligned} z_1 \cdot z_2 &= r_1(\cos\varphi_1 + i \sin\varphi_1) \cdot r_2(\cos\varphi_2 + i \sin\varphi_2) = \\ &= r_1 \cdot r_2 \cdot ((\cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2) + \\ &\quad + i \cdot (\cos\varphi_1 \cdot \sin\varphi_2 + \sin\varphi_1 \cdot \cos\varphi_2)) \end{aligned}$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 \cdot (\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2))$$

$$z_1 = 6(\cos 70^\circ + i \sin 70^\circ), \quad z_2 = 4(\cos 25^\circ + i \sin 25^\circ)$$

$$\begin{aligned} z_1 \cdot z_2 &= 6 \cdot 4(\cos(70^\circ + 25^\circ) + i \sin(70^\circ + 25^\circ)) = \\ &= 24(\cos 95^\circ + i \sin 95^\circ) \end{aligned}$$

KOMPLEKS SONLAR USTIDA ARIFMETIK AMALLAR

Trigonometrik ko'inishdagi kompleks sonlarni bo'lishni qaraymiz.

$$z_1 = r_1(\cos\varphi_1 + i \sin\varphi_1), \quad z_2 = r_2(\cos\varphi_2 + i \sin\varphi_2),$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{r_1(\cos\varphi_1 + i \sin\varphi_1)}{r_2(\cos\varphi_2 + i \sin\varphi_2)} = \frac{r_1}{r_2} \cdot \frac{(\cos\varphi_1 + i \sin\varphi_1)(\cos\varphi_2 - i \sin\varphi_2)}{(\cos\varphi_2 + i \sin\varphi_2)(\cos\varphi_2 - i \sin\varphi_2)} = \\ &= \frac{r_1}{r_2} \cdot \frac{(\cos\varphi_1 \cdot \cos\varphi_2 + \sin\varphi_1 \cdot \sin\varphi_2) + i(\sin\varphi_1 \cdot \cos\varphi_2 - \cos\varphi_1 \cdot \sin\varphi_2)}{\cos^2\varphi_2 + \sin^2\varphi_2} \end{aligned}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2))$$

$$z_1 = 6(\cos 70^\circ + i \sin 70^\circ), \quad z_2 = 4(\cos 25^\circ + i \sin 25^\circ),$$

$$\frac{z_1}{z_2} = \frac{6}{4} (\cos(70^\circ - 25^\circ) + i \sin(70^\circ - 25^\circ)) = 1,5 (\cos 45^\circ - i \sin 45^\circ)$$

Kompleks sonni natural darajaga ko'tarish

$z = r(\cos\varphi + i \sin\varphi)$ kompleks son uchun $\forall n \in \mathbb{N}$ soni uchun Muavr formulasi:

$$z^n = r^n (\cos\varphi + i \sin\varphi)^n = r^n (\cos(n\varphi) + i \sin(n\varphi))$$

$z = 3(\cos 15^\circ + i \sin 15^\circ)$ kompleks son 4-darajasini toping:

$$\begin{aligned} z^4 &= 3^4 (\cos(4 \cdot 15^\circ) + i \sin(4 \cdot 15^\circ)) = 81 \cdot (\cos 60^\circ + i \sin 60^\circ) = \\ &= 81 \cdot \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = \frac{81}{2} (1 + \sqrt{3}i) \end{aligned}$$

$$\text{Javob: } \frac{81}{2} (1 + \sqrt{3}i)$$

Masalalar yechish

2 masala

$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ kompleks sonning 10-darajasini toping.

Yechish: Dastlab trigonometrik ko'rinishda yozib olamiz:

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1, \quad \varphi = \frac{5\pi}{3} = 300^\circ$$

$$z = 1 \cdot (\cos 300^\circ + i \sin 300^\circ)$$

$$z^{10} = 1^{10} (\cos 3000^\circ + i \sin 3000^\circ) = 1 \cdot (\cos 120^\circ + i \sin 120^\circ) =$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Javob: $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Masalalar yechish

3 masala

$(1 + i)^{100}$ ifodaning qiymatini toping.

$$\begin{aligned} \text{Yechish: } (1 + i)^{100} &= ((1 + i)^2)^{50} = (\cancel{1} + 2i + \cancel{i^2})^{50} = \\ &= (2i)^{50} = 2^{50} \cdot i^{50} = 2^{50} \cdot (i^2)^{25} = \\ &= 2^{50} \cdot (-1)^{25} = -2^{50} \end{aligned}$$

Javob: -2^{50}

Kompleks sondan ildiz chiqarish

Kompleks sondan natural tartibli ildiz olishni ildiz tartibiga teskari qiymatli darajaga oshirish sifatida qabul qilish mumkin. Bunda ham Muavr formulasidan foydalanib quyidagicha formula xosil qilish mumkin

$$\begin{aligned} \sqrt[n]{z} &= z^{\frac{1}{n}} = r^{\frac{1}{n}} (\cos \varphi + i \sin \varphi)^{\frac{1}{n}} = \\ &= r^{\frac{1}{n}} \left(\cos \left(\frac{\varphi + 2\pi k}{n} \right) + i \sin \left(\frac{\varphi + 2\pi k}{n} \right) \right) \end{aligned}$$

Bu formulada k o'rniga $0, \pm 1, \pm 2, \dots$ qiymatlarni qo'yib turli ildizlarni topamiz.

Masalalar yechish

5 masala

$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ kompleks sonidan 4-darajali ildiz chiqaring.

Yechish: Dastlab trigonometrik ko'rinishda yozib olamiz:

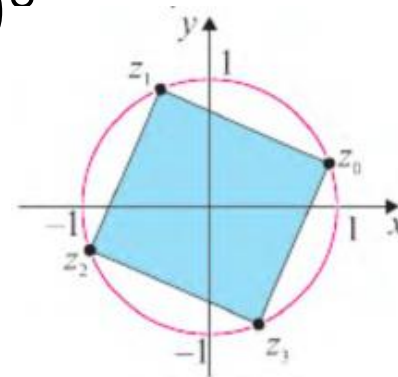
$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1, \quad \varphi = \frac{\pi}{3} = 60^\circ$$

$$z = 1 \cdot (\cos 60^\circ + i \sin 60^\circ)$$

$$\sqrt[4]{z} = z^{\frac{1}{4}} = 1^{\frac{1}{4}} \left(\cos \frac{60^\circ + 2\pi k}{4} + i \sin \frac{60^\circ + 2\pi k}{4} \right)$$

$$k = 0 \quad \underline{z_0 = \cos 15^\circ + i \sin 15^\circ} \quad k = 2 \quad z_2 = \cos 195^\circ + i \sin 195^\circ$$

$$k = 1 \quad z_1 = \cos 105^\circ + i \sin 105^\circ \quad k = 3 \quad z_3 = \cos 285^\circ + i \sin 285^\circ$$



MUSTAQIL YECHISH UCHUN MISOLLAR

1 $z_1 = 1 + i\sqrt{3}$ $z_2 = 1 - i\sqrt{3}$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$

$\frac{z_1}{z_2} = ?$

Uyga vazifa

2 $z = \frac{1}{(1 - i\sqrt{3})^6}$

4 $(-1)^{\sqrt{3}}$

3 $z = (1 + i\sqrt{3})^{15}$