Topic 8.

Derivative and differentiation rules

$y = f(x)$ function (a,b) in the interval determined let it be (a,b) interval belongs to x_0 and $x_0 + \Delta x$ points is taken . In this Δx argument gain and function gain Δy as follows will be :

$$\Delta y = f(x_0 + \Delta x) - f(x_0)$$

Function the product to the product of the argument ratio considered :

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

this ratio $\Delta x \to 0$ in limit is determined . If this limit exists if , it is given $f(x)$ of the function x_0 **at the point derivative** is called and $f'(x_0)$ with is determined . So by doing

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \quad \text{or} \quad f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Given $f(x)$ from the function derivative to find practice this function **differentiation** is called

- **Differentiate main rules :**

1. If $y = c$ if $(c = const), y' = 0$ will be
2. $y = cu(x)$ if $y' = cu'(x)$ will be
3. $y = U(x) + V(x) + W(x); \quad y' = U'(x) + V'(x) + W'(x)$
4. $y = u\vartheta$ if $y' = u'\vartheta + u\vartheta'.$
5. $y = \frac{u}{\vartheta}$ if $y' = \frac{u'\vartheta - u\vartheta'}{\vartheta^2}$
6. let's say $y = F(u)$ complicated function be , that is $y = F(u), u = \varphi(x)$ differentiable functions let it be In it derivative as follows found :

$$y'_x = F'_u(u) \cdot u'_x(x)$$

➢ Differential formulas

$c' = 0, c = const$	$(tgx)' = \frac{1}{\cos^2 x}$
$(x^n)' = nx^{n-1}$	$(ctgx)' = -\frac{1}{\sin^2 x}$
$(a^x)' = a^x \cdot \ln a$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(e^x)' = e^x$	$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\log_a x)' = \frac{1}{x \ln a}$	$(arctgx)' = \frac{1}{1+x^2}$
$(\ln x)' = \frac{1}{x}$	$(arcctgx)' = -\frac{1}{1+x^2}$
$(\sin x)' = \cos x$	$(shx)' = chx$
$(\cos x)' = -\sin x$	$(chx)' = shx$
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(thx)' = \frac{1}{ch^2 x}$

Solved from examples samples

Example 8.1. $f(x) = \cos x$ function given This of the function $f'(x)$ derivative $\lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ formula according to count

Solution:

This issue at work $\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$ from the formula is used .

$$\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\sin\left(\frac{2x+\Delta x}{2}\right) \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0}\left(2\sin\left(\frac{2x+\Delta x}{2}\right)\right) \cdot \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} = \lim_{\Delta x \to 0}\left(2\sin\left(\frac{2x+\Delta x}{2}\right)\right) \cdot \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{2 \cdot \frac{\Delta x}{2}} =$$

* In this from the excellent limit formula used :

$$\left\{ \lim_{\Delta x\to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{2\cdot\frac{\Delta x}{2}} = \frac{1}{2}\cdot \lim_{\Delta x\to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} = \frac{1}{2}\cdot 1 = \frac{1}{2} \right\} *$$

$$= \lim_{\Delta x\to 0}\left(2\sin\left(\frac{2x+\Delta x}{2}\right)\right)\cdot\frac{1}{2} = 2\sin x\cdot\frac{1}{2} = \sin x\ .$$

Example 8.2. Given y of the function derivative differentiation from the rules find $y = 4x^3 - 2\cdot 5^x + 9x - 6\log_2 x - \ln 7$ using .

Solution:

Function derivative count for differentiation rules and differentiation from the formulas used :

$$y' = \left(4x^3\right)' - \left(2\cdot 5^x\right)' + (9x)' - \left(6\log_2 x\right)' - (\ln 7)' =$$

$$= 4\cdot\left(x^3\right)' - 2\cdot\left(5^x\right)' + 9\cdot(x)' - 6\cdot\left(\log_2 x\right)' - 0 =$$

$$= 4\cdot 3x^2 - 2\cdot 5^x\cdot\ln 5 + 9 - 6\cdot\frac{1}{x\ln 2} - 0 =$$

$$= 12x^2 - 2\cdot 5^x\ln 5 + 9 - \frac{6}{x\ln 2}$$

Example 8.3. $y = \cos x\cdot 3^x$ function find the derivative .

Solution:

Given $y(x)$ function two $f_1(x) = \cos x$ and $f_2(x) = 3^x$ of the function multiple that it was for , $y' = (uv)' = u'\cdot v + u\cdot v'$ differentiation from the rule used :

$$y' = \left(\cos x\cdot 3^x\right)' = (\cos x)'\cdot 3^x + \cos x\cdot\left(3^x\right)'\ .$$

From this differentiation in the table $(\cos x)' = -\sin x,\ \left(3^x\right)' = 3^x\ln 3$ from the formulas when using , general derivative will be as follows ;

$$y' = -\sin x \cdot 3^x + \cos x \cdot 3^x \ln 3 = 3^x \left(\ln 3 \cos x - \sin x\right).$$

Example 8.4. $y = \dfrac{x^2 + \sqrt{x}}{tgx}$ of the function find the derivative .

<u>Solution:</u>

This function derivative in the calculation $y = \dfrac{u}{v}; y' = \left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$ division for used differentiation from the rule used :

$$y' = \left(\frac{x^2 + \sqrt{x}}{tgx}\right)' = \frac{\left(x^2 + \sqrt{x}\right)' tgx - \left(x^2 + \sqrt{x}\right)\left(tgx\right)'}{tg^2 x} =$$

$$= \frac{\left(2x + \dfrac{1}{2\sqrt{x}}\right) tgx - \left(x^2 + \sqrt{x}\right)\dfrac{1}{\cos^2 x}}{tg^2 x}.$$

Example 8.5 . $y = ctg^2 x$ of the function find the derivative .

<u>Solution:</u>

Given function complicated function because $u = ctgx$ it was is determined and from this $y = u^2$ to be known . Now differentiation formulas from the table can be written as :

$$y'_x = 2u \cdot u' = 2ctgx \cdot \left(ctgx\right)';$$

$$\left(ctgx\right)' = -\frac{1}{\sin^2 x};$$

From this

$$y' = 2ctgx \cdot \left(-\frac{1}{\sin^2 x}\right)$$

will be

Example 8.6. $f(x)=\sqrt[5]{x}$ function of the derivative $x_0=32$ at the point calculate the value of

Solution:

Initially , the function derivative $f'(x)$ found :

$$f'(x)=\left(\sqrt[5]{x}\right)'=\frac{1}{5\sqrt[5]{x^4}};$$

Now function in the derivative x instead of $x_0=32$ put :

$$f'(x_0)=\frac{1}{5\sqrt[5]{x_0^{\ 4}}}=\frac{1}{5\sqrt[5]{32^4}}=\frac{1}{5\cdot 2^4}=\frac{1}{80}.$$

Example 8.7. $f(x)=\frac{3-2x}{x^2+4}$ function derivative values positive to be x of values define :

Solution:

Initially , the function derivative $f'(x)$ found :

$$f'(x)=\frac{(3-2x)'\cdot\left(x^2+4\right)-(3-2x)\cdot\left(x^2+4\right)'}{\left(x^2+4\right)^2}=$$

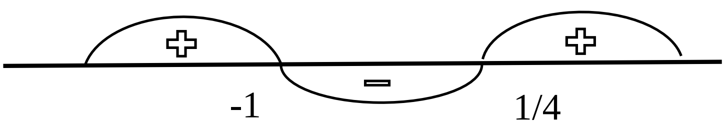
$$=\frac{-2\cdot\left(x^2+4\right)-(3-2x)\cdot 2x}{\left(x^2+4\right)^2}=\frac{-2x^2-8-6x+4x^2}{\left(x^2+4\right)^2}=\frac{2x^2-6x-8}{\left(x^2+4\right)^2};$$

This found derivative values positive to be for $f'(x)\geq 0$ to be need

$$\frac{2x^2-6x-8}{\left(x^2+4\right)^2}\geq 0;$$

In this fraction the denominator $\left(x^2+4\right)^2>0$ positive the fact that known . Therefore, the range of the positive image of the fraction is found:

$$2x^2-6x-8\geq 0\Rightarrow(x+1)(2x-8)\geq 0;$$



As a result function derivative values positive to be x of values range as follows will be : $(-\infty;-1]\cup\left[\frac{1}{4};+\infty\right)$.

Issues for independent work

$f(x)$ function given This of the function $f'(x)$ derivative $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ formula according to calculate (8.1-8.4).

8.1 $f(x)=\frac{1}{x^2}$;

8.2 $f(x)=\sin 3x$;

8.3 $f(x)=x^3$;

8.4 $f(x)=\frac{1}{x}$;

Given y of the function derivative differentiation from the rules find using (8.5-8.26).

8.5 $y=\sin 4x$;

8.6 $y=3x^5-\sin x$;

8.7 $y=2\ln x$;

8.8 $y=\left(x^3+4x+7\right)^4$;

8.9 $y=ctg\,2x$;

8.10 $y=e^{3x}$;

8.11 $y=3x^2-\frac{1}{x^3}$;

8.12 $y=\left(\frac{x}{3}+7\right)^6$;

8.13 $y = e^x \cos x$;

8.14 $y = (\cos 2x + 2\sin 2x)e^x$;

8.15 $y = 5^x - 7tgx + 3ctgx + arctgx$;

8.16 $y = 2\sqrt{x} - 4\cos x + 2\sin x + \log_3 x - \ln 5$;

8.17 $y = \dfrac{3x+2}{2x+3}$;

8.18 $y = \dfrac{2^x}{\sin x}$;

8.19 $y = \ln(1+\cos x)$;

8.20 $y = x^7 - 2x^5 + 5 - \dfrac{8}{x^3} + \dfrac{5}{6}x^5\sqrt{x}$;

8.21 $y = \sqrt{2-3x^4}$;

8.22 $y = e^{\sqrt{2x}}(2x-1)$;

8.23 $y = 3^{\arccos 2x}$;

8.24 $y = \ln \ln x$;

8.25 $y = e^{\sin 5x}$;

8.26 $y = x^x$;

8.27 $f(x) = 1 - 6\sqrt[3]{x}$ function of the derivative $x_0 = 8$ at the point calculate the value of

8.28 $f(x) = 2 - \dfrac{1}{\sqrt{x}}$ function of the derivative $x_0 = \dfrac{1}{4}$ at the point calculate the value of

8.29 $f(x)$ function derivative values negative to be x of values define :

$f(x) = \dfrac{1-x}{x^2+8}$;

8.30 $f(x)$ function derivative values negative to be x of values define :

$f(x) = \dfrac{x+1}{x^2+3}$;

Answers

8.5. $y' = 4\cos 4x$

8.6. $y' = 15x^4 - \cos x$

8.7. $y' = \frac{2}{x}$

8.8. $y' = 4\left(x^3 + 4x + 7\right)\left(3x^2 + 4\right)$

8.9. $y' = -\frac{2}{\sin^2 2x}$

8.10. $y' = 3e^{3x}$

8.11. $y' = 6x + \frac{3}{x^4}$

8.12. $y' = 2\left(\frac{x}{3} + 7\right)^5$

8.13. $y' = e^x \cos x - e^x \sin x$

8.14. $y' = 5e^x \cos 2x$

8.15. $y' = 5^x \ln 5 - \frac{7}{\cos^2 x} - \frac{3}{\sin^2 x} + \frac{1}{1 + x^2}$

8.16. $y' = \frac{1}{\sqrt{x}} + 4\sin x + 2\cos x + \frac{1}{x \ln 3}$

8.17. $y' = \frac{5}{\left(2x + 3\right)^2}$

8.18. $y' = \frac{2^x\left(\sin x \cdot \ln 2 - \cos x\right)}{\sin^2 x}$

8.19. $y' = -\frac{\sin x}{1 + \cos x}$

8.20. $y' = 7x^6 - 10x^4 + \frac{24}{x^4} + \frac{55}{12}x^{\frac{9}{2}}$

8.21. $y' = -\frac{6x^3}{\sqrt{\left(2 - 3x^4\right)}}$

8.22. $\sqrt{2}e^{\sqrt{2x}}\sqrt{x} + 2e^{\sqrt{2x}} - \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

8.23. $-\dfrac{3^{\arccos(2x)}\ln(9)}{\sqrt{1-4x^2}}$

8.24. $\dfrac{1}{x\ln x}$

8.25. $5e^{\sin 5x}\cos 5x$

8.26. $e^{x\ln x}\ln x+e^{x\ln x}$

8.27. $-\dfrac{1}{2}$

8.28. $-\dfrac{1}{16}$

8.29. $x\in(-2;4)$

8.30. $x\in(-\infty;-3)\cup(1;+\infty)$.