“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

Limitlar

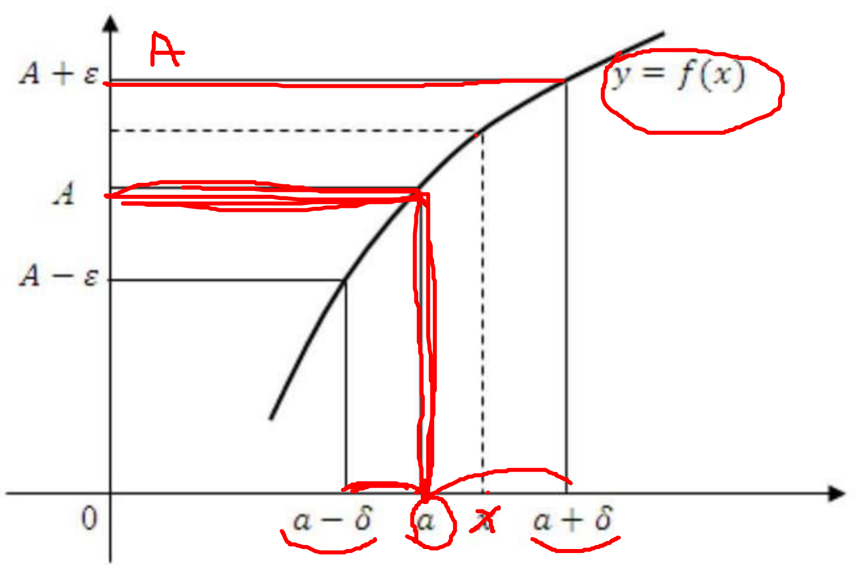
Fan nomi: Hisob (Calculus)

Reja:

1. Limit tushunchasi
2. Cheksiz kichik va cheksiz kata funktsiyalar
3. Limit xossalari
4. Ajoyib limitlar

1.Funksiyaning nuqtadagi limiti.

1-ta'rif. Agar $y=f(x)$ funksiya $x=a$ nuqtaning biror atrofida aniqlangan bo'lib ($x=a$ nuqtaning o'zida aniqlanmagan bo'lishi mumkin) istalgan $\varepsilon>0$ son uchun shunday $\delta>0$ son mavjud bo'lsaki, $|x-a|<\delta$ tengsizlikni qanoatlantiradigan barcha $x\neq a$ nuqtalar uchun $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa, A chekli son $y=f(x)$ funksiyaning $x=a$ **nuqtadagi** (yoki $x\to a$ dagi) **limiti** deb ataladi.



$x\neq a$

$x\to a$

$$\lim_{x\to a} f(x)=A$$

$\frac{5}{x}$

1) $\lim_{x\to a(x\to\infty)} f(x) = 0$ → Chekdiz kichik

2) $\lim_{x\to a(x\to 0)} f(x) = \infty$ → Chekdiz katta

$\frac{0}{0}$; $\frac{3}{0}$; $\frac{\infty}{\infty}$; $-\frac{3}{0}$; $\frac{5}{\infty}$;

$x=2$

$\lim_{x\to\infty} \frac{5}{x} = 5\cdot\lim_{x\to\infty} \frac{1}{x} = 5\cdot 0 = 0$

$\lim_{x\to\infty} \frac{1}{x} = 0$

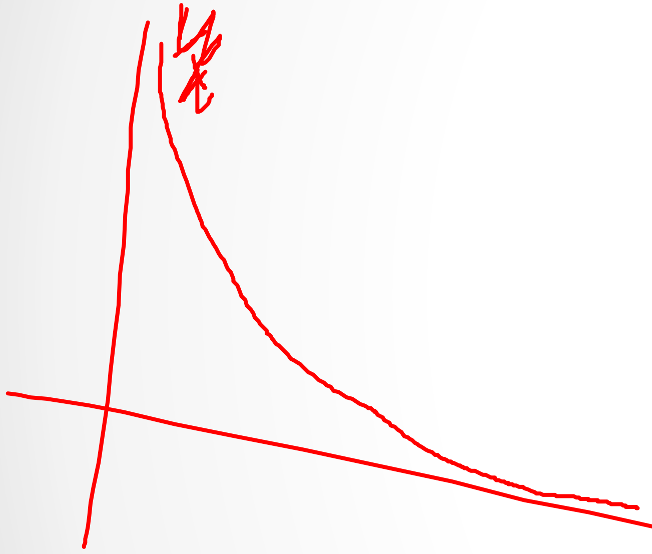
$\frac{1}{\infty} \neq 0$

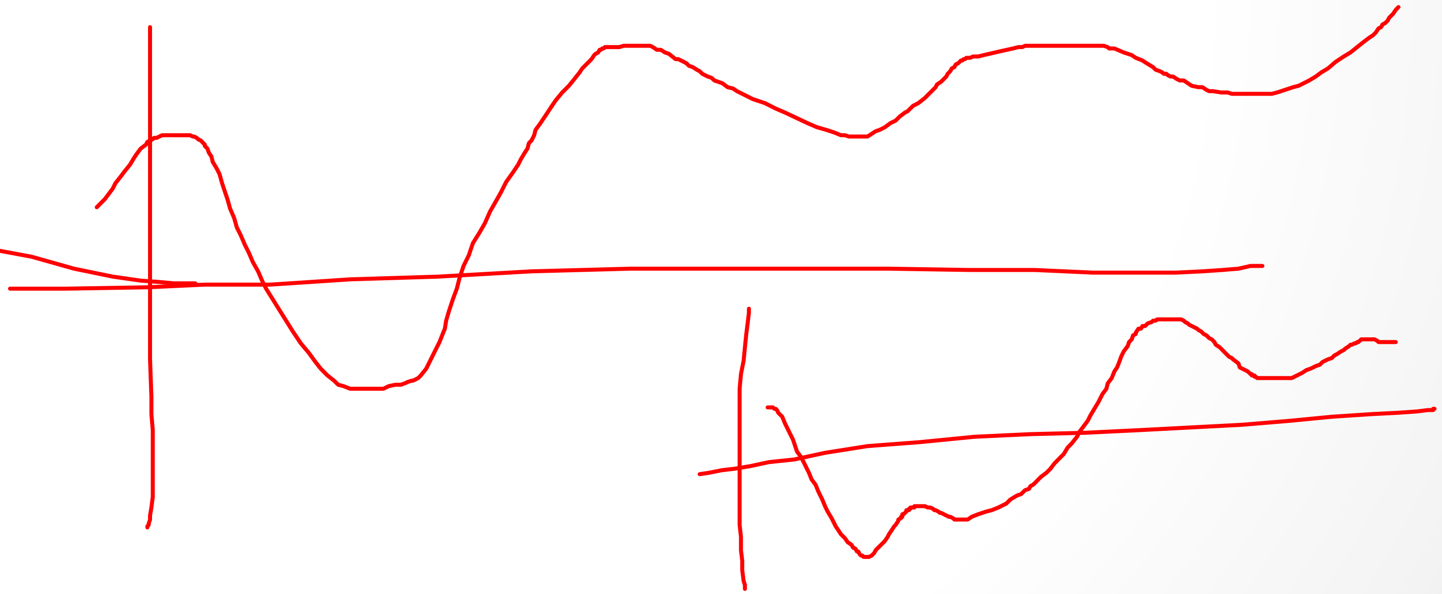
$\lim_{x\to 2} (x+3) = 5$

$$\lim_{x \to 0} x = 0 \qquad \lim_{x \to 0} \frac{1}{x} = \infty$$

$$\lim_{x \to \infty} x = \infty \qquad \lim_{x \to \infty} \frac{1}{x} = 0$$

→ cheksiz kichik





V. 1 - teorema. O‘zgarmas funksiyaning limiti o‘ziga teng. $\lim c = c\ (c = const)$.

2 - teorema. Limitga ega bo‘lgan chekli sondagi o‘zgaruvchi miqdorlaralgebraik yig‘ndisining limiti, bu o‘zgaruvchilar limitining yig‘indisiga teng.

$$\lim(U_1 + U_2 + ... + U_k) = \lim U_1 + \lim U_2 + ... + \lim U_k$$

3 - teorema. Limitga ega bo‘lgan chekli sondagi o‘xgaruvchi miqdorlar ko‘paytmasining limiti bu o‘zgaruvchilar limitlarini ko‘paytmasiga teng

$$\lim(U_1 \cdot U_2 \cdot ... \cdot U_k) = \lim U_1 \cdot \lim U_2 \cdot ... \cdot \lim U_k$$

4 - teorema. Limitga ega bo‘lgan ikki o‘zgaruvchi miqdor bo‘linmasining limiti, agar bo‘luvchining limiti 0 ga teng bo‘lmasa, bo‘linuvchi va bo‘luvchi limitlarining nisbatiga teng,

ya’ni $\lim \frac{U}{V} = \frac{\lim U}{\lim V}$; $(\lim V \neq 0)$

$$\lim_{x\to\infty} \frac{\frac{x^2}{x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{x}{x^3}} = \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^3}}{1+\frac{1}{x^2}} = \frac{\lim_{x\to\infty}\left(\frac{1}{x}+\frac{2}{x^3}\right)}{\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)}$$

$$\frac{0+0}{1+0} = \frac{0}{1} = 0$$

1) $\lim_{x\to 0} 5 = 5$

2) $\lim_{x\to\infty}\left(\frac{x^2+2}{x^3+x}+\frac{1}{x}\right) = \lim_{x\to\infty}\frac{x^2+2}{x^3+x} + \lim_{x\to\infty}\frac{1}{x} =$

$= \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^3}}{1+\frac{1}{x^2}} + \lim_{x\to\infty}\frac{1}{x} = \frac{0+0}{1+0} + 0 = 0$ $\quad \frac{\infty}{\infty}$

2) $\lim\left(\frac{x^2+2}{x^3}\right.$

3)

4)

Misol

Limitni hisoblang. (вычислить предел)

$$\lim_{x\to 6}\boxed{\frac{x^2-8x+12}{x^2-7x+6}} = \lim_{\boxed{x\to 6}} \frac{\cancel{(x-6)}(x-2)}{\cancel{(x-6)}(x-1)} = \lim_{x\to 6}\frac{x-2}{x-1} = \frac{6-2}{6-1} = \frac{4}{5}.$$

$x=6$

$$\frac{6^2-8\cdot 6+12}{6^2-7\cdot 6+6} = \frac{0}{0}.$$

Ajoyib limitlar

Birinchi ajoyib limit

$$\lim_{x\to 0}\frac{\sin x}{x}=1.$$

Ikkinchi ajoyib limit

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e.$$

Ajoyib limitlar va limitlar haqidagi teoremalar asosida quyidagi formulalar hosil qilingan:

1. $\lim\limits_{x\to 0}\frac{\sin kx}{kx}=\lim\limits_{x\to 0}\frac{tg kx}{kx}=\lim\limits_{x\to 0}\frac{sh kx}{kx}=\lim\limits_{x\to 0}\frac{th kx}{kx}=1$ $k\in R$.

2. $\lim\limits_{x\to 0}\frac{(1+kx)^m-1}{kx}=m\ \ (m>0).$

3. $\lim\limits_{x\to 0}\frac{\ln(1+kx)}{kx}=1.$

4. $\lim\limits_{x\to 0}\frac{a^{kx}-1}{kx}=\ln a\ \ (a>0).$

5. $\lim\limits_{x\to 0}\frac{e^{kx}-1}{kx}=1.$

6. $\lim\limits_{x\to 0}x^a\ln x=\lim\limits_{x\to +\infty}x^{-a}\ln x=\lim\limits_{x\to +\infty}x^a e^{-x}=0\ \ (a>0).$

7. $\lim\limits_{x\to \infty}\left(1+\frac{k}{x}\right)^x=e^k$

8. $\lim\limits_{x\to 0}(1+x)^{\frac{k}{x}}=e^k$

9. $\lim\limits_{x\to \infty}\left(1+\frac{1}{f(x)}\right)^{f(x)}=e$, bu yerda $x\to\infty$ da $f(x)\to\infty$.

10. $\lim\limits_{x\to \infty}(1+f(x))^{\frac{1}{f(x)}}=e$, bu yerda $x\to 0$ da $f(x)\to 0$

$shx=\frac{e^x+e^{-x}}{2}$

giper.sinus

$chx=\frac{e^x-e^{-x}}{2}$

giper.kosinus

Uyga vazifa

1) $\lim\limits_{n\to\infty}\left(\sqrt{n^2+1}-\sqrt{n^2+n+1}\right)$ limit hisoblansin.

∞

$\frac{(+)}{(+)}$

2) $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x-1}$

4) $\lim\limits_{x\to 0}\dfrac{\sin 7x}{\sin 14x}$

3) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1+x^2}}{\sqrt{1+x}-1}$

5) $\lim\limits_{n\to\infty}\dfrac{3n^3+n-1}{n^4+2n}$