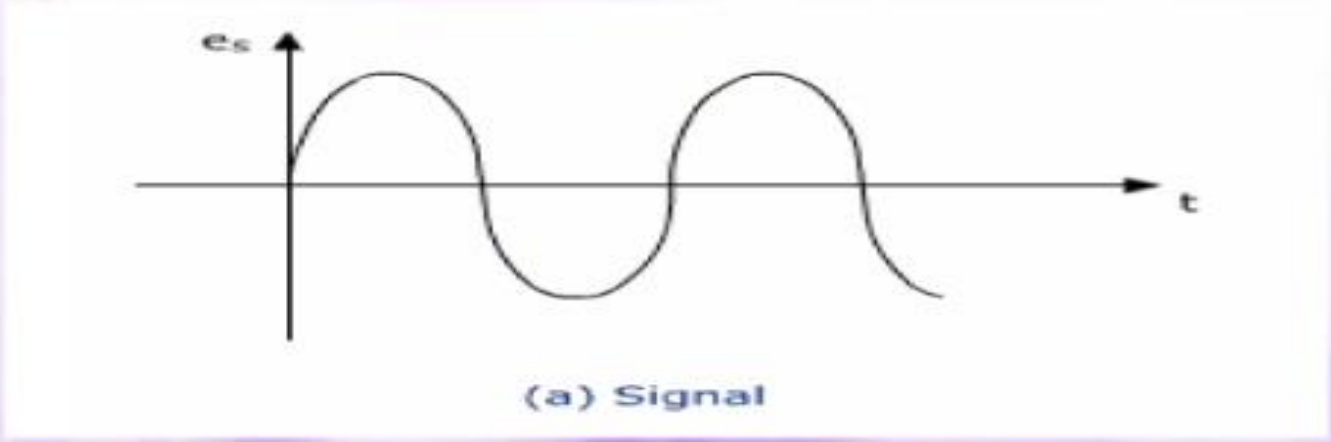
Signal processing

SIGNAL

- **What is Signal??**

Ans: Any physical phenomenon that carries or convey information from one place to other and represents as a function of independent variables such as time, distance, etc.

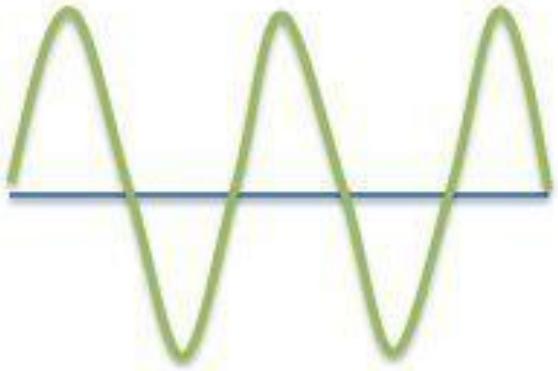


SIGNALS

- Analog signals
- Discrete signals
- Digital signals

Process

- Discretization
- Quantization

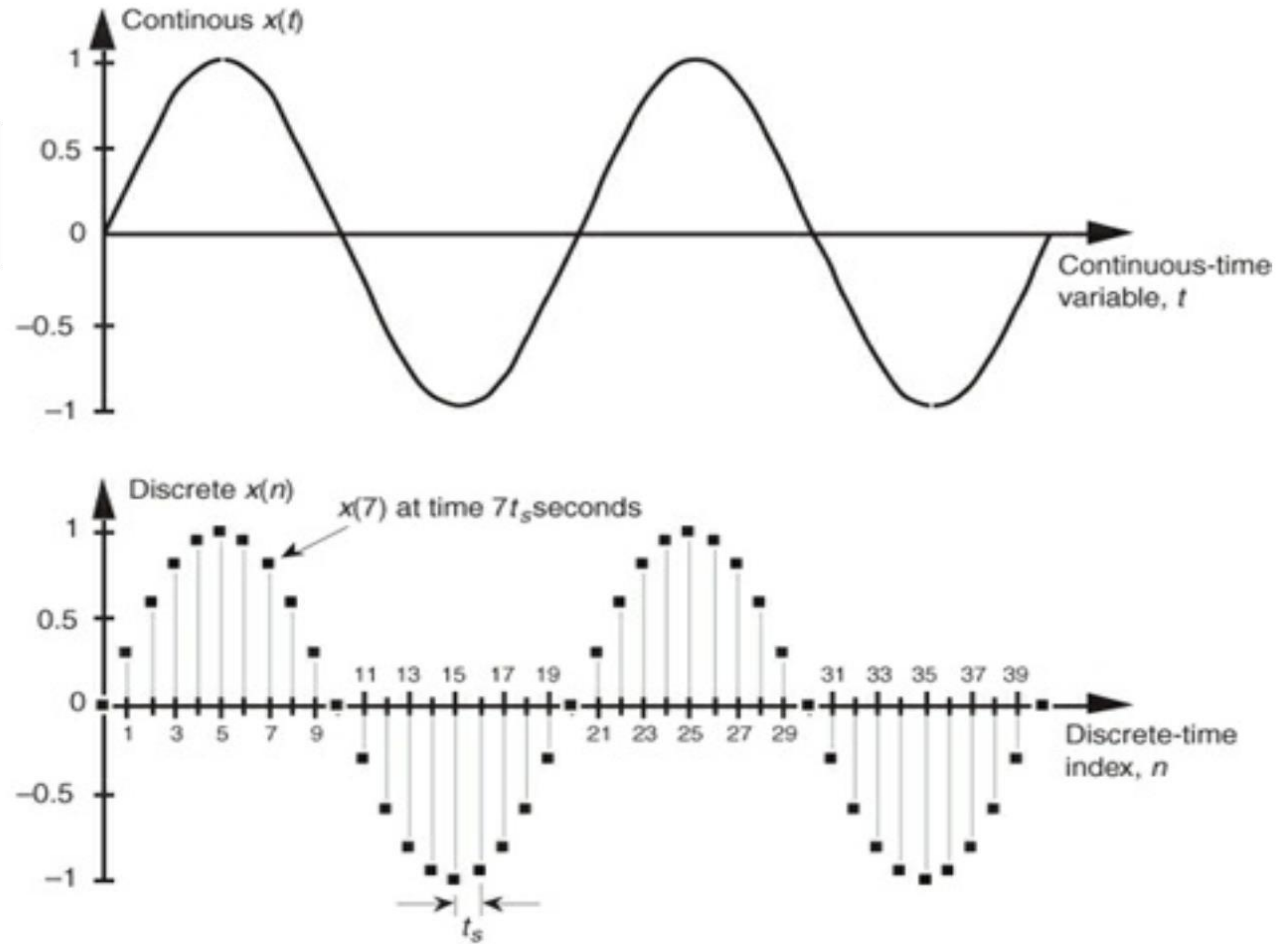


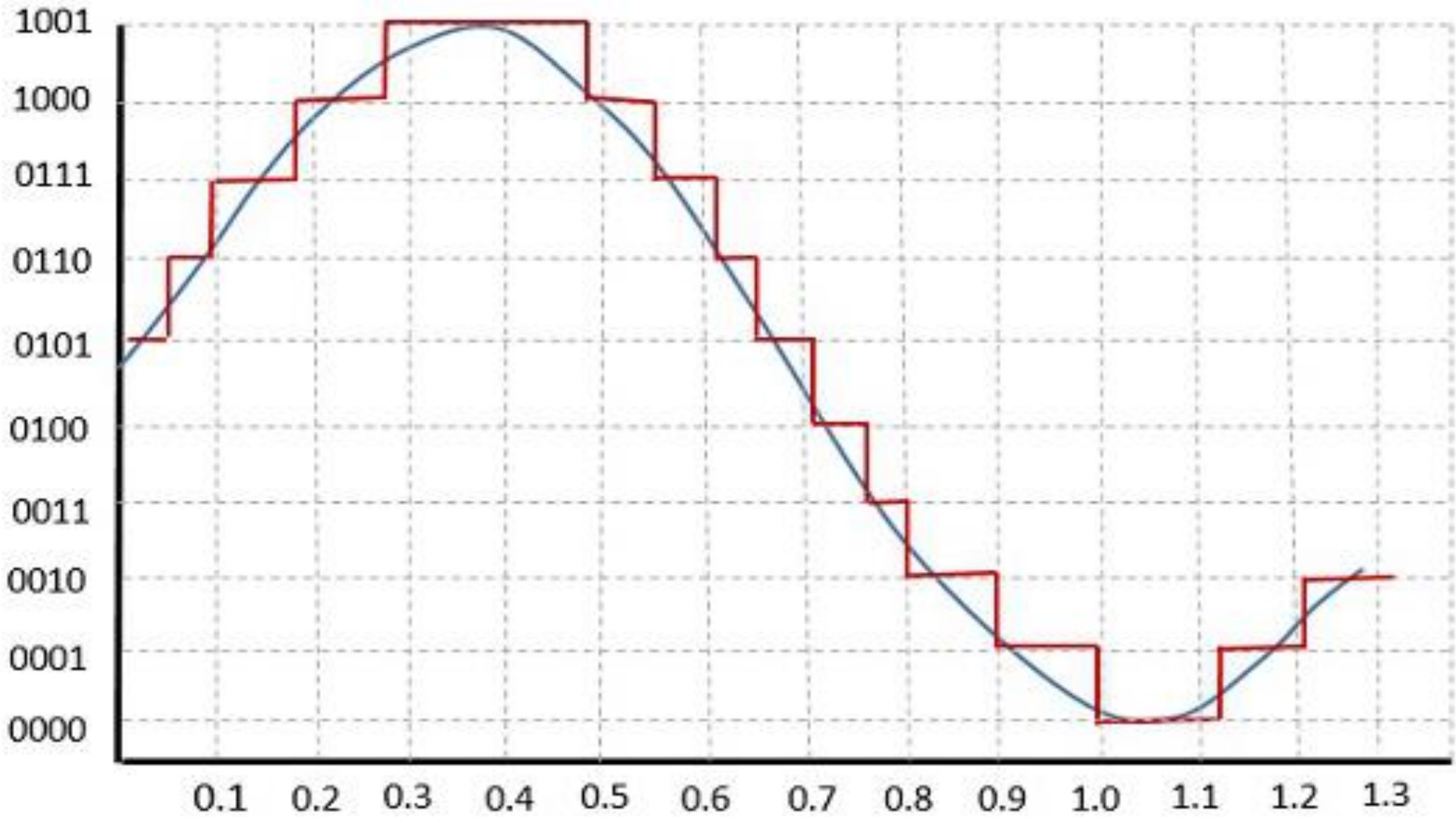
Vs

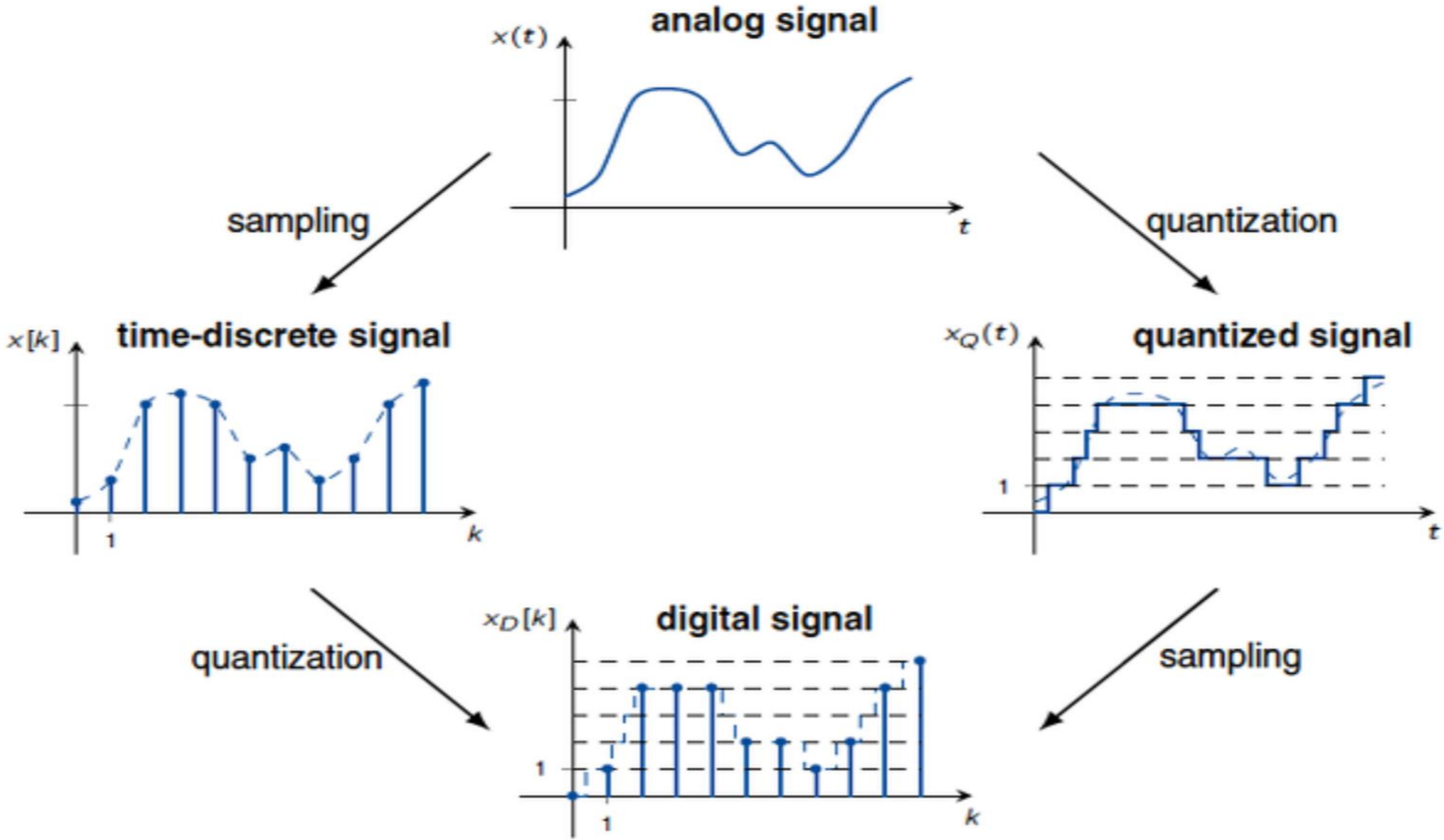
0100111101

Analog Signal

Digital Signal




1001
1000
0111
0110
0101
0100
0011
0010
0001
0000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3


analog signal
x(t)
t
sampling
quantization
time-discrete signal
x[k]
1
k
quantized signal
$x_Q(t)$
1
t
quantization
sampling
digital signal
$x_D[k]$
1
1
k

Jean Baptiste Joseph Fourier (1768-1830)

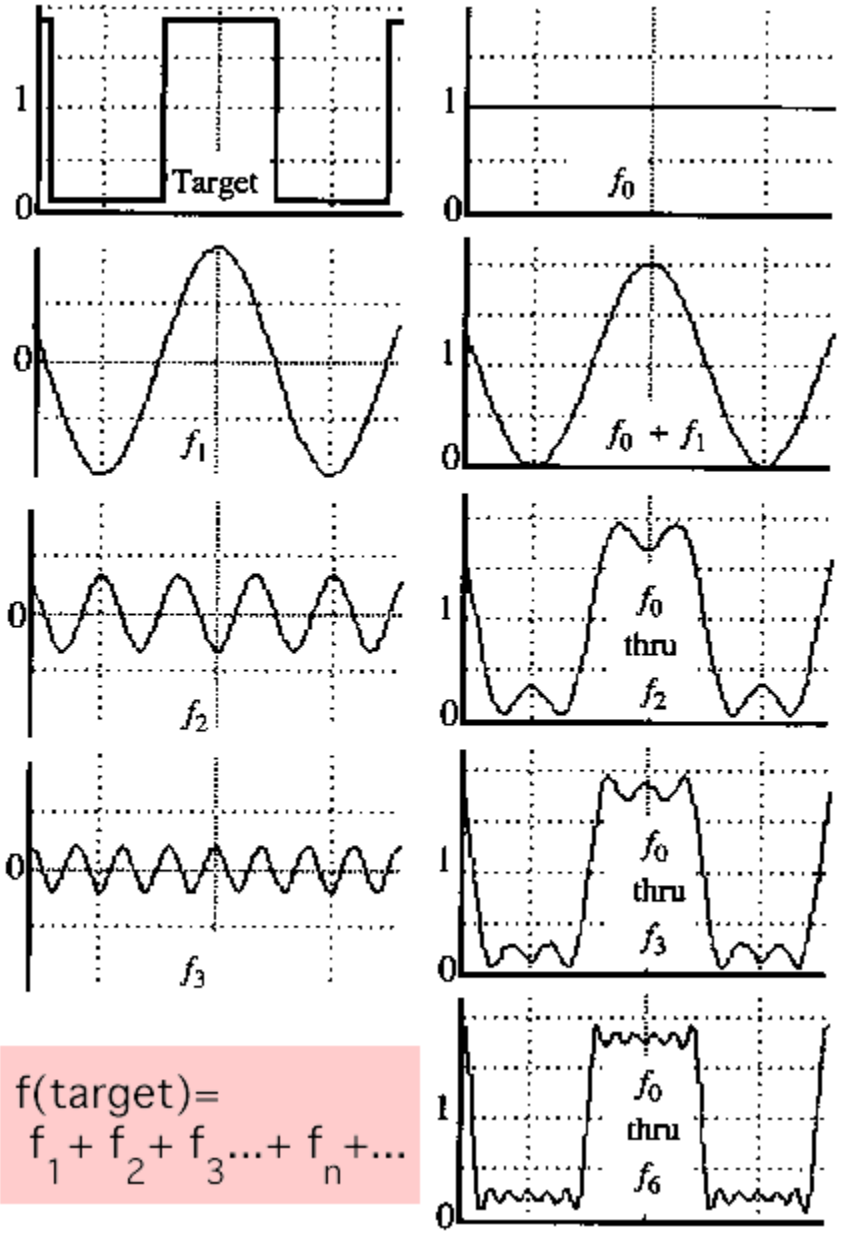
- Had crazy idea (1807):
- **Any** periodic function can be rewritten as a weighted sum of Sines and Cosines of different frequencies.
- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's true!
  - called Fourier Series
  - Possibly the greatest tool used in Engineering

A Sum of Sinusoids

- Our building block:

$$A\sin(\omega x + \phi)$$

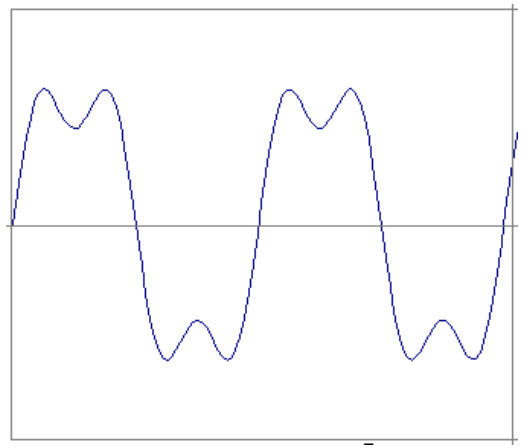
- Add enough of them to get any signal *f(x)* you want!
- How many degrees of freedom?
- What does each control?
- Which one encodes the coarse vs. fine structure of the signal?



f(target)=
$f_1 + f_2 + f_3 ... + f_n + ...$

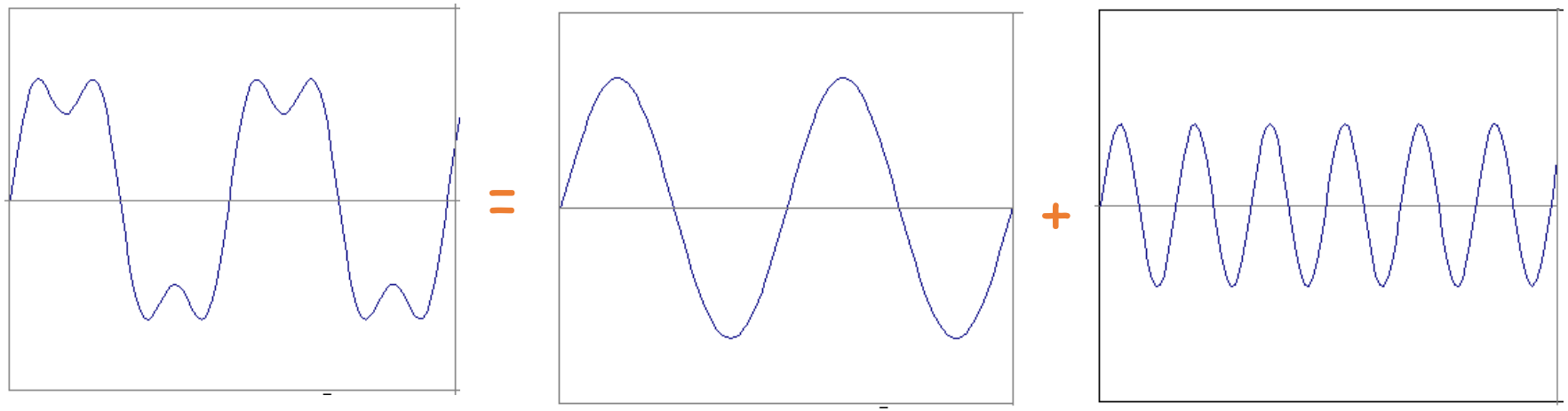
Time and Frequency

- example : $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi (3f) t)$



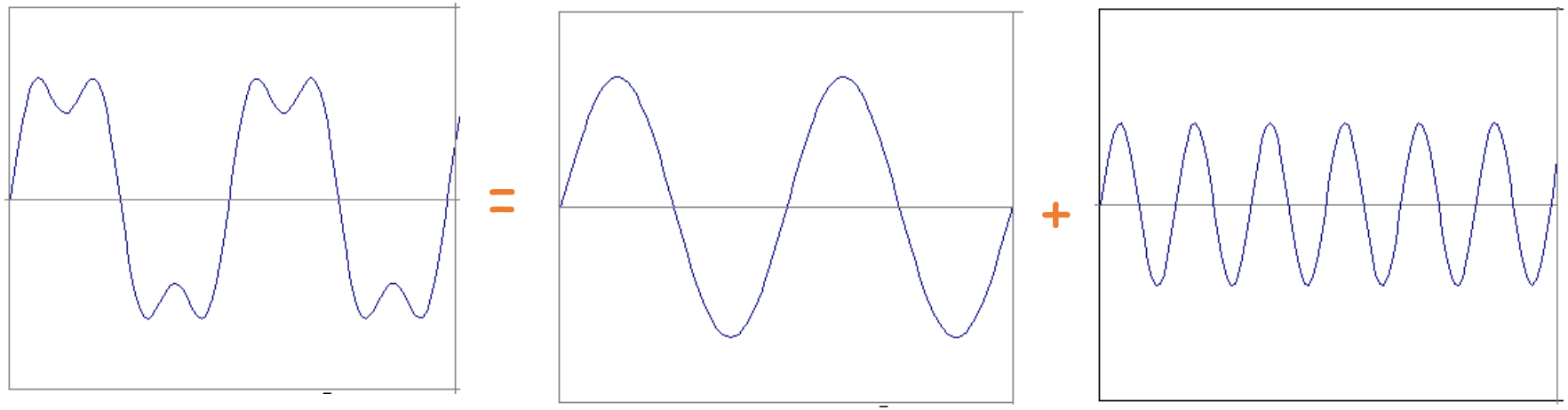
Time and Frequency

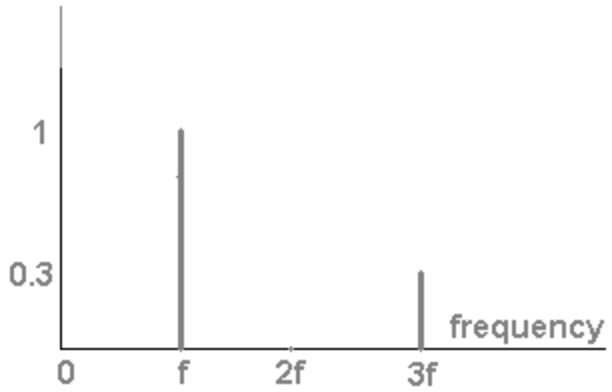
- example : $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi (3f) t)$



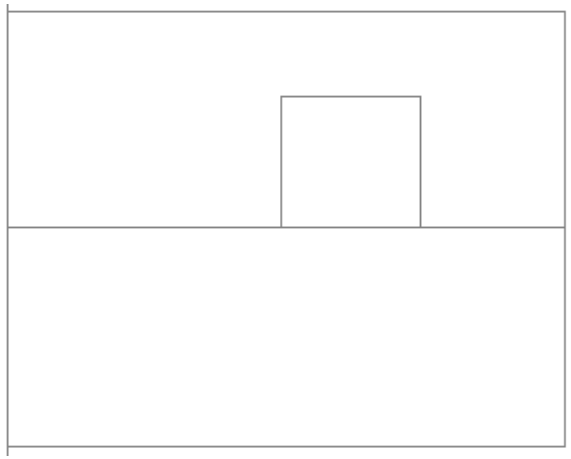
Frequency Spectra

- example : $g(t) = \sin(2pi\, f\, t) + (1/3)\sin(2pi\, (3f)\, t)$



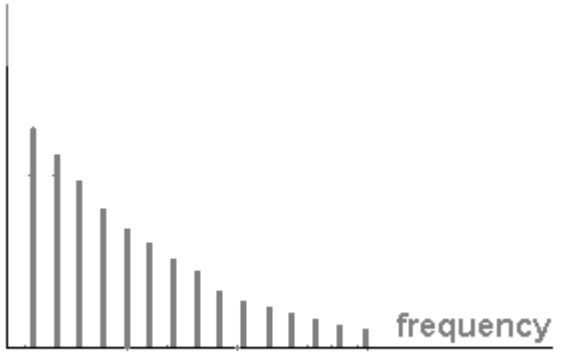


Frequency Spectra

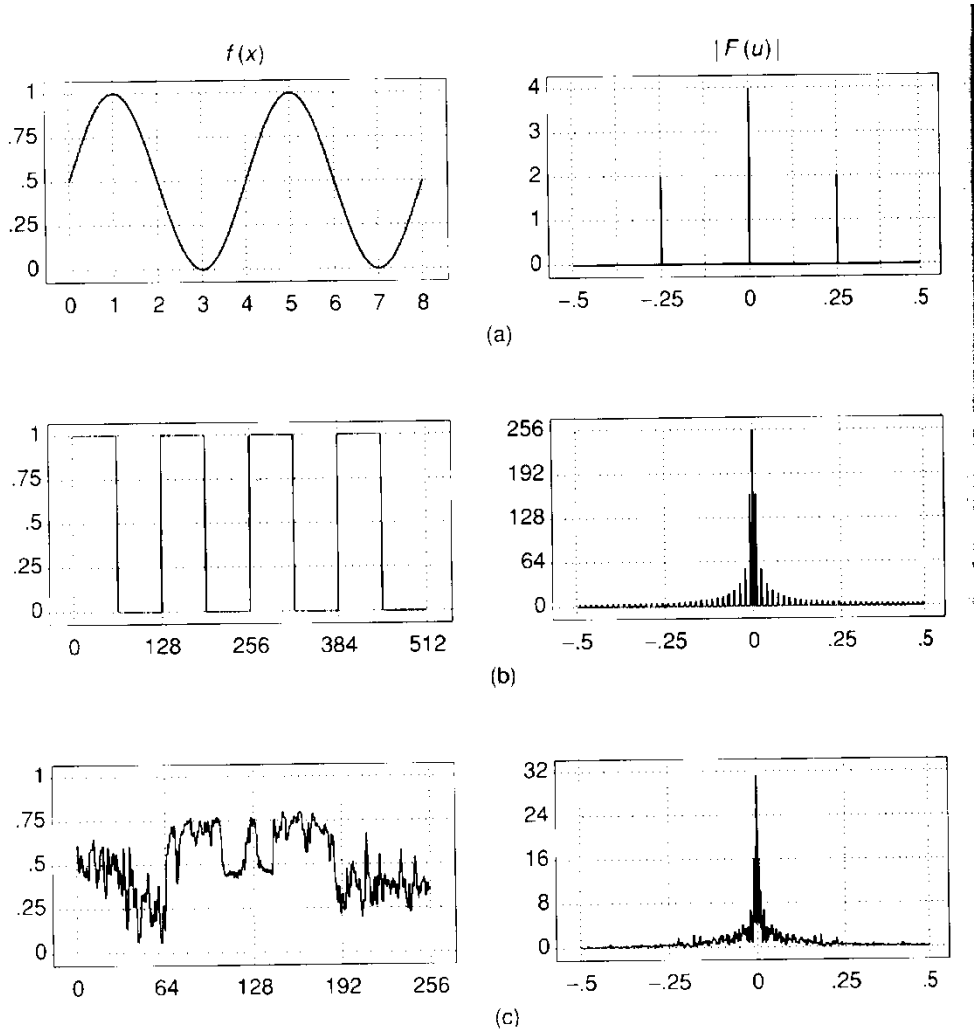


=

$$A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)$$



Frequency Spectra



Fourier Transform – more formally

Represent the signal as an infinite weighted sum of an infinite number of sinusoids

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i 2\pi u x} dx$$

Note: $e^{ik} = \cos k + i \sin k \qquad i = \sqrt{-1}$

Arbitrary function ⟶ Single Analytic Expression

Spatial Domain (*x*) ⟶ Frequency Domain (*u*)

(Frequency Spectrum *F(u)*)

Inverse Fourier Transform (IFT)

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{i 2\pi u x} dx$$

Fourier Transform

- Also, defined as:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-iux}\, dx$$

Note: $e^{ik} = \cos k + i \sin k \qquad i = \sqrt{-1}$

- Inverse Fourier Transform (IFT)

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(u) e^{iux}\, dx$$