“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

Integral

Fan nomi: Hisob (Calculus)

Reja:

1. Boshlang’ich funktsiya tushunchasi
2. Asosiy integrallar jadvali
3. Aniq integral va uning xossalari
4. Nyuton-Leybnits formulasi
5. Ikki karrali integral

Boshlang'ich funksiya va aniqmas integral

[a,b] kesmada aniqlangan y = f(x) funksiya uchun ushbu kesmaning barcha nuqtalarida

$$F'(x) = f(x)$$

tenglik bajarilsa, u holda F(x) funksiya shu kesmada f(x) funksiyaning **boshlang'ich funksiyasi** deyiladi.

x^2 dx

Masalan: 1/3 sin3x ning hosilasi cos3x ga teng. Shuning uchun 1/3 sin3x funksiya cos3x funksiya uchun boshlang'ich funksiya bo'ladi.

$f(x) = x^3$

$f'(x) = 3x^2$

$\int 3x^2 \, dx = x^3$

$f(x) = x^2$

$f'(x) = 2x$

$\int 2x \, dx = x^2$

Boshlang‘ich funksiya mavjudligi haqida teorema:

Har bir uzluksiz funksiya, bir – biridan ixtiyoriy o‘zgarmasga farq qiluvchi cheksiz ko‘p boshlang‘ich funksiyalarga ega bo‘ladi.

Boshlang‘ich funksiyaning umumiy F(x) + C ko‘rinishi berilgan

y = f(x) funksiyaning **aniqmas integrali** deyiladi.

Bu yerda C – ixtiyoriy o‘zgarmas son va

$$\boxed{\int f(x)dx} = F(x) + C$$

kabi belgilanadi.

Bunda $\int$ - integral belgisi,

f(x) – integral osti funksiyasi,

f(x)dx –integral ostidagi ifoda deyiladi.

Asosiy integrallar jadvali

Asosiy integrallar jadvali quyidagi formulalardan iborat:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$
2. $\int \frac{dx}{x} = \ln|x| + C;$
3. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C;$
4. $\int a^x dx = \frac{a^x}{\ln a} + C;$
5. $\int e^x dx = e^x + C;$
6. $\int \sin x dx = -\cos x + C;$
7. $\int \cos x dx = \sin x + C;$
8. $\int \frac{dx}{\cos^2 x} = tgx + C;$
9. $\int \frac{dx}{\sin^2 x} = -ctgx + C;$
10. $\int tgx dx = -\ln|\cos x| + C;$
11. $\int ctgx dx = \ln|\sin x| + C;$
12. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C;$
13. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg\frac{x}{a} + C;$
14. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C;$
15. $\int \frac{dx}{\sqrt{x^2 + \lambda}} = \ln\left|x + \sqrt{x^2 + \lambda}\right| + C.$

$\int x^4 dx = \frac{x^5}{5} + C$

$\int \frac{1}{x} dx = \int x^{-1} dx = \frac{x^0}{0}$

$\int e^{2x} dx = \frac{1}{2} e^{2x} + C$

$\int e^{2x} dx = \frac{1}{2} e^{2x} + C$

$2 \cdot e^{2x} + 0$

$\int \cos 5x dx = \frac{1}{5}\sin 5x + C$

$\int \cos \frac{1}{12}x dx = 12 \sin \frac{1}{12}x$

$\int \sin 3x dx = -\frac{1}{3}\cos 3x + C$

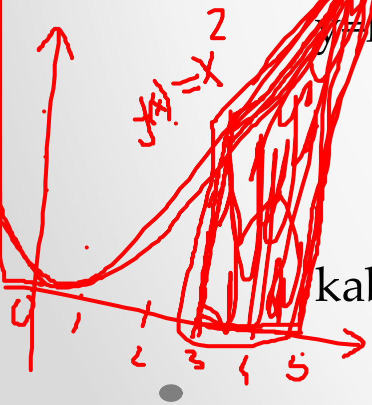
$\int e^{8x} dx = \frac{1}{8} e^{8x} + C$

Aniq integral tushunchasi

Aniq integral. Aniqmas integral ifodasiga ixtiyoriy C o'zgarmas kirgani uchun x ning berilgan qiymatida bu integralning qiymatini topib bo'lmaydi. Ammo berilgan b va a nuqtalarda integral qiymatlarining ayirmasini toppish mumkin:

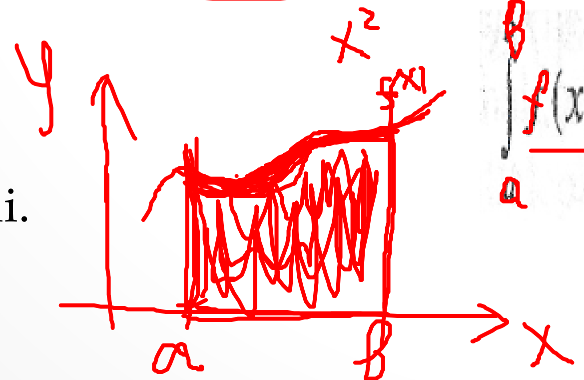
$$[F(b)+C]-[F(a)+C]=F(b)-F(a).$$

Bu tenglik y=f(x) funktsiyaning barcha boshlang'ichlari uchun b va a nuqtalardagi ular qiymatlarining ayirmasi bir xil va u C ning tanlanishiga bog'liq emasligini ko'rsatadi. Shuning uchun y=f(x) funktsiya b va a nuqtalardagi boshlang'ich qiymatlarining ayirmasi y=f(x) funktsiyaning [a;b] kesmadagi aniq integrali deyiladi. [a;b] kesmadagi aniq integral



$$\int_a^b f(x)dx =$$

kabi belgilanadi.



$$\int f(x)\,dx = \quad +C$$

$$\int_3^5 x^2dx = \frac{x^3}{3}\Big|_3^5 = \frac{5^3}{3} - \frac{3^3}{3} = \frac{98}{3}$$

3. Aniq integralning asosiy xossalari

1-xossa. O’zgarmas ko’paytuvchini aniq integral belgisidan tashqariga chiqarish mumkin: agar $A = const$ bo’lsa, u holda

$$\int_a^b Af(x)dx = A\int_a^b f(x)dx \qquad (1)$$

$$\int_0^{\pi/2} 2\sin x\,dx = 2\int_0^{\pi/2}\sin x\,dx = -2\cos x\Big|_0^{\pi/2} = -2\left[\cos\frac{\pi}{2} - \cos 0\right]$$

2-xossa. Bir necha funksiyalarning algebraic yig’indisidan olingan aniq integral qo’shiluvchilardan olingan integrallarning algebraic yig’indisiga teng. Ikki qo’shiluvchi bo’lgan holda

$$\int_a^b [f_1(x) + f_2(x)]dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx \qquad (2)$$

$$\int_0^{\pi/2}[\sin x + x^2]dx = \int_0^{\pi/2}\sin x\,dx + \int_0^{\pi/2} x^2\,dx$$

$$\int_0^{\pi/2} 3\cos 4x\,dx = 3\cdot\frac{1}{4}\sin 4x$$

3-xossa. Agar $[a,b]$ kesmada $(a < b)$ $f(x)$ va $\varphi(x)$ funksiyalar $f(x) \le \varphi(x)$ shartni qanoatlantirsa, u holda

$$\int_a^b f(x)dx \le \int_a^b \varphi(x)dx \qquad (3)$$

4-xossa. Agar m va M - $f(x)$ funksiyaning $[a,b]$ kesmadagi eng kichik va eng kata qiymatlari bo’lib, $a \le b$ bo’lsa, u holda

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a) \qquad (4)$$

5-xossa. (o’rta qiymat haqida teorema). Agar $f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo’lsa, u holda bu kesmada shunday ξ nuqta topiladiki,

$$\int_a^b f(x)dx = (b-a)f(\xi) \qquad (5)$$

4. Aniq integralni hisoblash. Nyuton-Leybnits formulasi

$$\int_a^b f(x)dx$$

Aniq integralda quyi a chegara mahkamlangan, yuqori b chegara esa o'zgraib tursin. U holda integralning qiymati ham o'zgarib turadi, ya'ni integral yuqori chegaraning funksiyasi bo'lib qoladi.

Yuqori chegarani x bilan, integral o'zgaruvchini t bilan belgilaymiz:

$$\int_a^x f(t)dt$$

Integralga ega bo'lamiz. a o'zgarmas son bo'lganda bu integral x yuqori chegaraning funksiyasi bo'ladi. Bu funksiyani biz $\Phi(x)$ bilan belgilaymiz:

$$\Phi(x) = \int_a^x f(t)dt$$

Agar $f(t)$ - nomanfiy funksiya bo'lsa, u holda $\Phi(x)$ miqdor son jihatdan $aAXx$ egri chiziqli trapetsiyaning yuziga teng (216-rasm). x o'zgarganda bu yuza o'zgarishi ochiq ravshan.

$$\int_a^b f(x)dx = F(b) - F(a)$$

$F(b) - F(a)$ ayirma F boshlang'ich funksiyaning tanlanishiga bog'liq emas.

Agar

$$F(b) - F(a) = F(x)\Big|_a^b$$

belgilash kiritsak

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Nyuton-Leybnits formula aniq integrallarni hisoblashning qulay usulidir.

$$-\frac{1}{3}\cos 3x$$

$$\int_0^{\pi/2} \sin 3x = -\frac{1}{3}\cos 3x\Big|_0^{\pi/2} =$$

- Вычислить интеграл $\iint\limits_{\Delta}(3x^2 - 2xy + y)dxdy$ если область интегрирования Δ ограничена линиями

  x = 0, x = y², y = 2.

- **Решение:**

$$\iint\limits_{\Delta}(3x^2 - 2xy + y)dxdy =$$

$$\int_0^2 dy \int_0^{y^2} (3x^2 - 2xy + y)dx$$

$$= \int_0^2 dy \int_0^{y^2} (3x^2 - 2xy + y)dx = \int_0^2 (x^3 - yx^2 + yx)\Big|_0^{y^2} dy =$$

$$= \int_0^2 (y^6 - y^5 + y^3)dy = \left(\frac{y^7}{7} - \frac{y^6}{6} + \frac{y^4}{4}\right)\Bigg|_0^2 = \frac{244}{21}$$

$$(x^3 - x^2 y + xy)\Big|_0^{y^2} =$$

$$= (y^2)^3 - (y^2)^2 y + y^3 - 0 = y^6 - y^5 + y^3$$

Uyga vazifa

Aniqmas integrallarni hisoblang.

1. a) $\int \frac{\sin^3 x}{\cos x}dx$; b) $\int \cos 2x \cdot \sin 10x dx$;

c) $\int tg^2 7x dx$.

2. a) $\int \frac{1}{x^2+2x+5}dx$; b) $\int \sin(7x-1)\sin 5x dx$;

c) $\int \frac{3x+2}{x^2+1}dx$.

3. a) $\int \frac{x^2-1}{x^2+1}dx$; b) $\int \sin^3(1-3x)dx$;

c) $\int \frac{x+3}{x+1}dx$.