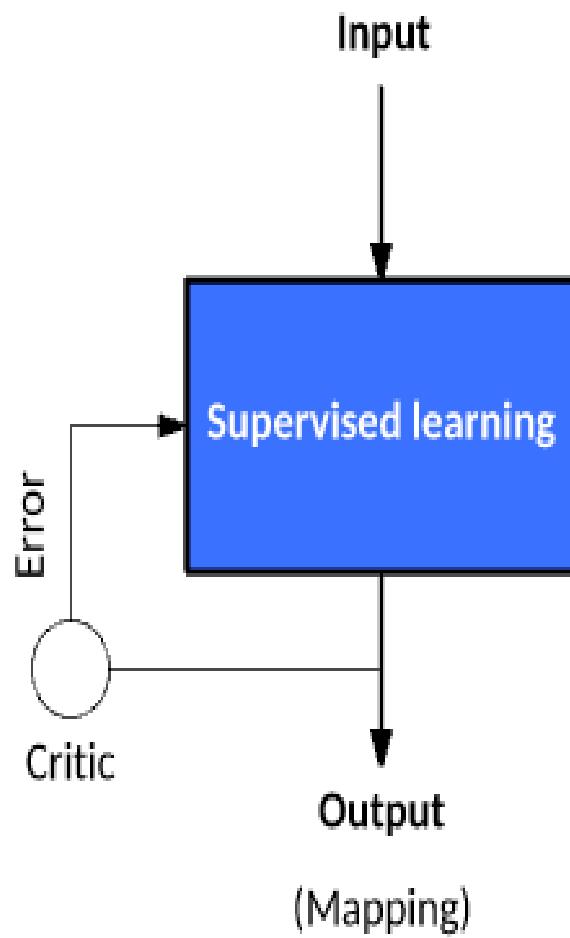


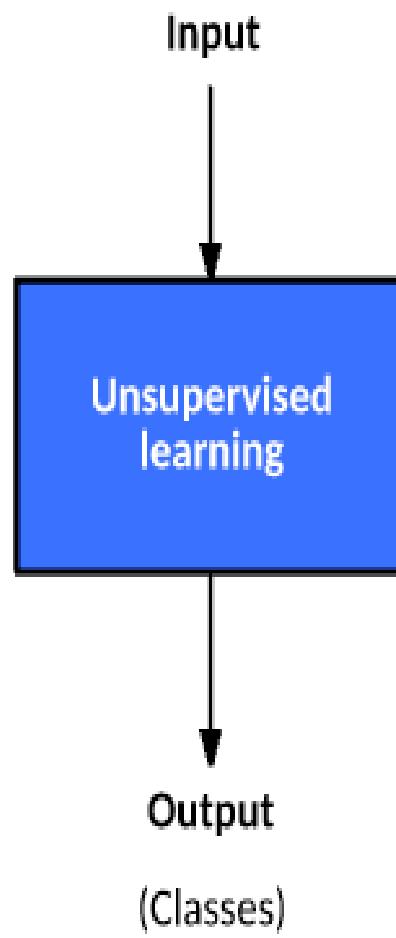
# Linear Regression

*Expected change in  $Y$  per unit  $X$*

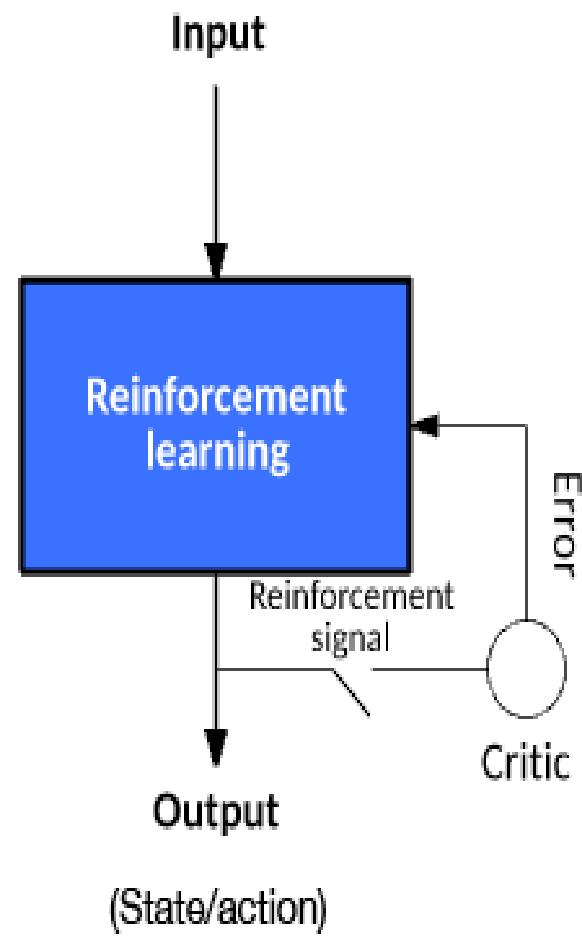
(Data with labels)

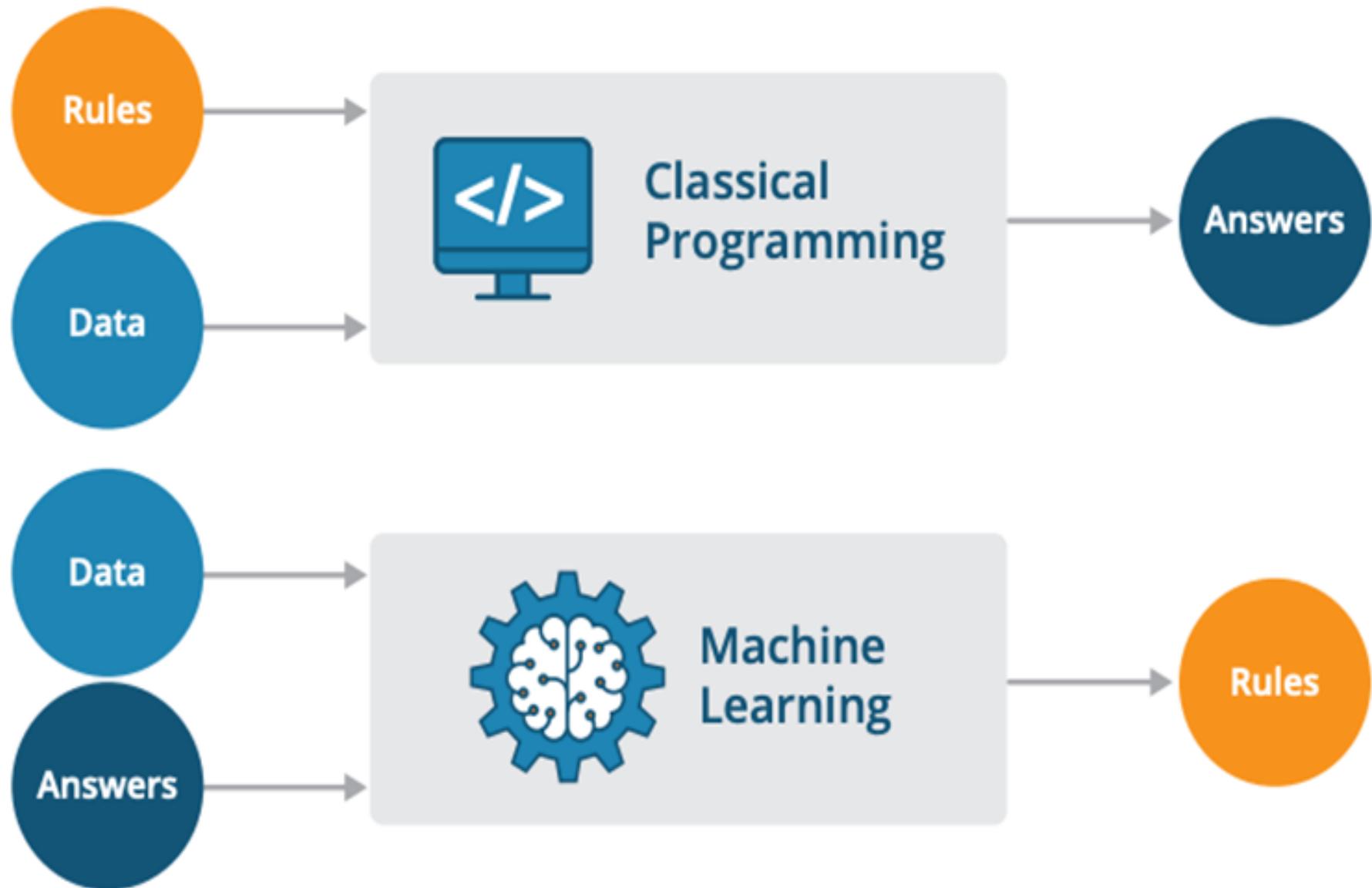


(Data without labels)

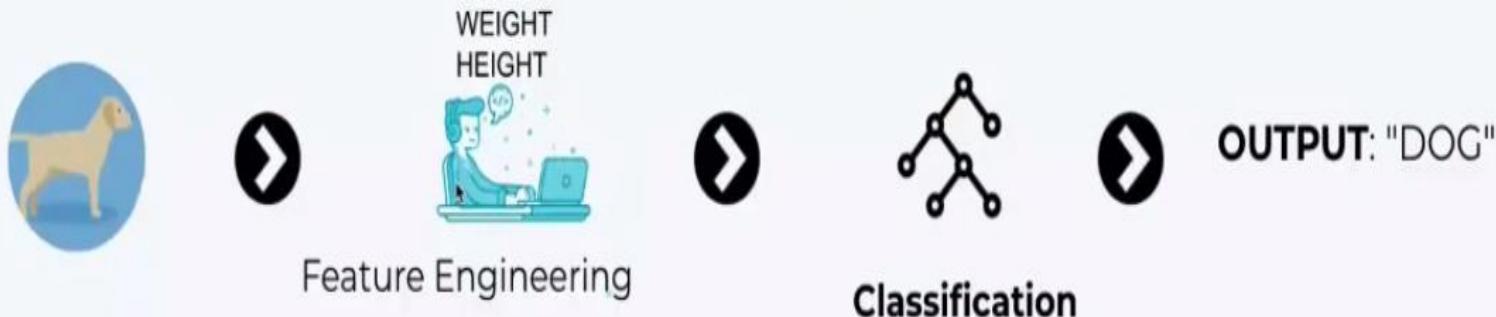


(States and actions)

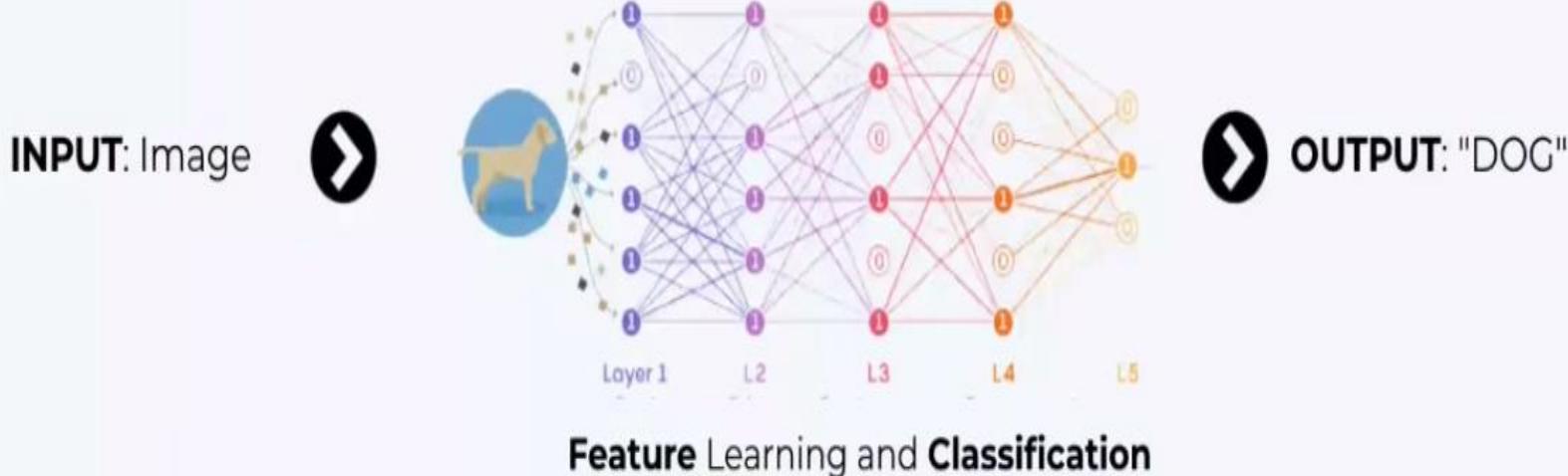




## Traditional Machine Learning

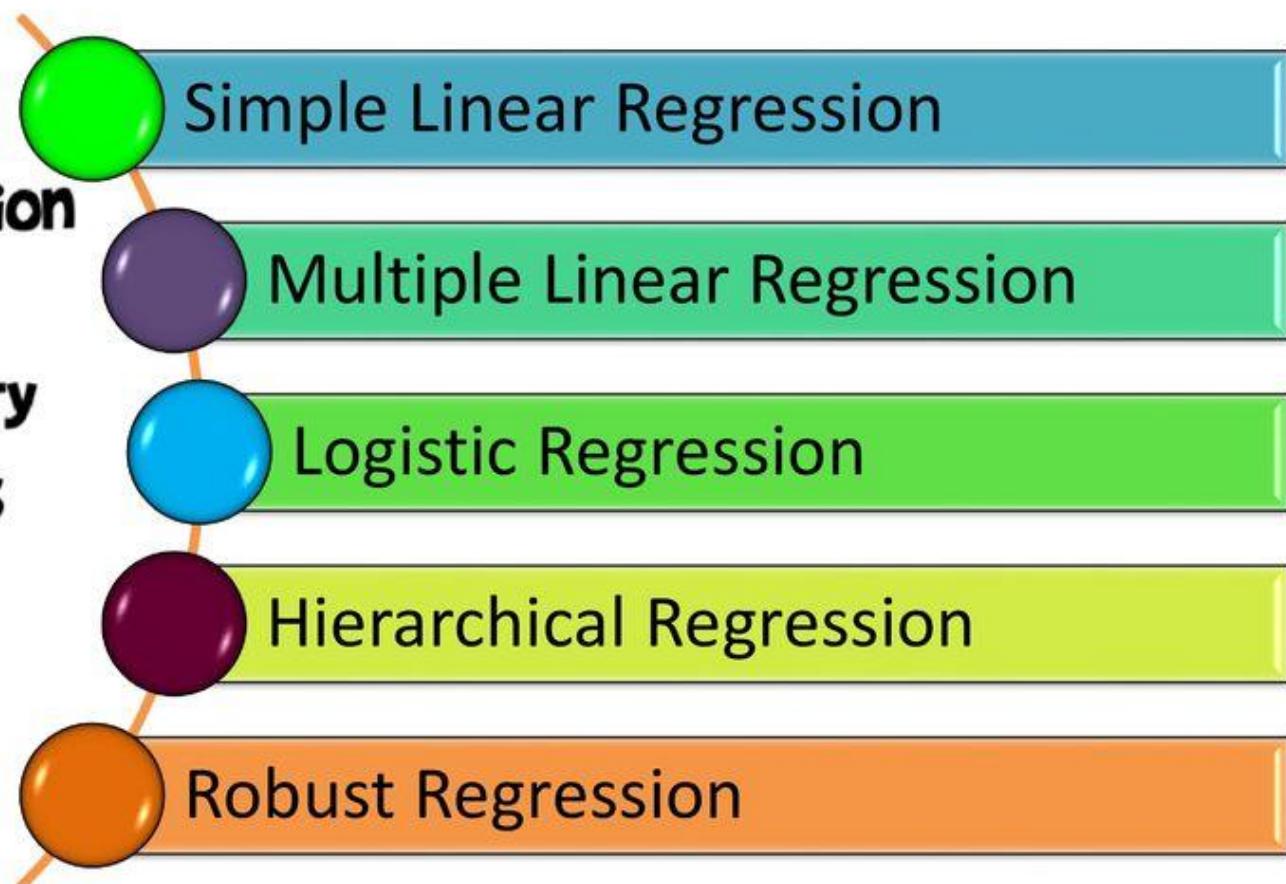


## DEEP LEARNING ARTIFICIAL NEURAL NETWORK (ANN)



# Regression Analysis

**Making a Prediction  
is when we use  
clues in the story  
to make a guess  
about what will  
happen next.**

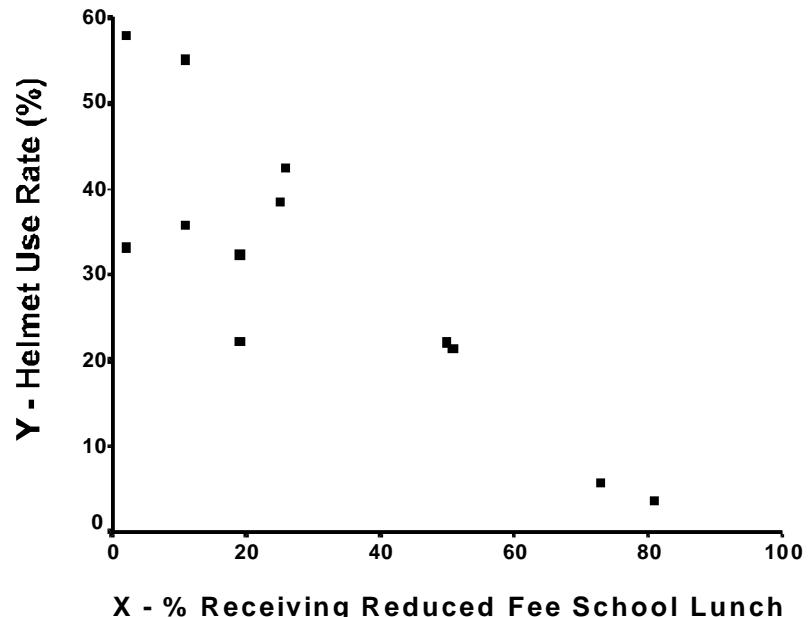


# Introduction

- $X$  = independent (explanatory) variable
- $Y$  = dependent (response) variable
- Use instead of correlation
  - when distribution of  $X$  is fixed by researcher (i.e., set number at each level of  $X$ )
  - studying functional dependency between  $X$  and  $Y$

# Illustrative data (bicycle.sav)

- Same as prior chapter
- $X$  = percent receiving reduce or free meal (RFM)
- $Y$  = percent using helmets (HELM)
- $n = 12$  (outlier removed to study linear relation)



# Regression Model (Equation)

$$\hat{y} = a + bX$$

“y hat”

where

$\hat{y}$  represents predicted average of  $Y$  at a given  $X$

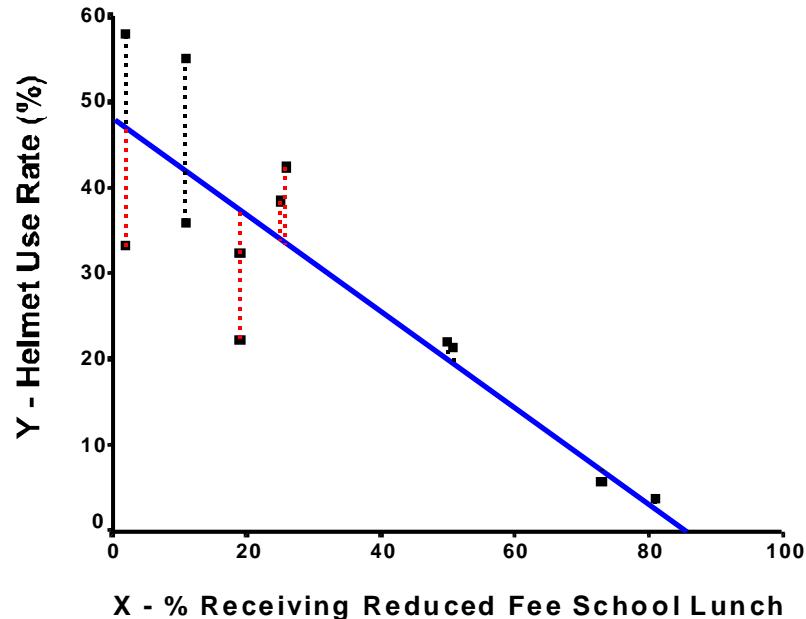
$a$  represents the line's intercept

$b$  represents the line's slope

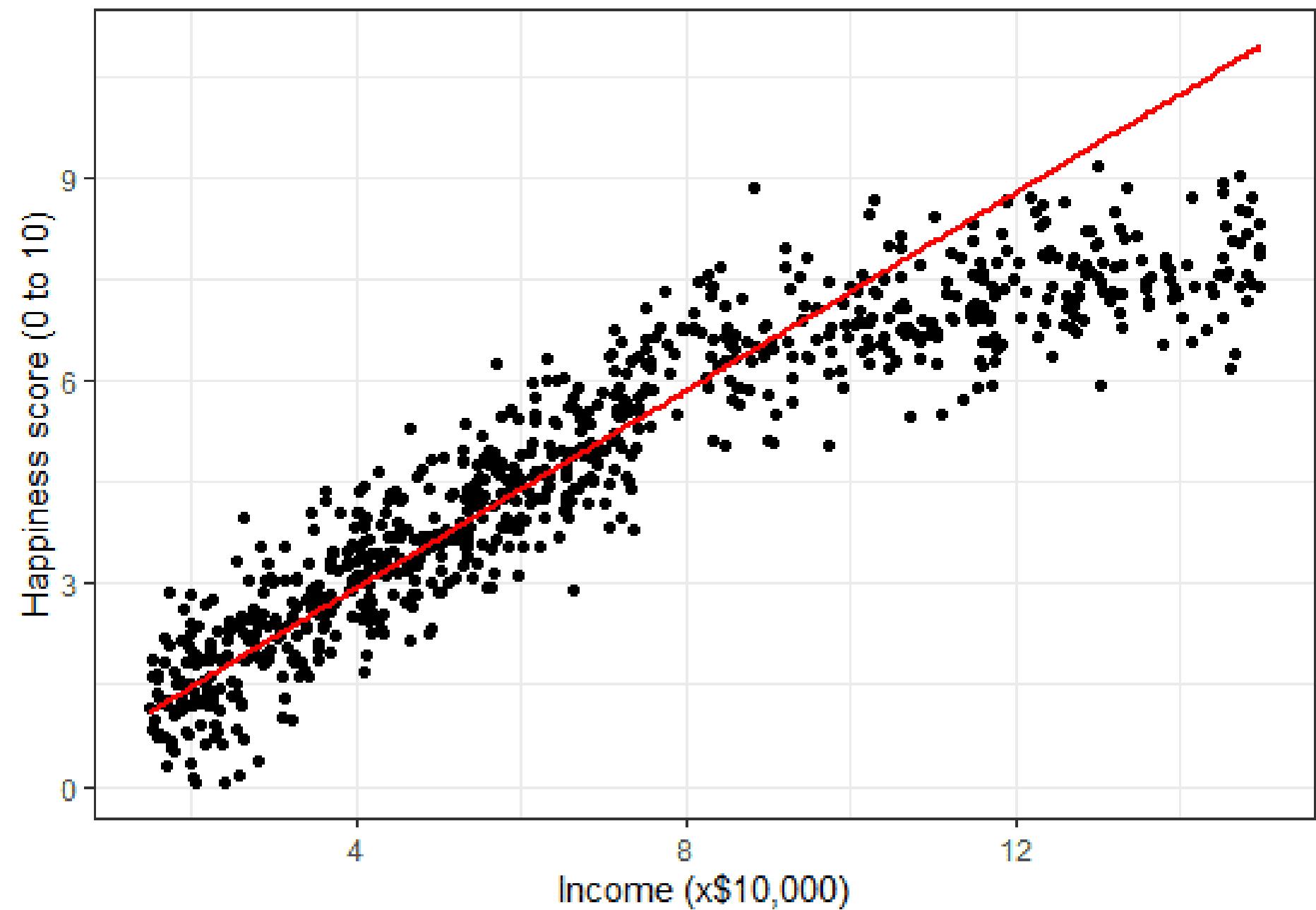
# How formulas determine best line

(p. 15.2)

- Distance of points from line = *residuals* (dotted)
- Minimizes sum of square residuals
- *Least squares regression line*



# Reported happiness as a function of income



# Formulas for Least Squares Coefficients with Illustrative Data

$$b = \frac{SS_{XY}}{SS_{XX}} = \frac{-4231 .1333}{7855 .67} = -0.539$$

$$a = \bar{y} - b\bar{x} = 30.8833 - (-0.539)(30.8333) = 47.49$$

SPSS output:

Model		Coefficients <sup>a</sup>			
		B	Std. Error	Standardized Coefficients Beta	t
1	(Constant)	47.490	4.242		11.194
	X (% children receiving school lunch)	-.539	.106	-.849	-5.087

a. Dependent Variable: Y (% bicycle riders wearing helmets)

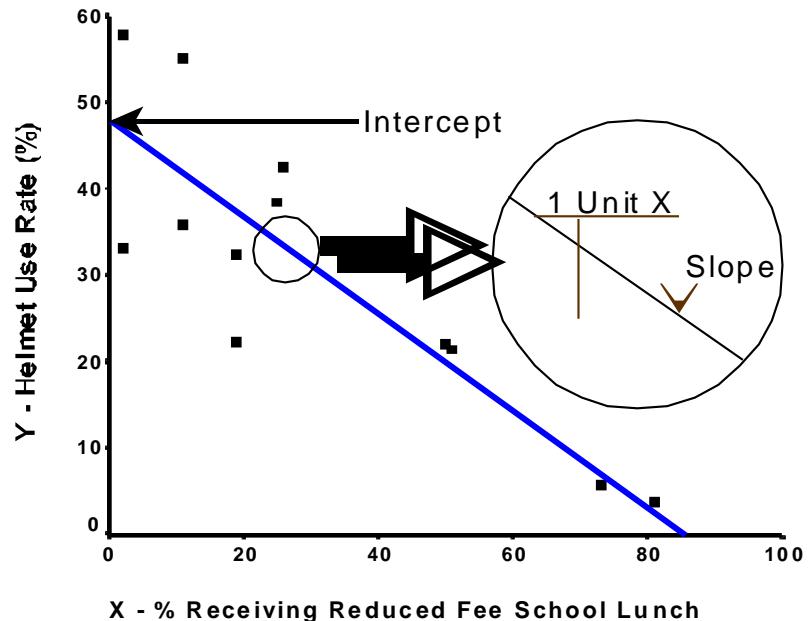
# Alternative formula for slope

$$b = \frac{s_Y r}{s_X}$$

# Interpretation of Slope ( $b$ )

(p. 15.3)

- $b = \text{expected change in } Y \text{ per unit } X$
- Keep track of units!
  - $Y = \text{helmet users per 100}$
  - $X = \% \text{ receiving free lunch}$
- e.g.,  $b$  of  $-0.54$  predicts *decrease* of  $0.54$  units of  $Y$  for each unit  $X$



Ma`lumki chiziqli funksiya tuzish uchun a va b koeffitsentlar aniqlanishi kerak, bir chiziqli regression usul uchun ularni toppish formulalari quyida keltirilgan:

$$b = r \frac{s_y}{s_x} \quad (2)$$

$$a = \hat{y} - b\hat{x} \quad (3)$$

bu yerda:

$r$  – chiziqli funksiya uchun korelyatsiya koeffitsenti;

$s_y$  - chiquvchi qiymatlar uchun o`rtacha kvadratik chetlanish;

$s_x$  - kiruvchi qiymatlar uchun o`rtacha kvadratik chetlanish;

$\hat{y}$  – y qiymatlarning o`rta arifmetigi;

$\hat{x}$  - x qiymatlarning o`rta arifmetigi.

Chiziqli funksiya uchun Korrelyatsion koeffitsent formulasi:

$$r = \frac{\sum(x - \hat{x})(y - \hat{y})}{\sqrt{\sum(x - \hat{x})^2 * \sum(y - \hat{y})^2}} \quad (2.2)$$

Chiziqli Dataset qiymatlari uchun o`rtacha kvadrat chetlanish:

$$S_y = \sqrt{\frac{\sum(y - \hat{y})^2}{n-1}} \quad (2.3)$$

$$S_x = \sqrt{\frac{\sum(x - \hat{x})^2}{n-1}} \quad (2.4)$$

Barcha koeffitsentlar topilgandan so`ng natija chiziqli funksiya tenglamasi hisoblanadi. Bashorat qilinishi kerak bo`lgan holat uchun argument qiymati ( $x$  data) kiritiladi va natija quyidagi funksiya yordamida hisoblanib chiqariladi:

$$F(x) = a + b x$$

