“TOSHKENT IRRIGATSIYA VA QISHLOQ XO’JALIGINI MEXANIZATSIYALASH MUXANDISLARI INSTITUTI” MILLIY TADQIQOT UNIVERSITETI

Funktsiyalar.

Funktsiya tushunchasi

Fan nomi: Hisob (Calculus)

Reja:

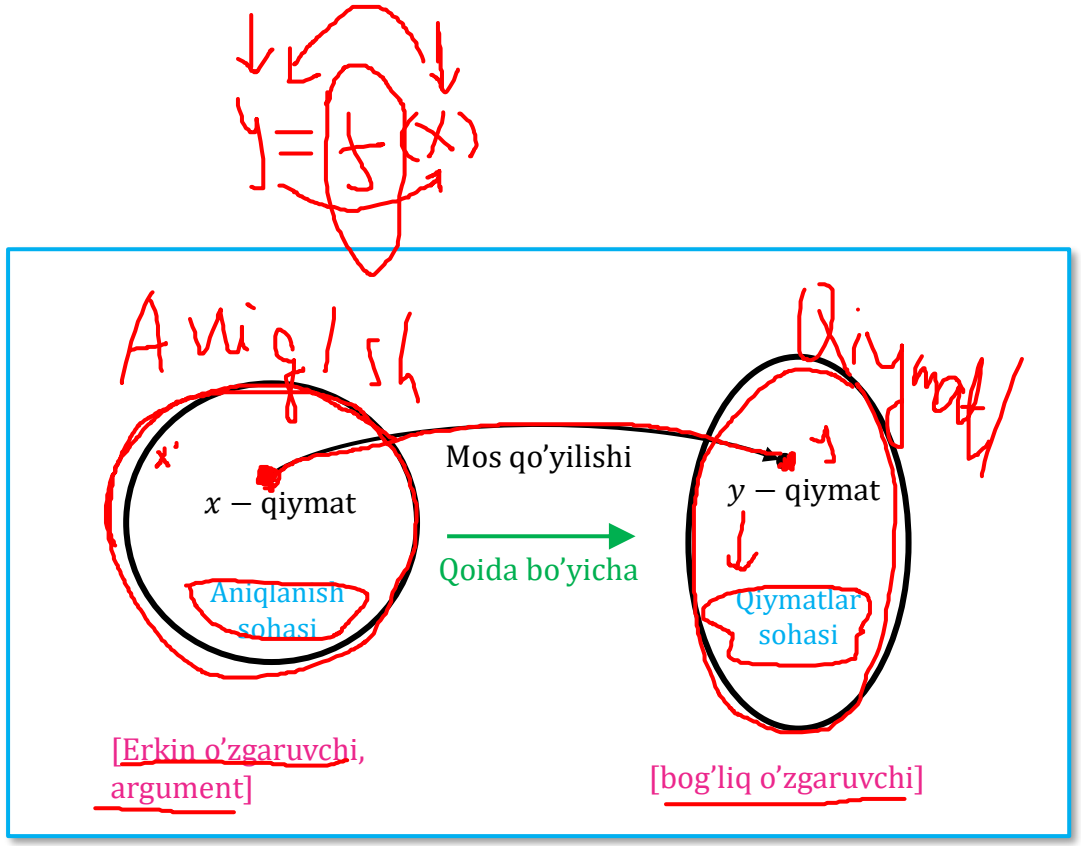
1. Funktsiya tushunchasi
2. Funktsiyaning aniqlanish va qiymatlar sohasi
3. Funktsiyaning berilish usullari
4. Funktsiya grafigi
5. Funktsiyaning juft-toqligi

1 Funktsiya tushunchasi

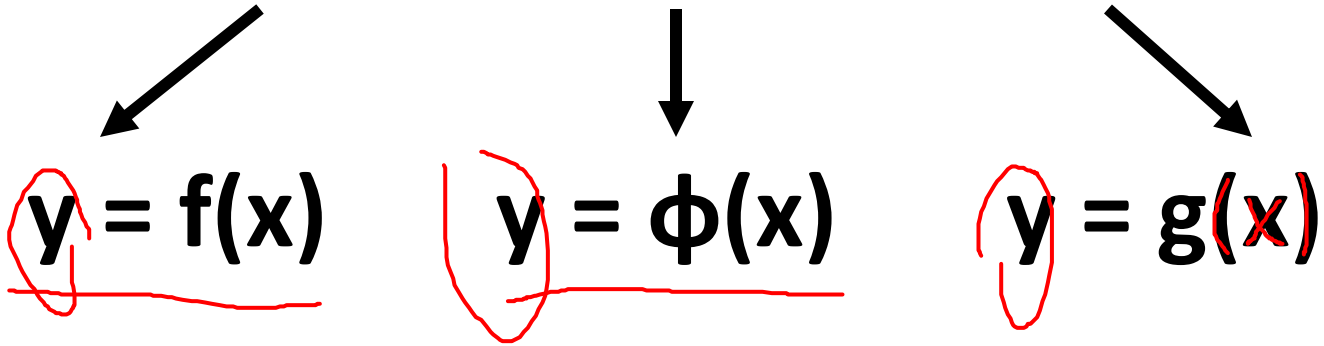
Funktsiya-u ikki to’plam elementlari orasida moslik o’rnatishga taaluqli bo’lgan asosiy matematik tushunchalardan biridir.

f qoida berilgan bo’lsin.

Ta’rif. Agar X to’plamning har bir x elementiga f qoida bo’yicha Y to’plamdagi yagona y element mos qo’yilsa, u holda X to’plamda y=f(x), x∈ $X, y \in Y$ funktsiya berilgan deyiladi.



Funktsiya:

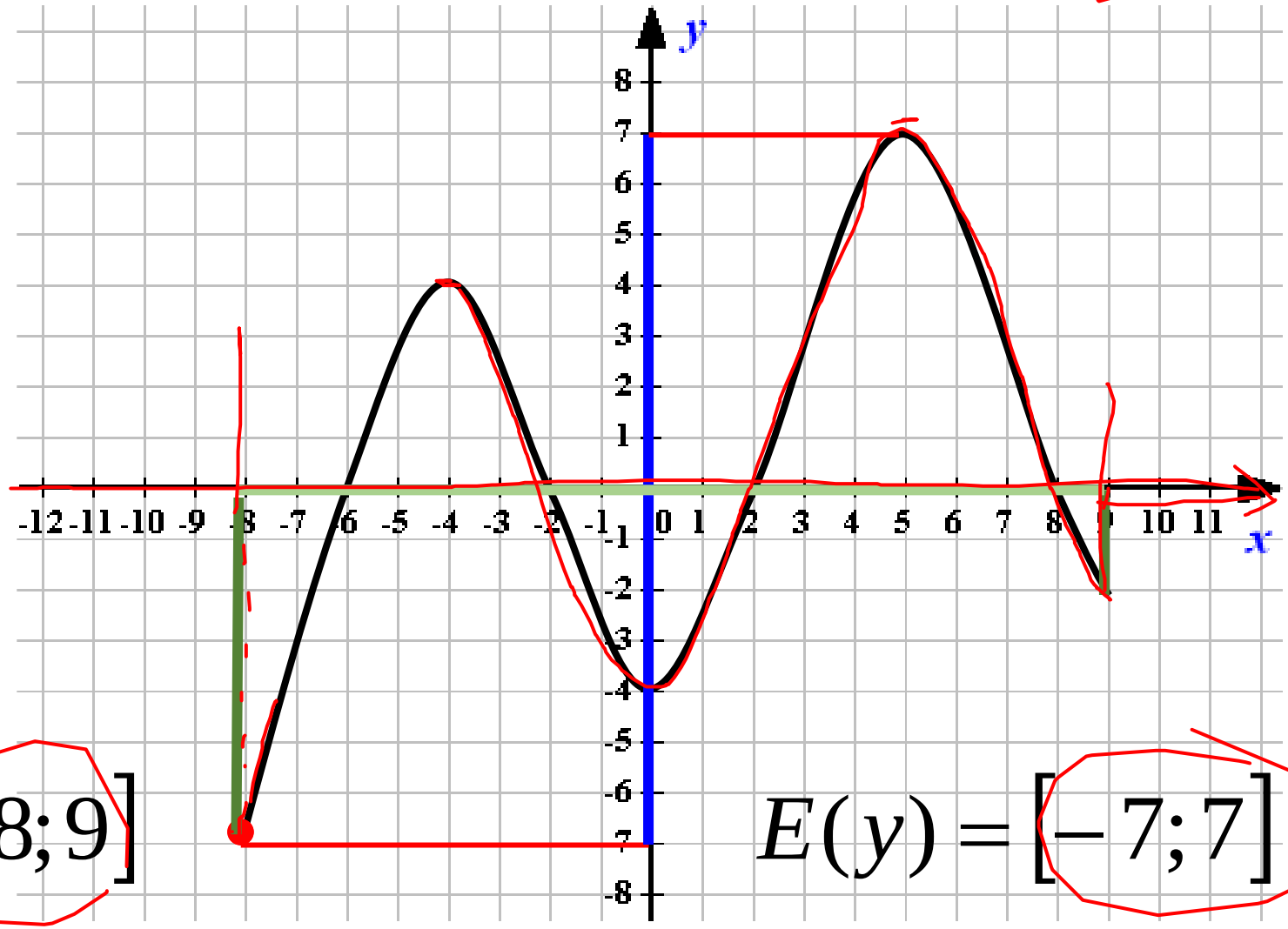


x - erkli o’zgaruvchi, yoki argument;

y – bog’liq o’zgaruvchi yoki funktsiya;

f, φ, g - qoida, yoki qonuniyat.

- X to’plam funktsiyaning aniqlanish sohasi deyiladi va D(f) kabi belgilanadi. Y funktsiyaning o’zgarishlar to’plami funktsiyaning qiymatlar sohasi deyiladi va E(f) kabi belgilanadi.



$$D(y) = [-8;9]$$

$$E(y) = [-7;7]$$

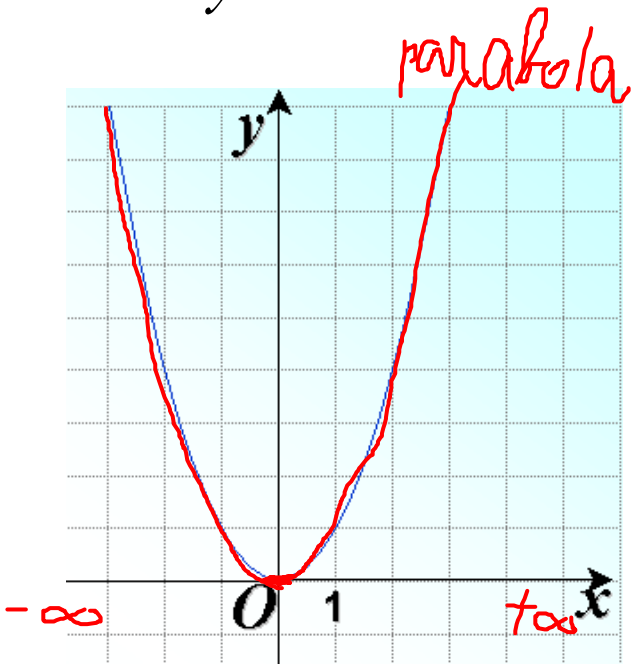
1- misol. $f(x)=\frac{x}{x^2-4}$ funksiyaning aniqlanish sohasini toping.

Yechilishi. $\frac{x}{x^2-4}$ funksiyaning maxraji nolga aylanadigan nuqtalarda funksiya ma‘noga ega emas. Demak, bu funksiyaning aniqlanish sohasini topishda quyidagi $x^2-4\neq 0$ yoki $x\neq\pm 2$ shartlar bajarilishini talab qilish kerak.

Shunday qilib, funksiyaning aniqlanish sohasi uchta oraliqning birlashmasidan iborat, ya‘ni

$$D(f)=(-\infty;-2)\cup(-2;2)\cup(2;+\infty).$$

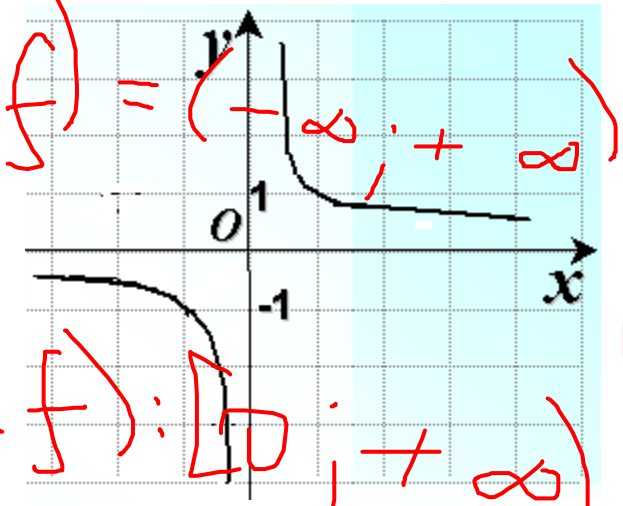
1. $y = x^2$



parabola

$$D(f) = (-\infty; +\infty)$$
$$E(f) = [0; \infty)$$

2. $y = \frac{1}{x}$



$D(f) = (-\infty; 0) \cup (0; +\infty)$

$E(f) = (-\infty; 0) \cup (0; +\infty)$

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$E(f) = (-\infty; 0) \cup (0; +\infty)$

Misollar: Funktsiyaning aniqlanish sohasini topishga doir masalalar.

Javoblar:

$$y=\frac{x-1}{(x+2)(x-3)}$$

$$D(f): x\neq -2;\ x\neq 3$$

$$y=x^2-3x+4$$

$$D(f)=(-\infty;+\infty)$$

$$y=\sqrt{\frac{x}{x-2}}$$

$$D(f)=(-\infty;0]\cup(2;+\infty)$$

$y=\sqrt{\frac{x}{x-2}}$

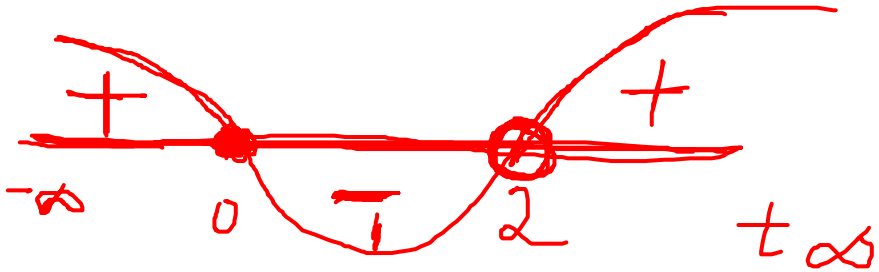
① $\frac{x}{x-2} \geq 0$

$x(x-2) \neq 0$

$x \neq 0; \quad x \neq 2$

$\frac{x}{x-2} = 1$

$x = x - 2$



$D(y): (-\infty; 0] \cup (2; +\infty)$

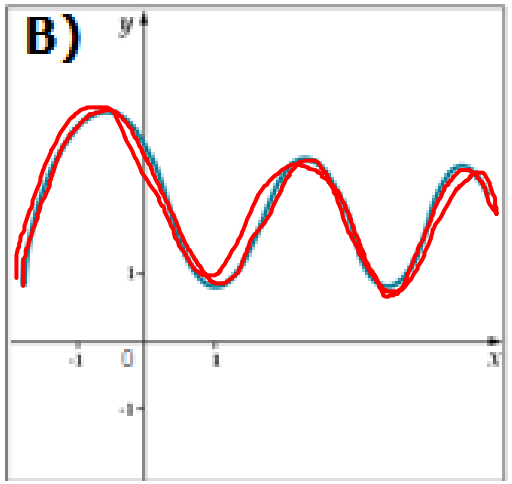
$(1; +\infty)$

Funktsiya berilishi

- 1. Analitik

$y = x^2$; $y = \frac{1}{x}$; $y = \sqrt{\frac{x}{x-2}}$

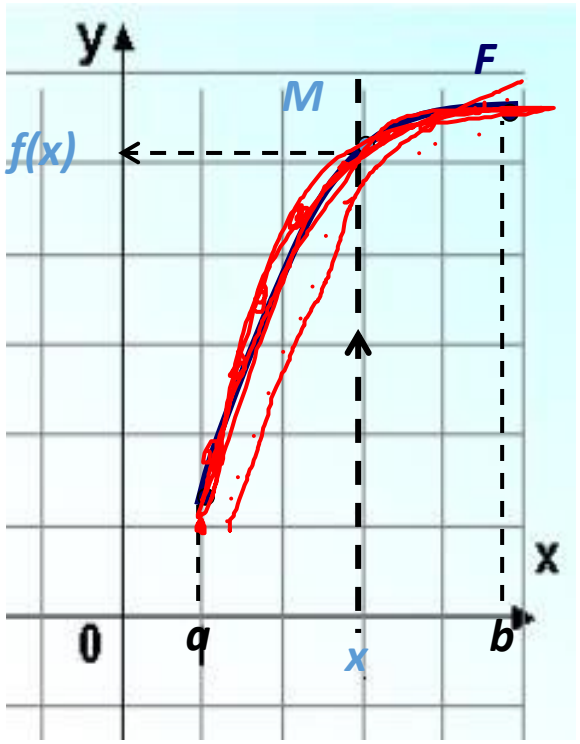
- 2. Grafik



3. Jadval

x	y
10	31
12	36
14	48
16	53
18	65

Funktsiya grafigi



$y = x^3 + 1$

F – funktsiya grafigi

x	1	-1			
y	2	-2	.	.	.

2- ta‘rif. Tekislikning $(x, f(x))$ kabi aniqlangan nuqtalaridan iborat ushbu

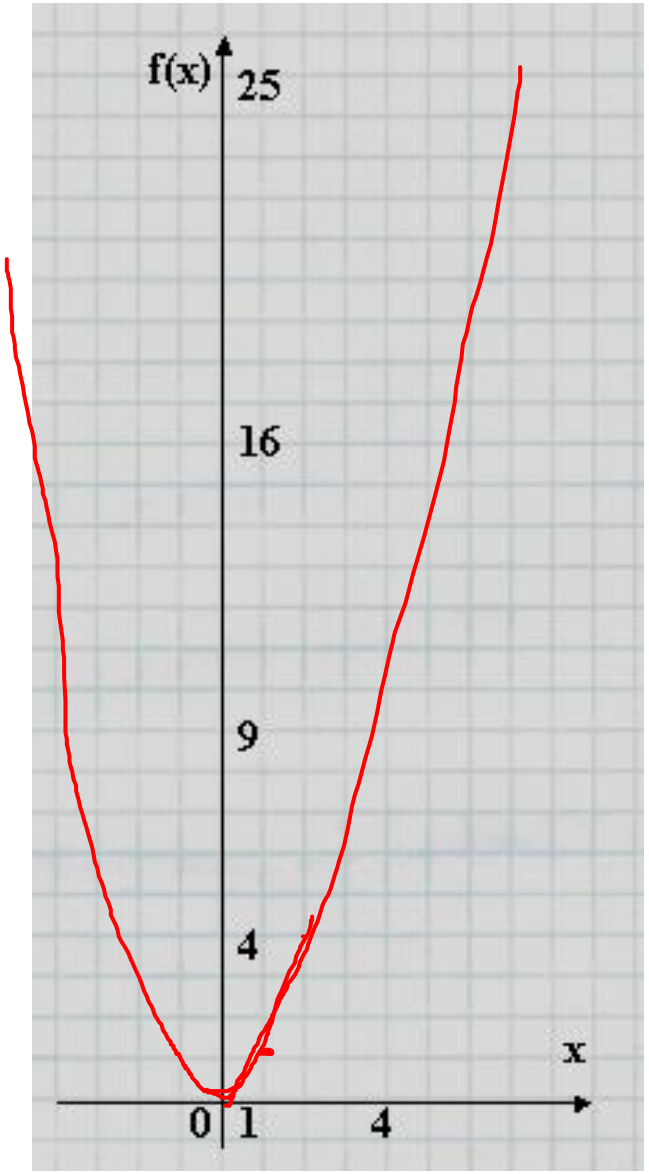
$$\{(x, f(x))\}=\{(x,f(x)):x \in X,\ y=f(x) \in Y\}$$

to‘plam, *funksiyaning grafigi* deb ataladi.

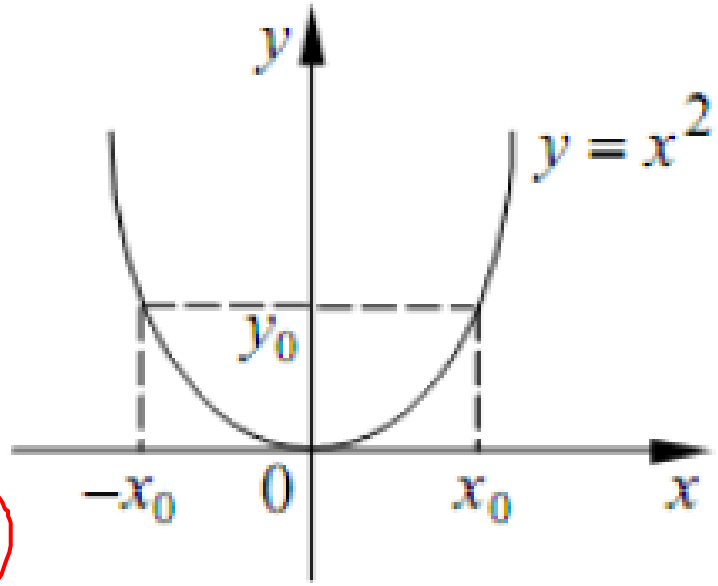
Misol. Funktsiya jadval shaklida berilgan:

x	1	2	3	4	5	6	7	8	9
f(x)	1	4	9	16	25	36	49	64	81

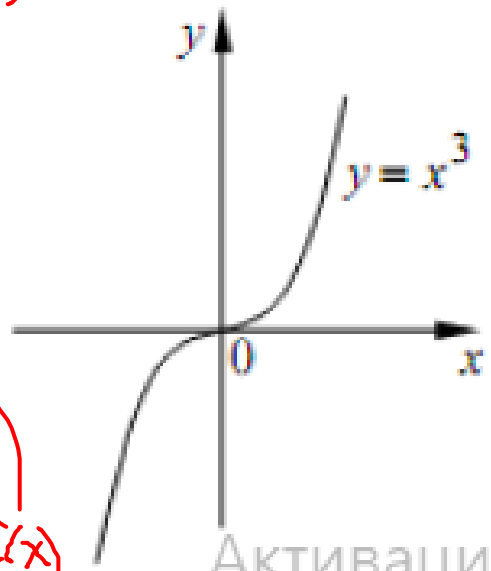
Funktsiya grafigini yasash lozim.



Agar f(x) funktsiya o’zining X aniqlanish sohasida nolga nisbatan simmetrik bo’lib, ixtiyoriy x∈ $X\ uchun\ f(x) = f(-x)$ $tenglik\ bajarilsa\ juft\ deyiladi.$



Agar f(x) funktsiya o’zining X aniqlanish sohasining ixtiyoriy ixtiyoriy x∈ $X\ uchun\ f(-x) = -f(x)$ $tenglik\ bajarilsa\ toq\ deyiladi.$



2. Davriy funksiyalar. $f(x)$ funksiya X $(X \subset R^1)$ to‘plamda aniqlangan bo‘lsin.

4- ta‘rif. Agar shunday o‘zgarmas T $(T \neq 0)$son mavjud bo‘lsaki, istalgan x, $x+T \in X$ lar uchun

$$f(x+T)=f(x)$$

tenglik o‘rinli bo‘lsa, $f(x)$ *davriy funksiya* deyiladi, bunda T — funksiyaning *davri* deb ataladi.

3. Bir qiymatli va ko‘p qiymatli funksiyalar. Agar X to‘plamdagi har bir x songa biror qoida yoki qonunga ko‘ra Y to‘plamdan bitta y son mos qo‘yilsa, u holda y funksiya *bir qiymatli* deyiladi, ya‘ni $\forall x_1, x_2 \in X, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

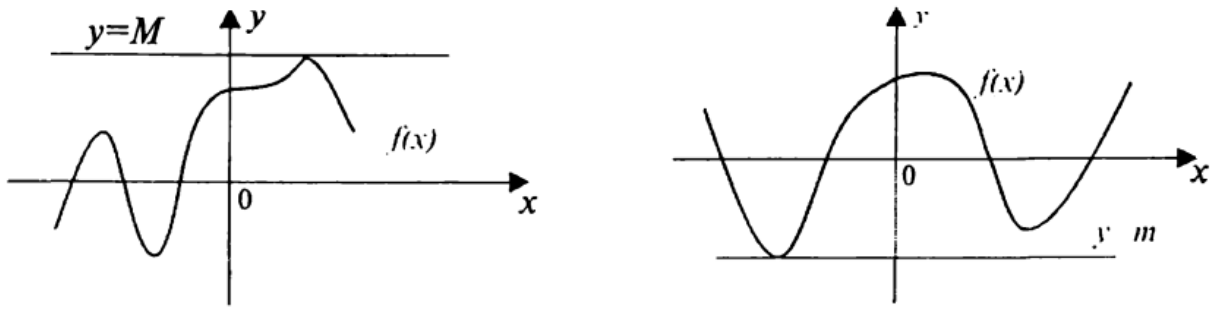
Agar X to‘plamdagi har bir x songa biror qoida yoki qonunga ko‘ra Y to‘plamdan bittadan ortiq yoki cheksiz ko‘p y son mos qo‘yilsa, u holda funksiya *ko‘p qiymatli* deyiladi. Masalan:

1) $y = \pm\sqrt{x}$ — ikki qiymatli funksiya;
2) $y=\operatorname{Arcsin} x$ — ko‘p qiymatli funksiya;
3) $y=3x+2$ — bir qiymatli funksiya.

4. Chegaralangan va chegaralanmagan funksiyalar. $y=f(x)$ funksiya X to‘plamda aniqlangan bo‘lsin.

6- ta‘rif. Agar shunday o‘zgarmas M(o‘zgarmas m) son topilib, istalgan $x\in X$ uchun $f(x)\le M$ $(f(x)\ge m)$ tengsizlik o‘rinli bo‘lsa, $f(x)$ funksiya X to‘plamda *yuqoridan* (*quyidan*) *chegaralangan* deyiladi, aks holda esa funksiya *yuqoridan* (*quyidan*) *chegaralanmagan* deyiladi (3.5- chizma).

7- ta‘rif. Agar $f(x)$ funksiya X to‘plamda ham yuqoridan, ham quyidan chegaralangan bo‘lsa, ya‘ni shunday o‘zgarmas M



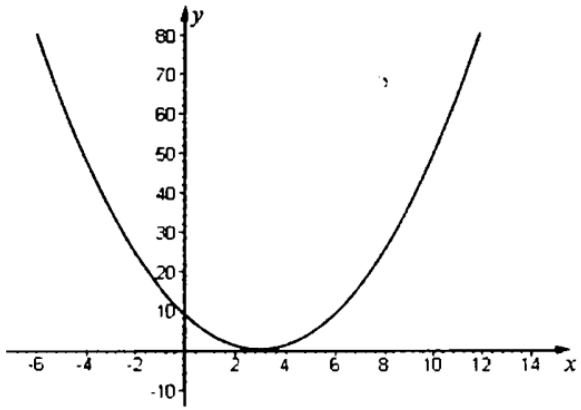
a) yuqoridan chegaralangan funksiya. *b)* quyidan chegaralangan funksiya.

3.5- chizma.

va m sonlar mavjud bo‘lib, istalgan $x\in X$ uchun

$$m\le f(x)\le M \qquad (1)$$

tengsizlik o‘rinli bo‘lsa, u holda $f(x)$ funksiya X to‘plamda *chegaralangan* deyiladi (3.6- chizma).

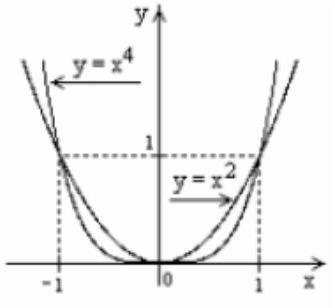


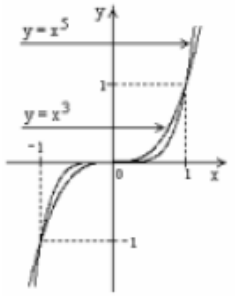
Quyidan chegaralangan funktsiya

Asosiy elementar funktsiyalar

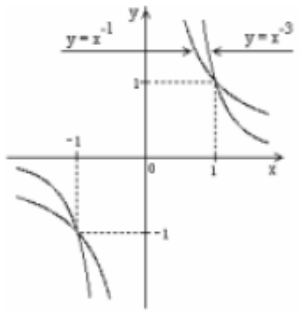
Darajali funktsiyalar

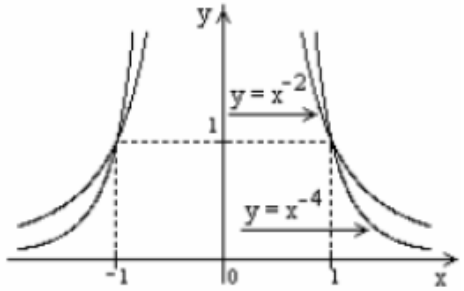
1.1. $y=x^n, n\in N.$





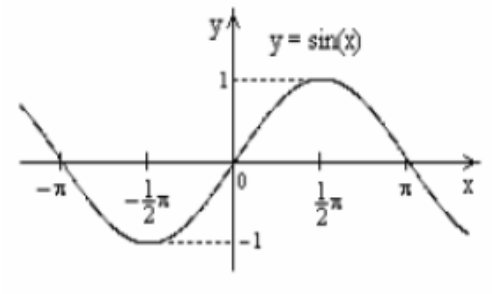
1.2. $y=\frac{1}{x^n}, x\neq 0.$



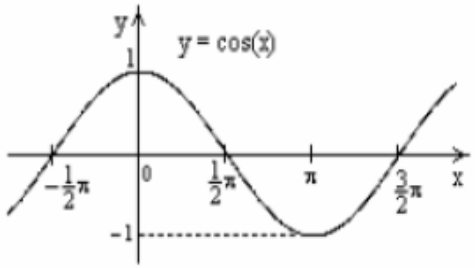


Trigonometrik funktsiyalar

3.1. $y=\sin x$



3.2. $y=\cos x$



Topshiriq

Funktsiya berilgan:

1. D(y)-aniqlanish sohasi;

2.E(y)-qiymatlar sohasi;

3.Funktsiyani juft toqlikka tekshirish;

1-variant

1. $y = 2x^2 - 4x + 5$
2. $y = \frac{1}{4x - 2}$
3. $y = log_{\frac{1}{2}} x$
4. $y = \sqrt{x^2 + 8x + 15}$
5. $y = x^3$

2-variant

1. $y = (x - 5)^2 + 3$
2. $y = \frac{x - 1}{x^2 - 9x + 20}$
3. $y = log_2 x$
4. $y = 2x^2 + 4x - 8$
5. $y = x^3 + 1$