Control of unstable water flow in open channels

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Abstract. The analysis and modeling of hydrodynamic processes in open channels are crucial for effective water resource management and hydraulic structure design. This study investigates unsteady water flow in main channels using the Saint-Venant equations. Analytical and numerical methods were employed to model various flow scenarios. The developed mathematical model effectively describes and analyzes flow characteristics such as discharge, free surface elevation, and flow velocity. The obtained results have practical applications in water resource management, flood forecasting, and infrastructure design for water conservation. Overall, this work contributes to the advancement of knowledge in open channel hydraulics and provides a basis for developing more accurate and reliable water system modeling methods. These equations allow for the simulation of hydrodynamic processes in rivers, channels, and reservoirs, enabling the prediction of water level fluctuations, flow velocities, and pressure distribution under various operating conditions. This plays a vital role in water resource management and the design of hydraulic structures.

1 Introduction

Water resources management and hydraulic structure design are crucial aspects of sustainable development in modern societies. With increasing pressure on water resources due to population growth, urbanization, and climate change, the need for accurate and reliable methods for modeling hydrodynamic processes is becoming increasingly important. The Saint-Venant equations, which describe unsteady water flow in open channels, are one of the most important tools used for this purpose. The Saint-Venant equations include the mass conservation equation and the momentum conservation equation, which express fundamental laws of physics applicable to water flow in rivers, channels, and other hydraulic structures. These equations allow engineers to consider important parameters such as water discharge, channel bed slope, flow cross-sectional area, average flow velocity, and others, making them indispensable for modeling and analyzing water flows.

The Saint-Venant equations remain relevant and important in modern hydraulic engineering design and water resource management. With the increasing frequency of

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extreme weather events, such as heavy rains and floods, models based on the Saint-Venant equations allow for predicting the behavior of water flows and developing effective strategies to prevent and mitigate the consequences of floods. When designing dams, channels, reservoirs, and other hydraulic structures, it is crucial to consider the dynamic behavior of water. The equations help engineers calculate and optimize designs, ensuring their safety and efficiency. These equations are used for modeling and analyzing the impact of hydrodynamic changes on the ecosystems of water bodies and rivers. This is particularly important for developing environmental protection measures and restoring aquatic ecosystems. To ensure safe and efficient navigation on waterways, as well as to manage water supply and irrigation systems, it is necessary to understand and predict the behavior of water flows. The equations provide the necessary tools for these purposes. In the context of climate change, which leads to changes in precipitation patterns and water levels, the importance of accurate hydrodynamic models is increasing. The Saint-Venant equations help adapt to these changes, providing reliable forecasts and the ability to develop adaptive measures. They play a critically important role in modern hydraulic engineering design, water resource management, and adaptation to changing climatic conditions, underscoring their relevance and significance in today's world.

The purpose of this work is to study and simulate unsteady water flow in open channels using the Saint-Venant equations. As part of this goal, the basic equations will be analyzed, key parameters and variables will be identified, mathematical models will be developed, and new solution algorithms will be proposed. Particular attention will be paid to the application of these models to practical problems such as flood management, hydraulic engineering design, and environmental impact assessment.

The scientific novelty of this work lies in the refinement of mathematical models, integration of additional factors, development of new solution algorithms, introduction of multi-scale modeling approaches, and exploration of the applicability of the Saint-Venant equations in new domains. Experimental investigations to validate the proposed models and algorithms will confirm their reliability and applicability under real-world conditions.

Modern information technologies are opening up new possibilities for modeling and managing water resources. These technologies make it possible to consider a multitude of factors and conditions that affect water management facilities, such as rivers, pumping stations, reservoirs, and main channels. GIS systems allow for the collection, analysis, and visualization of spatial data. With their help, it is possible to create detailed maps of water resources, track changes in real time, and model the consequences of various scenarios, such as floods or droughts. Computer models, such as HEC-RAS, SWAT, and MIKE SHE, are used to simulate hydrological processes. These models allow for predicting the movement of water in rivers and channels, as well as assessing the impact of various factors, such as precipitation, evaporation, and water intake. SCADA (Supervisory Control and Data Acquisition) systems are used for monitoring and managing water resource infrastructure. They allow for the automation of control processes for pumping stations and reservoirs, optimizing their operation and reducing the risk of emergencies.

Modern data analysis and machine learning methods allow us to predict water consumption, identify leaks, and detect inefficient resource use. These technologies can process vast amounts of data from various sensors and devices, providing more accurate forecasts and recommendations. A network of devices equipped with sensors and connected to the internet enables real-time data collection on the state of water bodies. IoT devices can measure water level, quality, flow rate, and other parameters, providing timely information for decision-making. Cloud platforms offer powerful tools for storing, processing, and analyzing big data. They allow for the integration of data from various sources and provide access to this data for all stakeholders, including government agencies, researchers, and local communities. Decision Support Systems (DSS) integrate various models and data to help managers make informed decisions regarding water resource management. These systems can suggest optimal strategies for water allocation, emergency response, and long-term resource management planning. Flooding is one of the most serious and widespread problems in many countries. Its occurrence is caused by various factors, including intense precipitation, blockages in drainage systems, obstacles in riverbeds, and dam failures. This issue has been widely addressed in scientific literature, with numerous studies dedicated to understanding the causes and consequences of floods, as well as developing methods and strategies for their prevention and management. The works of Chan et al [1,2] and Han et al [3] provide a detailed examination of various aspects of the flood problem, including geographical distribution, causes, and development mechanisms. The authors identify the influence of various factors, such as climate change and hydrological processes, on the scale and frequency of floods, and also consider the effectiveness of different risk management methods and strategies.

Furthermore, the research by Natasha et al. [4] highlights the role of channel overcapacity in causing flooding. They draw attention to the need for a more detailed analysis of the hydraulic characteristics of watercourses and rivers to effectively predict and prevent catastrophic consequences. Dubey et al. [5] highlight the importance of understanding the variability in channel physical and hydraulic parameters for developing effective flood management strategies. They offer methods and modeling to analyze and predict the behavior of water systems and develop appropriate engineering solutions. In the field of flood management, it is common to use hydrological systems models to analyze the dynamics of water level, discharge, and flow velocity. Numerous researchers, including Gharbi and colleagues [6], Kane et al. [7], Retzinis et al. [8], Dasallas et al. [9], Beyaztas and his group [10], and also Kay and his colleagues [11], devoted their work to studying this problem.

One approach to solving the problem of establishing flow velocity and water depth at each model site is using one-dimensional (1D) hydraulic models, as described in detail in [12]. However, such models mainly apply to regular and uniform channel sections such as rectangles, half-pipes or trapezoids, which limits their ability to account for the complexity of irregular shapes of natural river channels [12-13]. Thus, the main impetus for this study is the need to develop a mathematical model that can reliably simulate water flow in rivers with irregular channel patterns.

There are numerous mathematical models for studying phenomena in fluids, such as Boussinesq Type Equations (BTE), potential theory, and the Navier–Stokes equations. The initial set of extended Boussinesq-type equations, often referred to as the Standard Boussinesq Equations, was derived by Peregrine [14]. These equations were developed based on assumptions of weak nonlinearity and frequency dispersion, primarily applicable to relatively shallow water flows due to the assumption of weak dispersion. Subsequent efforts to extend the validity and applicability of these Standard Boussinesq Equations have significantly improved their features and applicability, as seen in the works of Madsen and Sorensen [15] and Nwogu [16]. These equations are widely utilized by researchers to study phenomena in fluids, as demonstrated by the works of Kazolea and Delis [17], Forbes and his colleagues [18], Jin and co-authors [19], and Magdalena and her co-authors [20].

However, working with Boussinesq-type equations presents a challenge in handling higher-order terms. Potential theory is not suitable for the numerical investigation of these phenomena, mainly because it involves a large number of equations. On the other hand, Navier-Stokes equations, as employed in the works of Darrigol [21], Wilcox [22], Menter [23], Durbin [24], and Shang [25], offer a comprehensive model but come with a high computational cost, resulting in slow computations.

In this study, we propose a model based on the Saint-Venant equations. These equations provide relatively simple solutions both analytically and numerically, which allows for faster results with lower computational cost. The works of Magdalena and her colleagues [26-28] extensively describe the methods for analyzing and numerically solving the Saint-Venant equations, as well as provide examples of their practical applications. Additionally, working with the Saint-Venant equations gives us flexibility in altering channel configurations, cross-sections, and surface characteristics. This flexibility allows the model to adapt to various river and watercourse conditions, which is critically important in the context of river management and water resource management tasks.

Through modeling, it is possible to assess the impact of dam construction or changes in river beds on ecosystems and water supply. Modern sensors can monitor pollution and other changes in water quality, allowing for prompt responses to issues. Analyzing data on water consumption in agriculture helps develop more efficient irrigation schemes, reducing water loss and increasing crop yield. The implementation of modern information technologies in water resource management significantly enhances the efficiency and reliability of these systems, ensuring sustainable development and safeguarding water resources for future generations. This work aims to deepen knowledge in the field of open channel hydrodynamics and provide new tools for solving current water resource management challenges and designing hydraulic structures, which is critically important for sustainable development and adaptation to changing climate conditions [29-32].

Methods

This work employs various methods to study and model unsteady water flow in open channels using the Saint-Venant equations. The Saint-Venant equations describe unsteady water flow in open channels and are an important tool for modeling hydrodynamic processes.

The mass conservation equation:

$$\frac{\partial Q}{\partial x}+\frac{\partial \omega}{\partial t}=0. \tag{1}$$

This equation expresses the law of conservation of mass, where Q - is the flow rate of water, and ω is the cross-sectional area of the flow.

Conservation of momentum equation:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{\omega}+g\omega h\right)=-g\omega\left(i_f-i\right)-\frac{Q}{K\left(x,z\right)}. \tag{2}$$

where i_f — is the slope of the free surface flow and i — is the channel bed slope.

Flow depth:

$$h\left(x,t\right)=z\left(x,t\right)-z_0\left(x\right).$$

where $z_0\left(x\right)$ — ordinate of the channel bottom.

Flow cross-sectional area:

$$\omega(x,h)=\int_0^h B(x,z)dz.$$

Average current speed:

$$v=\frac{Q}{\omega}.$$

Speed of propagation of small waves:

$$c=\sqrt{\frac{d(Q)}{d(\omega)}}.$$

Hydraulic radius of the channel:

$$R = \frac{\omega}{X}.$$

where X- Wetted perimeter of the channel, C - Chezy coefficient.

Flow module:

$$K(x,z) = CR^{1/2} = \frac{\omega}{X}.$$

where $Q(x,t)$ - Water flow (m³/s), depending on the longitudinal coordinate x and t time, $z(x,t)$ - The ordinate of the free water surface (m), depending on x and t, g - Gravity acceleration (gravitational constant), usually taken as 9.81 m/s², i - The slope of the channel bottom is defined as $i = -\frac{dz_0}{dx}$, $z_0(x)$ — the ordinate of the channel bottom, $B(z)$ - Flow width along the surface of the living section (m).

Flow $Q(x,t)$ - characterizes the volume of water passing through the cross-section of the channel per unit of time.

The ordinate of the free surface $z(x,t)$ shows the height of the water level in the channel.

The bottom slope i determines the slope of the channel and affects the gravitational acceleration of the flow.

The flow width $B(z)$ and cross-sectional area $\omega(z)$ depend on the channel geometry and play an important role in determining the hydraulic characteristics.

These equations are used to model the behavior of water flows in rivers, channels, reservoirs and other hydraulic structures. They predict changes in water level, flow velocity and pressure distribution under different operating conditions, which is critical for water resource management and hydraulic structure design.

The Chezy coefficient is determined by the Pavlovsky formula:

$$C = \frac{1}{n} R^{1/6},$$

where n — -channel roughness coefficient.

The characteristic form of equations (2) has the form:

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = B,$$

where

$$U = \begin{pmatrix} Q \\ z \end{pmatrix}, \qquad A = \begin{pmatrix} \frac{\partial Q}{\partial \omega} & \frac{\partial Q}{\partial z} \\ \frac{\partial z}{\partial \omega} & \frac{\partial z}{\partial z} \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ -gw\left(i_f - i\right) - \frac{Q}{K(x,z)} \end{pmatrix}.$$

To solve the equations, it is necessary to set the initial and boundary conditions:

Initial conditions:

$$Q(x,0) = Q_0(x), \quad z(x,0) = z_0(x).$$

Boundary conditions at points $x_1 = 0$ and $x_2 = l$:

$$Q(0,t) = Q_1(t), \quad Q(l,t) = Q_2(t)$$

A water supply plan is set $q(x,t)$ for a period of time $[0,T]$.

So, having the initial and boundary conditions, as well as expressions for all parameters, we can begin to solve the system of equations (1)-(2) taking into account all additional conditions.

Solution algorithm:

Setting the initial conditions: $Q_0(x)$ and $z_0(x)$.

Setting boundary conditions: $Q_1(t)$ and $Q_2(t)$.

Dividing the area into a grid: $[0,l]$ by x and $[0,T]$ by t into small intervals.

These methods, when combined, enable a comprehensive study of unsteady water flow in open channels, ensuring high accuracy, reliability, and applicability of the developed models and algorithms in real-world conditions.

Results

Let's consider a numerical example of solving equation (1)-(2) using the characteristics method:

Step 1: Setting initial and boundary conditions

$$Q_0(x) = \sin\left(\frac{\pi x}{L}\right), \qquad Q_1(t) = \cos\left(\frac{\pi t}{T}\right).$$

Step 2: Dividing the area into a grid

We use a grid with steps Δx and Δt.

Step 3: Solution by the method of characteristics

For each time step t_n:

$$Q(x_i, t_{n+1}) = Q(x_i - v\Delta t, t_n),$$

where x_i — coordinate of the grid node.

We examined a mathematical model of unsteady water movement in main channels, including basic equations, initial and boundary conditions, as well as methods for their analytical and numerical solution.

Integration: At each time step we solve a system of equations for new values $Q(x,t)$ and $z(x,t)$.

Having solved equations (1)-(2) with given initial and boundary conditions, it is possible to determine the functions $Q(x,t)$ and $z(x,t)$ that describe the water flow and the ordinate of the free surface at any time and any point in the channel. This allows you to simulate the behavior of water flow and optimize the management of water facilities.

A Python program has been developed that solves the equation using the method of characteristics.

The graph that is created by the program is a contour graph (contour map) of the distribution of water flow $Q(x,t)$ along the length of the channel and in time (Fig. 1-Fig. 2).

The x axis (horizontal axis) represents the distance along the channel, measured from the starting point $x=0$ to the end of the channel $x=L$. The t axis (vertical axis) represents the time starting from $t=0$ to the ending time $t=T$. The color scale indicates the water consumption values $Q(x,t)$ at each grid point. Typically color gradients are used, where different colors correspond to different Q values. For example, darker colors may indicate lower flow rates, while lighter colors may indicate higher flow rates.

At time $t=0$, the water flow rate $Q(x,0)$ along the channel is determined by the initial function $Q_0(x)$. On the graph, this is represented by the distribution of colors along the x axis at $t=0$. At the beginning of the channel ($x=0$), the water flow changes over time according to the $Q_1(t)$ function. On the graph, this is reflected by the change in colors along the t axis at $x=0$.

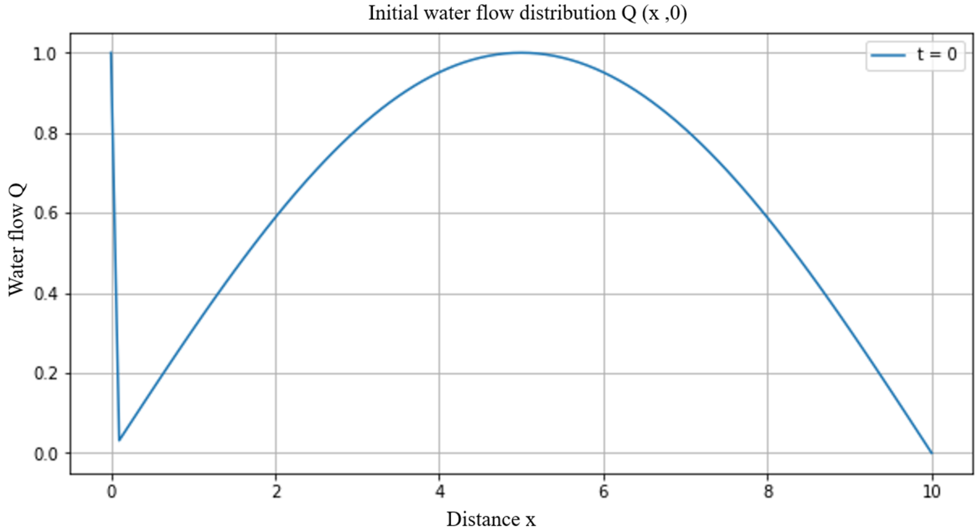

Fig. 1. Initial water flow distribution Q (x, 0).

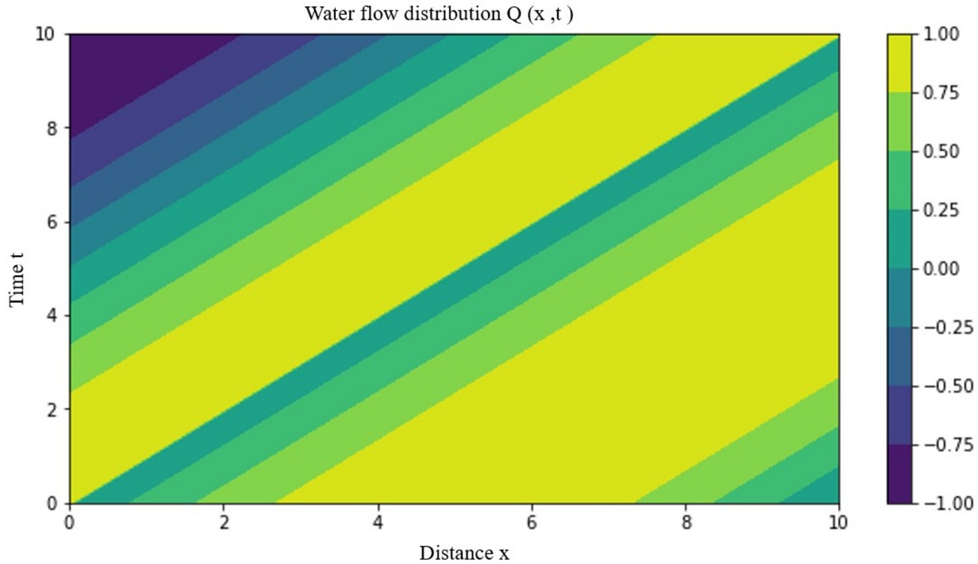

Fig. 2. Water flow distribution Q (x, t).

Equation (1)-(2) takes into account the delay effect, that is, the change in water flow at the initial section propagates along the channel at a certain speed v. This means that changes made at the beginning of the channel will be visible at other points in the channel after a certain time. This is represented on the graph by diagonal color-changing lines, which show how waves of flow changes propagate along the channel.

Let us assume that the initial condition $Q_0(x)$ specifies a sinusoidal distribution of water flow along the channel, and the boundary condition $Q_1(t)$ specifies a cosine change in the flow rate at the beginning of the channel. In this case, when $t = 0$, the sinusoidal distribution will be visible along the x axis.

As time increases, changes in the flow rate at the beginning of the channel ($x = 0$) will propagate along the channel and appear on the graph as diagonal stripes that move to the right at a speed of v.

The contour graph offers a clear visualization of how water flow changes over space and time. It is an essential tool for analyzing the behavior of water flows in the channel, enabling the identification of time and place where critical changes in water flow may occur. In practical applications, such graphs assist engineers and researchers in making informed decisions about water resource management.

The graph of the solution to equation (1)-(2) shows how the initial and boundary conditions, as well as the lag effect, affect the distribution of water flow along the main channel over time. This visual tool is useful for understanding the dynamics of water flows and optimizing the management of water systems. When you start the program, a graph will be displayed showing the distribution of water consumption $Q(x,t)$ at various time values t from 0 to 1.

Each curve on the graph will correspond to a specific point in time, allowing you to see changes in water flow along the channel over time.

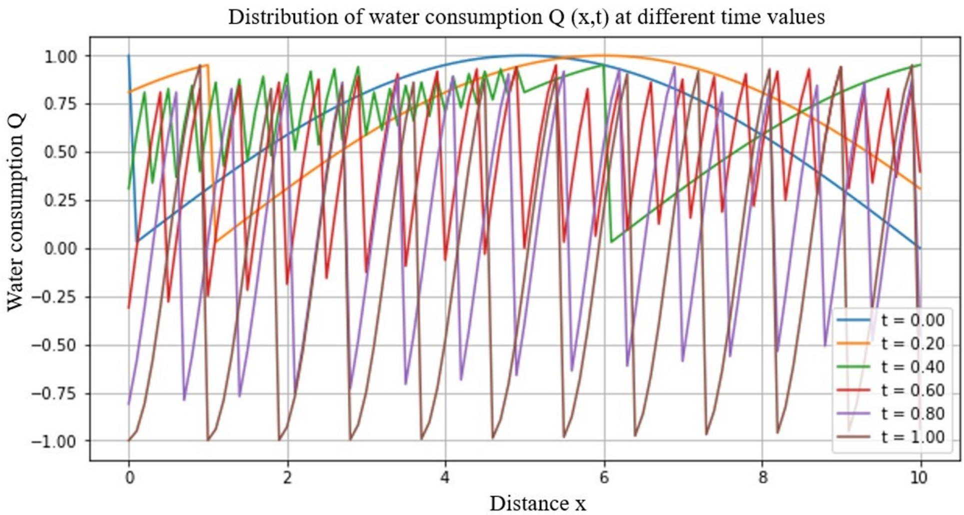

Fig. 3. Distribution of water consumption Q (x, t) at different time values.

Each curve on the graph will correspond to a specific point in time, allowing you to see changes in water flow along the channel over time.

Conclusion

In this work, unsteady water flows in open channels were investigated and modeled using the Saint-Venant equations. The developed mathematical model, which includes the fundamental equations of mass and momentum conservation, allows for the effective description and analysis of hydrodynamic processes in various types of channels. Throughout the study, various methods of analysis and solution were applied, including analytical methods, numerical methods, and modeling techniques. These methods enabled both theoretical research and practical modeling of different water flow scenarios, including the impact of various parameters on flow characteristics. The main results of the work include the development of new mathematical models, the improvement of numerical solution methods, and the analysis of the influence of different factors on the behavior of water flows. The obtained results can be used for more effective water resource management, the design of hydraulic structures, and the prediction of water system behavior under various conditions. Overall, this work makes an important contribution to the field of open channel hydrodynamics and provides a foundation for further research in this area. The developed models and methods can be applied to various engineering and environmental tasks related to the use and management of water resources.

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