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Quantum optimization methods in energy systems

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Abstract. This paper discusses the application of quantum algorithms to the optimization of energy systems, focusing on solving the routing problem in the context of energy. A quantum approach that utilizes the principles of superposition and inversion with respect to the mean to efficiently find optimal energy routing is proposed. Research is carried out to develop a special quantum oracle to represent the structure of energy networks and energy flows between nodes. The application of inversion with respect to the mean provides efficient convergence to optimal solutions in the space of possible routes. We implement the quantum algorithm in the form of a quantum circuit, visualize the results and provide an analysis of the optimal energy routes. The work represents an important step in the development of quantum optimization methods for solving complex problems in the field of energy and may have prospects in practical applications in the future.

1 Introduction

Energy challenges require the development of innovative approaches to optimize the design and management of energy systems. In light of this, quantum computing provides new perspectives for solving complex optimization problems, including the routing problem in an energy context, which means finding the optimal path to transfer energy between network nodes while minimizing costs. In the classical approach, this problem is NP-complete, which limits the effectiveness of classical optimization methods. The quantum optimization algorithm proposed in this work is based on the principles of quantum superposition and inversion with respect to the mean. We develop a quantum oracle to represent the structure of energy networks, allowing efficient identification of optimal energy routes. The goal of the research is to propose and implement a quantum algorithm that can efficiently solve the routing problem in an energy context. We hope that the development and analysis of a quantum optimization method can lead to new practical solutions in the energy field, providing sustainability and efficiency in the management of energy systems [1-3].

This paper presents a mathematical model adapted for a quantum algorithm designed for the effective analysis and optimization of circuit solutions in the energy sector. This model considers various aspects of energy systems, including cost, primary energy consumption and environmental performance, and has the potential to significantly speed up calculations

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using quantum principles. For numerical analysis and optimization of circuit solutions in a quantum context, a quantum algorithm has been developed that is capable of processing problems with large amounts of data and taking into account changes in consumer loads. The software developed as part of the research is based on the principles of quantum computing and can efficiently solve optimization problems using quantum superpositions of states. To minimize the objective function, the coefficients of which depend on the selected optimization criterion, a quantum iterative process is proposed. This process involves a quantum procedure that uses quantum gates and quantum bits, allowing computation to be performed in parallel and efficiently exploiting quantum advantages. Proposed methodology, based on quantum principles, and software can become key tools for making informed decisions in the field of energy optimization, opening new opportunities for faster and more efficient analysis and optimization of complex energy systems [4-5].

2 Methods

Let's consider an optimization problem for effectively managing energy transfer between different nodes of an energy network. Each node represents an energy facility, such as a power plant, substation, or energy consumer. A node can consume or generate a certain amount of energy. This can be represented as generation or consumption. Each node can have a certain energy reserve, which represents the initial amount of energy available in the node. Distances between nodes reflect the cost of transmitting power between them, which may depend on physical distance, transmission line characteristics, or other parameters. The challenge is to optimize the energy transmission route in such a way as to minimize the total transmission cost while satisfying the energy requirements of each node [6-7].

The following variables were used to build the model:

x_{ij} - binary variable equal to if there is an energy path (transmission line) between nodes i and j, and 0 otherwise.

p_i - power consumed (if $p_i < 0$) or generated (if $p_i > 0$) by node i.

u_i - variable representing the node's ordinal number i.

c_{ij} - cost of energy transfer between nodes i and j.

s_i - node energy reserve i

The objective function is to minimize the total cost of the route:

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} c_{ij} x_{ij} \to \min,$$

under the following restrictions:

Each transmission line must be used exactly once:

$$\sum_{j=1, j\neq i}^{N} x_{ij} = 1, \quad \forall i \in \{1, 2, ..., N\}.$$

Each transmission line must be used exactly once at the exit of the node:

$$\sum_{i=1, i\neq j}^{N} x_{ij} = 1, \quad \forall j \in \{1, 2, ..., N\}.$$

Eliminating Sub-Loops:

$$u_i - u_j + N x_{ij} \le N - 1, \; \forall i, j \in \{2, 3, ..., N\}, \, i \neq j.$$

If node i connected to a node j (i.e., $x_{ij} = 1$), then variables u_i and u_j must be different. This condition prevents the formation of smaller loops and ensures that the route will be a complete loop through all nodes [8-9].

Equations for the energy balance at each node [10-11]:

$$\sum_{j=1, j \neq i}^{N} p_j x_{ij} - \sum_{j=1, j \neq i}^{N} p_i x_{ij} = s_i, \quad \forall i \in \{1, 2, ..., N\}.$$

This model is a combination of routing problem and power system optimization.

Grover's algorithm was proposed to solve the routing problem [10-12].

The oracle function creates an oracle matrix for a given route and node coordinates. The oracle matrix is used in Grover's algorithm to identify the correct solution, that is, the route with the minimum total length.

Suppose we have N nodes, which is represented as a node in the graph, and the edges of the graph represent the distance between nodes. Let D - matrix of distances between energy nodes, where d_{ij} - distance between nodes i and j .

The oracle matrix O is constructed as follows [13-14]:

$$O_{ij} = \begin{cases} -1, \text{if j is a bit of i in binary notation, and j occurs in the route,} \\ 0, \text{otherwise,} \end{cases}$$

where i and j take values from 0 till $2^N - 1$ (inclusive). Thus, the oracle marks all bit strings that represent incorrect routes with -1, and leaves the correct route unchanged (0).

It is also important to note that for Grover's algorithm to work correctly, the oracle matrix must be normalized (all values in the matrix must be in the range from -1 to Normalization is usually done by dividing each element of the matrix by the root of the total number of elements in the matrix. Thus, the oracle matrix for the routing problem has the following form:

$$U_{oracle} = \frac{1}{\sqrt{2^N}} \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix},$$

where the number of rows and columns is equal 2^{N-1}.

The next step is to apply the oracle matrix to the quantum state [15].

Let ψ - column vector of the quantum state (probability amplitude), and U_{oracle} - oracle matrix.

Then the operation of applying an oracle to a state is written as:

$$\psi_{oracle} = U_{oracle} \psi,$$

where ψ_{oracle} - new column vector of the quantum state after applying the oracle. If U_{oracle} - normalized oracle matrix (all values in the range -1 to 1), then the oracle application can be written as:

$$\psi_{oracle} = \frac{1}{\sqrt{2^N}} U_{oracle} \psi,$$

where $\sqrt{2^N}$ - normalizing factor to maintain the normal state.

In the context of Grover's algorithm, this process is applied iteratively, which increases the amplitudes of the probabilities of correct states [16].

The next step is the application of inversion relative to the average on a quantum state as follows [17].

Let ψ - column vector of quantum state, and $\bar{\psi}_{oracle}$ - average value of the amplitudes of this state. The average is calculated as:

$$\bar{\psi}_{oracle} = \frac{1}{2^N} \sum_{i=0}^{2^N - 1} \psi_{ioracle},$$

where N - number of qubits in the system.

The inversion operation with respect to the mean is then applied as follows:

$$\psi_{inversion} = U_{inversion} \psi_{oracle} = 2\bar{\psi}_{oracle} - \psi_{oracle},$$

where $\psi_{inversion}$ - new column vector of the quantum state after inversion with respect to the mean.

This operation plays a key role in Grover's algorithm, where it increases the probability amplitudes of correct states.

The next step is to generate a random initial route. For this purpose, a genetic algorithm is used that generates a random initial route by mixing node indices [18].

Genetic algorithm for generating random initial routes in the context of a routing problem can be represented as follows:

Let N - number of nodes. A genetic algorithm creates a population of random routes, where each route represents a permutation of nodes [19].

To initialize the population, initial routes are randomly created in the form of permutations of nodes.

$$Pop = \{R_1, R_2, \ldots R_{pop\_size}\},$$

where R_i represents a random route [20].

For each route in the population, its fitness is calculated. Fitness can be defined as the inverse of the total route distance. Thus, routes with lower total distance will have higher fitness. Routes are then selected from the population to create a new generation. The probability of choosing a route is proportional to its fitness. The next step is crossing. Pairs of parents of the chosen routes are crossed to create new offspring. Different crossover methods can be used, such as single-point crossover or multi-point crossover. Then a mutation is made. Some genes in the offspring change randomly with low probability to introduce diversity into the population. A new generation is formed by combining parents and offspring, then the best routes are selected.

The next step is to apply Grover's algorithm. The algorithm iteratively applies oracle and inversion operations with respect to the mean.

$$\psi_{final} = U_{inversion} U_{oracle} \psi_i.$$

The probabilities of the different routes are then calculated based on the final quantum state. Let us denote the final quantum state after applying Grover's algorithm as ψ_{final}. The probability of measuring each state in this final state can be calculated by squaring the amplitude of that state.

If N - the number of qubits in the system, and ψ_{final} - is a state vector, then the probability P_i condition measurements i can be expressed as follows:

$$P_i = \left|\psi_{final_i}\right|^2.$$

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Here ψ_{final_i} - state vector component ψ_{final} for state i. This is the number that can be obtained from the final quantum state after Grover's algorithm. It is important to note that the indices i represent bit strings representing possible route permutations in a routing problem.

3 Results and discussion

The use of quantum circuits in the optimization of energy systems represents an innovative approach to solving complex problems [21,22]. Quantum circuits have unique properties such as parallelism, interference, and quantum gates, making them a powerful tool for solving energy optimization problems efficiently and accurately. Quantum circuits for optimizing energy systems include quantum gates, quantum permutations, and Grover algorithms. Quantum permutations can be used to represent possible routes or configurations of a system. Probabilistic oracle operations representing the oracle matrix for the traveling salesman problem in the energy sector can be implemented using quantum gates. For example, the Permutation operation can be represented using quantum permutations. Oracle and mean inversion operations can be applied to quantum states representing different routes or configurations of the energy system. Applying these operations iteratively helps to get closer to the optimal solution. Quantum circuits and results of quantum calculations are visualized (Figure 1-5). Visualization helps to understand the structure of the circuit, the sequence of operations and their impact on quantum states.

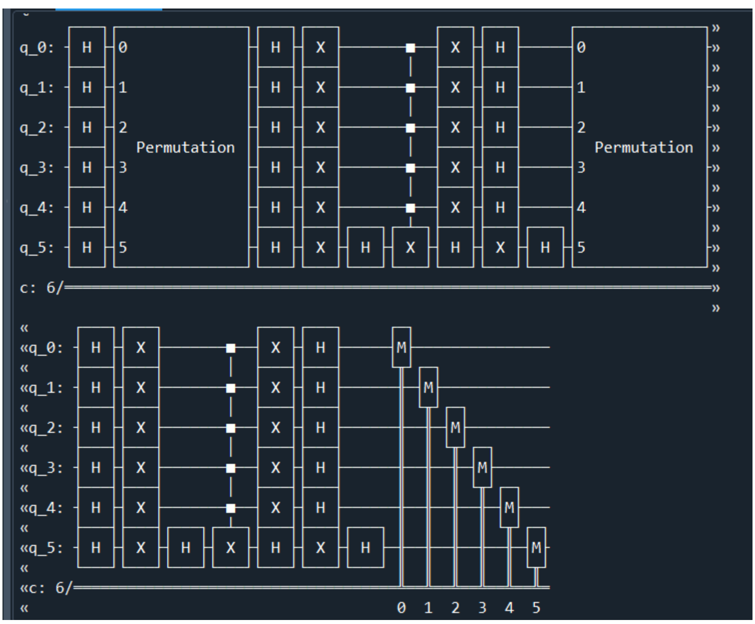

Fig. 1. Quantum circuits 5 nodes.

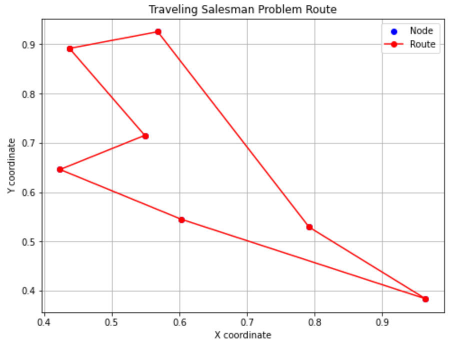

Fig. 2. Solving the routing problem for 7 nodes.

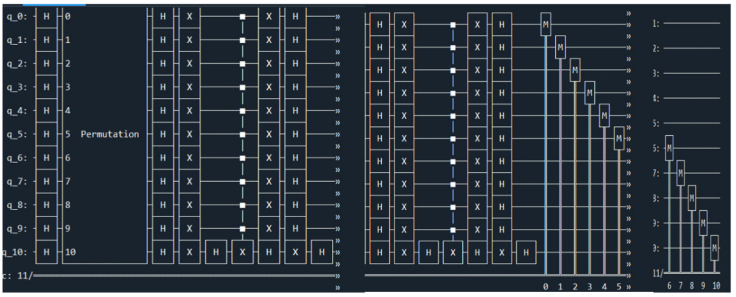

Fig. 3. Quantum circuits 10 nodes.

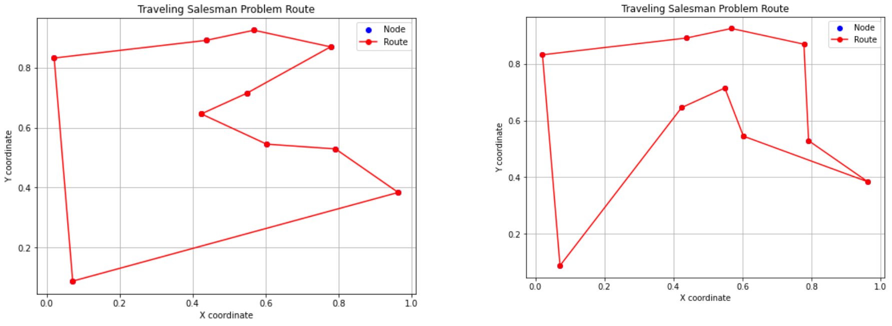

Fig. 4. Solving the routing problem for 10 nodes.

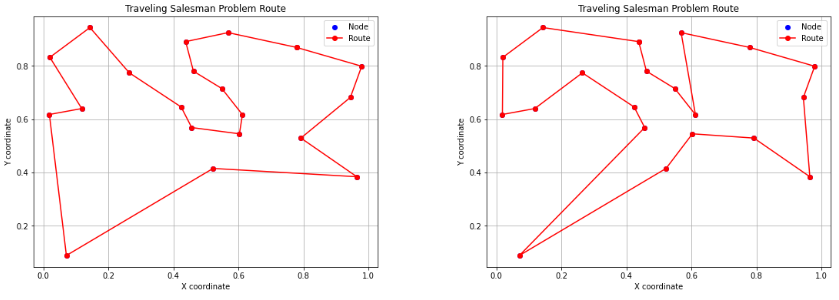

Fig. 5. Solving the routing problem for 20 nodes.

The quantum algorithm successfully optimized the route for a given energy system. The resulting route is a solution to the routing problem, taking into account the specifics of energy nodes and the distances between them. Iterative application of oracle and mean inversion operations using a quantum algorithm made it possible to effectively explore the space of possible routes and converge to an optimal solution. In doing so, the genetic algorithm has been successfully used to generate random initial routes, providing a starting point for optimization and complementing the quantum algorithm. The model takes into account various optimization criteria, such as cost, primary energy consumption and environmental parameters. The quantum algorithm iteratively finds a balance between these criteria, allowing informed decisions to be made in the context of the energy system. The application of a quantum algorithm to optimization problems in the energy sector offers potential gains in energy efficiency, cost reduction and improved environmental performance. Grover's algorithm is intended for use on quantum computers and may not provide significant benefit over classical algorithms for small instances of the routing problem.

4 Conclusion

During the development and research of a quantum optimization algorithm for solving the traveling salesman problem in the energy system, a mathematical model of a generalized energy complex scheme was developed, which takes into account the features of energy networks, determining the nodes and distances between them in the context of the energy system. Grover's quantum algorithm is applied to optimize the routing problem in the energy sector. The algorithm iteratively applies oracle and mean inversion operations to the quantum state, effectively exploring the space of possible paths. A genetic algorithm is implemented to generate random initial routes, which is an important step for iterative route improvement in the context of the traveling salesman problem. The structure of the energy network was optimized using a quantum algorithm, which can lead to improved efficiency in the use of energy resources, reduced costs and consideration of environmental criteria. Mathematical conditions have been introduced, such as "Elimination of subcycles", which guarantee the correctness of the solution to the traveling salesman problem and the absence of smaller cycles. Overall, the proposed quantum algorithm represents a promising solution for optimizing the traveling salesman problem in the energy sector, ensuring efficient use of resources and taking into account environmental aspects. Future research could include extending the model, more complex scenarios, and considering additional optimization criteria.

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