Model for forest ecosystems based on quantum optimization

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Abstract. This paper presents a mathematical model developed to describe the dynamics of forest ecosystems. The model is based on the principles of cross-diffusion, consider the interaction between two plant species in a forest environment. The model considers various parameters, like diffusion, growth and interaction coefficients and environmental capacities between species. Factors of influence of external conditions on each species of plants are also introduced. The differential equations are solved numerically using the finite difference method. This paper studies cross-diffusion dynamics by combining classical differential equations and quantum-inspired optimization techniques. The focus is on cross-diffusion processes, where populations interact through complex mechanisms of diffusion and reaction. The study uses a hybrid approach that combines classical methods for solving differential equations with quantum optimization, a quantum computing platform. Visualization of the results is presented in the form of 3D graphs reflecting the spatial distribution of plant populations in a forest ecosystem at various time steps. The resulting mathematical model and its visualization provide a tool for a deeper understanding of the influence of various factors on the dynamics of forest ecosystems. Analysis of such a model could be useful for predicting long-term changes in forests and developing sustainable forest management strategies.

1 Introduction

Forest conservation is a set of measures aimed at sustainable management of forests in order to preserve their biodiversity, environmental functions, and ensure the sus-tainable use of forest resources. This is an important part of environmental manage-ment and sustainable development, as forests play a key role in maintaining the health of the planet. Protecting the diversity of plants, animals and microorganisms in the forest ecosystem includes the protection of rare and endangered species, the development and implementation of sustainable methods of logging, exploitation of timber and other forest resources, taking into account the needs of current and future generations, the prevention of soil erosion, pollution of water sources and the preservation of quality soils in the forest

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zone, protection from fires and pests, forest restoration and rehabilitation, conducting observations, studying changes in forest ecosystems, assessing the effectiveness of forest conservation measures. Forest conservation requires the joint efforts of governments, organizations, scientific institutions and the public. Effective forest management today ensures sustainable use of resources and conservation of forests for future generations [1-3].

Modern forest ecosystems are affected by a variety of factors, including climate change, anthropogenic impacts and other variable external influences. Understanding the long-term consequences of these changes on the structure and sustainability of forest communities is an important goal in forest ecology and management. Mathe-matical models play a key role in analyzing and predicting the dynamics of forest ecosystems. In this article we will consider the development of a mathematical model of cross-diffusion for forest ecosystems. The cross-diffusion model allows us to de-scribe the interaction of different species in conditions of spatial heterogeneity, taking into account the influence of factors such as climatic conditions, soil properties and water availability. We will also look at various coefficients in the model, including diffusion, growth and species interaction coefficients, and their interaction with exter-nal factors. Application of the model to real forest ecosystems, including tugay forests of Uzbekistan, will allow us to assess its applicability and importance in analyzing forest dynamics in a changing environment. This work represents an important step in understanding the processes shaping forest ecosystems and can serve as a basis for the development of sustainable forest management strategies in the face of modern environmental challenges [4-7].

Modern forest ecosystems face a number of serious challenges, including climate change, anthropogenic impacts, deforestation and other factors that can significantly affect their sustainability and biodiversity. In this regard, there is a need to develop and apply new methods of analysis and management of forest resources. Mathematical models, especially cross-diffusion models, provide powerful tools for studying complex interactions between different species in forest ecosystems. They make it possible to take into account spatial heterogeneity, which is especially important when analyzing forest dynamics in a changing environment. The relevance of this work is that it can provide new insights into the impact of various factors on forest ecosystems, as well as help in the development of more effective forest management strategies. Application of the model to real forest ecosystems, including tugay forests of Uzbekistan, will allow it to be adapted to the specific conditions of the region and make more accurate predictions of the impact of changes on forest communities. The article introduces the concept of cross-diffusion dynamics and identifies problems in optimizing system parameters. The motivation for using quantum optimization and classical differential equation solving methods to solve these problems is discussed. Thus, the development and analysis of a mathematical model of cross-diffusion for forest ecosystems is an urgent task that meets the challenges of modern ecology and forest management [7-9].

2 Materials and methods

A mathematical model for monitoring forest ecosystems may include a number of equations and variables that reflect key aspects of forest health. However, it is worth noting that creating an accurate mathematical model can be challenging due to the complexity of biological processes and interactions in an ecosystem. Rather than present a complete mathematical model, I will provide a general framework that can be extended and customized depending on the specific needs and characteristics of the forest ecosystem. The study uses classical numerical methods to solve differential cross-diffusion equations, providing insight into population dynamics. In addition, quantum optimization is being introduced into the optimization process, demonstrating the potential benefits of hybrid computing [10-12].

A mathematical model of cross-diffusion can be represented by a system of equations that describe changes in the concentration of species in space and time. For simplicity, let's consider two interacting species in a forest ecosystem. Let us denote the population concentrations of two species as $u(x, y, t)$ and $v(x, y, t)$, where x and y - are coordinates in space, and t - is time.

The system of equations can have the following form:

$$\begin{aligned} \frac{\partial u}{\partial t} &= D_u \nabla^2 u + r_u u\left(1 - \frac{u}{K_u}\right) - \alpha_{uv} uv - \beta_u u, \\ \frac{\partial v}{\partial t} &= D_v \nabla^2 v + r_v v\left(1 - \frac{v}{K_v}\right) - \alpha_{vu} vu - \beta_v v, \end{aligned} \tag{1}$$

Where: u and v - population concentrations of trees of species u and v, respectively; D_u and D_v - diffusion coefficients for species u and v, respectively, describing the distribution of species in space; r_u and r_v - growth coefficients for species u and v respectively, describing how quickly populations can reproduce; K_u and K_v - environmental capacities for species u and v, respectively, limiting population growth; α_{uv} and α_{vu} - interaction coefficients between species u and v, describing competition for resources; β_u and β_v - coefficients of impact of factors on types u and v, such as climate change.

The equations describe diffusion, growth, interactions, and the effects of external factors on populations in space and time. This model can be adapted and expanded depending on the specific characteristics of the forest ecosystem in Uzbekistan and the data provided. Model coefficients depend on specific species characteristics, environmental conditions, and interaction factors. Their values can be determined based on research data or expert assessments

3 Results and Discussion

An example of a real forest ecosystem in Uzbekistan can be found in the country's reserves and national parks. One example is the Kizilkum Nature Reserve. The eco-system of the Kizilkum Nature Reserve is characterized by harsh desert conditions, and here the interaction of species directly depends on the availability of water and resources in the desert environment.

Mathematical modeling of such an ecosystem may include taking into ac-count the influence of dry periods, changes in soil composition, water resources and the impact of human activity on the natural environment.

Plants:

Haloxylon ammodendron

Tamarix spp.

Calligonum spp.

Let's look at example parameters for two plant species typical of the tugay forest in Uzbekistan: the tugay tree and the herbaceous plant.

A tugay tree (for example, Haloxylon ammodendron):

Diffusion coefficient $D_u = 0.08$, $D_v = 0.04$.

Growth rate $r_u = 0.07$, $r_v = 0.03$.

Environment capacity $K_u = 0.7$, $K_v = 0.3$.

Interaction coefficients $\alpha_{uv} = 0.012$, $\alpha_{vu} = 0.008$.

Coefficients of influence of external factors $\beta_u = 0.002$, $\beta_v = 0.001$.

An herbaceous plant (for example, Artemisia):

Diffusion coefficient $D_u = 0.05$, $D_v = 0.02$.

Growth rate $r_u = 0.05$, $r_v = 0.02$.

Environment capacity $K_u = 0.6$, $K_v = 0.2$.

Interaction coefficients $\alpha_{uv} = 0.01$, $\alpha_{vu} = 0.006$.

Coefficients of influence of external factors $\beta_u = 0.001$, $\beta_v = 0.005$.

These values can be used to construct a mathematical cross-diffusion model for the interaction between tugay tree and herbaceous plant in a given ecosystem. How-ever, it should be remembered that the exact values of the coefficients might vary depending on the specific conditions and species in the tugay forest of Uzbekistan.

The diffusion coefficient in the mathematical model of cross-diffusion refers to the extent to which species (in this context, populations) are distributed in space. This coefficient determines how quickly individuals spread relative to their current location. In cross-diffusion models for two species (e.g. plants u and v), each species has its own diffusion coefficient (D_u for u and D_v for v). The values of these coefficients affect how quickly each species spreads through space based on its concentration gradient. The higher the diffusion coefficient, the faster the species spreads in space. For example, a high diffusion coefficient may correspond to a species that spreads easily across an area or has good mobility. On the contrary, a low diffusion coefficient indicates a slow spread of the species. In the context of forest ecosystems, this may reflect, for example, the ability of plants to disperse their seeds or the ability of certain species to migrate in the face of environ-mental change.

The growth rate in a mathematical cross-diffusion model is a parameter that de-termines how quickly a species' population grows over time in the absence of external factors (e.g., competition with other species, resource limitations, etc.). Cross-diffusion models typically use different growth factors for each species. Maintaining a growth rate (r_u for plant u and r_v for plant v) indicates that the species tends to increase its numbers when it is exposed to favorable conditions. This may include reproduction, growth, survival of young individuals, etc. In practice, the growth rate may depend on resource availability, environmental quality, influence of competitors, and other factors. In the context of forest ecosystems, the growth rate may reflect, for example, the ability of plants to efficiently photosynthesize, utilize nutrients, or other mechanisms that promote their growth and development.

Environmental capacity (K) in a mathematical cross-diffusion model is a parameter that describes the maximum amount of resources or maximum population size that an environment can support. This parameter determines the extent to which the environment is capable of providing resources for the life and development of the species. In the context of ecosystems and forests, environmental carrying capacity may reflect, for example, the availability of water, nutrients, light, growing space, and other factors influencing plant growth and development. When a species' population approaches this carrying capacity, growth and reproduction may slow down as available resources limit them. If the population size exceeds the carrying capacity of the environment, this can lead to reduced survival, competition for resources, deterioration of environmental conditions, and possibly a decline in the species' abundance. In the context of a cross-diffusion model for two species (e.g. plants u and v), each species will have its own environmental capacity parameter (K_u for plant u and K_v for plant v).

The interaction coefficients in the mathematical model of cross-diffusion reflect the influence of one species on another during their interaction. In the context of ecosystems and forest populations, these coefficients can be parameters that describe how one species affects the growth, development or survival of another species, and vice versa. In a cross-diffusion model for two species, for example plants u and v, the interaction coefficients (α_{uv} and α_{vu}) can influence the following aspects. Interaction coefficients can determine how strongly one species competes with another for available resources such as nutrients, water, or light. The model may also include species interactions through external factors, such as climate conditions, to which both species may respond differently. Coefficients can reflect types of interaction, such as positive affect (symbiosis), negative affect (an-tagonism), or other forms of interaction. Changes in interaction coefficients can af-fect population dynamics; by studying them in a model, you can understand how changes in one population can affect another, and vice versa.

Coefficients of interaction with external factors in a mathematical model of cross-diffusion for forest ecosystems can represent parameters that describe the in-fluence of external factors on the dynamics of growth and interaction of various species in forests. These factors may include climate conditions, soil properties, water availability and other environmental aspects. Studying the influence of these external factors in the context of a mathematical model can help to understand how changes in the environment can influence the dynamics of forest ecosystems.

A program has been developed that implements numerical modeling of a system of differential equations that describe the dynamics of two plant species (u and v) in space. The code uses the finite difference method to approximate spatial derivatives and solve the equations numerically. The program works according to the following algorithm:

Determination of model parameters (diffusion coefficients, growth coefficients, environmental capacity, etc.).

Creating a spatial grid with dimensions *nx* and *ny*.

Setting initial conditions for plant populations (*u0* and *v0*).

Define a step function that performs one simulation time step. This function uses the Laplace operator to approximate spatial derivatives and changes the values of the variables and according to a system of differential equations.

Execute a simulation loop for a specified number of time steps. Visualization of the results in the form of 3D graphs of populations u and v at the last time step is shown in Figure 1.

Thus, this code provides a tool to study the interaction dynamics of two plant spe-cies in a spatial environment using a cross-diffusion model. Optimized system param-eters obtained using a hybrid approach are presented, highlighting the synergy of classical and quantum-inspired methods. 3D visualization of populations offers new insights into population dynamics, enriching our understanding of system behavior. Figure 2 shows a quantum circuit that represents a two-qubit quantum circuit [13-15].

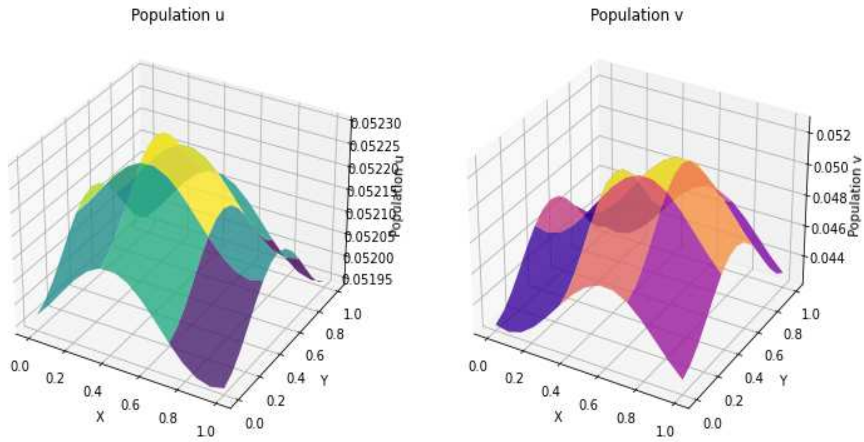

Fig. 1. Visualization of results in the form of 3D graphs of populations u and v.

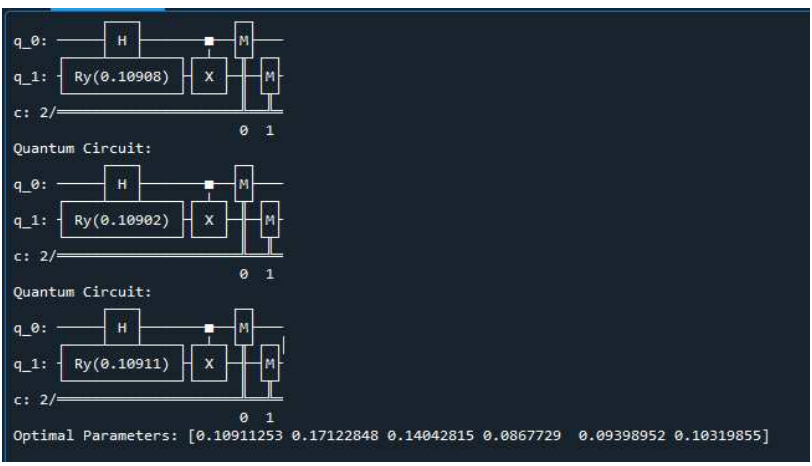

Fig. 2. Quantum variational optimization scheme.

The first qubit (q_0) passes through the Hadamard gate, designated H. It interacts with qubit q_1 through the X gate ($CNOT$ gate), which performs a controlled NOT operation. The H gate and X gate are the basic elements of a quantum circuit, used to prepare states and interact between qubits. The second qubit (q_1) undergoes a rotation operation (Ry gate) with a parameter of 0.10911. Interacts with qubit q_0 through gate X. Both qubits undergo measurement (M), which leads to quantum superposition of states and quantum interaction. The measurement results are stored in the classical register (classical bits) c, which has two bits (c: 2). The measurement results are stored in the classical register (classical bits) c, which has two bits (c: 2). A quantum circuit is a key tool in quantum computing, allowing the preparation, interaction and measurement of quantum states. In this case, the quantum circuit demonstrates the interaction and correlation between two qubits [16].

The proposed mathematical model, based on the principles of cross-diffusion to describe the dynamics of forest ecosystems, represents an innovative approach to studying the interaction of different plant species in a forest environment. In this section, we discuss key aspects of the work, its potential applications, as well as limitations and directions for future research. The model successfully integrates the principles of cross-diffusion while taking into account parameters that influence ecosystem dynamics. This allows a more complete description of complex processes in forests, taking into account the interaction of plants and external factors. The use of numerical methods such as the finite difference method provides efficient numerical solution of differential equations, which is an important step in understanding the dynamics of a system. The research aims to use quantum-inspired optimization techniques to refine model parameters. This provides new insights into the use of quantum computing in environmental research. Quantum variational optimization makes it possible to improve the accuracy of model parameters, which can be a key factor in increasing the reliability of the results [17].

3D graphs representing the spatial distribution of plant populations enrich the visual experience of the results. This is useful for intuitively understanding how changes in parameters affect forest ecosystems. Limitations of the model may include simplified assumptions about interactions and the omission of certain factors such as cli-mate change. Further research can be aimed at improving the accuracy of the model, taking into account more complex environmental influences, as well as adapting it to specific types of forests. The developed model can be used to predict changes in forests under the influence of various factors, such as climate change or economic activity. This can be useful in developing forest management strategies and maintaining their sustainability. The use of quantum methods opens up new prospects for optimizing parameters and more accurately reproducing the dynamics of natural systems [18-20].

4 Conclusion

This paper presented a mathematical model that describes the dynamics of forest ecosystems using the principles of cross-diffusion and numerical methods for solving differential equations. The model takes into account the interaction between different plant species, as well as the impact of external conditions on the ecosystem. One of the key contributions of the work is the integration of quantum-inspired optimization methods to refine model parameters. This represents a new approach to environmental research where quantum computing is used to improve the accuracy and efficiency of optimization. Visualizing the results in the form of 3D plots of the spatial distribution of plant populations at different time steps enriches the understanding of the dynamics of forest ecosystems. An important direction for future research is to adapt the model to specific forest types and take into account more complex interaction factors. The results obtained and the developed model provide important tools for a deeper understanding of the dynamics of forest ecosystems and their response to various impacts. This can be useful in developing strategies for sustainable forest management and predicting long-term changes in the natural environment.

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