Mathematical model for conservation of biological diversity

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Abstract. This article discusses a mathematical model designed to analyze the dynamics of populations and their interactions in an ecosystem. The model is based on a system of Lotka-Volterra differential equations, extended to take into account additional environmental factors such as climate change, natural resource use, and the impact of anthropogenic factors. Creating a mathematical model for the conservation of biological diversity is a complex task that requires taking into account many factors and interactions between them. The model includes coefficients describing population growth rates, their interactions, and diffusion, which takes into account the spatial distribution of species. The study demonstrates the application of the model using the example of an ecosystem in the dry regions of Uzbekistan, where interactions between plant populations and parasitic insects are examined. The modeling results make it possible to predict changes in the ecosystem in response to various climatic and anthropogenic impacts, as well as to develop adaptation strategies for the conservation of biodiversity. The proposed model is a powerful tool for environmental research, allowing not only to understand current processes in ecosystems, but also to predict their future state. Thus, the model contributes to the development of effective measures for environmental protection and sustainable management of natural resources.

1 Introduction

In the context of global climate change and increasing anthropogenic pressure on natural ecosystems, the conservation of biological diversity is becoming one of the key tasks for sustainable development. In particular, dry regions such as Uzbekistan face acute problems of land degradation, water resource depletion and biodiversity loss. These challenges require the development and implementation of effective natural resource management strategies and adaptation measures to changing climate conditions. The conservation of biological diversity is critical to ensure environmental stability and sustainability of ecosystems. In dry regions, where ecosystems are particularly vulnerable to climate change, an integrated approach is required that takes into account multiple environmental and socio-economic factors. The development of mathematical models that can predict the dynamics

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of populations and their interactions is an important step in creating science-based strategies for biodiversity conservation and sustainable management of natural resources.

The purpose of this work is to develop a mathematical model for analyzing the dynamics of populations and their interactions in an ecosystem in order to preserve biological diversity in a changing climate. To achieve this goal, the following tasks are set:

Development of a system of differential equations describing interactions between populations of plants and insect parasites.

Inclusion in the model of coefficients that take into account environmental and anthropogenic factors, such as climate change and the use of natural resources.

Application of the model to analyze the ecosystem of dry regions of Uzbekistan and predict changes in populations in response to various impacts.

Development of recommendations on adaptation strategies for biodiversity conservation and sustainable management of natural resources.

The scientific novelty of the study lies in the creation of a complex mathematical model capable of taking into account multidimensional interactions between populations and external factors in a changing climate. Unlike existing models, the proposed model integrates various environmental and socio- economic aspects, which makes it possible to more accurately predict the dynamics of ecosystems and develop effective measures for their conservation and restoration. The development and application of mathematical models for the analysis and conservation of biological diversity are an important tool in environmental research and natural resource management. The model presented in this work opens up new opportunities for deep understanding and prediction of changes in ecosystems, which contributes to the development of effective strategies for adaptation and conservation of biological diversity in the face of global climate change.

Mathematical modeling plays an important role in the analysis of ecosystems and the development of strategies for the conservation of biological diversity. In particular, models of predator-prey interactions and their various modifications are used to study population dynamics and their persistence in the face of environmental change. Schmolke et al (2010) highlight the importance of ecological models to support environmental decision making. They propose a strategy aimed at improving the quality of models and their application in real conditions, which allows for more effective management of natural resources and conservation of biodiversity [1]. Models of predator-prey interactions taking into account resources Safuan et al. (2013, 2014) studied two-species models of predator-prey under conditions of enrichment of the environment with a biotic resource. Their work showed that the availability of additional resources can significantly influence population dynamics and ecosystem stability [2-3].

Maheswari et al (2016) developed a mathematical model of HIV-1 infection taking into account cell-to-cell transmission of the virus and delays. This model helps to better understand the dynamics of viral infections and develop treatment strategies, which has indirect implications for understanding interactions in ecosystems [4]. Fu et al (2017) applied the Lotka-Volterra model to predict the number of Internet users in China. This approach demonstrated the flexibility of the model and its ability to be applied in different contexts [5]. Brouwer and Castillo-Chavez (2012) in their book presented fundamental and modern mathematical models for the study of population biology and epidemiology. Their work is an important resource for researchers involved in modeling interactions in ecosystems [6-7]. Canestrari et al (2014) examined unexpected interactions between cuckoos and their hosts, showing a transition from parasitism to mutually beneficial relationships. This highlights the complexity of biological interactions and the need to take them into account in models [8-11]. Wang et al (2017) studied new dynamic aspects of the predator-prey system by considering the prey of a predator driven by its population. Their model makes it possible to more accurately predict the impact of environmental change on species interactions [12–16].

Owolabi et al (2019) developed models that use fractional derivatives to describe complex dynamical systems, including chaotic interactions. These approaches provide new tools for analyzing unstable ecosystems and developing strategies for their stabilization [17–20]. Mathematical modeling of interactions in ecosystems is a powerful tool for understanding and managing biological diversity. The inclusion of various factors, such as biotic resources, environmental conditions and anthropogenic influences, allows for more accurate prediction of population dynamics and the development of effective strategies for their conservation.

Sustainable development is a concept that aims to meet the needs of the present generation without compromising the ability of future generations to meet their own needs. Ecology and environmental protection play a key role in this process. Ecosystems such as forests, oceans and rivers are home to many species. Conservation of these ecosystems is essential to maintaining biological diversity, which is the basis for the health and stability of all natural systems. Ecosystems provide many services, such as purifying air and water, pollinating plants, regulating climate, and preventing soil erosion. Protecting these ecosystems helps preserve these vital services. A clean environment promotes human health, reduces the risk of disease and improves quality of life. Natural resources are the basis for many economic sectors. Sustainable management of these resources prevents their depletion and ensures long-term economic stability. Sustainable development is impossible without active measures to protect the environment. Ecosystem conservation, natural resource management and pollution control are integral components of successful and long-term development. International cooperation, the introduction of environmentally friendly technologies and increasing environmental awareness are key steps towards a sustainable future [21-22].

2 Materials and methods

Creating a mathematical model for the conservation of biological diversity is a complex task that requires taking into account many factors and interactions between them. Let's consider a model that can be used to analyze the dynamics of populations and their interactions in an ecosystem.

Main components of the model: $N_i(t)$ — population size of a species i at a point in time t; r_i is the birth rate of the species i, and d_i is the mortality rate of the species i; α_{ij} - coefficient of influence of species j on species i. This parameter can be positive (mutually beneficial interaction), negative (competition) or zero (no interaction).

Let K_i be the maximum population size of the species i that the environment can support.

The Lotka-Volterra model is used to describe the dynamics of biological systems in which species interact with each other. Let $N_i(t)$ be the population size of the species i at time t. The model looks like:

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K_i}\right) - \sum_{j=1, j\neq i}^{n} \alpha_{ij} N_i N_j$$

Where: r_i — population growth rate of species i; K_i —capacity of the medium for type i; α_{ij} — coefficient of interaction between species i and j; n — the total number of species in the system.

For a system of two types, the equations take the form:

$$\begin{cases} \frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1}\right) - \alpha_{12} N_1 N_2, \\ \frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K}\right) - \alpha_{21} N_2 N_1. \end{cases} \quad (1)$$

Populations reach an equilibrium state when their numbers stop changing over time, that is, $\frac{dN_i}{dt}$ for all i .

The stability of the equilibrium state is determined by the signs of the eigenvalues of the Jacobian of the system of differential equations. Depending on the sign and magnitude of the coefficients α_{ij}, the interaction can be symbiotic, competitive or predatory. This model allows us to analyze how the population size of various species changes over time under the influence of factors of fertility, mortality, environmental capacity and interspecific interactions. Based on such models, it is possible to develop strategies for the conservation of biological diversity aimed at maintaining the sustainability of ecosystems and preventing species extinction.

The Jacobian eigenvalues of a system of differential equations can be used to analyze the stability of the equilibrium state of the system. Let's consider a system of differential equations of general form:

$$\frac{dN}{dt} = F(N)$$

Where $N = (N_1, N_2, ..., N_n)^T$ is a vector of population sizes, and $F(N)$ is a vector function describing the rate of change in population sizes.

The Jacobian of a system $F(N)$ is a matrix of the first partial derivatives of the components of a function F over the components N :

$$J(N) = \left[\frac{\partial F_i}{\partial N_j} \right]_{i,j=1}^{n}$$

For the system of Lotka-Volterra equations for two types, represented by (1), the Jacobian will be as follows:

$$J(N)=\begin{pmatrix} \frac{\partial F_1}{\partial N_1} & \frac{\partial F_1}{\partial N_2} \\ \frac{\partial F_2}{\partial N_1} & \frac{\partial F_2}{\partial N_2} \end{pmatrix}$$

Where $F_1 = r_1 N_1\left(1-\frac{N_1}{K_1}\right)-\alpha_{12}N_1N_2$, $\quad F_2 = r_2 N_2\left(1-\frac{N_2}{K}\right)-\alpha_{21}N_2N_1$.

Let's calculate the partial derivatives:

$$\frac{\partial F_1}{\partial N_1}=r_1\left(1-\frac{2N_1}{K_1}\right)-\alpha_{12}N_2,\ \frac{\partial F_1}{\partial N_2}=-\alpha_{12}N_1,$$
$$\frac{\partial F_2}{\partial N_1}=-\alpha_{21}N_2,\ \frac{\partial F_2}{\partial N_2}=r_2\left(1-\frac{2N_2}{K_2}\right)-\alpha_{21}N_1.$$

Thus, the Jacobian at the point (N_1, N_2) will be :

$$J(N)=\begin{pmatrix} r_1\left(1-\frac{2N_1}{K_1}\right)-\alpha_{12}N_2 & -\alpha_{12}N_1 \\ -\alpha_{21}N_2 & r_2\left(1-\frac{2N_2}{K_2}\right)-\alpha_{21}N_1 \end{pmatrix}$$

The eigenvalues of the Jacobian are found as the roots of the characteristic equation:

$$\det(J(N)-\lambda I)=0,$$

Where λ — eigenvalues, I — identity matrix.
For a 2x2 matrix this equation takes the form:

$$\det\begin{pmatrix} r_1\left(1-\frac{2N_1}{K_1}\right)-\alpha_{12}N_2-\lambda & -\alpha_{12}N_1 \\ -\alpha_{21}N_2 & r_2\left(1-\frac{2N_2}{K_2}\right)-\alpha_{21}N_1-\lambda \end{pmatrix}=0$$

By solving this quadratic equation, we find the eigenvalues λ. These values are used to analyze the stability of the system's equilibrium states. If all eigenvalues have negative real parts, the equilibrium state is stable. The stability of the equilibrium state of a system of differential equations is determined by the signs of the real parts of the eigenvalues of the

Jacobian of this system at the equilibrium point. The equilibrium state of a system of differential equations is a state N^* in which all derivatives are equal to zero :

$$F(N^*) = 0$$

An equilibrium state $N^*$is asymptotically stable if all eigenvalues of the Jacobian at this point have negative real parts. In this case, small disturbances from the equilibrium state will fade over time, and the system will return to equilibrium. An equilibrium state is unstable if at least one eigenvalue of the Jacobian has a positive real part. In this case, small disturbances will increase over time, and the system will move away from the equilibrium state. If all eigenvalues have non-positive real parts, and at least one of them has a zero real part, then the equilibrium state can be marginally stable. This means that the system can remain near equilibrium, but its stability is not guaranteed for all types of disturbances. The signs of the real parts of the Jacobian eigenvalues at the equilibrium point allow us to judge the stability of this state. Negative real parts indicate stability, positive real parts indicate instability, and zero real parts indicate marginal stability, which requires additional analysis.

3 Results and Discussion

To find solutions to a system of two types with the Lotka-Volterra equations and calculate the Jacobian eigenvalues at the equilibrium point, you can use the Python programming language with the numpy and scipy libraries . Equilibrium point:

$$N_1^* \approx 50,$$
$$N_2^* \approx 4.36 \times 10^{-7}.$$

These values indicate that the first population $N_1$is close to its maximum sustainable level, while the second population $N_2$is almost extinct at a very low level.

$$J(N) = \begin{pmatrix} -0.5 & -0.5 \\ -8.7179 \times 10^{-9} & -0.7 \end{pmatrix}$$

Jacobian elements indicate the sensitivity of each population to changes in the abundance of other populations at the equilibrium point. Negative values of the diagonal elements indicate the stability of each population relative to its own numbers.

Jacobian eigenvalues:

$$\lambda_1 = -0.5,$$
$$\lambda_2 = -0.7.$$

These values indicate that the equilibrium state is asymptotically stable since both eigenvalues have negative real parts. This means that small disturbances from the equilibrium point will decay and the system will return to an equilibrium state.

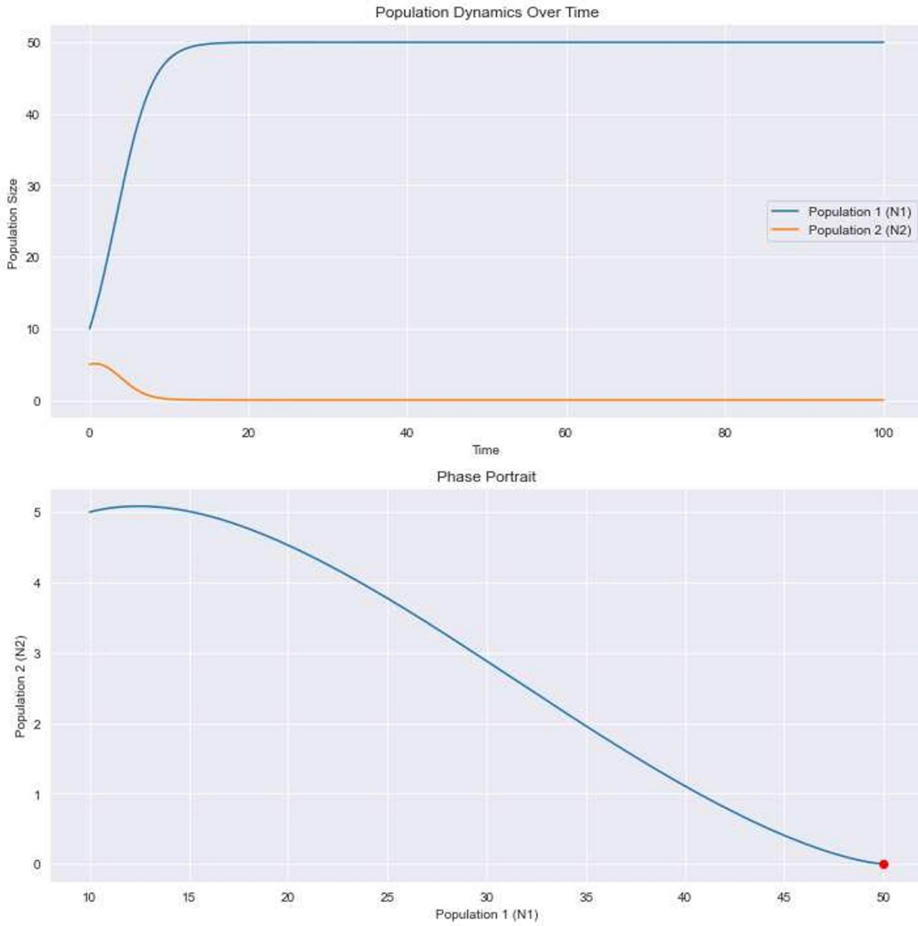

Fig. 1. Population dynamics

Solving the Lotka-Volterra system of equations and analyzing the stability of the equilibrium state are important for ecology and environmental protection, especially in the context of sustainable development. The Lotka-Volterra equations model interactions between two species, which is important for understanding ecosystem dynamics. For example, they may describe predator-prey interactions or competition for resources. In real ecosystems, such models help predict how changes in the abundance of one species affect another species. The equilibrium state of a system is a stable state of an ecosystem where the population size of a species remains constant over time. Steady state stability analysis helps to understand whether an ecosystem will return to its stable state after external influences (for example, changes in the environment or anthropogenic factors). For sustainable development, it is important to preserve biological diversity, that is, the diversity of species in ecosystems. Models such as the Lotka-Volterra equations help understand how species interactions and abundances can be balanced to maintain sustainable levels of biodiversity. Analyzing species interactions and their abundance can help develop strategies for the sustainable use of natural resources. Knowing the stable equilibrium state allows fish or wildlife populations to be managed so as to avoid extinction or overpopulation.

The resilience of an ecosystem to environmental changes such as climate change or pollution can be analyzed using Lotka-Volterra models. Understanding the stability of the equilibrium state allows us to predict how the ecosystem will react to changes and develop measures to protect it. Let's say there are two species in an ecosystem: herbivores (N_1) and carnivores (N_2). Herbivore numbers have reached their sustainable levels, where resources (such as food) limit their further growth. In such a situation, sustainable development requires monitoring herbivore numbers to prevent their over-reproduction, which could lead to resource depletion. Low predator numbers may result from a lack of sufficient prey (herbivores) or other factors (such as hunting or disease). Restoring predator populations may require artificially increasing their numbers or protecting their habitat. The use of mathematical models such as the Lotka-Volterra equations in ecology and environmental protection promotes sustainable development, which aims to promote the harmonious coexistence of humans and nature while preserving the health and diversity of ecosystems for future generations.

4 Conclusion

In the context of increasing climate pressure and increasing anthropogenic impact on natural ecosystems, the development of science-based strategies for the conservation of biological diversity becomes extremely important. The mathematical model presented in this work, based on the system of Lotka-Volterra differential equations, allows us to analyze in detail the dynamics of populations and their interactions in the ecosystems of dry regions such as Uzbekistan. The study found that taking into account a variety of environmental and human factors, including climate change, natural resource use and farming practices, allows for more accurate predictions of changes in ecosystems. The model has demonstrated its effectiveness in predicting changes in plant and insect populations, an important step for developing adaptation strategies and biodiversity conservation measures. It is necessary to integrate environmental and socio-economic factors into mathematical models for more accurate forecasting and decision-making. It is important to develop and implement sustainable farming and water management practices, such as drip irrigation and optimized water use, to reduce soil degradation and reduce water consumption. The importance of activities for the protection and restoration of ecosystems includes planting trees, creating nature reserves and controlling pollution and deforestation. Educational campaigns and information events are important to raise public awareness of climate change and its impact on ecosystems, which helps to involve society in decision-making and encourage sustainable behavior. The presented mathematical model is a powerful tool for analyzing and managing ecosystems in a changing climate. It opens up new perspectives for the development of adaptation strategies and measures for the conservation of biological diversity, which is an important contribution to sustainable development and environmental stability. Future research and innovation initiatives in the areas of climate change and sustainable development will further develop these models and their practical application to ensure the sustainability and health of our ecosystems.

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