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Nonlinear vibrations of earth structures

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Abstract. This article offers a detailed analysis of the current state of plane structure dynamics, addressing the complexities posed by non-homogeneous materials exhibiting nonlinear elastic and viscoelastic properties during real-life operations. The study proposes a comprehensive mathematical model and algorithm to investigate the dynamic behavior of such structures, employing the non-linear hereditary Boltzmann-Volterra theory to describe viscoelastic material properties accurately. Nonlinear oscillatory systems are analyzed using Lagrange equations based on the d'Alembert principle. The problem is approached through a multi-step process. Initially, the linear elastic problem of the structure's natural oscillations is solved to determine its natural frequencies and modes of oscillations. Subsequently, these eigenmodes are employed as coordinate functions to address forced nonlinear oscillations in viscoelastic non-homogeneous systems. The complexity of the problem necessitates solving a Cauchy problem comprising a system of nonlinear integrodifferential equations. To illustrate the methodology, the study examines the Gissarak earth dam, considering real operational conditions and nonlinear, viscoelastic material properties near resonant modes of vibrations. Utilizing numerical methods, the dynamic behavior of the structure is analyzed, assessing displacements and stress components at different time points under non-stationary kinematic action. Stress concentration regions within the structure are identified for resonant vibrations, allowing the evaluation of its strength. Furthermore, the impact of nonlinear elasticity and viscoelasticity on the structural dynamics is quantified. This research provides valuable insights into the behavior of plane non-homogeneous structures, considering real-world scenarios and material complexities, ultimately contributing to an improved understanding of structural dynamics and facilitating the identification and mitigation of potential structural challenges.

1 Introduction

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Recently, the construction of various hydro-technical structures has been conducted on a large scale in areas with high seismicity and rich in hydro-energy resources; it requires the creation of a reliable methodology and algorithms for dynamic calculation of earth structures, taking into account the properties of elasticity, viscoelasticity of the material for the real nature of the structure operation.

There are a large number of publications concerning the dynamic calculations of structures of this type, however, the study of the parameters of oscillatory processes, taking into account elastic nonlinearity and viscoelasticity of the material were studied with less completeness. The relevance of solving such problems is primarily due to the demands of the design, construction, and operation of various types of earth structures.

The purpose of this study is to develop a methodology, an algorithm for solving the problem of forced vibrations of non-homogeneous structures that have nonlinear elastic and viscoelastic properties of the material; study of the dynamics of specific problems and analysis of the results obtained from the point of view of discovering new mechanical effects.

To solve the problem, the following scheme was accepted. The finite element method (FEM) first solves the linear elastic problem of natural vibrations of plane non-homogeneous structures (the first few natural modes of vibrations are determined). The problem of forced oscillations of non-homogeneous structures, taking into account the nonlinear viscoelastic properties of the material is solved in the form of an expansion in terms of the found natural modes of oscillations obtained for the elastic problem, and then reduced to solving a system of nonlinear integrodifferential equations.

A particular difficulty arises when choosing the coordinate functions in which the sought-for solution is expanded. In this case, the functions for bodies of a simple shape (beam, plate, cylindrical shell, etc.) are quite simple. For bodies of a more complex shape, the choice of coordinate functions that reduce the original system to a system with a finite number of degrees of freedom is a difficult task. This explains the fact that for the coordinate functions, it is proposed to use the eigenmodes of the elastic structure.

In recent years, studies have been published where the stress-strain state and dynamic behavior of various structures were studied.

These scientific works include the following:

- in [4], vibrations of dams using various methods are considered. Various types of dam oscillations, such as free, forced oscillations and cases of oscillations in resonant modes, were studied. Methods for controlling dam vibrations by installing sensors, analyzing data, mathematical modeling and applying special control methods were described. Examples of practical applications of methods for controlling dam vibrations in various countries were given.
- the application of the finite element method for calculating the oscillations of dams is described in [5]. A new approach to the calculation of dam oscillations is given, based on the use of the finite element method. A dam model based on the finite element method is presented, and its dynamic characteristics are calculated. A new control method is considered, based on the use of resonant sensors. In conclusion, the results of the study and its practical significance are given.
- in [6], an analysis of dam vibrations that occur during the operation of hydro-technical structures is given, the most optimal conditions for their operation are determined. Methods for mathematical modeling of dam dynamics are described and a comparative analysis of various operating modes is given, the influence of the following factors are analyzed: water level change, frequency and amplitude of oscillations, and consideration of the influence of hydraulic forces on dam dynamics. It is shown that under certain modes of operation of dams, critical conditions arise that lead to damage to the structure.

- in [7], the main causes of dam oscillations (wind and hydrodynamic loads) and the influence of the relief on the location of dams, were studied. The monitoring methods used to assess dam vibrations, including the use of vibrometric and infrasonic sensors, were studied. The analysis of monitoring data recorded at several dams is given. The main causes of dam oscillations on each of the dams, and the influence of dam parameters, such as height and length, on the intensity of oscillations are highlighted.
- in [8], modeling of the dynamics of dams under conditions of resonant interaction with water is considered. In a numerical study, the influence of various factors on the vibrations of dams, such as water velocity, water depth, dam parameters, and others, was studied. For modeling, the methods of computational fluid dynamics and the finite element method were used. The results show the influence of resonant interaction with water on the dynamics of dams, which can lead to significant vibrations and damage to dams. Several methods were proposed to reduce oscillations and improve the stability of dams, such as the change in the geometry of the dam and the choice of optimal dam parameters.
- studies conducted in [9] were related to the choice of parameters for models of dam oscillations. A principal component method was proposed, which makes it possible to determine the most important parameters influencing dam vibrations and use them as input data for modeling. The data obtained from the monitoring of dam vibrations were used. It was shown that the principal component method is an effective tool for determining the parameters of dam oscillation models.
- the study in [10] proposed a new approach to the analysis of the dynamic characteristics of dams based on their modeling and experimental studies. A technique for modeling dam oscillations using the finite element method is given. Experiments were conducted to obtain data on the dynamic characteristics of dams under various loading conditions. Various parameters of dams that affect their dynamic stability - geometric parameters, material properties and loading conditions were also considered. An analysis of the influence of these parameters on the dynamic characteristics of dams and recommendations for optimizing the design of the dam were given.
- in [11], the possibility of using artificial neural networks to determine the parameters of models of dam oscillations is explored. The problem of determining the dynamic parameters of the dam based on the available data on its oscillations is considered using artificial neural networks that can be trained on the basis of the available data and make predictions. The process of data acquisition on dam vibrations is described. Several experiments were conducted to evaluate the accuracy and efficiency of the method. It is shown that the use of artificial neural networks makes it possible to achieve high accuracy in determining the parameters of dam oscillation models.
- in [12], using unmanned aerial vehicles, a system for monitoring and controlling dam vibrations was developed. Modeling of the monitoring process of dam vibrations was conducted. Inertial measurement devices and high-resolution cameras were used to monitor dam vibrations. The software was used to analyze the data and characterize the oscillations of the dams. The system was tested on several dams of various designs.
- in [13], a method for mathematical modeling of dam oscillations using the FEM is given, considering the dissipative properties of materials. A numerical study of dam oscillations is given in the example of a simulated hydroelectric power plant, taking into account the effects of external loads such as seismic waves, hydrodynamic shocks, and the dissipative properties of materials. The dissipative properties of the material definitely affect the dynamics of dam oscillations; in particular, an account for the dissipation makes it possible to significantly reduce the oscillation amplitude, which in turn increases the dynamic stability of dams.
- a method for determining the dynamic characteristics of dams based on the Fourier series, which makes it possible to determine the frequency spectrum of dam vibrations and

evaluate its dynamic characteristics, such as natural vibration frequencies and damping, is presented in [14]. The results of the numerical simulation of dam oscillations are based on the theory of thin-walled shells. The proposed method is applied to analyze the dynamic characteristics of dams on the example of several types of dams and to determine with high accuracy natural oscillation frequencies and damping of dams. It was revealed that the geometric parameters of dams have a significant impact on their dynamic stability.

- in [15], a method for assessing the impact of natural disasters on dam vibrations is proposed, which can be used to assess risks and prevent catastrophic situations in this area. Various factors influencing the behavior of dams are described, such as water levels, earthquakes and other natural phenomena. A mathematical model and numerical methods for assessing the impact of natural disasters on dam vibrations are considered. The research based on this technique allows us to determine the possible risks and aftermaths in the event of catastrophic situations.
- in [16], dynamic processes occurring in dams during the operation of hydroelectric power plants were studied. A new mathematical model is given that takes into account the free edge effect in dams. The results of numerical simulation for various operating conditions of dams are presented, considering various parameters, such as flow velocity, water depth, etc., on the behavior of the dam.
- the study given in [17] showed the possibility of using the finite element method (FEM) to simulate the vibrations of dams. The efficiency and accuracy of the FEM in solving problems related to the dynamic behavior of dams were studied. Various aspects of the FEM application in modeling dam oscillations are considered, including the choice of a mathematical model, the formulation of boundary conditions, and the analysis of the results. Conclusions are drawn about the applicability of the FEM for modeling dam oscillations and its advantages and limitations are indicated.
- reference [18] is devoted to the analysis of the influence of dynamic loads on the operation of dams during their design and operation, taking into account various factors that cause dynamic loads on dams, such as earthquakes and other natural phenomena. An overview of various methods and approaches to the analysis of dynamic loads on dams is presented, and analytical and numerical methods and the use of computer models and simulations to analyze the dynamic behavior of dams are considered.
- in [19], the influence of micro-seismic vibrations on the operation of dams is considered, taking into account the safety and stability of dams. An overview of micro-seismic oscillations and their characteristics, such as geological faults, seismic activity and anthropogenic factors, is given. The main attention is paid to the analysis of the influence of micro-seismic vibrations on the structural elements of dams; their influence on the strength and stability of structures is analyzed. A technique for assessing the impact of micro-seismic vibrations on dams, including modeling and numerical analysis, is presented.
- calculation of the dynamic characteristics of dams is given in [20], taking into account the interaction with the medium and soil. The influence of soil conditions on the dynamic behavior of dams is investigated and a method for calculating these characteristics is given. A review of methods for calculating the dynamic characteristics of dams and their interaction with soil is presented. Various aspects of interaction modeling are considered, including the choice of mathematical models, taking into account the geometry of dams, soil properties and boundary conditions. A method for calculating the dynamic characteristics of dams is given, with an account for the interaction with soil.
- in [21], the dynamics of dams under the influence of seismic loads is modeled using the finite element method. A review of methods for modeling the dynamics of dams under seismic loads is presented. The main aspects of using the finite element method are considered, such as the choice of dam geometry, material properties, determination of boundary conditions and consideration of seismic effects. Numerical models are given

using the finite element method, which makes it possible to evaluate the dynamic characteristics of dams, such as vibration amplitudes, resonant frequencies and strains, as well as the influence of various factors - dam geometry, materials and boundary conditions.
- in [22], the issues of the impact of dynamic loads, such as seismic and wind effects, on the operation of dams and methods for their strengthening are considered. The results of the analysis of various cases of the impact of dynamic loads on dams are presented, and methods for modeling such loads and assessing their impact on the design of dams are described. Methods for strengthening dams are given, including the use of various types of reinforcement, the raising of a dump, installing additional supports, and using damping systems.
- the study in [23] considers a mathematical model used to describe the dynamics of dams, taking into account the non-linearity in the operation of dams, including nonlinearities in materials, hydrodynamic effects and non-linear effects of the dam-soil contact. The results of numerical simulation of the dynamic behavior of dams are given, taking into account nonlinear effects. The influence of nonlinearities on the characteristics of dam oscillations, such as amplitude, frequency and modes of oscillations, was studied. An analysis is given of the influence of nonlinearities on the operation of dams under various types of dynamic loads (seismic and water loads).
- in [24], the dynamic parameters of the dam were studied using two main approaches - monitoring and mathematical modeling. Monitoring methods are used to collect data on the dynamic pattern of the dam. Mathematical models are used to evaluate the dynamic parameters of the dam. The results of the study are given, in which the data obtained from monitoring are compared with the results of mathematical modeling. The importance of a combination of monitoring and mathematical modeling for an accurate assessment of the dynamic parameters of dams is emphasized.
- in [25], numerical methods and mathematical modeling are used to analyze and obtain quantitative estimates, taking into account the influence of soil parameters on the dynamics of dams. Soil models are selected that take into account the physical and mechanical properties of soil, as well as parameters that change to assess their impact on the dynamics of dams. The results of studies given show the effect of changing soil parameters on various dynamic characteristics of dams, such as oscillation amplitude, resonant frequencies, forces of interaction with soil, and others. Quantitative estimates of these changes and their impact on the reliability and stability of dams are presented.
- the influence of the geometric characteristics of dams (height, width of the base, slope gradient, etc.) on their dynamics is studied in [26], using a mathematical model and numerical methods. The results of studies of the influence of various geometric parameters on the dynamics of dams, such as the amplitude of oscillations, resonant frequencies, strains and stresses, quantitative estimates of these changes and their consequences for the reliability and stability of dams are presented.
- in [27], the influence of ground base on the dynamics of dams is considered; the models of ground base are selected taking into account the physical and mechanical properties of soil, as well as the parameters used to analyze the influence of ground base on the dynamics of dams. The results are obtained in the form of oscillation amplitudes, resonant frequencies, strains and stresses in dams considering the influence of ground base on the dynamics of dams. Quantitative estimates of these changes and aftermaths are given for the reliability and stability of dam operation.
- in [28], methods of mathematical modeling and numerical analysis are used to assess the consideration of geometric parameters in modeling the dynamics of dams. In modeling the dynamics of dams, an approach based on the finite element method was chosen. A mathematical model and numerical methods to simulate the dynamics of dams are given, taking into account the influence of changes in geometric parameters, such as height, width,

slope gradient, and others, on the dynamic behavior of dams. The results of numerical experiments are given, and the effect of changing these parameters on the amplitudes of vibrations, stresses and strains of dams is analyzed.
- the dynamics of dams was studied in [29] taking into account nonlinear effects. A mathematical model was chosen that considers nonlinear effects and numerical methods were used to study the dynamics of dams. Nonlinear phenomena were considered, such as non-linear stiffness, non-linear damping and non-linear forces that can arise in the operation of dams. Results are given in which the influence of nonlinear effects on the dynamic behavior of dams is analyzed. The amplitudes of oscillations, stresses and strains of dams under various conditions and parameters are studied. Recommendations are proposed for optimizing the operation of dams, taking into account these effects.
- in [30-35], some results are given concerning the study of the stress-strain state and the dynamic behavior of an earth dam as a multilayer structure interacting with a ground base. The internal force factors and the strength of these structures are evaluated under various types of impacts.

As the above analysis of well-known scientific publications shows, each of these methods has its advantages and disadvantages; despite this, they are used in solving specific practical problems.

Based on this analysis, it can be noted that the development of mathematical models, solution methods, and the study of dynamics for an earth structure is currently a very relevant and urgent problem all over the world that needs to be solved.

2 Methods

2.1 Mathematical model

Nonlinear forced vibrations of earth structures (Fig. 1) are considered under the action of a non-periodic kinematic impact $\vec{x} \in \Sigma_u : \vec{u}_0(\vec{x},t) = \vec{\psi}_1(t)$ applied to its lower part. The hydrostatic pressure of water $\vec{p}_c = \rho_o g \ (h - x_2)$ acts on surface S_p of the upper slope of the dam. The rest of the surface of the structure is stress-free. Here, the material of the structure is considered linearly elastic for domain v_1, linearly viscoelastic for domain v_2, nonlinearly elastic for domain v_3, and nonlinearly viscoelastic for domain v_4. The actual geometry of the structure and the non-homogeneous mechanical properties of the material for its individual sections are taken into account.

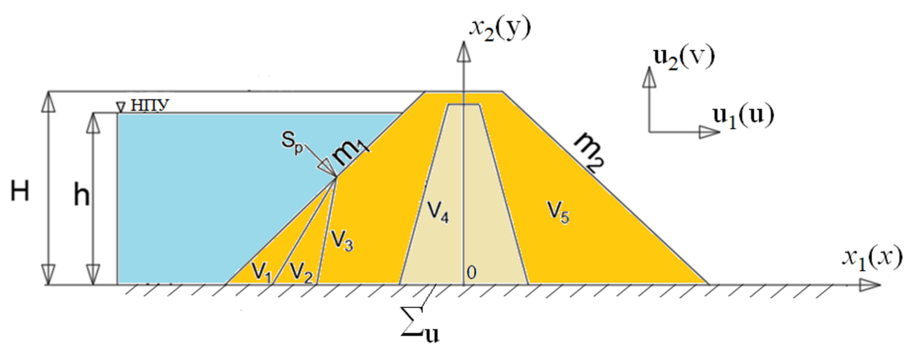

Fig.1. Plane scheme of a non-homogeneous earth dam

The dynamic behavior and the stress-strain state of the structure (Fig. 1) under unsteady forced vibrations and the action of non-periodic kinematic influences on it, i.e. $\vec{x} \in \Sigma_u$: $\vec{u}_0(\vec{x},t) = \vec{\psi}_1(t)$ are to be determined in the plane-deformed state.

To set the task, the following are used:

the linear Cauchy relations:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i,j = 1,2 \tag{1}$$

the stress-strain relations in the following form:

$$\begin{aligned}
&\vec{x} \in \mathrm{v}_1 : \ \sigma_{ij}^1 = k_1 \varepsilon_{ij}(t)\delta_{ij} + 2G_1 e_{ij}(t) \\
&\vec{x} \in \mathrm{v}_2 : \ \sigma_{ij}^2 = k_2 \varepsilon_{ij}(t)\delta_{ij} + 2G_2 \left\{ e_{ij}(t) - \int_o^t R_1(t-\tau) e_{ij}(\tau)\, d\tau \right\} \\
&\vec{x} \in \mathrm{v}_3 : \ \sigma_{ij}^3 = k_3 \varepsilon_{ij}(t)\delta_{ij} + 2G_3 \left\{ e_{ij}(t) + \gamma\, e_{ij}(t) e(t) \right\} \\
&\vec{x} \in \mathrm{v}_4 : \ \sigma_{ij}^4 = k_4 \varepsilon_{ij}(t)\delta_{ij} + 2G_4 \left\{ \left[e_{ij}(t) - \int_o^t R_1(t-\tau) e_{ij}(\tau)\, d\tau \right] + \right. \\
&\left. + \gamma \left[e_{ij}(t) e(t) - \int_o^t R_3(t-\tau) e_{ij}(\tau) e(\tau)\, d\tau \right] \right\} \\
&e_{ij} = \varepsilon_{ij} - \frac{1}{3}\mathrm{v}\delta_{ij}, \quad e = e_{k\ell} e_{\ell k}, \quad \mathrm{v} = \varepsilon_{ii}, \qquad i,j,k,\ell = 1,2
\end{aligned} \tag{2}$$

the kinematic boundary conditions:

$$\vec{x} \in \sum\nolimits_u : \vec{u}_0 \ (\vec{x},t) = \vec{\Psi}_1(t) \tag{3}$$

the initial conditions:

$$\begin{aligned}
&\vec{x} \in \mathrm{v} : \vec{u}(\vec{x},0) = \vec{\Psi}_2(\vec{x}) \\
&\frac{\partial \vec{u}(\vec{x},o)}{\partial t} = \vec{\Psi}_3(\vec{x})
\end{aligned} \tag{4}$$

the principle of virtual displacements, which takes into account the work of inertia forces:

$$\delta A_n = -\int_{\mathrm{v}} \rho_n \ \frac{\partial^2 \vec{u}}{\partial t^2} \delta \vec{u} \, d\,\mathrm{v} \quad , \tag{5}$$

internal stress work:

$$\delta A_\sigma = -\int_{\mathrm{v}} \sigma_{ij}^n \delta\, \varepsilon_{ij} d\,\mathrm{v} \tag{6}$$

and hydrostatic pressure work:

$$\delta A_p = \int_{S_p} \vec{p}_c \delta\vec{u} dS \tag{7}$$

In accordance with the principle of virtual displacements, the work of all active forces, taking into account the forces of inertia, on a virtual displacement is zero, i.e.

$$\delta A = \delta A_n + \delta A_G + \delta A_p = 0 \tag{8}$$

Here $\vec{u}$, ε_{ij}, σ_{ij} are the components of the displacement vector, strain and stress tensors; $\delta\vec{u}$, $\delta\varepsilon_{ij}$ are the isochronous variations of displacements and strains, respectively; ρ_n is the density of the material of the elements of this system; $V=V_1+V_2+V_3+V_4+V_5$ is the volume occupied by the structure elements; $\vec{\Psi}_1$ is the given vector-function of time; $\vec{\Psi}_2$, $\vec{\Psi}_3$ are the given vector-functions of coordinates; K is the volumetric modulus of elasticity; G is the instantaneous shear modulus; R_1, R_3 are the relaxation kernels; ρ_0 is the density of water; g is the free fall acceleration; $(h\text{-}x_2)$ is the depth of a point on the upstream face of the dam. In all problems considered, the displacement vector has two components $\vec{u}=\{u_1,u_2\}=\{u,\mathrm{v}\}$ in the coordinate frame $\vec{x}=\{x_1,x_2\}=\{x,y\}$. $i, j = 1,2$; $n = 1,2,3,4,5$.

Displacements $\vec{u}(\vec{x},t)$ and stresses $\sigma_{ij}(\vec{x},t)$ that satisfy variational equations (5) - (8), (1) - (2) and conditions (3) - (4) on any virtual displacement $\delta\vec{u}$ are to be determined.

2.2 Solution method

Approximate solution to the variational problem (5) - (8) and (1) - (4) is sought in the form of expansion in terms of natural modes of vibrations of elastic non-homogeneous structures.

2.2.1 Natural oscillations

To solve the problem of unsteady forced vibrations of a structure, a linear elastic problem of natural vibrations is first solved in the absence of external forces, assuming that the mechanical properties of its individual sections are non-homogeneous. It is also assumed that homogeneous kinematic boundary conditions are set on surface $\sum_u$, i.e.:

$$\vec{x} \in \sum\nolimits_u : \vec{u} = 0,\ \delta\vec{u} = 0 \tag{9}$$

The task is to determine the eigenfrequencies and the modes of vibrations of structures (Fig. 1), i.e.:

$$\vec{u}(\vec{x},t) = \vec{u}^*(\vec{x})\cos\omega t \tag{10}$$

where ω is the eigenfrequency and $\vec{u}^*$ is the natural mode of oscillations of the structure.

In the mathematical formulation of the problem, the principle of virtual displacements (5)-(8), (1), and homogeneous kinematic boundary conditions (8) are used. In expression (2), parameters $\gamma=0, R_1=0,\ R_3=0$ are assumed zero.

After substituting (10) in (1), (5)-(8), the problem of natural vibrations of a non-homogeneous elastic structure (Fig. 1) for $\gamma = 0, R_1 = 0,\ R_3 = 0$ is reduced to a variational equation with respect to natural modes $\vec{u}^*(\vec{x})$, i.e.

$$-\int_{\mathrm{v}} \sigma_{ij}^* \delta\varepsilon_{ij} d\,\mathrm{v} + \omega^2 \rho \int_{\mathrm{v}} \vec{u}^* \delta u^* d\,\mathrm{v} = 0 \tag{11}$$

$$\vec{x} \in \sum\nolimits_u : \vec{u}^* = 0,\ \delta\vec{u}^* = 0$$

where $V = V_1 + V_2 + V_3 + V_4 + V_5$ is the volume occupied by the structure; σ_{ij}^* are the stress amplitudes corresponding to $\delta\vec{u}^*$.

When searching for eigenforms, an additional normalization condition is also set

$$\int_{\mathrm{v}} \vec{u}^{*2} d\,\mathrm{v} = 1 \tag{12}$$

Now the problem of natural oscillations is reduced to determining ω_n^2 and $\vec{u}^*(x)$, which satisfy the variational equation (11), conditions (8) on any virtual displacement $\delta\vec{u}^*$.

Using the procedure of the finite element method (FEM), the variational problem (11) is reduced to a system of homogeneous algebraic equations for eigenvalues of the following form:

$$\left([K] - \omega^2 [M]\right)\{\vec{u}^*\} = 0\,, \tag{13}$$

here $[K],[M]$ are the matrices of stiffness and mass; $\omega, \{\vec{u}^*\}$ are the eigenfrequency and natural vector of the structure (Fig. 1).

As a result of solving equations (13), several lower eigenfrequencies of the structure $\omega_1, \omega_2, \varpi_3, \omega_4, \omega_5$ and the corresponding eigenvectors (u, υ) for each grid node of the structure (Fig. 1) are determined, obtained after finite element discretization, i.e. for each value of $\omega_1, \omega_2, \varpi_3, \omega_4, \omega_5$:

$$\begin{gathered} \omega_1 : \{u^*{}_1\} = \{u_1^1,\ \upsilon_1^1, u_1^2, \upsilon_1^2, \ldots, u_1^m, \upsilon_1^m\} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \omega_5 : \{u^*{}_5\} = \{u_5^1,\ \upsilon_5^1, u_5^2, \upsilon_5^2, \ldots, u_5^m, \upsilon_5^m\} \end{gathered} \tag{14}$$

Based on this, it is possible to write the displacement functions inside each finite element in the following form:

$$\begin{aligned} u_n &= \varphi_1^e u_{i,n}^e + \varphi_2^e u_{j,n}^e + \varphi_3^e u_{k,n}^e \\ \upsilon_n &= \varphi_1^e \upsilon_{i,n}^e + \varphi_2^e \upsilon_{j,n}^e + \varphi_3^e \upsilon_{k,n}^e \end{aligned}\ . \tag{15}$$

The last expressions represent the field of displacements inside the ***e***-th element at the ***n***-th natural mode (***n***=1, 2, …, ***N***) of vibrations.

2.2.2 Nonlinear forced oscillations

The solution to problem (5) - (7), (1) - (2) under conditions (3) - (4) is sought in the form of an expansion in terms of natural modes of vibrations:

$$\vec{u}(\vec{x},t)=\vec{u}_0(\vec{x},t)+\sum_{k=1}^{N}\vec{u}_k^*(\vec{x})y_k(t),$$
$$\delta\vec{u}=\sum_{k=1}^{N}\vec{u}_k^*(\vec{x})\delta y_k(t), \tag{16}$$

where y_n(t) are the sought-for time functions; $\{\vec{u}^*_k(\vec{x})\}$ are the natural modes of oscillations of the structure found by the FEM; $\{\vec{u}_0(\vec{x},t)\}$ are the given functions (3); δy_k are the independent isochronous variations of the sought-for function $y_k(t)$; N is the number of eigenforms held in expansion (16).

When obtaining a system of resolving equations for function $y_k(t)$, it is necessary to calculate the elementary work (5) - (7) of all active forces acting on the system for virtual displacements, and then substitute the resulting expressions into variational equation (8). Equating the factors to zero with independent variations δy_k, we obtain N nonlinear integro-differential equations with respect to the sought-for function $y_k(t)$ of the following form:

$$\sum_{n=1}^{N}\Big[M_{ij}\ddot{Y}_j+Q_{ij}Y_j-C_{ij}\int_o^t R_1(t-\tau)Y_i(\tau)\,d\tau=\Phi_i\ddot{U}_o(t)-$$
$$-\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}L_{ijkm}Y_jY_kY_m+\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}H_{ijkm}\int_o^t R_3(t-\tau)Y_j(\tau)Y_k(\tau)Y_m(\tau)\,d\tau \tag{17}$$

with initial conditions

$$Y_n(0)=Y_{on},\quad \dot{Y}_n(0)=\dot{Y}_{on} \tag{18}$$

The coefficients of system (17) are integrals, taken over volume V of the structure, from combinations of products of eigenmodes and their derivatives.

System (21) is written in the following form:

$$\ddot{Y}_i+\omega_i^2Y_i=B_i\ddot{U}_o(t)+\sum_{j=1}^{N}\lambda_{ij}^2\int_o^t R_1(t-\tau)-$$
$$-\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}L_{ijkm}Y_jY_kY_m+\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}H_{ijkm}\int_o^t R(t-\tau)Y_j(\tau)Y_k(\tau)Y_m(\tau)\,d\tau \tag{19}$$
$$Y_i(0)=Y_{io},\quad \dot{Y}_i(0)=\dot{Y}_{io},\quad i=1,2,\ldots$$

where the following designations are taken:

$$\omega_i^2=\frac{Q_{ii}}{M_{ii}},\ \lambda_{ij}^2=\frac{C_{ij}}{M_{ij}},\ B_i=\frac{\Phi}{M_{ii}}$$
$$C_{ijkm}=\frac{L_{ijkm}}{M_{ij}},\ P_{ijkm}=\frac{H_{ijkm}}{M_{ij}} \tag{20}$$

Let us apply the freezing method [2] to system (19). By the following substitution

$$
\begin{aligned}
&Y_i(t) = C_{1i}\cos\omega_i t + C_{2i}\sin\omega_i t + \frac{B_i}{\omega_i}\int_o^t \ddot{u}(\tau)\sin\omega_i(t-\tau)\,d\tau \\
&\dot{Y}_i(t) = \omega_i\left[-C_{1i}\sin\omega_i t + C_{2i}\cos\omega_i t + \frac{B_i}{\omega_i}\int_o^t \ddot{u}(\tau)\cos\omega_i(t-\tau)\,d\tau\right.
\end{aligned}
, \qquad (21)
$$

system (19) is reduced to the standard form:

$$
\begin{aligned}
C_{1i} = -\frac{1}{\omega_i} = \Bigg\{ &\sum_{j=1}^{N}\lambda_{ij}^2\int_o^t R_1(t-\tau)\Big[\ C_{1j}(\tau)\cos\omega_j\tau + C_{2j}(\tau)\sin\omega_j\tau + \\
&+\frac{B_i}{\omega_j}\int_o^\tau \ddot{u}_o(S)\sin\omega_j(\tau-s)ds\ \Big]d\tau - \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}C_{ijkm}\Big[\ C_{1j}\cos\omega_j t + \\
&+C_{2j}\sin\omega_j t + \frac{B_i}{\omega_j}\int_o^t \ddot{u}_o(\tau)\sin\omega_i(t-\tau)d\tau\ \Big]\Big[\ C_{1k}\cos\omega_k t + \\
&+C_{2k}\sin\omega_k t + \frac{B_k}{\omega_k}\int_o^t \ddot{u}_o(\tau)\sin\omega_k(t-\tau)d\tau\ \Big]\Big[\ C_{1m}\cos\omega_m t + \\
&+C_{2m}\sin\omega_m t + \frac{B_m}{\omega_m}\int_o^t \ddot{u}_o(\tau)\sin\omega_m(t-\tau)d\tau\ \Big] + \\
&+\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}P_{ijkm}\int_o^t R_3(t-\tau)\Big[\ C_{1j}(\tau)\cos\omega_j\tau + C_{2j}(\tau)\sin\omega_j\tau + \\
&+\frac{B_i}{\omega_j}\int_o^\tau \ddot{u}_o(s)\sin\omega_j(\tau-s)ds\ \Big]\Big[\ C_{1k}(\tau)\cos\omega_k\tau + C_{2k}(\tau)\sin\omega_k\tau + \\
&+\frac{B_k}{\omega_k}\int_o^\tau \ddot{u}_o(s)\sin\omega_k(\tau-s)ds\ \Big]\Big[\ C_{1m}(\tau)\cos\omega_m\tau + C_{2m}(\tau)\sin\omega_m\tau + \\
&+\frac{B_m}{\omega_m}\int_o^\tau \ddot{u}_o(s)\sin\omega_m(\tau-s)ds\ \Big]d\tau - \Bigg\}\sin\omega_i t,
\end{aligned}
$$

$$C_{2i} = -\frac{1}{\omega_i} = \left\{ \sum_{j=1}^{N} \lambda_{ij}^2 \int_o^t R_1(t-\tau) \Big[C_{1j}(\tau)\cos\omega_j\tau + C_{2j}(\tau)\sin\omega_j\tau + \right.$$
$$+ \frac{B_i}{\omega_j}\int_o^\tau \ddot{u}_o(s)\sin\omega_j(\tau-s)ds \Big] d\tau - \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N} C_{ijkm} \Big[C_{1j}\cos\omega_j t +$$
$$+ C_{2j}\sin\omega_j t + \frac{B_i}{\omega_j}\int_o^t \ddot{u}_o(\tau)\sin\omega_i(t-\tau)d\tau \Big]\Big[C_{1k}\cos\omega_k t +$$
$$+ C_{2k}\sin\omega_k t + \frac{B_k}{\omega_k}\int_o^t \ddot{u}_o(\tau)\sin\omega_k(t-\tau)d\tau \Big]\Big[C_{1m}\cos\omega_m t +$$
$$+ C_{2m}\sin\omega_m t + \frac{B_m}{\omega_m}\int_o^t \ddot{u}_o(\tau)\sin\omega_m(t-\tau)d\tau \Big] +$$
$$+ \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N} P_{ijkm}\int_o^t R_3(t-\tau)\Big[C_{1j}(\tau)\cos\omega_j\tau + C_{2j}(\tau)\sin\omega_j\tau +$$
$$+ \frac{B_i}{\omega_j}\int_o^\tau \ddot{u}_o(s)\sin\omega_j(\tau-s)ds \Big]\Big[C_{1k}(\tau)\cos\omega_k\tau + C_{2k}(\tau)\sin\omega_k\tau +$$
$$+ \frac{B_k}{\omega_k}\int_o^\tau \ddot{u}_o(s)\sin\omega_k(\tau-s)ds \Big]\Big[C_{1m}(\tau)\cos\omega_m\tau + C_{2m}(\tau)\sin\omega_m\tau +$$
$$\left. + \frac{B_m}{\omega_m}\int_o^\tau \ddot{u}_o(s)\sin\omega_m(\tau-s)ds \Big] d\tau - \right\} \cos\omega_i t,$$

By performing the freezing procedure [2] and taking into account (21), the above system of differential equations can finally be reduced to the following form:

$$\ddot{Y}_i + \omega_i^2 Y_i = B_i\ddot{u}_o(t) + \sum_{j=1}^{N}\lambda_{ij}^2\left(R_{1j}^C Y_j - \frac{1}{\omega_j} R_{1j}^S Y_j \right) - \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N} C_{ijkm}\Big[C_{1j} Y_j Y_k Y_m -$$
$$- B_i \sum_{j=1}^{N}\frac{\lambda_{ij}^2}{\omega_j}\int_o^t R_1(\tau)\left[\int_{t-\tau}^{t}\ddot{u}_o(s)\sin\omega_j(t-\tau-s)ds \right] d\tau +$$
$$+ \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N} P_{ijkm}\int_o^t R_3(\tau)\left[Y_j(t)\cos\omega_j\tau - \frac{1}{\omega_j}\dot{Y}_j(t)\sin\omega_j\tau - \right.$$
$$\left. - \frac{B_j}{\omega_j}\int_{t-\tau}^{t}\ddot{u}_o(s)\sin\omega_j(t-\tau-s)ds \right]\times\left[Y_k(t)\cos\omega_k\tau - \frac{1}{\omega_k}\dot{Y}_k(t)\sin\omega_k\tau - \right.$$
$$\left. - \frac{B_k}{\omega_k}\int_{t-\tau}^{t}\ddot{u}_o(s)\sin\omega_k(t-\tau-s)ds \right]\times\left[Y_m(t)\cos\omega_m\tau - \frac{1}{\omega_m}\dot{Y}_m(t)\sin\omega_m\tau - \right.$$
$$\left. - \frac{B_m}{\omega_m}\int_{t-\tau}^{t}\ddot{u}_o(s)\sin\omega_m(t-\tau-s)ds \right] d\tau$$
$$Y_i(0) = Y_{io}, \quad \dot{Y}_i(0) = \dot{Y}_{io}$$

where $R_{1i}^S = \int_o^t R_1(\tau)\sin\omega_i\tau\, d\tau,\ R_{1i}^C = \int_o^t R_1(\tau)\cos\omega_i\tau\, d\tau$.

System (22) is solved using the Runge-Kutta numerical method [3].

3 Results and Discussion

In this study, we investigated the natural and unsteady forced oscillations of the Gissarak earth dam, considering its real geometry and the non-homogeneous features of the structure. The dam has a height (H) of 138.5 m, crest width of 16 m, upstream slope (m_1) of 2.2, downstream slope (m_2) of 1.9, and a crest length of 660 m. The core of the dam is made of loam with a soil elasticity modulus (E) of 2400 MPa, specific gravity (γ) of 1.7 tf/m^3, and Poisson's ratio (μ) of 0.35. The retaining prisms are made of rocks with an elasticity modulus (E) of 3600 MPa, specific gravity (γ) of 1.9 tf/m^3, and Poisson's ratio (μ) of 0.35.

Figure 2 displays the calculation results obtained using the above method. It presents the first natural frequencies and modes of vibrations of the Gissarak earth dam, taking into account the non-homogeneous features of the structure and materials.

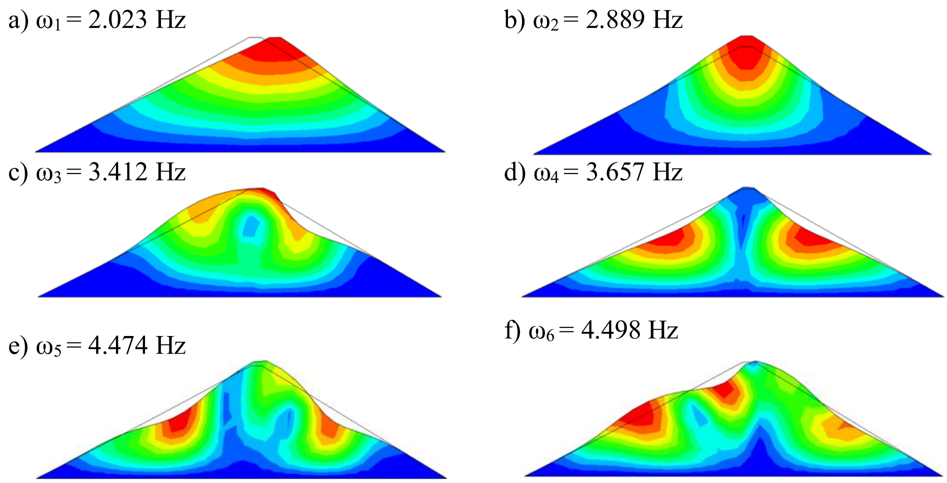

Fig. 2. First six eigenfrequencies and modes of vibrations of the Gissarak Dam

The study of the eigenfrequencies obtained indicates that for the dam under consideration, some higher frequencies are located quite densely in a narrow spectrum.

The main low-frequency modes of natural vibrations for the dam are (Fig. 2) - the central section shear (the first mode); vertical displacements of the central section (the second mode); and complex deformation of slopes (the third and subsequent modes).

Further, using the developed methodology and algorithm, the problem of unsteady vibrations of the dam is solved under kinematic excitation of the dam base of the following form:

$$x \in \sum\nolimits_u : \ddot{u}_o(t) = \ddot{\upsilon}_o(t) = F \sin pt, \quad 0 < t \le t^* \tag{23}$$

with initial conditions (24)

$$y_i(0) = y_{io}, \quad \dot{y}_i(0) = \dot{y}_{io} \quad . \tag{24}$$

To describe the viscoelastic properties of the dam material, a kernel of the form

$$\Gamma(t) = Ae^{-\beta t} t^{\alpha - 1} \qquad (0 < \alpha < 1) \tag{25}$$

is used with parameters: A=0,074, $\alpha = 0{,}25,\ \beta = 0{,}00243$, obtained from experimental creep curves for some types of soils. The coefficient of nonlinearity is $\gamma = 50000$.

In specific calculations, the acceleration amplitude in the base of the dam is F = 0.2 m/sec^2, the frequency of the kinematic action p in (23) is taken close to the second natural frequency of the structure, i.e. p=ω_2 =3.4 Hz (close to the resonance mode of oscillations).

Coefficients (20) of the system of integro-differential equations (19) are determined using the values of natural modes of oscillations shown in Fig. 2.

Figure 3 shows the changes in horizontal displacements u obtained for the dam point with coordinates (x_1 = 70.6 m, x_2 = 14.2 m) under a kinematic action of the form (23) with a duration t*=4 sec. As seen from the results, the forced oscillations of the system occur with a noticeably increasing amplitude. After the load is removed (i.e. after t=4 sec), the system performs free oscillations, and for the linear case, these oscillations are almost harmonic. For a nonlinearly elastic material of the dam, the amplitude begins to grow to certain values. This explains the non-linear relationship between stresses and strains, in this case, a rigid diagram □□□ for soil. For the viscoelastic case, the damping of oscillations with time is characteristic.

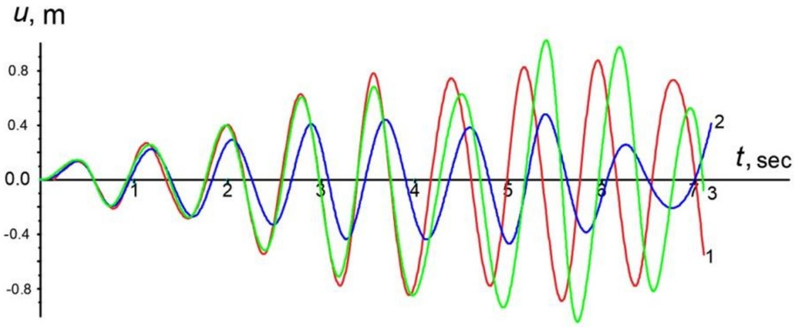

Fig. 3. Horizontal displacement of dam points (x_1=70.6 m, x_2=14.2 m) under unsteady vibrations

Figure 4 shows the change in stresses σ_{11} at the point of the dam (x_1 =310.8 m, x_2=39.4 m) under the action of an impulse with a duration of t*=4sec. Designations 1 and 2 refer to the case of linear and nonlinear elastic solutions to the problem, 3 - to the case when the problems are solved in a nonlinear viscoelastic formulation. Here, the maximum values of stresses that occur after a certain time after the termination of the pulse are given. The stresses for the linear and nonlinear elastic cases are harmonic in nature, and for the nonlinear viscoelastic case, the stresses decrease, which is a manifestation of viscosity.

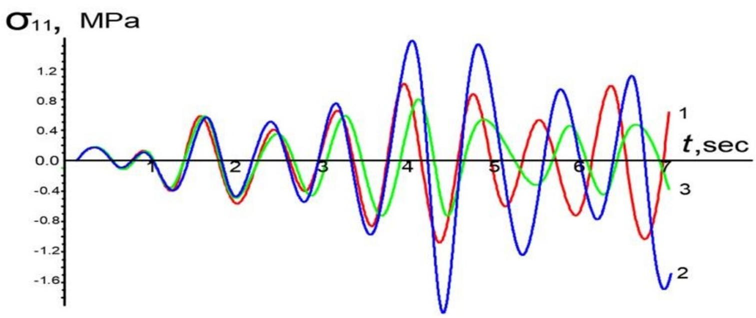

Fig. 4. Horizontal normal stresses of dam points (x_1=310.8 m, x_2=39.4 m) under unsteady oscillations

The conducted studies of forced oscillations of the dam under non-stationary kinematic effects showed that:
- under a prolonged kinematic impact, individual sections of the structure experience tensile dynamic stresses, i.e. the soil of the dam works in tension;
- an account for nonlinear elastic properties leads to a noticeable decrease in stresses arising in the body of the dam, depending on the acceleration values;

- an account for viscosity leads to gradual damping of oscillations over time, limiting the amplitude of oscillations throughout the body of the dam in resonant modes;
- near the resonant mode of oscillations, in order to take into account the physical nonlinearity of the material of the structure, the occurrence of large stresses in some individual sections of the structure is observed, which can lead to a violation of the strength of the dam.

4 Conclusions

1. The article provides a detailed review of well-known publications related to the study of the dynamic behavior of a number of plane structures.
2. A mathematical model, method, and algorithm are given for solving a dynamic plane problem for bodies, considering the nonlinear elastic and viscoelastic properties of the material under the action of various kinematic impacts.
3. The eigenfrequencies and vibration modes of an earth dam are determined on the example; nonlinear steady-state forced vibrations of a real dam are studied, taking into account the nonlinear elastic and viscoelastic properties of the structure material under various kinematic effects near the resonant modes.
4. Unsteady forced oscillations of an earth dam near the resonant mode of oscillations are studied, taking into account the physical nonlinearity of the material of the structure; the occurrence of large stresses in some individual sections of the structure is shown, which can lead to a violation of the strength of the dam.

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