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ABSTRACTS

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I. SECTION. SCIENTIFIC HERITAGE OF AL-KHWARIZMI

Muhammad ibn Musa al-Khorezmiy-great thinker

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The great mathematician, astronomer and geographer Muhammad al-Khwarizmi lived and worked in the late eighth and first half of the 9th century. At the beginning of the 9th century, Muhammad al-Khwarizmi served at the court of Al-Mamun al-Rashid in Marw and, on his orders, traveled to the west of India and became acquainted with their mathematics. As a result of this, he wrote "on Indian numbers"(Khisab al-Hind). The work has been important in the history of science, causing the spread of Indian numerals and the decimal counting system, which are called "Arabic numerals"all over the world.

Only 10 of the more than 20 works belonging to the Khwarizmian pen have come down to us. Of these, "a short book on the account of al-Jabr and al-Muqabala"is an algebraic work, "Book about Indian accounting"or "Book on addition and subtraction"is an arithmetic work. Khwarizmi develops it, giving an idea of the Indian account in the introduction to his arithmetic treatise.

With this work, Khwarizmi cites algebra as an independent and separate science.

The work consists mainly of three sections:

1) solving equations with one unknown of order 1 and 2 using al-Jabr and al-Muqobala, perform actions with rational and irrational expressions and ways of solving numerical tasks are given using the equation;

2) dedicated to geometry, in this case, some applications of algebra are shown to tasks related to the measurement of quantities;

3) issues of practical application of algebra, i.e. inheritance, are given.

Khorezmi's "Zij"(astronomy tables) and comparing Ptolemy's works on geography, he writes "Kitab surat al-arz". It is an important work dedicated to geography and geodesy.

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Effective use of al-Khwarizmi's heritage in mathematics lessons

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This article discusses the discoveries and innovations of ancient and modern scientists in mathematics, their application to life, mathematical laws, mathematical talent, and similarities in the research of Eastern and Western scientists, as well as information about the activities of life.

Mathematics is an important field in the system of sciences and means "Science"in Greek. Also math - is one of the ancient sciences, other arithmetic, geometry, later algebra, forming mathematical analysis, analytical geometry, one after the other, improved them. The fields of mathematics are diverse despite the fact that they belong to one subject under the sign of commonality combined. We can clearly see this commonality from the following definition of mathematics. "Mathematics is the real thing quantitative relations and spatial forms".

A great encyclopedic scholar is Al-Khwarizmi. Al-Khorazmi He made many achievements by conducting research in the field of mathematics. Al-Khorazmi Abu Abdullah Muhammad ibn Musa (783-850) was a great mathematician, astronomer, geographer. Despite the dominance of idealistic ideas in his time, Al-Khorazmi was a painstaking scientist following his ways, he opened a wide path to advanced social and philosophical thinking and created immortal discoveries in mathematics. He dedicated his creativity to the development of science and advanced ideas sought to convey to the peoples of the world. Ten of the works written by Al-Khorazmi are up to us has arrived. Two of them are dedicated to algebra and arithmetic, science played an important role in its history. In the words of the well-known researcher D. Sarton, he:

"The greatest mathematician of his time and, in many ways, all He is one of the greatest mathematicians of all times"and is important in the history of science became an unquenchable torch with his discoveries. Khorezm to Latin in the 12th century The translated work "Arfmetika"("Kitab filhisab al hind") made the Europeans Indian numbers, introduced the positional decimal number system and its other numbers showed its advantage over systems. Performing operations on whole and fractional numbers and introduced the square root method. Khorezmi says: Imam Ma'mun's interest in science and the difficulties encountered in the work of scientists in this field qualities such as helpfulness encouraged me to write a short work on calculation. In writing this work, it is understandable, easy, useful and interpersonal for the students especially inheritance, which helps facilitate computational work in problems in distribution, making a deal, in trade, land surveying and other similar things I wrote that I intended it to be a guide for calculations.

Only 10 of more than 20 works belonging to Khorezmi's pen have reached us. These are: "A short book about algebra and al-muqabala calculus"is an algebraic work; "A Book on Indian Arithmetic" or "A Book on Addition and Subtraction" is an arithmetical work; "Kitab surat-ul-arz"is a work on geography. Also "Zij "Book about working with Asturlob "Book about making Asturlob "About determining azimuth using Asturlob "Kitab ar-ruhoma "Kitab at-tarikh "About determining the Jewish calendar and holidays pamphlet". Four of these works have been preserved in Arabic, one in Farghani's work, two in Latin translation, and the remaining three have not yet been found.

The scientific heritage of our Great Ancestor

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The aim of this study is to introduce Muḥammad ibn Mûsâ alKhwârizmî and his works in terms of history of mathematics and mathematics education. Muḥammad ibn Musa al-Khwârizmî an Iraqi Muslim scholar and it is the first of the Muslim mathematicians who have contributed to this field by taking an important role in the progress of mathematics in his own period. He found the concept of Algorithm in mathematics. In some circles, he was given the nickname Abu Ilmi'lHâsûb (the father of the account). He carried out important studies in algebra, triangle, astronomy, geography and map drawing. Algebra has carried out systematic and logical studies on the solution of inequalities at second level in the development of the algebra. He with all these studies have contributed to mathematical science and today was a guide to the works done in the field of mathematics.

Muhammad al-Khorazmi is a scientist who made an invaluable contribution to world science with his genius discoveries. Information about his age has not been preserved. However, taking into account that he was invited to the court of Ma'mun ibn Harun al-Rashid, the governor of Khurasan in Baghdad in the early 800s, he studied in his native Khorezm and became a famous scholar in his twenties. In 813, Mamun took the caliphate crown and moved to Baghdad with the scholars gathered around Marwda. A great lover of science, Ma'mun founded the "Bayt al-hikma"("House of Wisdom"), which is called "Baghdad Academy"in the history of science. Muhammad al-Khorazmi was the director of this scientific center until the end of his life. Many scientists of the Arab East from the countries of Central Asia worked here. A library full of ancient manuscripts and a specially built observatory served them.

Khorezmi's "Zij"consists of 37 chapters and 116 tables. The first five chapters of the work are devoted to chronology, in which "flood "iskandar "safar"and the rules for converting AD dates into Hijri are presented. Chapter 6 describes the division of a circle into 12 signs, a sign into 30 degrees, a degree into 60 minutes, a minute into 60 seconds, and so on. Chapters 7–22 deal with the motions of the Sun, the Moon, and the five planets. In these chapters, Khorezmi skillfully used ancient and early medieval Indian astronomical data, Iranian and Greek sources, and based on Ptolemy's geocentric system, described the movements of the planets. Chapter 23 is devoted to trigonometry: in it Khorezmi introduces the concepts of "plane"and "reflected sine"and gives tables of these functions. Chapters 25–27 are devoted to mathematical geography. Here are the rules for determining the longitude and latitude of geographical places, and it is shown that the change of these coordinates is related to the change of the ecliptic and equatorial coordinates in the annual and nightly movement of the Sun.

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II. SECTION. MATHEMATICAL MODELING

Dual-phase-lag theory reflection of thermoelastic waves from a rotating half-space with gravitation

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In this research, the generalised thermoelasticity theory is used to investigate the reflection of plane harmonic waves from a semi-infinite rotating elastic solid with a gravity field. The reflection coefficients, which are the ratios of reflected wave amplitudes to incident wave amplitudes, are calculated. The numerical solution is carried out, and the graphic representation of the influence of the rotation and gravity fields on the reflection coefficient is shown. In the presence and absence of rotation and gravity, comparisons are conducted with the predictions of the dual-phase-lag model and Lord-Shulman theory. The rotational and gravitational effects explain the reflection coefficient of waves on the free surface of half-space.

Model equations for stresses

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The development of mathematical models for stresses, from the point of view of determining the safety margins of structures and their elements, is an important and urgent problem of mathematical modeling.

Usually, a two-dimensional boundary value problem in stresses consists of two equilibrium equations and a strain compatibility condition and is reduced to solving a biharmonic equation concerning the additional Airy function [3]. In this paper, a modified version of the two-dimensional model equation is considered, consisting of differentiated equilibrium equations and the Beltrami-Mitchell equation [2] for stresses, i.e.

$$\begin{array}{l} \frac{\partial^2\sigma_{11}}{\partial x^2}+\frac{\partial^2\sigma_{12}}{\partial x\partial y}=0, \frac{\partial^2\sigma_{21}}{\partial x\partial y}+\frac{\partial^2\sigma_{22}}{\partial y^2}=0 \\ \frac{\partial^2\sigma_{12}}{\partial x^2}+\frac{\partial^2\sigma_{12}}{\partial y^2}+\frac{1}{1+\upsilon}\frac{\partial^2(\sigma_{11}+\sigma_{22})}{\partial x\partial y}=0 \end{array} \tag{1}$$

with boundary conditions

$$\begin{array}{ll} x=0,\ell_1: & \sigma_{11}|_{x=0,\ell_1}=1-\frac{y^2}{\ell_1}, \quad \sigma_{12}|_{x=0,\ell_1}=0, \\ y=0,\ell_2: & \sigma_{22}|_{y=0,\ell_2}=0, \quad \sigma_{12}|_{y=0,\ell_2}=0 \end{array} \tag{2}$$

and, additional boundary conditions[3]:

$$\begin{array}{ll} x=0,\ell_1: & \left.\left(\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}\right)\right|_{x=0,\ell_1}=0, \\ x=0,\ell_2: & \left.\left(\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}\right)\right|_{y=0,\ell_2}=0. \end{array} \tag{3}$$

For the boundary value problem (1-3), finite-difference equations are compiled, which, unlike the classical ones, are symmetric and have a second order of approximation. Using the method of alternating directions and the method of iteration, the problem of a free rectangle stretched by a parabolic load from opposite sides is solved numerically. Comparison of the results of the two methods, with the well-known solutions of Timoshenko [1], ensures the validity of the formulated model equation and their numerical results.

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Disclosure of the hierarchy analysis method in the mpriority program using the example of buying a house

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Decision-making processes are at the heart of any purposeful activity. The need to make decisions under conditions of uncertainty arises in all areas of technology, economics and social life. Therefore, it is necessary to strive for the optimal use of the available information and, after weighing all possible solutions, try to find the best one among them. The Hierarchy Analysis Method (HAM) was developed by the American scientist T. Saaty [1]. It provides, with the help of simple and reasonable rules, the solution of multicriteria problems that contain qualitative and quantitative factors, while quantitative factors can have different dimensions. HAM is used to solve semi-structured and unstructured problems. The method is based on the decomposition of the task and its presentation in the form of a hierarchical structure. This allows you to include in the hierarchy all the knowledge on the problem being solved. As a result of the decision, the numerically expressed relative degree of interaction of elements in the hierarchy is determined. The solution of the problem with the help of HAM is carried out in stages.

The first stage involves the representation of the problem in the form of a hierarchy [2]. In the simplest case, the hierarchy is built starting from the goal, which is placed at the top of the hierarchy, through intermediate levels, on which the criteria are placed and on which subsequent levels depend, to the lowest level, which contains a list of alternatives.

Second stage. At this stage, it is necessary to prioritize the criteria and evaluate each alternative against the criteria to select the most important one.

Third stage. After the formation of the matrices of paired comparisons for all criteria and alternatives, it is necessary to determine the eigenvectors of the matrices, check the consistency of the matrices using their eigenvalues, and synthesize the global priorities of alternative solutions relative to the main goal.

The method of analysis of hierarchies allows modeling the psychological features of making expert decisions in multicriteria tasks. The proposed technique reveals the possibilities of the hierarchy analysis method and is quite simply implemented in the MPriority program.

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Coexistence of chaotic attractors for two-dimensional logistic map

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P_n is a representation of the proportion of the total population that exist in generation n. The population is then persumed to meet the conditions of the given difference equation $P_{n+1} = \lambda P_n(1 - P_n)$ where λ is positive constant. We may write $x = P_0$ and $f(x) = \lambda x(1 - x)$. We have a few iterates $P_1 = f(x)$, $P_2 = f(f(x))$, $P_3 = f(f(f(x)))$ and so on. This function is called Logistic map and the mathematical investigation of its dynamics has been a significant focus of research over the last half-century.

In this paper we try to find strange attractors for two-dimensional quadratic map, which are given by the following map

$$y = x^2 + c$$

As we know the point x is a fixed point for f if $f(x) = x$ and the point x is a periodic point of period n if $f^n(x) = x$

Definition. Let p be a periodic point of prime n. The point p is hyperbolic if $|(f^{(n)})'(p)| \neq 1$ and the fixed point p with $|f'(p) < 1$ is called attracting periodic point (an attractor)

An attractor can be a point, a finite set of points, a curve a manifold. It is called strange attractor (or chaotic) if it has a fractal structure.

Let $x = (x_1, x_2, ..., x_n) \in R^n, c = (c_1, c_2, ..., c_n) \in R^n, I = 1, 2, ..., n$ and $\pi : I \to I$ some permutations. We learn mapping

$$x'_k = x^2_{\pi(k)} + c_{\pi(k)} \tag{4}$$

on R^n to itself. It is multidimensional case of quadratic map. First, let's learn $n = 2$ case where mapping (4) is

$$\begin{cases} x' = y^2 + c_1 \\ y' = x^2 + c_2 \end{cases} \tag{5}$$

here $(x, y) \in R^2$ and $(c_1, c_2) \in R^2$. Our main interest is defining conditions for the mapping (5) to have strange attractor by (c_1, c_2) parametrs

Theorem. The map (5) has strange attractors when $c_1 = c_2 = -2$.

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On the behaviors of solutions of a nonlinear diffusion system with a source and nonlinear boundary conditions

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We considered the doubly nonlinear degenerate parabolic equations with the source

$$\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x}\left(\left| \frac{\partial u_i^k}{\partial x} \right|^{m-1} \frac{\partial u_i^k}{\partial x} \right) + u_i^{p_i},\ x \in R_+, t > 0,\ i = 1,\, 2,$$

coupled through nonlinear boundary conditions:

$$-\left| \frac{\partial u_i^k}{\partial x} \right|^{m-1} \frac{\partial u_i^k}{\partial x} \bigg|_{x=0} = u_{3-i}^{q_i}(0,t), \quad t > 0,\ i = 1,\, 2,$$

where $m > 1$, $k \geq 1$ and q_i, $p_i > 0$ are numerical parameters. The following initial data should be taken into account

$$u_i|_{t=0} = u_{i0}(x),\ i = 1,\, 2.$$

It is expected that the function and its corresponding first- and second-order derivatives conform to a set of criteria. Specifically, these derivatives should exhibit a degree of continuity, non-negativity, and compactness within the domain of R_+. Through the creation of numerous self-similar supersolutions and subsolutions, researchers have been able to derive the curve of critical global existence and Fujita type critical exponent. These self-similar solutions have proved to be invaluable in understanding the behavior of nonlinear diffusion systems.

Theorem 1. If $q_1 q_2 \leq \left(\dfrac{m}{m+1} \right)^2 (k+1-p_1)\,(k+1-p_2)$, then every nonnegative solution of the problem ()-() is global in time.

Theorem 2. If $0 < p_i \leq 1$ and $q_i \geq \dfrac{m(p_{3-i}-1)(p_i+k)}{(p_i-1)(m+1)}$ or $p_i > 1$ and $q_i \leq \dfrac{m(p_{3-i}-1)}{(p_i-1)(m+1)} \times (p_i+k)$ then, each of the solutions to ()-() blows up.

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Mathematical modeling of work synchronizer of the planetary mechanism

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In the apparatus of cotton pickers, planetary mechanisms are used to drive the spindles. By type, planetary gears are divided into: belt-friction and gears. According to the nature of the movement of the satellites, planetary gears are divided into epicyclic and hypocyclic. According to the frequency of the technological process, planetary gears are divided into cyclic and bicyclic. In bicyclic planetary gears, the satellites are continuously reversed, i.e. for one revolution of the carrier, the satellites change their direction of movement around their axis twice. Structurally, planetary gears are divided into two types: without opening and with opening of the kinematic chain. In the first case, the entire dynamic load falls on the gearing in the satellite reverse zone. In our case, we consider a cycle with opening of the kinematic chain. Therefore, in the transition zones, a synchronizer is installed for a smooth entry into the gearing with the segment [1].

The purpose of the study is to determine the frequencies of natural oscillations and the stability of the synchronizer in the operating mode. With a disturbing force F_0 from the side of the satellite tooth, we obtain forced oscillations of the link (figure) in the form of a straightened sinusoid:

$$F(t) = F_0 \sin \pi t / t_0$$

We determine the non-periodic forced oscillation of the synchronizer of the planetary mechanism:

$$y = F_0/c \left[\frac{4t\,(t - t_0)}{t_0^2} + 8/p^2 t^2 \left(1 + \cos pt - 4/p^2 t^2 \cdot \sin pt\right) \right]$$
in case of $0 < t < t_0$

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Information processing of the operation parameters of the synchronizer of the planetary mechanism

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Planetary mechanisms with reversing satellites are divided into two types: with opening and without opening the kinematic chain. In the first case, a synchronizer is used, which is installed between two toothed segments of internal and external gearing. The synchronizer of the planetary mechanism is designed for smooth entry of the satellites into gearing with the segments of the stationary wheels. Structurally, synchronizers are divided into two types: with angular and linear movement. The angular displacement synchronizer is an arc-shaped toothed segment consisting of several teeth, one end is located on a fixed support, and the other free end rests on a compression spring with a limiting device. The angle of movement of the synchronizer is determined by the formula

$$\varphi = \arccos\left(2l^2 - h^2\right)/2l^2$$

The linear synchronizer consists of a single tooth mounted on a slide with radial movement to the depth of the tooth height of the gears.

Under the influence of the disturbing force F_0 from the satellite tooth of the synchronizer of the planetary mechanism, we obtain a graph (drawing) of the movement of the link in the form of a straightened sinusoid according to the formula

$$F = F_0|\sin\omega t|$$

The law of the periodic perturbing force from the satellite tooth of the planetary mechanism is determined from the expression:

$$y = F_0/c\left[p\omega/\left(\omega^2 - p^2\right)\left(\sin pt + \operatorname{ctg}\pi p/2\omega \cdot \cos pt\right) - p^2\sin\omega t/\left(\omega^2 - p^2\right)\right]$$

Formulation of the mathematical model of the machine unit of the coulisse mechanism of the mortar pump

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The article is devoted to the study of the crank-rocker mechanism of the mortar pump, in which the leading link - the crank produces a continuously rotational motion, and the driven link - the piston performs a reciprocating motion. Here, the intermediate link of the device is a rocker mechanism that performs an oscillatory (rocking) movement, which allows for the transformation of continuous rotation of the crank of the rocker into a rocking motion with different intervals of forward (working) and reverse (idle) movement, due to the asymmetric phase angle φ_W of the working and idle φ_I stroke of the rocker mechanism. Therefore, the coulisse mechanism allows you to get a reduced speed V_W in the piston stroke zone than V_I the idle speed: $V_W < V_I$, because the phase angle φ_W of the working stroke of the coulisse is greater than the phase angle φ_I of the idle stroke: $\varphi_W = 240° > \varphi_I = 120°$.

When constructing a mathematical model of the machine unit of the crank-rocker mechanism of the mortar pump, we will assume that this is a dynamic model of this device. So in the mechanism under study, we load the initial and final forces F_1 and F_6, the given moments of inertia J^g_Σ, the reduced torque M^{re}_Σ, as well as the angular velocities of the initial ω_1 and final ω_6 links. The angular velocity ω_1 of the initial link is a dynamic model ω_m of the machine unit: $\omega_1 = \omega_m$

When constructing a model of the mechanism, all the forces and moments applied to it turn out to be reduced to one link and they are replaced by the given moment M^{re}_Σ. Their algebraic sum gives the value of the total reduced moment:

$$M^{re}_\Sigma = M^{re}_F + M^{re}_{F_3} + M^{re}_{F_4} + M^{re}_{F_5} + M^{re}_{F_6},$$

applied to the dynamic model.

The total reduced moment is defined as:

$$M^{re}_\Sigma = M^{re}_d + M^{re}_r,$$

where: M^{re}_d - reduced driving torque; M^{re}_r - the reduced torque of the resistance.

Thus, due to the reduction of forces, the entire main load applied to the crankrocker mechanism was measured by one total reduced moment M^{re}_Σ.

Determination of the border length of certain parts of relief surfaces using fractal measurement

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Objects with fractal properties have a fractal dimension, and the main and important approach to this concept is considered geometric dimension relations. Geometric measurement relations in Euclidean space are derived from the principle of dimensional consistency [1]. A fractal dimension (such as length) is not proportional to another dimension (such as area), meaning they do not have the same spatial dimension. We can derive the Euclidean geometric measure relation, which can be generalized to the fractal geometric measure relation From the principle of uniformity of measure. Fractal scale relationships can be used to obtain fractal scale and map scale indicators [3]. The map scaling relation can be thought of as a generalized fractal scaling relation. The commonality is available between the various geometric scaling relationships, which are used to obtain the boundary dimension of self-similar shapes embedded in two-dimensional space. The boundary length scaling parameter of relief surfaces and the boundary dimension fractal dimension can be obtained from relations. If enough data is available, various fractal parameters can be calculated. If we take the border of an area as an example, it is possible to study the spatial structure using the grid method and determine the degree of expansion of the area using fractal measurement. If there is not enough information to determine the boundary length of the area, the boundary length of the city can be calculated by determining the fractal dimension [2]. Referring to the Euclidean space, it is possible to construct a number of formulas for calculating the fractal dimension. In this case, the obtained image of the area is divided into simple triangle, square, regular hexagon and circle parts, and the fractal dimension is determined [3].

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The properties of the solution to Cauchy problem for a degenerate equation with absorption and variable density

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In the domain $Q = \left\{(t,x) : t \geq t_0 > 0, x \in R^N\right\}$, we consider the following problem:

$$|x|^{-n}\,\partial_t u = u^q div\left(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - |x|^{-n}\,t^l u^\beta,\ (x,t) \in Q \tag{1}$$

$$u\left(0,x\right) = u_0\left(x\right) \geq 0, \qquad x \in R^N \tag{2}$$

where $k, m \geq 1, p \geq 2, 0 < q < 1$, and non-negative n, l, β are given numerical parameters. The (1)-(2) arises in different applications [1]. The equation (1) might degenerate at the points where $u = 0$ and $\nabla u = 0$. Therefore, in this case, we need to consider a weak solution from having a physical sense class. The problem (1)-(2) has been intensively studied by many authors (see [2]-[4] and the literature therein).

Let us take the function

$$z\left(t,x\right) = l_1 t^{\frac{1+l}{1-\beta_2}}\left(t\right)\overline{f}\left(\xi\right) \tag{3}$$

$$\overline{f}\left(\xi\right) = \begin{cases} \left(a - b\xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{m_2+k_2(p-2)-1}}, & \text{if } m_2 + k_2\left(p-2\right) \neq 1, \\ e^{-b\xi^{\frac{p}{p-1}}}, & \text{if } m_2 + k_2\left(p-2\right) = 1, \end{cases} \tag{4}$$

where $m_2 = \frac{m}{1-q}, k_2 = \frac{k}{1-q}, \beta_2 = \frac{\beta-q}{1-q}, l_1 = \left[\frac{(1-q)(1-\beta_2)}{1+l}\right]^{\frac{1}{1-\beta_2}}, a = const > 0$ and $b = \frac{p\left[pk_2^{p-2}\right]^{-\frac{1}{p-1}}}{m_2+k_2(p-2)-1}$ are constants.

Theorem 1. Let us $m_2 + k_2\left(p-2\right) - 1 \geq 0$, $p > n$, $u\left(0,x\right) \leq z\left(0,x\right),\ x \in R^N$. Then for solution of the problem (1)- (2) an estimate

$$u\left(t,x\right) \leq z\left(t,x\right) \quad \text{ in } Q$$

hold.

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To the qualitative properties of the solution of a nonlinear parabolic equation with a time-dependent damping term

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Consider in $Q = \{(t,x) : t > 0,\ x \in R^N\}$ the following Cauchy problem

$$\frac{\partial u}{\partial t} = div(u^{l-1}\left|\nabla u^k\right|^{p-2}\nabla u) - g(t,x)u^{q_1}|\nabla u^m|^{p_1} \tag{1}$$

$$u(0,x) = u_0(x),\ x \in R^N \tag{2}$$

where $l,\ m,\ k\ \geq\ 1,\ p\ \geq\ 2,\ p_1,\ q_1\ \geq\ 0$ are given numerical parameters characterizing nonlinear media, $\nabla(\cdot) = grad_x(\cdot)$, function $u = u(t,x) \geq 0$ is a solution. In this paper, we consider cases $g(t,x) = t^{\sigma},\ \sigma > 0$. Equation (1) describes the processes of nonlinear heat conduction, diffusion, biological population and various other processes [1-4].

In this paper, based on the study of a self-similar equation of the form

$$u(t,x) = u(t,r)$$

$$\frac{\partial u}{\partial t} = r^{1-N}\frac{\partial}{\partial r}(r^{N-1}u^{l-1}\left|\frac{\partial u^k}{\partial r}\right|^{p-2}\frac{\partial u}{\partial r}) - t^{\sigma}u^{q_1}\left|\frac{\partial u^m}{\partial r}\right|^{p_1} \tag{3}$$

where $\overline{u}(t) = (T+t)^{\alpha}$, $\xi = |x|\,(T+t)^{-\beta}$, $T \geq 0$, $\alpha, \beta > 0$ and function $f(\xi)$ satisfies the following self-similar equation

$$\xi^{1-N}\frac{d}{d\xi}(\xi^{N-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df^l}{d\xi}) + \beta\xi\frac{df}{d\xi} + f^{q_1}\left|\frac{df^m}{d\xi}\right|^{p_1} + \alpha f = 0 \tag{4}$$

Let function $f\,(\xi)$ satisfy the following conditions:

$$f(0) = M,\ M \in R,\ f(d) = 0,\ 0 < d < \infty \qquad (5).$$

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Modeling and numerical calculations of chemically reacting turbulent jets based on three-dimensional Navier-Stocks equations

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This work is devoted to numerical modeling and research of reacting turbulent jets flowing from a rectangular nozzle with side lengths $2a$ and $2b$ into a cocurrent (flooded) oxidizer flow.

For mathematical modeling of this flow, three-dimensional parabolic systems of Navier-Stokes equations are used for multicomponent and chemically reacting gas mixtures. Algebraic and "$k-\varepsilon$" turbulence models were used to calculate the turbulent viscosity coefficient.

In the numerical integration of the system of equations with initial and boundary conditions, a two-layer implicit finite-difference scheme was used with an accuracy of the order of $O(\Delta x, \Delta y^2, \Delta z^2)$

As a method and algorithm for solving the system of Navier-Stokes equations, a modification of the SIMPLE method (Semi-Implicit Method for Pressure-Linked Equations) was used. To eliminate the mass imbalance at each calculated point of the grid, a difference continuity equation is used.

Corrections to velocities at each calculated point in space are calculated after solving the Poisson difference equation with respect to the potential-gradient function which expresses the corrections. In order to save computer time, the calculated amount of the grid increased with the expansion of the jet relative to the initial amount of the grid, i.e. in advance, the maximum number of grids were not allocated.

The main characteristics of the diffusion flame were studied, such as the length, shape when the propane-butane gas mixture flows out of a rectangular nozzle and propagates into the flooded (co-current) air flow.

- from the very beginning, the expansion for the free air jet and the reacting jet in the direction of the OZ axis, the boundary of the jet decreases while in the direction of the OY axis it grows;
- at the beginning of the jet propagation, it passes to the shape of an ellipse, and then, as it moves downstream, the shape of the jet takes a round shape, and such a pattern is observed for the torch farm;
- in the initial sections of the jet at the boundary of the displacement zone, the values of the transverse velocities ϑ and ω become comparable in absolute value with the value of the longitudinal velocity, and as we move away from the nozzle exit, the maximum value of the velocity ϑ and ω tends to zero, i.e. further, the jet behaves as a direct flow and can be considered as an axisymmetric problem;
- with a successful selection of the empirical parameters of the algebraic and two-parameter turbulence models, one can obtain well-matched parameters of the diffusion plume;
- changes in the speed of the fuel and oxidizer significantly affect the parameters of the torch.

To the properties solutions of the problem of nonlinear thermal conductivity with absorption

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This talk devoted to establishing new nonlinear effects such as effects of a finite velocity of propagation of thermal disturbances, spatial heat localization and finite time localization solution $(0 < q < 1)$, estimates of solutions and free boundary depending value of the numerical parameters, asymptotic solution in the critical value of the numerical parameters to the following the Cauchy problem for a degenerate double nonlinear parabolic equation in not divergent form

$$\frac{\partial u}{\partial x} = u^{n\nabla}\left(u^{m-1}\left|\nabla u^{k}\right|^{p-2}\nabla u\right) + \operatorname{div}(v(t)u) - b(t)u^{q}, \left(t > 0, x \in R^{N}\right),$$
$$\mathrm{u}(0,\mathrm{x}) = \mathrm{u}_0(x) \geq 0, x \in R^{N}\big] \ (\mathbf{1})$$

Here, u(x, t)-temperature, $\mathrm{m}, k \geq 1, p \geq 2, 0 < n < 1, 0 < v(t), b(t) \in c(0,\infty), \nabla() = .\operatorname{grad}_x(.) the parameters characterizing nonlinear processes$ b > 0, b(t)u^q - is the power of volumetric heat absorption. To investigating different qualitative properties of the solutions of the problem (1) for particular value of the numerical parameters intensively studied by many authors (see [1-3] and literature therein).

Based on the establishing qualitative properties of solutions the numerical analysis keeping nonlinear effects of solution of the problem (1), (2) carried out

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Mathematical model of the railway plan

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A railway track plan is a projection of the track axis onto a horizontal plane. The railway in the plan is a combination of alternating straight and curved sections [1].

The conjugation of straight sections with each other is carried out using curves. The specific proportion of all curvilinear sections of the track on the railway depends on the terrain. The mathematical plan of a railway track can be considered as a curve. Moreover, a railway track with a curved section is expressed as a spatial curve even in the case, when the relief appears to be an ideal horizontal plane. The spatiality of the curve of the railway track is associated with the overload of the outer rail and the requirement to reduce the centrifugal force.

The curvature of the road from a geometric point of view generates the curvature of the curve, expressing the plan of the path.

Obviously, since the curvature depends on the radius of the curve, the position of the railway line within the curved part of the track largely depends on the radius. Geometrically, this position generates the curvature σ_R of the curve under consideration.

When designing new railways, especially in conditions of complex terrain, the number of curves in the plan increases. This in turn affects the torsion of the curve representing the path plan. Therefore, the curvature of this curve depends on the elevation of the road in a horizontal plane, although the road is generally flat. So the total curvature of the path is the sum of the two curvatures: $\sigma = \sigma_R + \sigma_H$ where σ_H curvature depending on the angle of the touching plane with the horizontal plane.

If the path plan is considered as a spatial curve, then its Frenier formula expressing the dynamics of a curve with a differential form has the form:

$$\begin{cases} \dot{\tau} = k\nu; \\ \dot{\nu} = k\tau + (\sigma_R + \sigma_H)\beta; \\ \dot{\beta} = (\sigma_R + \sigma_H)\nu, \end{cases}$$

where are the vectors of the accompanying trihedron, and $(\dot{\tau})$ means the derivative with respect to the path length [2].

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Calculation of fractal dimension of blood vessels in the retina

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The issue of determining the fractal dimension of body organs which have fractal structure of the human body in medicine, which is currently developing rapidly, is considered in this article. In addition, it is important to provide practical support for early detection and treatment of various diseases in humans [1].

The clear idea of what parameters should be used to describe the structure of blood vessels is not given due to the complexity and multilevel repetition of the human vascular system. In addition, the criterion of normal development is necessary for the diagnosis of diseases. The several fractal analyzes have been performed to evaluate the circulatory systems of various healthy and diseased individuals in order to solve these problems [2]. Human vascular systems do not have a strictly fractal structure, as they are infinitely distributed, but they have similar properties, so the diffusion process is uniform. Therefore, human blood vessels have fractal properties and can be considered to form a fractal structure [3].

Blood vessel systems in the retina which have a fractal structure, serve to improve blood circulation in the eye, nourish it, and prevent functional disorders. Human eye blood vessels also have a fractal structure like the above fractal images, and the fractal dimension is determined. The blood vessels in the retina are separated and removed in order to do this. It is extracted on the basis of the following scheme, namely, an existing image which is loaded and the boundary measurements, are determined.

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On the mathematical model of the concentration of pollutants and their impact on the population of the river

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Toxic substances in the aquatic environment negatively affect all levels of the ecological hierarchy, including the behavior of organisms, population growth, community structure, and ecosystem integrity [1]. Many water quality criteria and effective control measures have been proposed to prevent species loss and maintain ecosystem functions .

Most of the environmental risks of toxicants are assessed in terms of individual responses. However, the interest of nature management lies in the long-term health of populations and ecosystems [2]. In order to draw conclusions that are of greater importance for ecosystems, it is necessary to assess the impact of environmental toxicants on the dynamics of exposed natural populations.

Mathematical modeling suggests a possible intermediate option by extrapolating toxic effects at the individual level to significant effects at the population level. Several types of mathematical models have been developed [3], including population matrix models and the ODE model to study influence of environmental toxicants on the dynamics of exposed populations in polluted aquatic ecosystems.

Consider a population and a toxicant in a polluted river or stream. Let $u(x,t)$ be the population density, and $w(x,t)$ be the toxicant concentration at point x and time t. The mathematical model describing the interaction between the population and the toxicant in a polluted river of length L has the form

$$
\begin{gathered}
u_t = d_1 u_{xx} - a_1 u_x + u[r - cu - mw], 0 < x < L, t > 0, \\
w_t = d_2 w_{xx} - a_2 w_x + [H(x) - puw - qw], 0 < x < L, t > 0, \\
d_1 u_x(0,t) - a_1 u(0,t) = u_x(L,t) = 0, t > 0, \\
d_2 w_x(0,t) - a_2 w(0,t) = w_x(L,t) = 0, t > 0, \\
u(x,0) = u_0(x) > 0, w(x,0) = w_0(x) > 0,
\end{gathered}
$$

where the model parameters d_1, d_2, a_1, a_2, r, c, m, p, and q are all positive constants.

First, we explore some mathematical issues related to the existence of a solution to a mathematical problem. Next, we establish the existence of a stationary state only for toxicants and a stationary state of coexistence of toxicants and the population. We obtain some sufficient conditions under which a population persists or dies out. Unlike traditional competition models for stream populations, the two reaction-diffusion-advection equations in our model have different forms and are asymmetric.

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Dynamics of the predator-prey system, taking into account the lower critical density of prey populations and intraspecific competition

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The presented work is devoted to a qualitative-numerical study of the generalized Volterra-Lotka model of the predator-prey system.

We additionally introduce intrapopulation and interpopulation factors of population interaction into the classical Volterra model. It is known that taking into account the nonlinearity factor of reproduction of isolated populations at low density, along with the factor of natural mortality, leads to the effect of the lower critical density of populations, called the Ollie effect. Taking into account the above factors, the model is described by a system of two ordinary differential equations depending on four parameters in dimensionless variables:

$$\begin{cases} \dot{x} \ = x\,(x-l)\,(1-x) - xy \\ \dot{y} = -\gamma y(m - x + \varepsilon y) \end{cases}$$

A complete qualitative study of the system at $\varepsilon = 0$ is being carried out (at ε=0 it was carried out in the book by A.D. Bazykin [1]).

We prove the existence of a region in the plane of parameters (l, m) corresponding to the joint existence of nontrivial singular points and the existence of a region whose points correspond to the existence of a limit cycle. The study of the system showed that the model predicts the possibility of four different modes of behavior of the system dynamics:

1). With a low adaptability of the predator to the prey, the predator population always dies out;

2). With an increase in fitness, a stable stationary coexistence of a predator with a prey is possible;

3). Coexistence of populations is possible only in a self-oscillatory mode, and the amplitude of oscillations is the greater, the higher the adaptability of the predator,

4). If the adaptability of the predator is too high, both populations are doomed to extinction under any initial conditions.

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Mathematical modeling of two-dimensional multiphase filtration in a deformable porous medium

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For many years, accurate modeling and simulation of two-phase flows has been a challenge for the safety and design of many industrial plant components. The homogeneous equilibrium model, the drift flow model [1], and the two-fluid model [2] are the most commonly used models for two-phase flows. The one-dimensional model of two-phase filtration in a porous medium, taking into account the deformation of the medium, was considered in [3].

The article considers the model of two-dimensional two-phase filtration in a deformable porous medium. An element of an oil reservoir with 4 oil wells is considered. Oil and water phases are involved in the filtration process.

In this case, the mathematical model can be represented as follows:
mass conservation law for phases

$$\frac{\partial}{\partial t}\left(m\rho_o s_o\right)+\frac{\partial \rho_o u_o}{\partial x}+\frac{\partial \rho_o v_o}{\partial y}=0$$
$$\frac{\partial}{\partial t}\left(m\rho_w s_w\right)+\frac{\partial \rho_w u_w}{\partial x}+\frac{\partial \rho_w v_w}{\partial y}=0$$

where $m = m_0+\beta_m\left(p-p_0\right)$ is dependence of porosity, m_0 is initial porosity, β_m is reservoir elasticity coefficient, p is reservoir pressure, p_0 is fixed pressure, s_o is oil saturation, ρ_o is density of oil phase, u_0 and v_o are the oil phase velocities in the direction of x and y, s_w is water saturation, ρ_w is water phase density, u_w and v_w are the water phase velocities in the direction of x and y.

Adding initial and boundary conditions, we obtain a mathematical model of two-dimensional two-phase filtration in a deformable porous medium. To solve the problem, the method of "large particles"was used. Based on the obtained results, the effect of medium deformation on the filtration process was studied.

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Relaxational fractional differential model of fluid filtration in a porous medium

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Many natural porous media have a fractal structure, the modeling of filtration processes in which requires the use of new approaches, methodologies and methods of analysis that differ significantly from traditional ones. The use of the classical theory of filtration of homogeneous fluids under an elastic regime based on Darcy's law sometimes leads to discrepancies between real and theoretical data [1].

The relaxation theory of fluid filtration as a non-classical, anomalous filtration was developed in [2] . In relaxation models of filtration the fractional differentiation tools were used in [3].

In this paper, in contrast to [3], we consider a generalized relaxation fractional differential model, where both relaxation phenomena with respect to filtration velocity and pressure gradient are used. On the basis of this generalized model, filtration equations are derived. A filtration problem for this equation is posed and numerically solved. The influence of the orders of fractional derivatives on the distribution of pressure in the medium at different moments of time is estimated.

The filtration model with double relaxation in one-dimensional case has the form

$$v + \lambda_v \frac{\partial v}{\partial t} = -\frac{k}{\mu} \frac{\partial}{\partial x} \left(p + \lambda_p D_t^{\alpha} p\right) \qquad (1),$$

where λ_v , λ_p are the relaxation times of the filtration velocity v and pressure p, respectively, k is the permeability of the medium, μ is the viscosity of the fluid, D_t^{α} are the Caputo fractional derivative operator.

For (1) the following piezoconductivity equation is derived

$$\frac{\partial p}{\partial t} + \lambda_v \frac{\partial^2 p}{\partial x^2} = \kappa \left(\frac{\partial^2 p}{\partial x^2} + \lambda_p D_t^{\alpha} \left(\frac{\partial^2 p}{\partial x^2} \right) \right) \qquad (2),$$

where $\kappa = \frac{k}{\mu \beta^*}$ is the coefficient of piezoconductivity, $0 < \alpha \leq 1$.

To solve the equation (2) with the appropriate initial and boundary conditions, the finite difference method was used. Numerical analysis shows that when α decreases from 1, a more intense pressure distribution p is observed. The influence of λ_v and λ_p on the distribution of pressure and filtration rate for various values of α was also evaluated. Conclusions about the joint influence of α, λ_v and λ_p on the characteristics of filtration are given.

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Modelling equations in strains

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Mathematical modelling equations with respect to deformations allows to more adequately investigate the safety margins and reliability of structures and their elements.

This paper is devoted to the formulation a two-dimensional boundary value problem in strains, consisting of two differentiated equilibrium equations (expressed with respect to deformations) and a compatibility condition in strains of the Beltrami-Mitchell type[1]

$$\begin{array}{l}(\lambda+2\mu)\frac{\partial^2\varepsilon_{11}}{\partial x^2}+\lambda\frac{\partial^2\varepsilon_{22}}{\partial x^2}+2\mu\frac{\partial^2\varepsilon_{12}}{\partial y^2}=0,(\lambda+2\mu)\frac{\partial^2\varepsilon_{22}}{\partial y^2}+\lambda\frac{\partial^2\varepsilon_{11}}{\partial y^2}+2\mu\frac{\partial^2\varepsilon_{12}}{\partial x^2}=0,\\ \mu(\frac{\partial^2\varepsilon_{12}}{\partial x^2}+\frac{\partial^2\varepsilon_{12}}{\partial y^2})+(\lambda+\mu)(\frac{\partial^2\varepsilon_{11}}{\partial x\partial y}+\frac{\partial^2\varepsilon_{22}}{\partial x\partial y})=0,\end{array} \tag{1}$$

with boundary

$$\begin{array}{l}\varepsilon_{22}|_{y=0}=\frac{1}{E_1}\sigma_{22},\varepsilon_{12}|_{y=0}=0,\varepsilon_{22}|_{y=l_2}=-\frac{1}{E_1}\sigma_{22},\varepsilon_{12}|_{y=l_2}=0,\\ \varepsilon_{11}|_{x=0}=0,\varepsilon_{21}|_{x=0}=0,\varepsilon_{11}|_{x=l_1}=0,\varepsilon_{21}|_{x=l_1}=0,\end{array} \tag{2}$$

and, additional boundary conditions

$$\left[\frac{\partial\varepsilon_{11}}{\partial y}\right]|_{y=0,l_2}=-\left[\frac{2\mu}{\lambda}\frac{\partial\varepsilon_{21}}{\partial x}\right]|_{y=0,l_2},\left[\frac{\partial\varepsilon_{22}}{\partial x}\right]|_{x=0,l_1}=-\left[\frac{2\mu}{\lambda}\frac{\partial\varepsilon_{12}}{\partial y}\right]|_{x=0,l_1} \tag{3}$$

for a rectangular area (l_1, l_2). The discrete analogue of the boundary value problem (1-3) is compiled by the finite difference method. According to the method of alternating directions, the solution of grid equations is reduced to the sequential application of the sweep method [2].

The tension problem of a rectangular plate by a parabolic load applied on opposite sides is solved. Numerical results are compared with the well-known solution of Timoshenko [3], thereby ensuring the validity of the formulated boundary value problem.

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Modeling of the biogen cycle in an ecosystem

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In this paper, we consider Lotka–Volterra mappings acting in a four-dimensional simplex with homogeneous tournaments. These mappings are interesting in that they can be offered as a model for studying the elements of living matter coming from the environment, which, having passed through a number of organisms, return to the external environment, and then are again included in the composition of living matter. For a more detailed study of the considered mappings, cards of fixed points are constructed and studied. According to the structure of the card, it is possible to represent the cycles of carbon, phosphorus and nitrogen .

Consider the Lotka–Volterra mapping, acting in a four-dimensional simplex having the form:

$$V : \begin{cases} x_1^{'} = x_1(1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4 + a_{15}x_5) \\ x_2^{'} = x_2(1 + a_{12}x_1 - a_{23}x_3 - a_{24}x_4 + a_{25}x_5) \\ x_3^{'} = x_3(1 + a_{13}x_1 + a_{23}x_2 - a_{34}x_4 - a_{35}x_5) \\ x_4^{'} = x_4(1 + a_{14}x_1 + a_{24}x_2 + a_{34}x_3 - a_{45}x_5) \\ x_5^{'} = x_5(1 - a_{15}x_1 - a_{25}x_2 + a_{35}x_3 + a_{45}x_4), \end{cases} \tag{1}$$

where the coefficients $0 < a_{ki} \leq 1$.

We know the following Theorem from [1].

Theorem 1.[1] Let $A = (a_{ki})$ is a skew-symmetric matrix, in this case

$$P = \left\{x \in S^{m-1} : Ax \geq 0\right\} \neq \varnothing, \ Q = \left\{x \in S^{m-1} : Ax \leq 0\right\} \neq \varnothing$$

consist of fixed points.

Here the simplex S^4 has four strong faces – $\overline{135}$, $\overline{145}$, $\overline{235}$ and $\overline{245}$. It is known [2] that two-dimensional faces corresponding to a strong subtournament they have one internal fixed point each. Then we get the following results:

Theorem 2. If the fixed points C_* and C_{**} form a pair (p, q), and in some neighborhood of the point C_* the mapping V is a local diffeomorphism, then there exists an invariant curve connecting C_* and C_{**}.

Theorem 3. Let there be four pairwise adjacent fixed points, which are both p points and q points for the faces containing them. Then there is an invariant surface passing through these fixed points.

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Dynamics of the composition of degenerate Lotka-Volterra mappings-as a model for studying the course of sexually transmitted viruses

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The paper is devoted to the study of the dynamics of the asymptotic behavior of the internal points of the composition of two discrete Lotka-Volterra mappings with partially oriented graphs. We propose to consider this composition as a discrete model of sexually transmitted viral diseases with a non-recurrent character, i.e. as a composition of two models SIR.

Let the Lotka–Volterra mappings acting in S^2 have the form [1], [2]:

$$V_1 : \begin{cases} x_1' = x_1[1 + ax_3], \\ x_2' = x_2[1 - bx_3], \\ x_3' = x_3[1 - ax_1 + bx_2], \end{cases} , \quad V_2 : \begin{cases} x_1' = x_1[1 - cx_3], \\ x_2' = x_2[1 + dx_3], \\ x_3' = x_3[1 + cx_1 - dx_2]. \end{cases}$$

The composition of these mappings has the form:

$$W = V_1 \circ V_2 : \begin{cases} x_1' = x_1[1 - cx_3][1 + ax_3[1 + cx_1 - dx_2]], \\ x_2' = x_2[1 + dx_3][1 - bx_3[1 + cx_1 - dx_2]], \\ x_3' = x_3[1 + cx_1 - dx_2][1 - ax_1[1 - cx_3] + bx_2[1 + dx_3]]. \end{cases}$$

Theorem 1. Let the composition $W = V_1 \circ V_2$. Then the fixed point is $A_1 \in \Gamma_{13}$:
– on condition $c = \frac{a}{a+1}$ will move to the top $e(0; 0; 1)$;
– on condition $c = \frac{a}{1-a}$ moves to the top of $e(1; 0; 0)$;
– in other cases, $A_1\left(\frac{\sqrt{a}(c-2)+\sqrt{c(ac+4)}}{2c\sqrt{a}}; 0; \frac{\sqrt{a}(c+2)-\sqrt{c(ac+4)}}{2c\sqrt{a}}\right)$ – repeller.
Fixed point $A_2 \in \Gamma_{23}$:
– on condition $d = \frac{b}{1-b}$ will move to the top $e(0; 0; 1)$;
– on condition $b_{23} = \frac{a_{23}}{1-a_{13}}$ moves to the top of $e(0; 1; 0)$;
– in other cases, $A_2\left(0; \frac{\sqrt{b}(d+2)-\sqrt{d(bd+4)}}{2b\sqrt{c}}; 0; \frac{\sqrt{b}(d-2)-\sqrt{d(bd+4)}}{2b\sqrt{c}}\right)$ – repeller.

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Mathematical models of traffic estimation and their types

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Mathematical models play a crucial role in traffic management by providing valuable insights, optimizing traffic flow, and aiding decision-making processes. Mathematical models can determine the capacity of road networks and identify bottlenecks or areas where traffic flow can be improved. By analyzing factors such as road geometry, traffic volume, and signal timings, these models can suggest optimal configurations and strategies for reducing congestion and maximizing throughput. Mathematical models form the foundation of ITS, which encompasses technologies such as traffic surveillance, dynamic message signs, and adaptive signal control systems. By integrating real-time data with mathematical models, ITS can monitor traffic conditions, detect incidents, and dynamically adjust traffic control strategies to optimize flow and enhance overall system performance. Also, mathematical models provide a systematic and quantitative approach to analyze and optimize traffic flow on roads. Using data, algorithms, and mathematical techniques, these models lead to improved mobility, reduced congestion, and improved overall transportation systems.

There are different approaches and models for traffic simulation, depending on the level of detail and complexity required. Here are a few commonly used types of traffic simulation models:

1. *Macroscopic Models;*
2. *Microscopic Models;*
3. *Hybrid Models;*
4. *Agent-based models.*

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Combined deep bed and cake filtration model

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At the initial value of time, we consider a layer of finite length with porosity m_2 and a homogeneous fluid (that is, a fluid without suspended particles). Starting from $t > 0$ at the point $x = 0$, an inhomogeneous liquid containing two different solid particles with a concentration of $c_0 = c_{10} + c_{20}$ begins to enter the layer. In this case, the large particles in the suspension sit on the surface of the porous medium and form a cake layer, while the small particles enter the porous medium and the deep layer participates in the filtration process. It is also observed that small particles settle in the cake layer.

Deep layer filtration equations for porous media $x \in [0, s_0]$

$$m_2\frac{\partial c_2}{\partial t} + v\frac{\partial c_2}{\partial x} + \frac{\partial \sigma_2}{\partial t} = D_2\frac{\partial^2 c_2}{\partial x^2}, \quad x \in [0, s_0],$$
$$\frac{\partial \sigma_2}{\partial t} = \begin{cases} k_{2r}v\sigma_2, & 0 < \sigma_2 \le \sigma_{2r}, \\ k_{2a}vc_2 - k_{2d}\sigma_2, & \sigma_{2r} < \sigma_2 < \sigma_{20}, \\ 0, & \sigma_2 = \sigma_{20}, \end{cases} \quad x \in [0, s_0],$$

where c_2 - the concentration of small particles in the suspension, v - the seepage rate, m_2 the porosity coefficient of the porous medium, σ_2 - the sediment concentration D_2 formed in the porous medium, - the diffusion coefficient in the porous medium, where σ_{20} is the total capacity of the filter, σ_{2r}- is the charging limit characterizing parameter of σ_2 , $k_{2r}-$ kinetics related to the "loading"effect in the porous medium coefficient, k_{2a} is the coefficient characterizing the deposition of solid particles in the porous medium, k_{2d} is the coefficient characterizing the detachment of solid particles.

A term that characterizes cake layer growth

$$\frac{ds}{dt} = \frac{c_1 v}{(1 - m_1)}, \quad x = s(t),$$

where c_1 is the concentration of large particles in the suspension, $s(t)$ is cake layer thickness, m_1 is cake layer porosity coefficient

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Mathematical model of solute transport in porous media with multistage deposition kinetics

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A two-zone porous medium with active and passive zones is considered. The transport equation has the form [1]

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2}-\frac{1}{\theta}\frac{\partial c_a}{\partial t}-\frac{1}{\theta}\frac{\partial c_p}{\partial t}-\lambda_e c$$

where c is the concentration of the substance in the liquid $(\mathrm{m}^3/\mathrm{m}^3)$, c_a is the concentration of the adsorbed substance in the active zone $(\mathrm{m}^3/\mathrm{m}^3)$, c_p- concentration of adsorbed substance in the passive zone, θ is porosity, λ_e is the first order decay (decomposition) coefficient, v is the component of the physical velocity of the liquid (solution).

For concentration in the active zone, we accept the following kinetic equations

$$\frac{\partial c_a}{\partial t}=\begin{cases}\beta_{ar}vc-\lambda_{ea}c_a, & \text{if} \quad 0<c_a\leq c_{ar}\\ \beta_{aa}vc-\beta_{ad}c_a-\lambda_{ea}c_a, & \text{if} \quad c_{ar}<c_a<c_{a0}\\ 0, & \text{if} \quad c_a=c_{a0}\end{cases}$$

where $\beta_{ar},\beta_{aa},\beta_{ad}$ are parameters, c_{ar} is the maximum concentration at which the charging effect ends, c_{a0} is the maximum concentration at achievement of which i.e. at $c_a=c_{a0}$ and $c=c_0$ is deposition of the substance in the core stops, λ_{ea} is the coefficient of decay (decomposition) of the deposited substance in the active zone.

The kinetics of deposition in the passive zone is taken as

$$\frac{\partial c_p}{\partial t}=\beta_n(c_p)\,vc-\lambda_{ep}c_p$$

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Construction of the automodel solution of the system of reaction-diffusion equations with bilinear nonlinear convective migration and diffusion

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Let us consider the Cauchy problem for the system of nonlinear equations of the parabolic type

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u^{m_1}}{\partial x}\right|^{p-2}\frac{du^{m_1}}{dx}\right) \pm cv_1(t)\frac{\partial y}{\partial x} - b_1 u^{q_1}\left|\frac{\partial v^{m_1}}{\partial x}\right|^{p_1} \\ \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial v^{m_2}}{\partial x}\right|^{p-2}\frac{dv^{m_2}}{dx}\right) \pm cv_1(t)\frac{\partial v}{\partial x} - b_1 v^{q_2}\left|\frac{\partial u^{m_2}}{\partial x}\right|^{p_2} \end{cases} \quad (1)$$

$$u(x,0) = u_0(x) \geq 0, v(x,0) = v_0(x) \geq 0,\ x \in \mathbb{R} \quad (2)$$

Where $m \geq 1$, $p_i > 1 + \frac{1}{m}$, $q_i > 0$, $p_i > 0$, $(i = 1,2)$, $p > 0$- numerical parameters of the environment, $u_0(x)$ and $v_0(x)$ positive definite continuous carrier functions, $b_1 u^{q_1}\left|\frac{\partial v^{m_i}}{\partial x}\right|^{p_1}$Damping condition, $cv_1\frac{\partial u}{\partial x}$ - convective migration.

All processes occurring in nature are represented by nonlinear problems. The equation under consideration, in particular, the value of the numerical parameters, consists of well-known equations: equations of porous media when $p = 2$ Buseneski equations, when $m = 2$, $p = 2$ Leibenson equations, er osti suvlari hused to describe action, when $k = m$ is called P-Laplace with equations.

(1) system of equations in various fields such as physics, chemistry, biology, ecology, neuroscience, etc., we observe phenomena that can be described as such systems. There are many problems that manifest themselves reaction diffusion systems [1-6]

The boundary problem (1), (2) plays an important role in mathematical modeling of the diffusion process in a non-linear medium, fluid flow in porous media, biological population dynamics, polytropic filtration, synergetics and solving problems in a number of other fields. For example, and represent the density of two biological populations or the temperature of two porous media in the process of heat propagation [1-4].

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Interval modeling in problems of the theory of electric circuits

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Many tasks of calculating electric power systems are characterized by non-deterministic values of parameters and modes of operation, due to their natural spread, variation in the process of operation, measurement errors of modes, or other factors. The non-determinism of network parameters has properties limited from below and from above by certain values, i.e. it is quite reasonable to assume that they are interval. As a criterion for solving such problems, a description of the limits of variation of the desired characteristics of electrical circuits is required. Since such problems are often encountered in practice and there is every reason to model them by interval methods [1]. The main reasons for considering interval models for calculation are parameters, the values of which can change due to seasonal, climatic and other reasons, in the calculation they are naturally set in intervals.

To build an interval model for the problem of calculating parameters, we will take a resistor, sources of EMF and current in the intervals:

$$R \in [\underline{R}, \overline{R}] \equiv \boldsymbol{R}, \qquad E \in [\underline{E}, \overline{E}] \equiv \boldsymbol{E}, \qquad I \in [\underline{I}, \overline{I}] \equiv \boldsymbol{I}, \tag{1}$$

where in the formulas according to [2], the interval values are indicated in bold type. The set of possible modes of an interval element will be called an interval current-voltage characteristic. For ideal interval sources of EMF and a resistor, we have interval current-voltage characteristics, respectively:

$$\begin{gathered} \Omega_E = \{U \mid U \in [\underline{E}, \overline{E}] = \boldsymbol{E}\}, \qquad \Omega_I = \{I \mid I \in [\underline{I}, \overline{I}] = \boldsymbol{I}\}, \\ \Omega_R = \big\{U \mid U \in \big[\min\{\underline{R}I, \overline{R}I\}, \max\{\underline{R}I, \overline{R}I\}\big] = \boldsymbol{U} = \boldsymbol{R}I\big\} = \\ = \big\{I \mid I \in \big[\min\{\underline{G}U, \overline{G}U\}, \max\{\underline{G}U, \overline{G}U\}\big] = \boldsymbol{I} = \boldsymbol{G}U\big\}, \end{gathered} \tag{2}$$

where Ω_E, Ω_I and Ω_R are sets containing point values of the corresponding interval current-voltage characteristics. If there are no a priori restrictions on voltage or current, then the branches of the interval current-voltage characteristics extend to infinity. If they are given in intervals as

$$\Omega_U = \big\{U \in (\underline{U} \le U \le \overline{U}\big\}, \quad \Omega_I = \big\{I \in (\underline{I} \le I \le \overline{I}\big\}, \quad \Omega_U \cap \Omega_I = \Omega_{UI}, \tag{3}$$

then the interval current-voltage characteristics are limited.

Next, we consider Ohm's law in interval form for the generalized branch. It is shown that, in contrast to the classical theory of electrical circuits, in the general case, in the interval form, equivalent transformation of the current source in EMF sources is impossible.

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Propagation of a non-stationary longitudinal wave from a thick-wall elastic spherical shell in acoustic space

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The tasks related to the problems of modeling and studying nonstationary wave processes in continuous media, as well as the interaction of deformable bodies with the environment, are relevant and are of practical and theoretical interest. Various issues related to the propagation and diffraction of non-stationary waves in acoustic and elastic media are described in scientific papers [1, 2, 3] and others. This work is devoted to the study of the problem of the propagation of a non-stationary longitudinal wave from a thick-walled elastic spherical shell in acoustic space. The aim of the work is to develop an algorithm for solving the problem and study non-stationary wave processes in an elastic shell and in the environment.

Let a thick-walled spherical elastic shell with an inner radius R_1 and an outer radius R_2 be placed in an infinite acoustic medium. At the initial moment of time $\tau = 0$, a normal surface load is applied $p_1(\tau, \theta)$ to the inner surface of the thick-walled shell. Taking into account the axial symmetry of the problem, the motion of the shell and the acoustic medium relative to the scalar displacement and velocity φ_2 potentials φ_1 are described by wave equations, the initial conditions are homogeneous, and there are no perturbations at infinity. The shell and the acoustic environment are in contact.

The initial-boundary value problem is solved using the integral Laplace transform in time τ and using the method of incomplete separation of variables. Given the axial symmetry of the problem, the given load $p_1(\tau, \theta)$ and the desired functions are expanded into series in Legendre polynomials $P_n(\cos\theta)$ [2]. In the image space, the problem is reduced to solving an infinite system of linear algebraic equations. The solution of the system is represented in the form of infinite series in terms of exponents. For the coefficients of infinite series, initial conditions and recurrence relations are obtained that do not require the use of the reduction method. The coefficients of the series of the desired functions are defined as rational functions of the Laplace transform parameter, which makes it possible to find the originals using the theory of residues. Formulas for the parameters of the environment and the shell have been obtained. Numerical experiments have been carried out. The obtained results of the work can be used in the field of design organizations in the construction of structures, as well as in the design of underwater reservoirs.

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Diffraction of non-stationary plane transversal waves on a hard sphere in a porous-elastic half-space

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Mathematical modeling and study of the difrakpropagation of non-stationary wave processes in continuous media is a complex and, at the same time, relevant direction in the wave dynamics of continuous media. The relevance of the problems of continuum dynamics is due to the development of various fields of technology, the creation of new structures operating under dynamic loads, as well as the problems of geophysics, seismology, gas exploration, oil exploration, the mining industry, the construction of civil and industrial structures. This work is devoted to the study of the problem of diffraction of a non-stationary plane transverse wave by a hard sphere in a porous-elastic half-space.

Let in a homogeneous isotropic porous elastic half-space $z \geq 0$ at a depth h from a flat boundary $z = 0$ on O_2 the O axis O_2z hard sphere of radius $R(\ R < h)$. At the initial moment of time, $\tau = 0$ the frontal point of the ball touches the front of a non-stationary plane shear wave (S -wave) with a given potential ψ_s, which forms a rotational motion of the medium around an axis Ox passing through the center of the sphere. On the surface of a hard sphere, the displacements are equal to zero. The flat boundary of the half-space is either a hard sphere or free surface

Taking into account the axial symmetry of the problem, the motion of the medium relative to the nonzero component of the displacement vector potential ψ is described by the wave equation, the initial conditions are homogeneous, and there are no perturbations at infinity. The initial-boundary value problem is solved using the integral Laplace transform in time τ and using the method of incomplete separation of variables. Taking into account the axial symmetry of the problem, the given potential ψ_s and the desired functions are expanded into series in Gegenbauer polynomials $C^{3/2}_{n-1}(\cos\theta)$[1]. In the space of images, the problem using the addition theorem [2] for spherical wave functions is reduced to solving an infinite system of linear algebraic equations. The solution of the system is represented in the form of infinite series in terms of exponents. For the coefficients of infinite series, initial conditions and recurrence relations are obtained that do not require the use of the reduction method. The coefficients of the series of the desired functions are defined as rational functions of the Laplace transform parameter, which makes it possible to find the originals using the theory of residues. Formulas for environmental parameters are obtained. Numerical experiments have been carried out. The obtained results of the work can be used in the field of geophysics, seismology, as well as in the design of underground reservoirs.

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Multiparametric mathematical model for the filtering of non-linear fluids in a three-layer hydrodynamic connected plasma

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The article deals with the use of multi-parametric mathematical models of the task of filtering fluids in a trilayer layer. The task of filtering Newton and non-Newton fluids in multilayered porosal environments, when these seams are hydrodynamically linked to the relevant tasks of building mathematical models and developing their solutions, which are used in determining the parameters for the development of multi-plastic fluid deposits (gas, oil, gascondensy, etc.). If the mean sea is based on its geologic data, it is assumed that the fluid moves in it in a horizontal direction and the two adjacent reservoir properties are such that they are vertical in nature, then it is possible to model the tasks of the Huntush type. Suppose that the medium plastic contains non-linear fluids, including structured ones, and in it we build a multi-parametric mathematical model comprising 13 types of mathematical models.

These models differ in the way the functional relationship between the filtration rate and the pressure gradient and the properties of the fluids and the seam.

The conditions at the mobile unknown disturbances and natural boundaries are determined by the continuity of flow and pressure functions with the choice of model parameters. Two adjacent low-permeability seams contain abnormal fluids with unknown disturbance boundaries. The mathematical model of this task is described by differential equations in private second-order derivatives with corresponding initial and boundary conditions.

The task is non-linear and uses methods to solve it: iterations by non-linear members of equations, and by variable t [1].

The grid run method for variable x is further used, and a computational multiparametric algorithm has been developed. A classification of models and solution algorithms has been performed based on their filtering behaviour.

The computational algorithms were tested on test data, the results for the various filtering laws were analysed, and patterns of divisional changes and disturbances were examined.

The flow from the upper and lower seams to the middle has been determined and conclusions have been drawn. The results of the calculations showed the applicability of the multi-parametric model and calculation algorithms for modelling multi-plastic real oil and gas deposits that are consistent with the model.

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The problem of stability of interval matrixes in the diagnostics of closed control systems

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The problem of checking the stability of a matrix is quite complex, and so far no simple algorithms for its solution have been obtained. Problems of this type often arise in the design of control systems, in solving economic problems, and also in the study of computational algorithms. For example, the problem of stability of interval matrices is widely studied in connection with its applications in control theory [1]. This is directly related to the stability analysis of state-space models with parametric uncertainties. Interval matrices have become an important choice in representing underdetermined control systems. Also, in the theory of linear control systems, an effective method for changing the dynamic response of a linear multidimensional system with inputs and states is to place the poles of a closed loop, i.e. the eigenvalues are placed at arbitrary predetermined locations in the complex plane.

Let us consider a system of first-order homogeneous differential equations, when all external influences are absent and the dynamics of the system is determined by its own structure. So, the interval mathematical model for a closed-loop control system in the state space has the following form:

$$\dot{X}(t) = \boldsymbol{A}X(t), \tag{1}$$

with a real interval state matrix

$$\boldsymbol{A} \in [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{nxn}; \underline{A} \leq A \leq \overline{A}\} \in \mathbb{IR}^{nxn},$$

whose elements are real intervals $\boldsymbol{a}_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}] \in \mathbb{IR}$ $(i, j = 1, 2, ..., n)$, and with a state vector $X(t) \in \mathbb{R}^n$. In this case, the desired interval characteristic polynomial is defined as

$$\boldsymbol{d}(\lambda) = det(\lambda E - \boldsymbol{A}) = \lambda^n + \boldsymbol{d}_1\lambda^{n-1} + \boldsymbol{d}_2\lambda^{n-2} + ... + \boldsymbol{d}_n, \tag{2}$$

where E is the identity matrix, $\boldsymbol{d}_i \in [\underline{d}_i, \overline{d}_i]$ $(i, j = 1, 2, ..., n)$ are the interval coefficients of the characteristic polynomial of the closed control system.

In this paper, we present a new approach for estimating the stability of interval matrices. The exact estimates obtained for the boundary of the eigenvalues of interval matrices are used in the analysis of the stability of continuous control systems with interval coefficients of stationary equations.

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Modeling divirgent-form cross-diffusion processes in different environments

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Consider in area $Q = \left\{(t,x):\ t > 0,\ x \in R^N\right\}$ the following problem

$$\begin{cases} \frac{\partial u}{\partial t} = div\left(v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - div\left(c(t)u\right) - \gamma_1(t)u \\ \frac{\partial v}{\partial t} = div\left(u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\right) - div\left(c(t)v\right) - \gamma_2(t)v \end{cases} \quad (1)$$

$$u\left(0,x\right) = u_0\left(x\right) \geq 0,\ v\left(0,x\right) = v_0\left(x\right) \geq 0,\quad x \in R^N, \quad (1)$$

where $k \geq 1, p, m_i, i = 1,2$ are numeric parameters, $\nabla(\cdot) = \underset{x}{grad}(\cdot)$, are $0 \leq u_0\left(x\right), v_0\left(x\right) \in C\left(R^N\right),\ \ c(t) > 0, 0 < \gamma_i(t) \in C(0,\infty), i = 1,2$ given functions.

The system (1) describes a number of physical processes in a two-component nonlinear medium, for example, it the processes of mutual reaction-diffusion, heat conduction, combustion, polytropic filtration of liquid and gas [1-2]. The system (1) is also called cross diffusion [1-2].

After the necessary calculations for functions $f(\xi),\ \ \psi(\xi)$, we have the following system of degenerate self-similar equations [2]

$$\begin{cases} \xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\psi^{m_1-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right) + \frac{\xi}{p}\frac{df}{d\xi} + b_1 f = 0 \\ \xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}f^{m_2-1}\left|\frac{d\psi^k}{d\xi}\right|^{p-2}\frac{d\psi}{d\xi}\right) + \frac{\xi}{p}\frac{d\psi}{d\xi} + b_2\psi = 0 \end{cases} \quad (1)$$

where $b_1 = \alpha_1/\left[1-(m_1-1)\alpha_2 - k(p-2)\alpha_1\right],\quad b_2 = \alpha_2/\left[1-(m_2-1)\alpha_1 - k(p-2)\alpha_2\right].$

Theorem 1. Let $q_i < 0,\quad N + kqq_i < 0,\quad i = 1,2$. Then the solution of the system () disappearing at infinity as $\eta \to \infty\quad \left(\eta = \ln\left(a + \xi^{p/(p-1)}\right)\right)$ has the asymptotic representation

$$\begin{cases} f(\xi) = A_1\left(a + \xi^\gamma\right)^{q_1}\left(1 + o(1)\right) \\ \psi(\xi) = A_2\left(a + \xi^\gamma\right)^{q_2}\left(1 + o(1)\right) \end{cases}$$

where the coefficients $A_i > 0$, $i = 1,2$ are the solution to the system of algebraic equations

$$\begin{cases} {A_1}^{k(p-2)}\,{A_2}^{m_1-1} = \left[-\frac{1}{\gamma^{p-1}p(N+k\gamma q_1)} + b_1\right]\left(\left|k\gamma q_1\right|\right)^{2-p} \\ {A_1}^{m_2-1}\,{A_2}^{k(p-2)} = \left[-\frac{1}{\gamma^{p-1}p(N+k\gamma q_2)} + b_2\right]\left(\left|k\gamma q_2\right|\right)^{2-p} \end{cases}$$

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Identification of the retardation coefficient and source in the equation of transport of substance in porous media

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In this work considered the problem of determining the retardation coefficient and the source in the model of the transfer of substances in porous media. In order to prepare additional information for solving the inverse problem, the corresponding direct problem with known values of the retardation coefficient was considered. Thus, the "initial data"for solving the inverse problem is prepared.

Consider the following equation for the transfer of matter in a porous medium [1, 2]

$$R\frac{\partial c}{\partial t} - D\frac{\partial^2 c}{\partial x^2} + \frac{v}{m}\frac{\partial c}{\partial x} + \lambda Rc - \frac{qc^*}{m} = 0, \tag{1}$$

where R retardation factor; c concentration of the mass of a substance in the liquid phase, i.e., the mass of the dissolved substance per unit volume of the solution (kg/m^2) m porosity(m^3/m^3), q is the volumetric injection rate of the initial fluid per unit volume.

t time (s), v is the fluid flow rate in a porous medium (m/s), c^* is the substance concentration in the injected fluid in a porous medium.

To solve equation (1), the following initial and boundary conditions are used. As an additional condition, we accept

$$c(0, x) = 0, \tag{2}$$

$$c(t, 0) = c_0, c(t, l) = 0, \tag{3}$$

$$c(t, x_k) = z_k(t), k = 1, 2, 3..., n, \tag{4}$$

where $z_k(t)$ are the values of the function $c(t, x_k)$ at points x_k ,usually obtained by measurements, $k = 1, 2, 3..., n,$.

Let us now consider the problem of determining the coefficient R and parameter q in the source term from the minimum condition for the following residual functional

$$J(R, q) = \sum_{k=1}^{n} \int_0^T [c(t, x_k) - z_k(t)]^2 \, dt \tag{5}$$

On the basis of numerical calculations, the coefficients are determined with sufficient accuracy. The optimal range of values of the regularization parameter is determined α

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Numerical solution of the stationary problem of optimal placement of minimum power heat sources in one-dimensional case

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In the work [1] of the non-stationary problem of optimal placement of heat sources in the processes described by the variable coefficient heat transfer equation was studied.

Is the minimum of a linear functional when the field D is one-dimensional

$$J\{f\} = \int_a^b f(x)dx \to \min, \tag{1}$$

and under the following conditions:

$$\begin{gathered} \frac{d}{dx}\left(\chi(x)\frac{du}{dx}\right) + f(x), \quad a < x < b, \\ \begin{cases} \chi(x)\dfrac{du}{dx}\Bigg|_{x=a} = \mu_1(a), \\ \chi(x)\dfrac{du}{dx}\Bigg|_{x=b} = \mu_2(b), \end{cases} \end{gathered} \tag{2}$$

$$m(x) \le u(x) \le M(x) \quad x \in D, \tag{3}$$

the function $f(x) \ge 0$ is required to be defined.

Here $u = u(x)$ – is the temperature of the rod at point x; $\chi(x) > 0$ – heat transfer coefficient; $\mu_1(a)$, $\mu_2(b)$, $m(x)$, $M(x)$ – are given functions. The functions $m(x)$ and $M(x)$ – are respectively, the minimum and maximum temperatures given in domain D. Heat sources are characterized by functions $f(x)$ that can be integrated with the square of $L_2(D)$ in space.

Since it is difficult to find a continuous solution to problem (1)-(3), we are looking for a numerical solution to the problem. In this case, using the difference scheme, we replace problem (2) with a finite difference equation. To obtain homogeneous conservative difference schemes, we use the integro-interpolation method. We approximate problem (1)-(3) in the form of a linear programming problem. After that, we get a linear programming problem and the problem is solved by the big M method [2].

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Modeling and system analysis in science and education

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Systems theory is a scientific and methodological concept of studying objects that are systems. System analysis is a problem-solving methodology based on the structuring of systems and the quantitative comparison of alternatives. System analysis provides for use in various sciences, systems, many different system methods and procedures. In our case, this is modeling and experiment, program control and regulation, abstraction and concretization, formalization and concretization, algorithmization.

In system analysis, modeling is considered as the main method of scientific knowledge, associated with the improvement of methods for obtaining and fixing information about the objects under study, as well as with the acquisition of new knowledge based on model experiments. System-level modeling is the best-known level of social modeling, considering the situation as a whole.

The classification of types of modeling can be carried out for various reasons. Considerable experience has been accumulated, which gives grounds to formulate some principles and approaches to building models [1,3].

The work is written on the basis of research and program implementations carried out within the framework of master's theses, as well as working training programs, such as "Modeling in the social sphere and system analysis", "Digital and information technologies" for the areas of education psychology and the social sphere.

Significant sections there are such as modeling and system analysis, the theory of concepts and the relationship of models, models of social systems and sociological processes, an introduction to modeling and forecasting social processes. Also processing of statistical data, application of mathematical and statistical methods, software packages for processing statistical data, statistical and graphical analysis of solutions. Modern programs for statistical data processing make it possible to apply complex modern methods of analysis.

Relative to other areas. These are differential equations as a mathematical tool for modeling and analyzing various phenomena and processes in science and technology. In training, the analysis of these models is mainly proposed to be carried out in tasks. Many differential equations encountered in various applications can be interpreted as equations on graphs [2,3].

Software was developed for modeling and calculations in the field of chemistry, taking into account the requirements for clarity in teaching [2]. In general, in chemistry, a huge set of different software has been developed for modeling and calculations in this area.

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Inverse problem of filtration of a homogeneous liquid in a two-layer porous medium

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In this paper, we consider the inverse problem of filtering a homogeneous liquid in a two-layer porous medium [1]. Let the first layer consist of a bi-porous structure, and the second layer be a purely porous medium. Filtration of a homogeneous liquid in a two-layer porous medium [1] is described by the following equations

$$\frac{\partial P_p}{\partial t} = \eta \frac{\partial^3 P_f}{\partial x^2 \partial t} + \chi_1 \frac{\partial^2 P_p}{\partial x^2}, 0 \leq x \leq x_0, 0 < t \leq T, \tag{1}$$

$$\frac{\partial P_f}{\partial t} = \chi_2 \frac{\partial^2 P_p}{\partial x^2}, L_0 \leq x \leq L, 0 < t \leq T, \tag{2}$$

where P_f, P_p-are the pressures of the first and second layers, respectively, MPa; x-coordinate, m; t-time,s; χ_1, χ_2-are the piezoconductivity coefficients of the first and second layers, respectively,m^2/s; η - is the delay time.

To solve equations (1), (2), we use the following initial boundary conditions

$$P_p(x,0) = P_f(x,0) = p_2 = const,$$

$$Q = Q_0 = const = -\frac{k_1}{\mu}\left[\frac{\partial P_f(0,t)}{\partial x} + \frac{\eta}{\chi_1}\frac{\partial^2 P_f(0,t)}{\partial x \partial t}\right],$$

$$P_p(L,T) = P_2,$$

$$\frac{k_1}{\mu}\left[\frac{\partial P_f}{\partial x} + \frac{\eta}{\chi_1}\frac{\partial^2 P_f}{\partial x \partial t}\right]\bigg|_{x=x_0-0} = \frac{k_2}{\mu}\frac{\partial P_p}{\partial x}\bigg|_{x=x_0+0}.$$

$$P_f(x_0,0) = P_p(x_0,t),$$

Let us use the following additional condition to solve the inverse problem

$$P_f(x_0,0) = z(t),$$

The inverse problem is posed as follows: determine the coefficients η, χ_1, χ_2 from the minimum condition for the following functional

$$J(\eta,\chi_1,\chi_2) = \int_0^T [P_f(0,t) - z(t)]^2 dt,$$

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Numerical solution of the problem for nonlinear systems of differential heat conductivity equations with nonlinear boundary conditions of the third kind

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There are a number of technical problems related to the distribution of heat in an inhomogeneous medium, which can be described using two-dimensional nonlinear systems of differential equations [1] in the area $\omega = \{a \le x \le b,\ c \le y \le d,\ t_o \le t \le T\ \}$ the following kind:

$$\frac{\partial u}{\partial t} = a_{11}\frac{\partial}{\partial x}\left(a_{12}\frac{\partial u}{\partial x}\right) + a_{13}\frac{\partial}{\partial y}\left(a_{14}\frac{\partial u}{\partial y}\right) + f_1\ , \qquad (1)$$

$$\frac{\partial v}{\partial t} = a_{21}\frac{\partial}{\partial x}\left(a_{22}\frac{\partial v}{\partial x}\right) + a_{23}\frac{\partial}{\partial y}\left(a_{24}\frac{\partial v}{\partial y}\right) + f_2 \qquad (2)$$

at initial

$$u_{|t=t_0} = \gamma_1, \qquad (3) \qquad\qquad v_{|t=t_0} = \gamma_2 \ (4)$$

and boundary conditions of the third kind:

$$\begin{array}{llll}
g_{11}\frac{\partial u}{\partial x} + g_{12}u_{|x=a} = \varphi_{11}\left(t, u_{|x=a}\ \right) & (5) & g_{13}\frac{\partial u}{\partial x} + g_{14}u_{|x=b} = \varphi_{12}\left(t, u_{|x=b}\ \right) & (6)\\
g_{21}\frac{\partial u}{\partial y} + g_{22}u_{|y=c} = \varphi_{21}\left(t, u_{|y=c}\ \right) & (7) & g_{23}\frac{\partial u}{\partial x} + g_{24}u_{|y=d} = \varphi_{22}\left(t, u_{|y=d}\ \right) & (8)\\
g_{31}\frac{\partial v}{\partial x} + g_{32}v_{|x=a} = \varphi_{31}\left(t, v_{|x=a}\ \right) & (9) & g_{33}\frac{\partial v}{\partial x} + g_{34}v_{|x=b} = \varphi_{32}\left(t, v_{|x=b}\ \right) & (10)\\
g_{41}\frac{\partial v}{\partial y} + g_{42}v_{|y=c} = \varphi_{41}\left(t, v_{|y=c}\ \right) & (11) & g_{43}\frac{\partial v}{\partial x} + g_{44}v_{|y=d} = \varphi_{42}\left(t, v_{|y=d}\ \right) & (12)
\end{array}$$

Where $u = u(x,y,t)$; $v = v(x,y,t)$; $a_{ij} = a_{ij}(x,y,u,v)$; $f_i = f_i(x,y,t,u,v)$;
$\gamma_k = \gamma_k(x,y)$; *and* φ_{ik} –continuous functions in their arguments; g_{ij}- constant coefficients that determine the type of problem $k = 1,2; i,j = \overline{1,4}$.

The report discusses various methods of linearization, explicit and implicit schemes[2] for solving the problem (1)-(12). Numerical results and their analyzes on a test example, obtained using the developed program in the language C#.

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A model of anomalous filtration of liquid in a one-dimensional homogeneous porous media

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Anomalous filtration of a homogeneous liquid in a porous medium with homogeneous permeability is considered.

The anomalous filtration law is taken as [1,2]

$$\vec{v} = -\frac{k_f}{\mu}\nabla\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \qquad (1)$$

where $\vec{v}$ is the filtration velocity, k_f is the pseudo-permeability, i.e. fractal permeability with dimension L^2T^α, L is dimension of length, T is dimension of time, μ is fluid viscosity, $\alpha\,(0<\alpha\leq 1)$ is order of the derivative, ∇ is Hamilton operator, p is pressure.

For homogeneous one-dimensional media the piezoconductivity equation based on (1) is written as

$$\frac{\partial p}{\partial t} = \chi\frac{\partial^2}{\partial x^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \qquad (2)$$

where $\chi = \frac{k_f}{\mu\beta^*}$ is the coefficient of piezoconductivity, β^* is the coefficient of elasticity of the medium.

For equation (2) in the finite bed, the following initial and boundary conditions are taken

$$p(0,x) = p_0\left(E\right), \qquad (3)$$

$$p(t,0) = p_1\left(t\right), \qquad p(t,l) = p_2\left(t\right), \qquad (4)$$

where $p_0(x)$, $p_1(t)$, $p_2(t)$ predefined functions, l- layer length.

Problem (2) - (4) is solved by the method of finite differences. Based on the numerical results, the influence of filtration anomaly on the distribution of pressure and filtration rate have been estimated.

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Solute transport in a two-zone medium with different characteristics

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Some mathematical models for solute transport in two-zone media were presented in [1,2]. In these models, a two-zone approach was used, where the mass transfer between the zones is modeled by a first-order kinetic equation.

Here we consider the process of transfer of colloidal particles under the assumption that in both zones there is a reversible retention of particles by different characteristics (parameters). We use a scheme similar to [4] for a medium with double porosity. The first zone with index (1) in the notation has a high permeability, and the second zone has a low one.

The equations of solute transport in the one-dimensional case are written in the form

$$\rho\frac{\partial S_{al}}{\partial t}+\theta_l\frac{\partial C_l}{\partial t}=\theta_1 D_1\frac{\partial^2 C_l}{\partial x^2}-\theta_1 v_l\frac{\partial C_l}{\partial x}+\alpha(C_m-C_l),(l=1;2;m=3-l), \qquad (1)$$

where t is the time, c, x-is the distance, m, D_l is the longitudinal dispersion coefficient, $m^2/s, v_l$ is the velocity of the fluid, $m/s, v_1<v_2, C_l$-is the volume concentration of the substance in the fluid, S_{al} are the concentrations of the deposited substance,$m^3/kg, \theta_1$-porosity of the zones, $m^3/m^3, \rho$ is the density of the medium, $kg/m^3, \alpha$ is the coefficient of mass transfer between the zones,s^{-1} .

Substance deposition in each of the zone sections occurs reversibly in accordance with the kinetic equations

$$\rho\frac{\partial S_{al}}{\partial t}=\theta_l k_{al}C_1-\rho k_{adl}S_{al},(l=1,2), \qquad (2)$$

where k_{al} are the coefficients of the substance deposition from the fluid phase l to the solid phase, s^{-1}, k_{adl} are the coefficients of separation of the substance from the solid phase and transition to the fluid,s^{-1}.

Let a fluid with a constant concentration c_0 of the substance be pumped into a medium initially saturated with a pure (without substance) fluid from the initial moment of time . Let us consider such periods of time, where the concentration field does not reach the right boundary of the medium,$x=\infty$. Under the noted assumptions, the initial and boundary conditions for the problem have the form

$$C_i(0,x)=0, S_{al}(0,x)=0, C_l(t,0)=c_0, \frac{\partial C_1}{\partial x}(t,\infty)=0, l=1,2. \qquad (3)$$

Problem (1)-(3) is solved by the finite difference method.

It is shown that with an increase in the values of the parameter α the two-zone medium acquires the properties of a homogeneous medium.

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Numerical solution of anomalous solute transport problem in an element of a fractured-porous medium

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This paper considers the anomalous solute transport in an element of a fractured-porous medium (FPM), which consists of a fracture and a porous block (matrix) adjacent to each other [1]. A fracture is a semi-infinite one-dimensional object so the distribution of the solute and the fluid flow over its cross-section is considered as uniform.

The equations for the solute transport in the FPM element are taken as

$$\frac{\partial^\alpha c_f}{\partial t^\alpha} + \bar{v}\frac{\partial c_f}{\partial x} = D_f \frac{\partial^\beta c_f}{\partial x^\beta} + m_0 D_m \frac{\partial^{1-\gamma}}{\partial t^{1-\gamma}} \left(\frac{\partial^\delta c_m}{\partial y^\delta} \right) \bigg|_{y=0}, \tag{1}$$

$$\frac{\partial^\gamma c_m}{\partial t^\gamma} = D_m \frac{\partial^{1+\delta} c_m}{\partial y^{1+\delta}}, \tag{2}$$

where $c_m = c_m(t,\, x,\, y)$ is the volume concentration in the porous block (matrix); $c_f = c_f(t,\, x)$ is the volume concentration of the solute in the fracture; $\bar{v}$ is the velocity of the fluid; $D_f,\ D_m$ are the coefficients of anomalous diffusion in the matrix and fracture, respectively; m_0 is the coefficient of porosity in the matrix, t is the time; $x,\, y$ are the space coordinates; α, γ and $\beta,\ \delta$ are the orders of the fractional derivatives with respect to time and space coordinates, respectively.

It is believed that concentration front c_m does not reach the matrix boundaries $y = \infty$ and $x = \infty$. Under these conditions, the initial and boundary conditions have the form:

$$c_f(0, x) = 0, c_m(0, x, y) = 0, \tag{3}$$

$$c_f(t, 0) = c_0, c_f(t, \infty) = 0, c_0 = const, \tag{4}$$

$$c_m(t, x, 0) = c_f(t, x), c_m(t, x, \infty) = 0. \tag{5}$$

The system of equations (1), (2) with (3) - (5) conditions is solved by the finite difference method [2].

Numerical results show that when anomalous phenomena are simultaneously taken into account in a fracture and a porous block, various variants of the interaction of the effects of “slow” and “fast” diffusion are observed. It has been established that “slow” diffusion in the fracture leads to “slow” diffusion in the porous block. Similarly, “fast” diffusion in the fracture leads to the same “fast” diffusion in the porous block. “Slow” diffusion in the porous block leads to “fast” diffusion in the fracture.

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Numerical solution of the axi-symmetric problem of suspensions filtering with formation of a cake layer

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We consider the process of filtering suspensions through a cylindrical filter. Suppose that the filtering occurs in the radial direction, which means that the direction of the velocities of the liquid and suspended particles coincide with the radial direction. Therefore, we ignore the angular flow of suspension in our adopted scheme (geometry) of the filtering and to consider it as one-dimensional [1].

A mathematical model of filtering with respect to the pressure can be written in the following form

$$\frac{\partial p_s}{\partial t} = \frac{p_A k^0}{\beta\mu}\left(1+\frac{p_s}{p_A}\right)^{1-\beta}\frac{1}{r}\frac{\partial}{\partial r}\left(\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta} r\frac{\partial p_s}{\partial r}\right) + \frac{q_{out}}{2\pi}\cdot\frac{1}{r}\frac{\partial p_s}{\partial r}. \quad (1)$$

$$p_s\left(0,r\right) = 0. \quad (2)$$

$$-2\pi\left[k^0\left(1+\frac{p_s}{p_0}\right)^{-\delta} r\frac{\partial p_s}{\partial r}\right]_{r=R} = \left.\frac{p_{\ell m}}{R_m}\right|_{r=R}, \quad (3)$$

$$p_s\left(t, R_L\left(t\right)\right) = 0, \quad (4)$$

In (1)-(4): p_s is the cake compressive stress, μ is fluid viscosity, k^0 denote the cake permeability k at $p_s = 0$, p_A is the specific stress, the indicators β, δ are constant values, R is outer radius of the filter element, $p_{\ell m}$ is the filtrate pressure at $r = R$, R_m is the medium resistance, $q_{out} = \left[2\pi r\frac{k^0}{\mu}\left(1+\frac{p_s}{p_0}\right)^{-\delta}\frac{\partial p_s}{\partial r}\right]_{r=R}$ is the instantaneous filtration velocity, t is time, r is radial coordinate.

The equation for the increasing radius $R_L\left(t\right)$, which expresses the thickness of the cylindrical cake, that is, the radius of the boundary between the suspension and the cake is given in the following form:

$$\frac{dR_L}{dt} = \frac{\varepsilon_s^0}{\varepsilon_s^0 - \varepsilon_{s0}}\left[\frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right]_{r=R_{L^-}} + \frac{1}{2\pi R_{L^-}} q_{out}, \quad (5)$$

On the surface $r = R_{L^-}$, the stresses of the compressing particles are zero. That is, $\varepsilon_s|_{R_{L^-}}$ can be taken equal to the solid content at zero stress ε_s^0. On the other hand, $\varepsilon_s|_{R_{L^+}}$ equals to the concentration of solids in suspension ε_{s_0}.

Using (1)-(5) the numerical results have been obtained corresponding. Distributions of compression pressure and the fluid pressure over the cake, as well as growth of the cake thickness are determined. The compression pressure decreases from the filter surface to the boundary of the cake and suspension.

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Anomalous transport equations with multi-term fractional time derivatives

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Recently, the issues of mathematical modeling of the processes of anomalous solute transport in porous media have attracted great attention. Equations in fractional order partial derivatives with an additional effect of time lag are, in general, a powerful tool for analyzing the processes of solute transport in porous media [1].

This paper considers the numerical solution of the diffusion equation with multi-term fractional time derivatives in a finite region.

The equation for the solute transport in a porous medium with multi-term time derivatives in the one-dimensional case is written as [2]

$$\frac{\partial^{\alpha} c}{\partial t^{\alpha}} + \sum_{s=1}^{n} r_s \frac{\partial_s^{\beta} c}{\partial t_s^{\beta}} = D \frac{\partial^{\gamma} c}{\partial x^{\gamma}} + f(t,x),$$

where $\alpha, \beta_s, s = 1, ..., n, \gamma$ are the orders of derivatives, $0 < \beta_s < \beta_s - 1 < ... < \beta_1 < \alpha < 1$, c is the volumetric concentration of the substance, D is the diffusion coefficient, r_s are the coefficients, $f(t,x)$ is the power of the sources of substances, t time, x coordinate. The orders of fractional derivatives α and γ vary in the following range: $0 < \alpha \leq 1, 1 \leq \gamma \leq 2$ and they are understood in the approach of Caputo. In (2) c is a dimensionless quantity and $[\frac{\partial^{\alpha} c}{\partial t^{\alpha}}] = T^{-\alpha}$, $[r_s] = T^{\beta_s - \alpha}$, $[D] = \frac{L^{\gamma}}{T^{\alpha}}$, $[f(t,x)] = T^{-\alpha}$, L is the dimension of length, T is the dimension of time.

An analysis of the obtained results shows that the use of differential equations with polynomial fractional time derivatives for modeling anomalous diffusion processes allows describe the effects of delayed development of concentration profiles. Taking into account the multi-term nature of the diffusion equation in comparison with the one-term equation leads to a slow spread of the solute concentration in the medium. It is shown that an increase in the value of constant coefficients (r_1, r_2,...,r_s) with local fractional time derivatives, it enhances the process of slowing down the distribution of concentration profiles. A similar enhancement of the retarded effects is also caused by a decrease in the orders of these local time derivatives.

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Mathematical modeling of the fractional Van der Pol-Duffing-Leonov oscillator

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In this paper, we study the Cauchy problem (1) for a fractional nonlinear Mathieu oscillator using a nonlocal explicit finite difference scheme.

$$\ddot{x}(t) + \lambda\left(x(t)^2 - 1\right) D_{0t}^{q(t)} x(t) + x^3(t) = mbg\,\mathrm{sign}\left(\dot{x}(t)\right), x(0) = a, \dot{x}(0) = b, \qquad (1)$$

where $x(t) \in C^2[0,T]$ is the displacement function to be determined, $\lambda > 0$ is the coefficient of friction, $b > 0$ is the scoring of the area under consideration, m is the building weight, g is the free fall acceleration. Fractional derivative operator $0 < q(t) < 1$ [1]:

$$D_{0t}^{q(t)} x(t) = \frac{1}{\Gamma(1 - q(t))} \frac{d}{dt} \int_0^t \frac{x(\tau)\, d\tau}{(t-\tau)^{q(t)}}, \qquad (2)$$

Next, a numerical analysis of problem (1) was carried out; by analogy with works [1,2,3], oscillograms and phase trajectories were constructed. An interpretation of the results is given.

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Fractional dynamical system S.V. Dubovsky in the study of economic crises and cycles

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Consider the following Cauchy problem:

$$\begin{cases} \partial_{0t}^{\alpha_1} x(\tau) = -\lambda n x(t)(x(t)-1)(y(t)-y^*) + \delta_1 \cos(\omega_1 t), x(0) = a \\ \partial_{0t}^{\alpha_2} y(\tau) = n(1-n) y^2(t)(x(t)-x^*) + \delta_2 \cos(\omega_2 t), y(0) = b \end{cases},$$

where $x(t)$ is the efficiency of innovation, the ratio of labor productivity at new jobs to the average productivity at all jobs of all ages, $y(t)$ is the efficiency of fixed assets (funds) of the organization, is the savings rate, gross capital formation in shares of GDP, λ is a parameter that determines the size and duration of cycles, a, b are positive constants that determine the initial conditions, $t \in [0, T]$ is the current time of the process under consideration, $T > 0$ is the simulation time, (x^*, y^*) is the coordinates of the equilibrium point of the system (), $\delta_1, \delta_2, \omega_1, \omega_2$ are given positive constants, fractional operators in system (1) are determined from (2):

$$\partial_{0t}^{\alpha_1} x(\tau) = \frac{1}{\Gamma(1-\alpha_1)} \int_0^t \frac{\dot{x}(\tau)\, d\tau}{(t-\tau)^{\alpha_1}}, 0 < \alpha_1 < 1,$$

$$\partial_{0t}^{\alpha_2} x(\tau) = \frac{1}{\Gamma(1-\alpha_2)} \int_0^t \frac{\dot{x}(\tau)\, d\tau}{(t-\tau)^{\alpha_2}}, 0 < \alpha_2 < 1.$$

System (1) describes long waves N.D. Kondratieff, taking into account heredity [1]. In the case when in the system () $\alpha_1 = \alpha_2 = 1$ and $\delta_1 = \delta_2 = 0$ we get the model of S.V. Dubovsky, which was proposed and studied by S.V. Dubovsky. Therefore, the dynamical system (1) will be called the generalized Dubovsky model (GDM) [1-3].

Further in the paper, a numerical analysis of model (1) is given.

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Numerical solution of the problem of anomalous filtration of a suspension in a porous medium with a fractal structure

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In this work, the problem of anomalous filtration taking into account the phenomenon of colmotation and suffusion in a two-dimensional porous medium consisting of two zones with inhomogeneous fluids is posed and numerically solved. It is considered that the medium has a fractal structure. The study area consists of two zones, one of which is highl permeable (zone $R_1\{0 \le x < \infty, 0 \le y \le l\}$) and the other is low permeable ($R_2\{0 \le x < \infty, l \le y \le \infty\}$). Initially the area R_1 and R_2 filled with liquid without substance [1].

$$\varepsilon_{0i}\frac{\partial c_i}{\partial t} = \varepsilon_{0i}\left\{D_{ix}\frac{\partial^{\beta_{ix}} c_i}{\partial x^{\beta_{ix}}} + D_{iy}\frac{\partial^{\beta_{iy}} c_i}{\partial y^{\beta_{iy}}}\right\} - \frac{\partial\left(v_{ix}c_i\right)}{\partial x} - \frac{\partial\left(v_{iy}c_i\right)}{\partial y} + \frac{\partial \varepsilon_i}{\partial t}, i = 1,2 \quad (1)$$

$$\frac{\partial \varepsilon_i}{\partial t} = \omega_1\left(\varepsilon_{0i} - \varepsilon_i\right)\left|\nabla p_i\right| - \omega_2 \varepsilon_i c_i, i = 1,2 \quad (2)$$

where C_i is the volumetric concentrations of solid particles in the liquid, $\varepsilon_{0i}, \varepsilon_i$ are the initial and current porosities, ω_1, ω_2 are the coefficients characterizing the intensity of colmotattion and suffusion of pores, $|\nabla p_i|$ are the absolute values of the pressure gradient p_i.

The filtration velocity components in R_1 and R_2 are defined as [2]

$$v_{ix} = -\frac{k_{ix}\left(\varepsilon_i\right)}{\mu}\frac{\partial^{\gamma_1} p_i}{\partial x^{\gamma_1}}, v_{iy} = -\frac{k_{iy}\left(\varepsilon_i\right)}{\mu}\frac{\partial^{\gamma_2} p_i}{\partial y^{\gamma_2}}, i = 1,2 \quad (3)$$

where μ - fluid viscosity, $k_i\left(\varepsilon_i\right)$ are the permeability coefficients of the regions R_1 and R_2, which are functions of ε_i due to colmotation and suffusion effects .

Piezoconductivity equation is defined as follows

$$\frac{\partial p_i}{\partial t} = \chi_i^*\left(p_i\right)\left(\frac{\partial^{\gamma_{ix}+1} p_i}{\partial x^{\gamma_{ix}+1}} + \frac{\partial^{\gamma_{iy}+1} p_i}{\partial y^{\gamma_{iy}+1}}\right), \chi_i^*\left(p_i\right) = \chi_i\left(\varepsilon_{0i} + \beta_i^*\left(p_i - p_0\right)\right), i = 1,2 \quad (4)$$

Problem (1), (2), (3), (4) is solved by the initial and boundary conditions by the finite difference method. Based on the numerical results, the concentration of suspended particles, the porosity of the medium, the fields of filtration velocity and pressure are determined. The influence of orders of the fractional derivative and model parameters on the filtration characteristics of the medium is analyzed.

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On one exact solution of non-stationary movement of liquid and gas in a pipeline in the presence of hydrodynamic resistance

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In this paper, the solution of the equation is presented taking into account the nonlinear term. The method of the gauge function is applied, integration using a simple wave, and the laws of damping in time and in coordinates for infinite and semi-infinite long pipes are obtained.

Let the system describe by equations:

$$\begin{cases} u_t + c^2\varphi_x = -\varepsilon u^2 \\ \varphi_t + u_x = 0 \end{cases}$$

Where u is the hydrodynamic velocity of the gas, $\varphi = \ln\frac{\rho}{\rho_0}, \rho$ is the density of the medium.

We multiply the second equation in (1) by $\pm e$, and add it to the first equation, and as a result we get the system and we introduce the calibration functions A and B : , i.e.

$$u + c\cdot\varphi = A, u - c\cdot\varphi = B$$

In the new notation:
$A_t + c\cdot A_x = -\varepsilon\cdot(A+B)^2, B_t + c\cdot B_x = -\varepsilon\cdot(A+B)^2$
We can write it using the total differential:
$\frac{dA}{dt} = -\varepsilon\cdot(A+B)^2, \frac{dB}{dt} = -\varepsilon\cdot(A+B)^2$
From (4) we have that
dA = dB
Its integral $A - B = [(A-B)] \quad (t=0)$
Let us substitute it into the first equation of system (4):
$\frac{dA}{dt} = -\varepsilon\cdot\left[2A - (A-B)_{t=0}\right]^2$
The change in the direct wave A, associated with friction, causes the appearance of a reverse wave B.

From it we have

$$\frac{dA}{\left[2A - (A-B)_{t=0}\right]^2} = dt\cdot(-2)$$

Integrating, we obtain the exact solution for the calibration functions.

The above method also makes it possible to study inhomogeneous problems. In particular, let us consider the distribution of the drag coefficient along the coordinate using an acoustic wave.

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Modeling divirgent-form cross-diffusion processes in different environments

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Consider in the domain $\theta = \{(t,x) : t \in R_+,\ x \in R^2\}$ parabolic system of two cross-diffusion equations

$$\begin{cases} \frac{\partial u}{\partial t} = \nabla\left(\sigma^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - \nabla\left(\ell(t)u\right) - \wp_1(t)u \\ \\ \frac{\partial \sigma}{\partial t} = \nabla\left(u^{m_2-1}\left|\nabla \sigma^k\right|^{p-2}\nabla \sigma\right) - \nabla\left(\ell(t)\sigma\right) - \wp_2(t)\sigma \end{cases} \quad (1)$$

$$u|_{t=0} = u_0\left(x\right),\ \sigma|_{t=0} = \sigma_0\left(x\right) \quad (2)$$

where numeric parameters $k \geq 1,\ p,\ m_i\ ,\ i = 1,2, \nabla(\cdot) = \underset{x}{grad}(\cdot),\ \ell(t) > 0, 0 < \wp_i(t) \in C(0,\infty),\ i = 1,2$ given functions.

Theorem 1. Let $b_1 \leq \frac{N}{p},\ \ i = 1,2,$ and $Z_1,\ Z_2\ -$ solving a system of algebraic equations

$$Z_1{}^{k(p-2)}\,Z_2{}^{m_1-1} = 1/p\left(\gamma k\gamma_1\right)^{p-1},\ Z_1{}^{m_2-1}\,Z_2{}^{k(p-2)} = 1/p\left(\gamma k\gamma_2\right)^{p-1}$$

and $u_0(x) = Z_1T^{-\alpha_1}\phi_1(\zeta)_{t=0},\ \ \sigma_0(x) = Z_2T^{-\alpha_2}\ \varphi_1(\zeta)_{t=0}\ ,\ \ x \in R^N$.Then the solution of system (1) has the property of finite cross-diffusion velocity [2].

Theorem 2. Let $b_1 \leq \frac{N}{p},\ \ i = 1,2,\ \ 1-[(m_1-1)\alpha_2 + k(p-2)\alpha_1] > 0,$ and $Z_1,\ Z_2-$solution of a system of algebraic equations (1)

$$Z_1{}^{k(p-2)}\,Z_2{}^{m_1-1} = 1/p\left(\gamma k\gamma_1\right)^{p-1},\ \ Z_1{}^{m_2-1}\,Z_2{}^{k(p-2)} = 1/p\left(\gamma k\gamma_2\right)^{p-1}$$

and $u_0(x) = Z_1T^{-\alpha_1}\phi_1(\zeta)_{t=0},\ \ \sigma_0(x) = Z_2T^{-\alpha_2}\ \varphi_1(\zeta)_{t=0}\ ,\ \ x \in R^N$. Then the solution of the system is spatially localized if, for $t > 0$, the conditions [1]

$$\int_0^t \ell(y)dy < \infty,\quad \tau(t) < \infty.$$

Theorem 3. Let $\psi_1 > 0,\ \psi_2 > 0$. Then the solution of system (1) with compact support for $\eta \to \infty\ \ \left(\eta = -\ln\left(a - \zeta^{p/(p-1)}\right)\right)$ has an asymptotic representation [2]

$$\begin{cases} \varphi(\zeta) = Z_1\varphi_1(\zeta)\left(1 + o(1)\right) \\ \phi(\zeta) = Z_2\phi_1(\zeta)\left(1 + o(1)\right) \end{cases}$$

where the coefficients $Z_i > 0\ ,\ \ i = 1,2$ are the solution to the system of algebraic equations

$$Z_1{}^{k(p-2)}\,Z_2{}^{m_1-1} = 1/p\left(\gamma k\gamma_1\right)^{p-1},\ \ Z_1{}^{m_2-1}\,Z_2{}^{k(p-2)} = 1/p\left(\gamma k\gamma_2\right)^{p-1}.$$

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Numerical solution of lanchester's quadratic law with delay argument

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Lanchester's quadratic law with a delay argument describes the delay of the motion dynamics of two opposite sides. The delay lies in the fact that the change in the number of sides does not occur instantly, but with a delay. For example, in the military field, support forces or intelligence information may be delayed due to some (such as technical problems, weather conditions, geographical obstacles, enemy actions, legal restrictions, etc.) factors.

Let $x(t)$ and $y(t)$ be the number of fighters on the first and second sides, respectively, at time t. Then the system of differential equations for this model has the following form:

$$\begin{cases} \frac{dx(t)}{dt} = -ay(t-\tau) + u(t), & x(t_0) = x_0, \\ \frac{dy(t)}{dt} = -bx(t-\tau) + \upsilon(t), & y(t_0) = y_0. \end{cases}$$

where $x(t)$ and $y(t)$ are the number of parties X and Y, respectively, t is the duration of the battle, τ is the delay time in the transfer of information between the parties to the conflict, $a > 0$ and $b > 0$ is the coefficient of losses due to the actions of opponents, $u(t) > 0$ and $\upsilon(t) > 0$ are control parameters, i.e. control of advancing auxiliary forces.

The numerical solution of the quadratic Lanchester law with delay using the Runge-Kutta method can be performed as follows:

1. Let us set the initial conditions $x(0)$ and $y(0)$, as well as the model parameters.

2. Calculate the values of k_1, k_2, k_3 and k_4 in the Runge-Kutta method for each equation at each time step:

$$k_1^x = -ay_{n-1} + u_n, \quad k_2^x = -a(y_{n-1} + \tfrac{h}{2}k_1^y) + u_n, \quad k_3^x = -a(y_{n-1} + \tfrac{h}{2}k_2^y) + u_n,$$
$$k_4^x = -a(y_{n-1} + hk_3^y) + u_n, \quad x_{n+1} = x_n + \tfrac{h}{6}(k_1^x + 2k_2^x + 2k_3^x + k_4^x).$$

$$k_1^y = -bx_{n-1} + \upsilon_n, \quad k_2^y = -b(x_{n-1} + \tfrac{h}{2}k_1^x) + \upsilon_n, \quad k_3^y = -b(x_{n-1} + \tfrac{h}{2}k_2^x) + \upsilon_n,$$
$$k_4^y = -b(x_{n-1} + hk_3^x) + \upsilon_n, \quad y_{n+1} = y_n + \tfrac{h}{6}(k_1^y + 2k_2^y + 2k_3^y + k_4^y).$$

where x_n and y_n are the number values of parties at the previous step, h is the time step.

3. Let's continue the calculations until the given end time.

If at some point in time the number of one of the parties is 0, then this party is considered the loser. Thus, we can obtain a numerical solution of the equations of the Lanchester quadratic law with delay using the Runge-Kutta method.

In conclusion, it should be emphasized that the theory of differential games can be used in planning combat operations of various types and scales, distributing enemy forces in several independent battle areas, and distributing limited resources of combat assets.

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Finding the thermophysical parameters of the material based on the hyperbolic equation of thermal conductivity

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With the development of technology, scientists have recently begun to use hyperbolic heat conduction equations more often. Here, a special place is occupied by the determination of the thermophysical parameters of the material, when the process of heating bodies by high-intensity energy flows is studied. Methods for solving the inverse problem for the hyperbolic heat conduction equation have not yet been sufficiently developed. For practical use, based on the measured statistical data, the development of methods for finding the thermophysical parameters of the heated material has a special place. Therefore, the development of methods for solving coefficient inverse problems for hyperbolic heat conduction equations becomes an urgent task.

This work is devoted to numerical methods for finding the thermophysical parameters of the material under the heat. The mathematical model on the basis of hyperbolic equation of thermal conductivity is presented as well as the relaxation of heat flow is taken into account.

The implementation of the Fourier method for solving the nonlinear hyperbolic problem of thermal conductivity in a one-dimensional approximation by spatial coordinate is considered on the assumption that the diameter of the heating region is much greater than the depth of heat penetration. The main idea of this method is that the solution of the problem for partial differential equations is reduced to solving auxiliary problems for equations with fewer independent variables.

Using the modified Newton method, numerical results were obtained for finding all the thermophysical parameters, as well as a comparison with the initially set values was carried out.

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Evaluation of explosion time in the problems of heat diffusion and combustion processes

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Currently, mathematical models of diffusion processes characterized by a system of non-divergent parabolic-type equations are widely researched in the world. Mathematical models of such diffusion processes, ignition of solid and liquid fuels in the theory of combustion, nonlinear heat transfer, gas and liquid filtration, is widely used to reveal the course of biological population processes [1,2]. In this case, the properties of solutions of nondivergent nonlinear equations are investigated, estimation of the explosion time in the heat diffusion and combustion processes was studied. The works of scientists dealing with similar issues were analyzed [1,2].

A mathematical model representing the processes of heat diffusion and combustion in a non-linear medium is considered. The source (absorption) power has also been studied as a function of time, coordinate, and temperature in general. An approximate automodel solution of the equation was obtained using the nonlinear separation algorithm. Properties of the solution of the problem were studied with the help of an approximate automodel solution, and the explosion time was estimated. An algorithm and a program for solving the problem have been created. The performance of the program was checked using a test case. A calculation experiment was carried out at different values of the parameters involved in the problem.

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Investigation of the asymptotic behavior of nonlinear parabolic equations in non-divergence form.

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At present, mathematical models of diffusion processes characterized by non-divergent systems of parabolic equations are widely studied in the world. Therefore, the determination of new qualitative properties of nonlinear models with a blow-up regime is one of the most relevant scientific research.

Consider the Cauchy problem for a no divergent equation in the domain $Q = (0, T) \times R:$

$$-\frac{\partial u}{\partial t} + u^{\gamma}\frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right) + \varepsilon u^{q-1} = 0 \tag{1}$$

$$u|_{t=o} = u_0(x) \geq 0,\ x \in \mathbb{R} \tag{2}$$

here $p > 2,\ \gamma > 1,\ q > 1$ are given numerical values. Finding the asymptotic in the case of $x \in \mathbb{R}$ was considered in [2-3], and in the case of $\gamma > 0$ in many papers [1-3].

Equation (1) represents many physical processes: reaction-diffusion, heat conduction, filtration, combustion processes. Here εu^{q-1} is a source with a power equal to ($\varepsilon = +1$) or in absorptions ($\varepsilon = -1$). Where $p > 2$ is a fixed parameter that takes different values in different processes.

In this paper, a self-similar solution is found and the asymptotic behavior of self-similar solutions in the case of fast and slow diffusion is proved. And also the results of numerical calculations are found confirming the presence of the properties of the finite speed of heat propagation and spatial localization of the solution of the Cauchy problem

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Estimate for blow-up solution of a non-divergent parabolic type equation describing the processes of heat diffusion and combustion

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In this work, in the domain Q=(0,T)×R we consider the following nonlinear equation not in divergence form:

$$-\frac{\partial u}{\partial t}+u^{\alpha}\frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)+u^{q-1}=0, \tag{1}$$

and initial value condition

$$u|_{t=0}=u_0\left(x\right)\geq 0, \quad x\in R_+, \tag{2}$$

where $p>2$, $\alpha>1$, $q>1$ – numerical parameters. In parameters , $q\geq 1$, $x\in R_+$ asymptotic representation of the solution of equation (1) has been found [1,2].

Main result.

The theorem on the upper solution of the problem (1), (2) has been proved.

Theorem. Let the following conditions hold

1. $q>1$ and $1<\alpha<2$ or $q<=1$ and $\alpha>2$,
2. $u\left(0,x\right)\leq u_+\left(0,x\right)$,

then the problem (1), (2) has a global solution with estimate

$$u\left(t,x\right)\leq B\left(\frac{1}{q-2}\right)^{\frac{1}{q-2}}(T-t)^{\frac{1}{q-2}}\left(a+\left(x(T-t)^{-\frac{\alpha+p-q}{p(2-q)}}\right)^{\frac{p}{p-1}}\right)^{\frac{p-1}{\alpha+p-2}},$$

where B=const.

Get estimates also a good result when you can’t find an exact solution. The found upper solution gives us preliminary information about the exact solution.

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Rayleigh-bénard chaotic convection in micropolar fluids

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In this research, the thermal instability and heat transport in a layer of micropolar fluids confined between permeable horizontal boundaries were investigate. Using normal mode analysis, the eigenvalue for the velocity and temperature boundary conditions was derived with the spin condition vanishes. The generalized Lorenz model was demonstrated by the energy conservation property, which can be reduce to classical Lorenz model under specific boundary conditions. With the assumption that the principle of the exchange of stabilities is valid for the problem considered, the linearized steady-state form for the generalized Lorenz model provides an analytical expression for the Rayleigh number. The onset of regular convective motion in form of rolls are determined for all boundary conditions by minimizing the expression concerning wave number and the critical Rayleigh number. This paper also examined the existence of chaotic motion in the generalized Lorenz system.

Mathematical modelling of the effects of seismic waves on buildings and structures in the epicentral zone of an earthquake

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The formulation of problems on determination of dynamic processes of seismic waves influence on above-ground buildings and underground structures and their numerical solution are connected with limitation of calculation area of half-space. In this case, on the boundaries of the bounded area it is required to set such boundary conditions of transparency so that seismic waves without distortion penetrated into this area, as well as the waves reflected from the studied objects completely escaped through the transparent boundaries of the computational area.

The first published work dealing with conditions of unreflected waves is [1] where formulas for the dependence of stresses on particle velocities at the boundary for plane waves are proposed. These formulas are used in many applied calculations. The second kind of equations for the stresses on the boundary include additionally the displacement relation. A third kind of boundary transparency is the introduction of a fictitious region with special wave absorption properties [2].

In this paper the exact conditions of the boundary transparency for seismic waves arriving parallel to the free surface of the half-space are proposed. The numerical methods for solving the problems under study are the crunch element method for discretisation in coordinates and the Newmark, Bathe [3] methods in time. In the proposed method of solving the posed problem we additionally solve the problem about propagation of plane waves in limited area without research objects, the solutions of which are used in boundary conditions. The programming tool used is FreeFem++ [4] that has its own language for description of the problem statement with appropriate boundary conditions, methods of partitioning into finite elements and description of the method for solution of the problem in terms of time variable.

As an example, the plane problem of elasticity theory for a rectangular half-plane with a recessed foundation and the body of a nine-storey building is solved

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Numerical modeling of the problem of mutual diffusion in fuzzy environment

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This study presents a numerical simulation of the problem of mutual diffusion in fuzzy environment. Fuzzy environments are complex systems characterized by uncertainty and fuzziness of the environment properties. The aim of this study is to develop an efficient numerical method capable of analyzing diffusion processes in such fuzzy environment. The model will take into account the uncertainty in the values of the medium parameters, as well as use fuzzy rules to describe the mass transfer through fuzzy boundaries. The results of numerical simulations will provide a deeper understanding of mutual diffusion in fuzzy environment and apply them in various fields such as the environment, medicine and engineering.

Let the following system of nonlinear differential equations representing mutual diffusion processes be given

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^{\sigma_1}\frac{\partial u}{\partial x}\right) - v^{\beta_1}\left|\frac{\partial u}{\partial x}\right|^{p_1} \\ \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(v^{\sigma_2}\frac{\partial v}{\partial x}\right) - u^{\beta_2}\left|\frac{\partial v}{\partial x}\right|^{p_2} \end{cases}, \tag{1}$$

$$\begin{cases} u\left(x,0\right) = u_0\left(x\right) \\ v\left(x,0\right) = v_0\left(x\right) \end{cases}, \quad x \in R \tag{2}$$

$$\begin{matrix} u(0,t) = u_1(t) \\ v(0,t) = v_1(t) \end{matrix}, \begin{matrix} u(1,t) = u_2(t) \\ v(1,t) = v_2(t) \end{matrix}, 0 \le t \le T, \tag{3}$$

here σ_1, σ_2, β_1, β_2, p_1, p_2 - are fuzzy numbers representing the environment and front parameters for diffusion processes.

Numerical simulation of the problem of mutual diffusion in fuzzy environment is of great importance for the development of fuzzy logic and fuzzy modeling, as well as for practical applications in various fields. This study contributes to an in-depth understanding of diffusion processes in fuzzy environment and the development of efficient numerical methods for their analysis.

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Resource selection in anomalous case detection in dlp systems

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The role of resources in detecting anomalous States in computer systems plays a large role. It is illuminated which parameters of the resources to obtain and give the intellectual systems anyan necessary.

Computer safety system in construction current in the day very many methods is being used. But among them, template search and anomalous search are the most used methods. Today, using these methods together with intellectual methods can get better results.

Of course, we know that intellectual methods work mainly based on data. The information provided also ensures the productivity of these methods. The more information, the better the result of the intellectual method, be it a self-taught method or a simple method.

- Active program processes;
- Active and new user;
- Information transmission channels;
- Registers;
- Temporarily or constant processes;
- The system settings to change movement to do;
- Files (System files and user documents files);
- computer parameters.

Other event information sources can be added to this list. From the mentioned sources of information, it is possible to obtain information that is used to determine the behavior of the user.

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Degenerate cases of Lotka-Volterra mappings acting in a simplex

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The paper considers the problem of studying the trajectories of points under the action of the Lotka – Volterra operator, i.e. determine the start and end points of the trajectories. In a word, it consists in studying the dynamics of points belonging to a simplex under the action of a given operator. The Lotka-Volterra mapping is uniquely determined by specifying a skew-symmetric matrix $A = (a_{ij})\,, a_{ij} = -a_{ji}, i,\ j = \overline{1, m}$ and acts on the simplex S^{m-1} according to the formula

$$x_k^{'} = x_k \left(1 + \sum_{i=1}^{m} a_{ki}x_i\right),\ k = 1, ..., m \tag{1}$$

given that $|a_{ki}| \leq 1$. Let $e_k = (0, ...\, 0, 1,\ 0, ..., 0\,)$, where (1) is in the wrong place k, then $S^{m-1} = co\,\{e_1, ..., e_m\}$ i.e. the simplex is the convex hull of the points e_k, which are called the vertices of the simplex S^{m-1}. If $\gamma \subset I = \{1,\, 2, ..., m\}$ is a non-empty subset, then $\Gamma_\gamma = co\,\{e_k :\ k \in \gamma\}$ is called the $|\gamma| - 1$-dimensional face of the simplex. Let $V : S^{m-1} \to S^{m-1}$ the Lotka – Volterra mapping defined by equality (1). It is known that for $x^0 \in S^{m-1}$ the trajectory $\left\{x^{(n)}\right\}$ is defined by the recursion relation

$$x^{(n+1)} = Vx^{(n)},\ n = 0,\, 1, ... \tag{2}$$

For any $x^0 \in S^{m-1}$ we set $\omega(x^0) = \left\{x^0,\, x^{(1)},\, ...\right\}'$ is the set of limit points of the positive trajectory and $\alpha(x^0) = \left\{x^0,\, x^{(-1)},\, x^{(-2)}, ...\right\}'$ is the set of limit points of the negative trajectory. It is also known that from $x^0 \in S^{m-1}$ and $x^0 \neq Vx^0$ follows $\omega(x^0) \in \partial S^{m-1}$, i.e. $\omega(x^0)$ belongs to the boundary of the simplex.

Definition 1. A skew-symmetric matrix is called a generic matrix if all principal minors of even order are positive.

Theorem 1. If x^0 is an interior point of the simplex, then $\alpha(x^0) \in P$ and $\omega(x^0) \in Q$, and both positive and negative trajectories converge.

Definition 2. Points $p \in P$ and $q \in Q$ form a $(p,\, q)$ pair if there exists $x^0 \in S^3$ such that $\omega(x^0) = q,\ \alpha(x^0) \in p$.

Theorem 2. Any trajectory converges, and $\alpha(x^0) \in P,\ \omega(x^0) \in Q$.

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Numerical simulation of a differential equation with a small parameter at the highest derivative by the method of preliminary integration

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A boundary value problem for a differential equation with a small parameter at the highest derivative is considered. The application of numerical methods to the solution of such equations encounters serious difficulties. These difficulties are associated with the presence of a small parameter at the highest derivative and, as a consequence, the appearance of sections with inhomogeneous gradients in the solution region. In this regard, the requirements for the approximation property of numerical methods increase sharply. Basically, for the numerical simulation of this class of problems, finite-difference [1], spectral [2] and spectral-grid methods [3] are used. In this paper, along with these methods, it is proposed to apply the method of preliminary integration (MPI). The essence of MPI is as follows. The highest derivative of the differential equation is expanded into a series in terms of Chebyshev polynomials of the first kind. Prior to solving the problem, this series is pre-integrated and thus, expressions are found for all lower derivatives and the desired solution. In this case, the constants appearing during the integration of the series are determined from the satisfaction of the corresponding boundary conditions. Only after this expression of series for derivatives and the desired solution are put into the main differential equation and a system of linear algebraic equations is obtained with respect to the expansion coefficients. By substituting these coefficients into the required series, the values of the solution and their derivatives are determined. The MPI excludes undesirable operations of term-by-term differentiation of series and improves the smoothness of the desired solution and their derivatives. Numerical calculations show high accuracy and efficiency of MPI.

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On some free boundary problems of the prey-predator model with nonlinear prey-taxis

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Population dynamics is one of the most widely discussed research topics in mathematical biology. New population dynamics applications have been developed that are used to model many natural phenomena. Population models, in particular competition models or predator-prey models, are important both in their original field and in their application to many other problems from other fields. Consequently, predator-prey systems have expanded their application and have given rise to systems that more accurately reflect the various biological problems that arise in the context of interacting species [1,2].

In many realistic simulation situations, both prey and predator tend to emigrate overseas to obtain new habitat and improve habitat. Then it is more reasonable to consider a region with a movable free boundary. It can be assumed that the free boundary is created only by the victim, and the propagation front expands at a rate proportional to the gradient of the prey population at the boundary[3].

This note considers the Beddington-DeAngelis diffusion predator-prey model with a nonlinear prey taxis and a free boundary:

$$u_t - u_{xx} + (u\chi(u)v_x)_x = f(u,v), t > 0, 0 < x < h(t),$$

$$v_t - dv_{xx} = g(u,v), t > 0, 0 < x < l,$$

$$u_x(t,0) = v_x(t,0) = u(t,h(t)) = v_x(t,l) = 0, t \geq 0,$$

$$h'(t) = -\mu u_x(t,h(t)), t \geq 0, h(0) = h_0,$$

$$u(0,x) = u_0(x), v(0,x) = v_0(x), 0 \leq x \leq h_0,$$

$$f(u,v) = buv/(c+u+mv) - au, g(u,v) = v(q-v) - ruv/(c+u+mv),$$

where u and v represent prey and predator densities, respectively. Constants a, b, c, m, q, r, d are positive; a is the mortality rate of the predator, which does not depend on the prey density; the function $rv/(c+u+mv)$ is the Beddington-DeAngelis functional response; and b/r is the conversion rate from prey to predator.

The main result of the work is the establishment of the global existence of a classical solution to the problem and the study of the behavior of the solution. A method is proposed for establishing a priori estimates of the Schauder type for a new class of problems with a free boundary for mixed-two-phase cross-diffusion systems. Certain sufficient conditions are also established for both reproduction and extinction.

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Mathematical modeling of the non-linear fractional oscillator Mathieu

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In this paper, we study the Cauchy problem (1) for a fractional nonlinear Mathieu oscillator using a nonlocal explicit finite difference scheme.

$$\partial_{0t}^{\alpha} x\left(t\right) + \gamma \partial_{0t}^{\beta} x\left(t\right) + x^3\left(t\right)\left(1 + \varepsilon \cos\left(t\right)\right) = 0, x\left(0\right) = a, \dot{x}\left(0\right) = b,$$

where $x\left(t\right) \in C^2\left[0, T\right]$ is the displacement function to be determined, $\gamma > 0$ is the coefficient of friction, $\varepsilon > 0$ is a parameter that has the meaning of an amplitude, a and b are given constants that define the initial conditions. Fractional derivative operators $1 < \alpha < 2$ and $0 < \beta < 1$ [1]:

$$\partial_{0t}^{\alpha} x\left(t\right) = \frac{1}{\Gamma\left(2-\alpha\right)} \int\limits_0^t \frac{\ddot{x}\left(\tau\right) d\tau}{\left(t-\tau\right)^{\alpha-1}}, \partial_{0t}^{\beta} x\left(t\right) = \frac{1}{\Gamma\left(1-\beta\right)} \int\limits_0^t \frac{\dot{x}\left(\tau\right) d\tau}{\left(t-\tau\right)^{\beta}},$$

Problem (1) is a Cauchy problem and for the values $\alpha = 2$ and $\beta = 1$ goes over to the classical Cauchy problem [2,3].

The paper proposes a numerical algorithm for solving the Cauchy problem based on a nonlocal explicit finite-difference scheme and investigates the issues of its stability and convergence.

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Implementation of the test 0-1 algorithm for the analysis of chaotic modes in fractional dynamical systems

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Information about chaotic regimes in various dynamical systems is of great practical importance. This is due to the fact that chaotic modes complicate the study of dynamic systems, it is very difficult to understand or predict their next states. Therefore, it is necessary to know the values of the dynamic parameters at which chaotic regimes arise. The values of the parameters of a dynamic system that lead to chaos are studied using various algorithms and tests, and maps of dynamic modes are built [1]. One such algorithm is Test 0-1. The algorithm was proposed in [2]. Basically, the 0-1 Test was used for discrete dynamic systems [2], there are works where it was used for time series analysis [3]. It should be noted that for continuous dynamical systems Test 0-1 can also be used [4], but here the oversampling problem arises. Discrete values of the system, obtained using a numerical scheme with a sufficiently large number of computing nodes of the computational grid, practically do not differ from the exact values.

The paper proposes an implementation of the Test 0-1 algorithm in the Matlab environment, which avoids the oversampling problem. The algorithm is used in the analysis of fractional dynamic systems - dynamic systems that are described using fractional derivatives.

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Computer simulation of axisymmetric problems of elasticity theory

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A computational algorithm to determine the stress state of axisymmetric bodies using the finite element method is presented in the article. On the basis of the computational algorithm and software developed, the influence of the configuration of structures was studied under the action of given loads.

The stress state of an axisymmetric structure with a circular notch is considered; on the left boundary of the structure, there is a central spherical cavity. The influence of the cavity on changes in the stress state in the vicinity of the notch is studied.

Table 1 shows the values of displacements and stresses at characteristic points of the structure. Analysis of the calculation results in the vicinity of the notch σ_{zz} indicates that the presence of a cavity redistributes stresses and reduces their maximum value by 5%.

Таблица 1: Values of displacements and stresses at characteristic points

Components		[1]	Without a hole	With a hole
On the right-hand side	u[cm]	$2.47 \cdot 10^{-2}$	$2.48 \cdot 10^{-2}$	$2.52 \cdot 10^{-2}$
	σ_{zz} $[\mathrm{N/cm^2}]$	$4.3993 \cdot 10^{4}$	$4.3993 \cdot 10^{4}$	$5.0709 \cdot 10^{4}$
On the notch σ_{zz} $[\mathrm{N/cm^2}]$		$6.4576 \cdot 10^{4}$	$6.4573 \cdot 10^{4}$	$6.1371 \cdot 10^{4}$

On the basis of a computational experiment, it was determined that the external load applied, due to the geometry of the structure, leads to stress concentration in the vicinity of the notch; the presence of a cavity in the center of the structure redistributes the stress field in the vicinity of the notch; a decrease in the curvature of the notch leads to a decrease in the stress state, as in the vicinity of the notch; in the case of the coincidence of the values of the radius of the cavity and the narrow part of the structure, a sharp increase in the stress state is observed in the vicinity of the notch [2-4].

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Numerical simulation of a non-stationary heat transfer process in contact interaction with the surrounding medium

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A non-stationary process of heat exchange is studied in the article, solving a 2D problem of heat conduction by the FEM; it is considered under the contact interaction between a structure heated to high temperatures and the ambient medium, and determining factors that affect the actual values of contact temperatures are defined. The effect of concentrators on the non-homogeneous thermal field formation is studied. The objectives of the study are to define the temperature field in the material layer adjacent to the heated surface and to quantify the surface temperature difference under adequate heat exchange conditions.

The authors consider a heat transfer process under the contact interaction of an infinite steel beam heated to a temperature of 1500^oC with the ground surface of a temperature of 20^oC. At a certain distance from the ground surface, there is a bounded area of rectangular shape filled with oil, water, or air. Over time, the beam cools due to heat exchange between the ambient medium ($T_\infty = 20^oC$) and the ground surface.

In solid soil, at the initial time, all thermal energy is concentrated in the steel beam. Then, over time, the thermal energy of the beam is partially distributed over the area of soil, and the partial heat exchange with the ambient medium (air) takes place. Since the thermal diffusivity of soil is less and it is more inert, heat gradually accumulates and spreads throughout the soil thickness, and then partially returns to the steel beam and partially exchanges between soil and air. Ultimately, the system comes to a thermal balance, as all thermal energy is transferred to the ambient medium (air). Thus, it was established that the temperature in the "beam-base"contact zone is an essentially non-stationary quantity.

Then, the process of heat transfer is considered; under the base of the steel beam at a distance of 0.15 m, there is a bounded area in the form of a square with dimensions of 0.2 m, occupied by air, oil, or water.

It was established that the maximum heating temperature of 74.2^oC was observed when the area was filled with water at t = 20 h, and the minimum temperature was 39.2^oC when the area was filled with oil at t = 40 h.

An analysis of the distribution curves of temperature values indicates that the presence of non-homogeneities in the structure significantly affects the temperature field. Since the thermal conductivity of air is much less, than the thermal conductivity of soil, in the vicinity of the upper surface of the bounded area filled with air, a concentration of higher temperatures is observed. The same pattern is observed when the area is filled with oil and water. Over time, the heat cools and spreads into the depth of the structure, where the maximum temperature value is observed inside the bounded area.

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Ecological-economic model of optimal use of water resources in agriculture

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Today, the issue of monitoring the use of water resources in agriculture is one of the urgent issues in the world. The need to apply information technologies to it and issues of its organization are being promoted [1]. In addition to this, in the use of information technologies in agriculture, remote sensors for measuring soil moisture or ambient temperature, sending accurate and fast information about the need for measures to maintain soil moisture can achieve several achievements in the irrigation system.

A two-criteria ecological-economic model of the placement and specialization of agricultural production aimed at the minimum of water consumption and the maximum of production [3] is considered.

A software complex for solving environmental and economic issues in decision-making has been developed [2]. Based on the structure and connection of the blocks that are the basis of this software complex, we can obtain the solutions of the two-criteria ecological-economic model in this case.

In relation to this [3] work, the parameter that takes into account the salinity level of the land and the use of chemical fertilizers and other parameters of the amount of harmful substances stored in the composition are added. From this, a new ecological-economic model of the placement and specialization of agricultural production, taking into account the salinity level of the land, will be formed.

In solving this problem, parametric simplex or discretization methods are used, and a Pareto set is developed and a set of efficient solutions is created. Alternative options for optimal solutions of the two-criteria problem aimed at the minimum of water consumption and the maximum of production are presented to the experts and the right of choice is left to them.

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Conditional Monte-Carlo scheme for stable Greeks of worst-of autocallable notes: multi-asset case

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It is well known that the application of Monte-Carlo method in pricing of products with early termination feature results in a high Monte-Carlo error and unstable Greeks, see [2].

We develop a Monte-Carlo scheme that utilizes a special structure of worst-of autocallable notes and produces stable Greeks. Algorithm uses the methods of linear algebra and apply orthogonal transformations in particular for using the measure, where autocallable note survives and not terminated. This scheme clearly demonstrates the variance reduction in Monte-Carlo scheme and can be used in pricing of multi-asset worst-of autocallable notes with any number of underlying assets. We suggest an algorithm and analyze its performance for an autocallable note on four assets. Suggested algorithm allows to calculate stable Greeks (Delta,Gamma,Vega and others) and substantially reduce the computational effort to achieve the desired accuracy in comparison to standard Monte-Carlo algorithm. As seen from the results, the computation of risk sensitivities using finite differences under this scheme produces substantial variance reduction compared to standard Monte-Carlo method.

	Effective algorithm		Standard approach	
# simulations	mean	st.error	mean	st.error
3000	$2.10 \cdot 10^{-1}$	$6.87 \cdot 10^{-3}$	$2.16 \cdot 10^{-1}$	$3.61 \cdot 10^{-2}$
5000	$2.09 \cdot 10^{-1}$	$8.38 \cdot 10^{-3}$	$2.04 \cdot 10^{-1}$	$5.64 \cdot 10^{-2}$
10000	$2.10 \cdot 10^{-1}$	$3.74 \cdot 10^{-3}$	$2.02 \cdot 10^{-1}$	$4.47 \cdot 10^{-2}$
20000	$2.09 \cdot 10^{-1}$	$4.04 \cdot 10^{-3}$	$2.13 \cdot 10^{-1}$	$2.73 \cdot 10^{-2}$
30000	$2.08 \cdot 10^{-1}$	$3.69 \cdot 10^{-3}$	$2.12 \cdot 10^{-1}$	$3.04 \cdot 10^{-2}$

Таблица 2: The results of calculation of delta with respect to the first base asset $S^{(1)}(0)$. “Standard approach” means the standard Monte-Carlo scheme, “Effective approach” means the algorithm given in our paper. The same results can be received for other greeks (sensitivity coefficients such as gamma,vega).

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On the asymptotics of solutions to the problem cross-diffusion with variable density and source

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In this paper, we study the properties of solutions of the nonlinear parabolic system of equations diffusion associated with a nonlinear boundary condition

$$|x|^n \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(v^{m_1-1}\frac{\partial u}{\partial x}\right) + u^\beta, \quad |x|^n \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(u^{m_2-1}\frac{\partial v}{\partial x}\right) + v^\beta, \quad x \in R_+, \ \ t > 0, \ (1)$$

$$-v^{m_1-1}\frac{\partial u}{\partial x}(0,t) = u^{q_1}(0,t), \quad -u^{m_2-1}\frac{\partial v}{\partial x}(0,t) = v^{q_2}(0,t), \ \ t > 0, \qquad (2)$$

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in R_+, \qquad (3)$$

Where $m_i > 1$, n, $q_i > 0 \ (i = 1, 2)$, u_0 and $v_0(x)$ non-negative continuous functions with compact carrier in.

Nonlinear parabolic equations (1) are found in various applications such as the model of biological populations, chemical reactions, heat propagation, cross diffusion, etc. For example, $u(x,t)$and $v(x,t) = 0$ are the densities of two biological populations in the process migration or temperature of two porous materials during heat propagation [1-3].

In recent years, questions of the global existence of solutions and the condition for the emergence of a blow-up regime have been intensively studied (see [1]). In particular, critical exponents of the Fujita type play an important role in the study of the properties of mathematical models of various nonlinear processes described by nonlinear parabolic equations and the system of such equations of mathematical physics (see [1-3] and References therein).

This work is devoted to the study of the conditions of global solvability and unsolvability in general in time of solutions to problem (1)-(3) based on self-similar analysis and the method of reference equations, as well as the influence of the inhomogeneity of the medium on the process under study. Various selfsimilar solutions of problem (1)-(3) are constructed in the case of slow diffusion with variable density, estimates and asymptotics of solutions are obtained, critical exponents of the Fujita type and critical exponents for the global existence of a solution are established.

Theorem 1. Let $q_1 > 1, q_2 > 1$ and $n = 0$ then any solution to the problem (1)-(3) is unbounded for sufficiently large initial data.

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Mathematical modeling for determining the rotation frequency of spindles in a cotton harvesting machine

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The article is devoted to determining the rotation frequency of the spindles in the working area of the harvesting machine in the stationary position of the spindle drum and the mobile movement of the cotton harvesting machine. Kinematic schemes of arrangement of fixed solar belt and friction tracks are considered.

In a stationary position, we obtain the number of revolutions of the spindle in the working area

$$n = \frac{(\varphi - \alpha)\,(R + r_{\mathrm{K}})}{2\pi \cdot r_{\mathrm{K}}}$$

where $\varphi-$ is the angle of wrapping of the roller rollers of the spindles by the direct rotation belt $\varphi = 120^\circ$:

$$\varphi = \frac{2\pi \cdot r_K}{R + r_K}$$

Determine the displacement X of the machine unit along the cotton axis:

$$X = \frac{V_{\mathrm{M}} \cdot 2\pi \cdot r_{\mathrm{K}}}{\omega_\sigma\,(R + r_{\mathrm{K}})}$$

We determine the trajectory of the curve during the mobile movement of the machine unit and obtain the transcendental curve S:

$$S = 2\pi \cdot r_{\mathrm{K}} - V_{\mathrm{M}} \frac{2\pi \cdot r_{\mathrm{K}}}{\omega_\sigma\,(R + r_{\mathrm{K}})}$$

Determine the required radius of the spindle roller

$$r_{\mathrm{K}} = \frac{\sqrt{(2\pi\omega_6 R - 2\pi V_{\mathrm{M}} - S\omega_\sigma) - 8\pi\omega_i^2 RS} - (2\pi\omega_6 R - 2\pi V_{\mathrm{M}} - S\omega_\sigma)}{4\pi\omega_6}$$

and angular speed of the drum

$$\omega_\sigma = \frac{2\pi \cdot V_{\mathrm{M}} \cdot r_{\mathrm{K}}}{(R + r_{\mathrm{K}})\,(2\pi \cdot r_{\mathrm{K}} - S)}$$

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Investigation of finite velocity and solution localization effects in cross-diffusion processes with convective transfer

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In the work considered in the domain $Q = \{(t, x) : t > 0, x \in R^n\}$ of the problem (1) – (2)

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - \operatorname{div}(c(t)u)$$
$$\frac{\partial v}{\partial t} = \operatorname{div}\left(u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\right) - \operatorname{div}(c(t)v)$$
$$u|_{t=0} = u_0(x) \geq 0,\ v|_{t=0} = v_0(x) \geq 0, x \in R^n$$

where $k \geq 1, p, m_i, i = 1, 2$ numerical parameters and $\nabla(\cdot) = \underset{x}{\operatorname{grad}}(\cdot), 0 \leqslant u_0, v_0 \in C(R^n)$.

The system (1) is degenerate in the area where it is degenerate, it may not have a classical solution in the area of degeneracy. Therefore, we are studied the generalized solution of the system with the property:

$$0 \leqslant u, v \in C(Q), v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u, \quad u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v \in C(Q).$$

Finite velocity properties of perturbation propagation and solution localization are extended for parabolic systems with double non-linearity, describing non-linear diffusion processes with convective transport. Such properties of solutions to problem (1), (2) in the case of one equation were given in detail in the works of the authors by constructing an exact self-similar solution. In this paper, by constructing an approximate solution for system (1), we prove the property of a finite perturbation propagation velocity and the spatial localization of the solution. The asymptotes of finite solutions of the selfsimilar equation, as well as the asymptotes of solutions of the self-similar equation, is established. The choice of an appropriate initial approximation for the iterative process in the numerical solution of the problem under consideration is proposed.

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Construction of resolving equilibrium equations based on the Vlasov-Kantorovich method and numerical analysis of convergence

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Modern operating conditions of spatial structural elements in the form of rods impose increased requirements on their strength calculation. In this regard, there is an increasing interest in the results of the calculation of spatial structural elements, taking into account plastic deformations. Great success in the study of physical nonlinear problems was achieved using the method of elastic solutions of A. A. Ilyushin based on the theory of small elastic-plastic deformations [1].

According to the Vlasov-Kantorovich method, the mathematical model for the solution of the three-dimensional Lame equation in the elastic-plastic stage is represented in a special form of an infinite series, the convergence of which depends on the initial approximations. The construction of the initial functions depends on the formulation of the problem and on the boundary conditions. This paper presents a method for constructing a numerical-analytical mathematical model of functions for an elastic rod in the form of a parallelepiped. Also, all numerical analyzes were carried out in the MatLab environment. The choice of the MatLab package is the convenience and simplicity of performing analytical and numerical operations.

In the work, a mathematical model is built in the form of systems of differential equations and a computational scheme is built. To test the developed scheme and the reliability of the results, as well as to study the stress-strain state of an elastic body in problems of constrained torsion, calculations were made with various geometric and mechanical characteristics of an elastic body. The obtained analytic-numerical solutions make it possible to construct a faster convergence of the solution in the future.

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Modeling the spread of a viral infection

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Consider the following parabolic problem in non-divergent form:

$$\frac{\partial u}{\partial t} = u^n \nabla(u^{m-1}\nabla u) + bu(1-u^{n-1}), \ \ (t>0,\, x \in R^N), \ u(0,x) = u_0(x), \ x \in R^N$$

Here $b > 0, m \geq 1, n > 1$. This problem was proposed in [1] as a mathematical model of the virus spreading process and the properties of solutions to this problem were obtained. In [2], the author considers the Cauchy problem for a degenerate parabolic equation in a non-divergent form, which is a diffusion approximation of the epidemic spread model in a closed population without remission. He proves the existence and uniqueness of a weak solution, suitably defined, and some qualitative properties. We have proved that the solution to this problem has the properties of a finite speed of the spread of the virus and spatial localization of the spread of the virus. Need, we choose the replacement

$$u(t,x) = exp(-bt)w(\tau(t),x)$$

In this case, a transformation was found that allows us to represent equation (1) to a self-similar equation by setting

$$w(\tau(t),x) = f(\xi), \ \ \xi = |x|/[b(n+m)\tau(t)-1]^{1/2}$$

Then we obtain the following self-similar equation

$$L(f) \equiv f^n \xi^{1-N}\frac{d}{d\xi}(\xi^{1-N} f^{m-1}\frac{df}{d\xi}) + \frac{\xi}{2}\frac{df}{d\xi} + f^n = 0$$

Consider the function

$$\overline{f}(\xi) = (a - (m+n-1)\frac{\xi^2}{4})_+^{1/(m+n-1)}, \ a > 0$$

Where the designation is used $(m)_+ = max(0,m)$. It is easy to calculate that

$$L(\overline{f}(\xi)) = -(N/2)\overline{f} + \overline{f}^n \leq 0$$

Therefore, the function $u_+(t,x) = exp(-bt)\overline{f}(\xi)$ has the property $u_+(t,x) \equiv 0$ at $|\, x \,| \geq 2(a/(n-1))^{1/2}\tau(t) < \infty, n > 1$. Because the $max\ \tau(t) = \frac{1}{b(n+m)}$, then the solution of problem (1) has the property of spatial localization. Viruses are concentrated in the area $|\, x \,| \leq [b(n+m)\tau(t)-1]^{1/2}$.

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Optimal management of water resources of large main canals with cascades of pumping stations

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In the process of water distribution management, the values of water discharge are mainly determined, which are set for consumers of canals of machine water lifting systems during a decade of the corresponding period and based on the solution of the problem of calculating the outflow and water discharge. Uzbekistan is considered as one of the largest irrigation farming countries in Central Asia. Proper use of existing water and land resources can increase crop production and yields in the agriculture sector [1]. Applications for head water intakes and canal sections for a decade determine the operating modes of canal sections that implement consumer requests in terms of costs. The choice of operating modes of the canal sections is carried out on the basis that all side water intakes are guaranteed to receive the planned flow rates of water resources with minimal water losses for filtration and evaporation. The choice of the operating mode of the control of canal is carried out on the basis that all lateral water intakes are guaranteed to receive the planned discharge of water resources with minimal water losses for filtration and consumption. When using canals, it is important to assess quantitative indicators of the state of reliability associated with such adverse effects as wear of canal dams under the influence of dangerous filtration currents, subsidence, and elevation of canal sections relative to the are. Based on the condition that the variable parameters remain constant over time, i.e., in equations (1) - (2) the partial time derivatives are equal to zero and given that the canal bed is prismatic, we obtain the following system of equations for the uneven movement of water in the canal section:

$$\frac{d\,Q}{d\,x} = q, \qquad \frac{dP}{dx} + \frac{d}{dx}\left(\frac{Q^2}{\omega}\right) = -g\omega\ \left(\frac{dz_0}{dx} + \frac{Q\,|Q|}{K^2}\right) + F,$$

Lateral outflows and inflows are concentrated or distributed. As concentrated inflows and outflows, lateral water outlets or concentrated inflows are considered, and as distributed outflows - losses due to filtration and evaporation. The task of determining the operating modes of the canal section in the presence of backwater from the lower barrier structure is reduced to determining such a value of the water level at the end of the canal section hk, which would minimize seepage and evaporation losses in the canal section. At the same time, the water levels in the canal sections, where the side outlets are located, satisfy the restrictions on the water pressure in front of the outlet structure and side outlets.

Development of a mathematical model of the production of grinding steel balls

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Nowadays, the demand for grinding steel balls in the mining and metallurgical industry is increasing day by day. Rolling mill rolls with variable pitch screw grooves are used in the production of grinding steel balls. These rolling mill rolls are manufactured using gauge technology. Gauge technology is a unique complex technological process. Taking this into account, based on the development of a mathematical model of this technological process, it is important to further improve the quality of the product and achieve economic efficiency.

In this thesis, a mathematical model of the problem of grinding steel balls production obtained by hot rolling method was developed and the main obtained results were analyzed. Based on the obtained data, cutting operation was carried out on the RT 117 screw-cutting lathe for the production of grinding steel balls. The screw-cutting lathe is equipped with a special copying device designed for cutting products.

The initial diameter of the grinding steel balls is determined as follows [1]:

$$d_b = \frac{D_k}{\eta_t}$$

where D_k- is the diameter of the gauge, η_t - is the coefficient of temperature expansion of metal during the rolling process.

The volume of the billets in the rolling process should be equal to the volume of the formed steel ball. Under the condition that the volume of the compressed metal does not change, the following expression is appropriate for the total volume of the rolling ball and bridge in an arbitrary section of the gauge:

$$V_{total} = V_\alpha + V_{C\alpha} + V_{S\alpha-360} + V_{\alpha-360} + V_{C\alpha-360}$$

Taking into account the constructed mathematical models and obtained numerical results, the following conclusions can be drawn: in solving the problem, accurate results were obtained regarding the necessary parameters for the production of grinding steel balls; the analysis of the constructed mathematical models shows that the solution of this problem is fully compatible with the technology of manufacturing rolling mill rolls with variable pitch screw grooves, used in the production of grinding steel balls obtained by hot rolling method; the developed mathematical model of a gauge technology for producing grinding steel balls enables the examination of changes in the ball shape during the rolling process; numerical parameters show an agreement with experimental results.

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On the problem for the ballistic-diffusion model of heat transfer in superlattices

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Semiconductor superlattices are of great interest because of their very low thermal conductivity and because of their potential applications in various devices, for example, for thermoelectric power generation, the development of intelligent coatings for temperature conditioning of cases, the design of information processing devices, thermal energy storage and etc. [1].

Heat transfer at the nanoscale differs from that predicted by the Fourier law, and as the thickness of the heat-conducting layer approaches the nanoscale, other non-Fourier transport modes appear, such as ballistic and wave [2].

Ballistic-diffusive models for heat transport are based on the assumption that there exist two kinds of heat carriers in the system. Those which propagate diffusively that are scattered in the core of the material, and those which propagate ballistically, interacting only with the boundaries or the interfaces of the system.

In thermodynamic theories of heat transport, ballistic-diffusive models are obtained assuming that the heat flow can be separated into the ballistic and diffusive components. It has been then assumed that the diffusive part is described by the Cattaneo equation and the ballistic part by Guyer-Krumhansl equation [3].

The system consists of two particle populations, namely, ballistic and diffusive heat carriers which have inner energies e_b and e_d, and heat fluxes q_b and q_d, respectively. In this way, the variable space becomes $\{e_b, e_d, q_b, q_d\}$. Then the balances of the component energies are:

$$C_b T_{bt} + q_{bx} = -Q - AT_b,$$

$$C_d T_{dt} + q_{dx} = Q + AT_d,$$

where the caloric equations $e_d = C_d T_d$ and $e_b = C_b T_b$ were used, where T_d and T_b are interpreted as quasi-temperatures and specific heats of C_d and C_b particles. We assumed that the heat exchange between the ballistic and diffusion parts breaks up into a dissipative part Q and a nondissipative part linear in the quasitemperature, A is a constant value.

The article demonstrates the theoretical results of the thermodynamic theory of heat transfer in nanometer one-dimensional multilayer systems, taking into account the separation of ballistic and diffusion heat fluxes described by the Guyer-Krumhansl constitutive equations.

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On the existence of classical solutions of the aggregation model with a free boundary

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The classical model of chemotaxis was first proposed by Keller and Segel [1] to describe the aggregation of some bacterial species. This model has been actively studied over the past twenty years [2]. The concept of chemotaxis is used to refer to the movement of cells towards or away from a chemical source, classified as positive and negative chemotaxis, respectively. According to the broad definition of chemotaxis, it is the study of any movement of a cell that is affected by a chemical gradient in such a way that it results in a net spread up the chemoattractant gradient or down the chemo-repellant gradient.

This work presents theoretical results on structural self-organization in biological systems, based on systems of reaction-diffusion equations, where nonlinear terms describe kinetics, and transfer processes are represented by isotropic diffusion. Cross-diffusion systems are a natural generalization of open dissipative structures described by models with nonlinear diffusion.

For more information on the application of free boundary problems to the study of biological problems, the reader may refer to [5]. We study a class of Keller-Segel systems with cross-diffusion and free boundary

$$
\begin{gathered}
u_t - (d_1(u)u_x)_x + (u\chi(u)v_x)_x = -au + bg(v)u, 0 < x < h(t), t > 0, \\
v_t - d_2 v_{xx} = k(v) - g(v)u, 0 < x < h(t), t > 0, \\
u(h(t),t) = 0, v(h(t),t) = 0, h'(t) = -\beta u_x(h(t),t), h(0) = h_0 > 0, t > 0, \\
u_x(0,t) = v_x(0,t) = 0, t > 0, u(x,0) = u_0(x), v(x,0) = v_0(x), 0 < x < h_0,
\end{gathered}
$$

where $u(t,x)$ is the density of cells in a given region Ω, and $v(t,x)$ is the concentration of the chemical signal. Cell dynamics is determined by the kinetics and movement of the population, the latter consisting of a diffusion flow that mimics non-directional cell migration and an advective flow with a velocity-dependent signal gradient that mimics the contribution of chemotaxis.

The main result of this article is the establishment of the existence and uniqueness of the classical solution of the system. First, we prove the result of local existence. A priori estimates of the Schauder type are established, on the basis of which the unique global solvability of the problem is proved.

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Problematic issues of modeling the customs clearance process by linear progarmming of variable coefficient

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The purpose of this study is to optimally manage the customs clearance process and minimize the time spent on this process. The following mathematical model is proposed for solving the problem of optimal control of the process of multi-stage customs clearance by linear programming with variable coefficients:

$$f(t) = \sum\nolimits_{k=1}^{n} r_k t_k \to min \tag{1}$$

$$\begin{cases} \sum\limits_{k=1}^{n} a_{jk} t_k \le b_1; \quad a_{jk} = 1, \quad if \quad j = (\overline{1, m_1}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_2; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_1 + 1, m_2}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_3; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_2 + 1, m_3}) \\ \sum\limits_{k=1}^{n} a_{jk} t_k \le b_4; \quad a_{jk} = 1, \quad if \quad j = (\overline{m_3 + 1, m_4}) \\ t_k > 0, k = (\overline{1, n}) \end{cases} \tag{2}$$

here: $n = 18$- number of stages of customs clearance process; $m_1 = 7$, $m_2 = 8$, $m_3 = 11$, $m_3 = 18$ - the stages of the customs clearance process, which are responsible for the participant in foreign trade, the carrier, the owner of the customs warehouse and the customs service, respectively; $a_{jk} = 0$at the values of the index j that are not included in the conditions of (); $r_k = r_k(X)$ - level of risk of execution of k-step of the customs clearance process; $X = X(x_1, x_2, ..., x_{58})$ -vector, x_i elements which are determined depending on the value of the columns of the cargo customs declaration.

The peculiarity of the problem of optimal management of the process of multistage customs clearance, which is described in (1) - (2) are variable coefficients of the target function [1]. The function $r_k(X)$, which represents the risk level of the k-stage of the process, is a function of the variables of the Cargo Customs Declaration. The implementation of these criteria made it possible to prevent in 2022 in 88 thousand 897 cases the risks of "determining the customs value of goods"and arrears to the state budget in the equivalent of more than 9 million 968.8 thousand US dollars.

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A mathematical model and program of treatment success and prediction using regression analysis in medicine

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In modern medicine, with the help of computers, it is often necessary to determine the comparative value of the general parameters of the differences between the selected objects that are observed or compared in experimental and clinical research. In solving this task, it is necessary to show clear and concrete values, that is, to determine the characteristic signs of the pathological process, to monitor the dynamic development of the disease, to evaluate the effective traditions of treatment, and others.

Parametric criteria have a greater value (power) than non-parametric criteria in the case of a normal distribution of characters (factors). Parametric criteria have the property of rejecting the null hypothesis without error if it is actually false. Therefore, in most cases, the samples to be compared are taken from a normally distributed population and treated with parametric criteria.

Prediction of treatment outcomes was performed using software based on multivariable regression analysis. It is based on a linear multivariate regression model

$$y^k = \beta_0 + \beta_1 x_1^k + \beta_2 x_2^k + ... + +\beta_m x_m^k + \varepsilon^k$$

If the distribution of characters (factors) is very different from the normal distribution, then non-parametric criteria can be used, and the non-parametric criterion is more powerful in these cases. We used the C++ Builder programming language to obtain these results.

Conclusion: With the help of the program created above, it is possible to identify and predict many factors affecting the treatment of patients in medicine. After that, it will be possible to determine and design the treatment process through the program.

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Multidimensional cross diffusion in chemical processes

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Chemical processes are complex systems that involve the transport of multiple species and the generation of new ones. In such cases, it is necessary to take into account the effects of cross-diffusion, which refers to the transport of one species that is influenced by the presence of another[1-3].

The concept of cross-diffusion is relatively new in the field of chemical engineering and has been introduced to capture the effects of non-ideal mixing in chemical systems. In a simple chemical reaction involving two species, the concentration of each species can be described by a single diffusion coefficient. However, in systems involving more than two species, the concentration of each species depends on the concentration of all the other species present, leading to the need for multiple diffusion coefficients. This model is based on the idea that the flux of each species is proportional to the concentration gradient of all the other species present[1-3].

The multidimensional cross-diffusion model is a set of partial differential equations that describe the transport of multiple species in a complex system. The equations take the form:

$$\begin{aligned} \partial C_1/_{\partial t} &= \nabla\left(D_1\nabla C_1\right) + \nabla\left(D_2C_2\nabla C_1\right) + \nabla\left(D_3C_3\nabla C_1\right) + ... \\ \partial C_2/_{\partial t} &= \nabla\left(D_2\nabla C_2\right) + \nabla\left(D_1C_1\nabla C_2\right) + \nabla\left(D_3C_3\nabla C_2\right) + ... \\ \partial C_3/_{\partial t} &= \nabla\left(D_3\nabla C_3\right) + \nabla\left(D_1C_1\nabla C_3\right) + \nabla\left(D_2C_2\nabla C_3\right) + ... \end{aligned} \qquad (1)$$

where C_1, C_2,C_3, ... are the concentrations of the different species D_1, D_2,D_3,... are the diffusion coefficients, and ∇ is the gradient operator. The second and third terms in each equation represent the cross-diffusion terms, which describe the influence of other species on the diffusion of each species. The cross-diffusion terms can be written in the form:

$$\nabla\cdot\ (D_2C_2\nabla C_1)\ =\ D_2\ (\partial C_2/\partial x)\,(\partial C_1/\partial x)\ +\ D_2\ (\partial C_2/\partial y)\,(\partial C_1/\partial y)\ +\ ...\ (2)$$

where x, y, ... are the spatial coordinates. This term represents the effect of the concentration gradient of species C_2 on the diffusion of species C_1. The other cross-diffusion terms can be written in a similar form.

In conclusion, the multidimensional cross-diffusion model is a powerful tool for simulating complex chemical systems. By taking into account the effects of cross-diffusion, this model can provide more accurate predictions of system behavior, leading to better understanding and control of chemical processes. However, its implementation requires careful consideration of the limitations and computational requirements of the model.

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Application of a mathematical model for cost optimization of waste water treatment processes to solve a specific problem

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A mathematical model reflecting the process of finding the best cleaning method with minimal costs is shown in [2.]. [1.] shows the technical and economic characteristics of the treatment facilities in Nukus: throughput capacity, actual, permissible and after cleaning the concentration of pollutants. The results of solving the formed task of calculating the PDS of substances and optimal water protection measures to achieve them, obtained using the software package, are given in table 4 [1.]. It follows from the obtained decision that in order to achieve the permissible values of the concentration of pollutants, it is necessary to carry out cleaning mainly according to the technological scheme 202, since during cleaning according to the existing typical scheme 101 some permissible values are not achieved (table 5) [1.]. The implementation of the results of the study with an economic effect shows that as a result of the application of these results, an improvement in the economic efficiency of wastewater treatment processes is expected. This can include reducing costs, optimizing resource utilization, or improving the overall performance of the cleaning system.

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Mathematical modeling and numerical solution of the problem of integral geometry on the family of parabolas

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The problems of integral geometry are to determine a function given (weighted) integrals of this function over family of manifolds. These problems are of importance in medical applications (tomography), and they are quite useful for dealing with inverse problems in hyperbolic differential equations (integral of unknown coefficients over ellipsoids or lines can be obtained from the first terms of the asymptotic expansion of rapidly oscillating solutions and information about first arrival times of a wave). Still there are many interesting open questions about the problem with local data and simultaneous recovery of density of a source and of attenuation.

We study problems of recovering a function given by weighted integrals over plane curves of a special shape. The curves and weight functions are piecewise smooth. Such problems of integral geometry are connected with the problem of reconstruction of internal structure of an object from the boundary data.

We reduce these problems to the investigation of Fredholm equations of first kind. Stability estimates for a solution to the considered problems in spaces of finite smoothness were obtained thereby demonstrating weak ill-posedness of the problem. We present also an efficient algorithm for stable solving of initial problem. And also the numerical results of the assigned tasks are carried out.

Reconstruction of a function from their integral data on a family of parabolas

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The reconstruction of a function from its integral data is a fundamental problem in various fields, including medical imaging, tomography, and signal processing. In this paper, we focus on the specific case of reconstructing a function from its integral data acquired on a family of parabolas.

We highlight the challenges and limitations associated with the reconstruction of a function from integral data, emphasizing the ill-posed nature of the problem and the need for regularization techniques. To address this problem, we propose a novel approach based on the theory of integral transforms and inverse problems. We develop algorithms that utilize the integral data obtained on the family of parabolas to reconstruct the underlying function.

This paper presents a comprehensive investigation into the reconstruction of a function from integral data on a family of parabolas. Our findings provide valuable contributions to the field of integral geometry and its applications, offering practical solutions for researchers and practitioners working in fields such as medical imaging and tomography.

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Regularization of the problem of integral geometry on families of semicircles and parabolas

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Integral geometry plays a crucial role in various fields, including medical imaging, computer vision, and remote sensing. The problem of integral geometry involves the study of geometric and analytic properties of objects based on their integrals over certain geometric sets. In this work, we focus on the regularization of the problem of integral geometry specifically on families of semicircles and parabolas.

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Modeling and optimization of electric energy transportation with minimization of losses in case of random demand

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One of the key considerations in electric energy transportation is the occurrence of losses within the transmission network. These losses not only impact the overall system efficiency but also result in inefficient utilization of resources. Additionally, electricity consumption exhibits a random nature, and the demand for electricity can fluctuate over time. Therefore, there is a need to develop a mathematical model that takes into account random demand and minimizes losses in the transportation of electric energy [1].

Now, we proceed to present the problem statement, which takes the following form:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ji}h_{ji}x_{ji} + \sum_{j=1}^{n}\sum_{k=1}^{l} b_{kj}z_{kj}y_{kj} \to \min \tag{1}$$

$$\sum_{j=1}^{n} a_{ji}x_{ji} \le S_i,\ i=\overline{1,m} \tag{2}$$

$$\sum_{k=1}^{l} b_{kj}y_{kj} \le T_j,\ j=\overline{1,n} \tag{3}$$

$$\sum_{j=1}^{n} b_{kj}y_{kj} = C_k(\theta),\ k=\overline{1,l} \tag{4}$$

$$\sum_{i=1}^{m} a_{ji}x_{ji} = \sum_{k=1}^{l} b_{kj}y_{kj},\ j=\overline{1,n} \tag{5}$$

$$x_{ji}\ge 0,\ y_{kj}\ge 0,\ i=\overline{1,m},\ j=\overline{1,n},\ k=\overline{1,l} \tag{6}$$

To solve problems (1)-(6), we employ the stochastic quasi-gradient method [2] and the simplex method.

By minimizing losses and optimizing energy transportation, we can enhance the overall reliability and sustainability of our electric power networks.

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Self-similar solutions of two-dimensional nonlinear cross-diffusion systems with boundary condition

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In the thesis, we studied qualitative properties of solutions of a nonlinear cross-diffusion system with variable density

$$|x^n|\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(v^{m_1-1}\frac{\partial u}{\partial x}\right),|x^n|\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\left(u^{m_1-1}\frac{\partial v}{\partial x}\right),x\in R_+,t\geq 0,$$

$$-v^{m_1-1}\frac{\partial u}{\partial x}(0,t)=u^{q_1}(0,t),-u^{m_2-1}\frac{\partial v}{\partial x}(0,t)=u^{q_2}(0,t),t\geq 0,$$

$$u(x,0)=u_0(x),v(x,0)=v_0(x),x\in R$$

where

$$m_i\geq 1,q_i\geq 0\,(i=1,2)\,,u_0\ and\ v_0(x)$$

-are the non-negative continuous functions with compact carrier in R_+

The system of equations at

$$m_i\geq 1,(i=1,2)$$

describes the processes with a finite velocity of disturbance propagation. Equations at

$$u(x,0)=0,v(x,0)=0,$$

are degenerate ones, therefore problem assumes a generalized solution that does not have the necessary smoothness at the points of degeneration [1-2].

The input unknown function is considered as a coordinate and time dependent function. In this way, the equation becomes an self-similar equation, and the solution of this equation is called an self-similar solution.

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Using the method of mathematical modeling in the study of the regulatory mechanisms of cardiac activity

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Due to the constant pumping function, the heart ejects about 10 tons of blood into the arterial system per day. Despite such a huge work, the heart always adequately responds to the needs of the body and maintains the necessary level of blood flow. Adaptation of the work of the heart to the changing needs of the body is provided by a number of regulatory mechanisms that have a complex morpho-functional basis. Intracardiac regulatory mechanisms include: intracellular regulatory mechanisms, regulation of intercellular interactions and nervous mechanisms - intracardiac reflexes. The study of the regulatory mechanisms of the heart allows to understand the normal and abnormal functionings of the heart, the study of various modes of the heart, to identify the causes of arrhythmias and sudden cardiac death. In the age of modern technology, the incidence of heart arrhythmia disease and sudden cardiac death has not decreased, due to the fact that the mechanisms for the occurrence of these abnormal conditions are not yet fully understood. For a deep study of these regulatory mechanisms, mathematical modeling can be carried out at the cellular level, intercellular level, as well as the heart at the level of the whole organ. In this research work, modeling of the heart at the organ level has been carried out. This article considers the problem of using a method of mathematical modeling in the research of regulatory mechanisms of human cardiac activity. Based on the methodology of mathematical modeling of excitation environments [1-3], the model equations were developed in the form of a system of functional-differential equations with delayed arguments based on the concept of OrAsta for researching regulatory laws [4]. The biofeedback mechanisms of the human heart and the spatio-temporal relations in the process of signal propagation between the elements of the cardiac conduction system are taken into account. The numerical and qualitative analysis of the developed mathematical model [4] was performed and the following 5 modes of cardiac regulatory mechanisms were determined: stationary state, regular periodic oscillation state (steady oscillatory solutions), dynamic chaos state (irregular oscillatory solutions), "black hole"mode (sudden cardiac death), fading mode (trivial attractor). The obtained results are fully compatible with the cardiac regulatorika, so model can interpret the normal and abnormal states of the heart and can be used to predict sudden cardiac death.

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Thermoplastic problem of parallelepiped compression between rigid plates

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It is known that numerous works devoted to the numerical solution of the nonlinear problem of thermal conductivity, electrostatics and the theory of elasticity are described by nonlinear systems of equations of elliptic type in three-dimensional space. The development of algorithms and programs for such general tasks [1] is always relevant and useful if it is accompanied by a computational experiment based on specific problems from the field of application [2].

This work is a continuation of the study, where an iterative method for solving a nonlinear spatial second boundary value problem for systems of second-order partial differential equations is proposed, based on the application of the establishment method and allowing finding a numerical solution [3]. With the help of a computational experiment, the influence of two-point and three-point approximations in boundary conditions of the second kind is numerically investigated [4].

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Three-dimensional numerical solution of the coupled dynamic problems based on the deformation and flow thermoplasticity theories

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Thermo-mechanical coupling is the most common class of coupled problems, in which the mechanical response of the structure depends on its thermal behaviour and vice versa. Investigations in the field of coupled thermoplasticity are enormously developed due to their many applications in the advanced structural design problems.

The coupled thermoplasticity problems depending on which plasticity theory, the deformation or incremental (flow) theory are used can be formulated two types of coupled thermo-mechanical boundary value problems. The coupled boundary value problems based on deformation theory plasticity [1] consists of motion equation, thermoplasticity constitutive relations and the heat conduction equation with a corresponding initial and boundary conditions. In formulating the coupled thermoplasticity boundary value problems using the flow theories, the motion and heat equations, initial and boundary conditions should be written with respect to the displacement and temperature increments.

This paper deals with the numerical solution of the 3D coupled dynamic thermoplasticity boundary value problems for an isotropic parallelepiped. In formulating the boundary value problems, the deformation [2] and strain space thermoplasticity theories [3] are used. Usually, in numerical solution of thermoplasticity boundary problems the original problem is partitioned into several smaller sub-problems, which are solved sequentially. In case of deformation theory of thermoplasticity, the partition of the problem is not required. The explicit and implicit finite difference equations are constructed [4]. For numerical solution the elimination method and recurrence formulas, in case of explicit schemes, are used. Comparison the numerical results, obtained on the basis of explicit and implicit schemes shows that the results are quite close.

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Numerical study of subsonic jets in a cylinder with obstacle

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A turbulent separated flow of an incompressible gas (air) in the vicinity of a plate installed in a cylinder at subsonic speed is considered. The flow is isothermal, external and surface forces are absent. The gas velocity at infinity V_∞ is parallel to the Oy axis. It is assumed that the source with a flow rate per second q is located at point A. In the same formulation, an analytical solution was obtained for the flow of an ideal gas in a cylinder [1,2].

For convenience, we write the system of equations for an incompressible medium in the following form:

$$\frac{\partial \Phi}{\partial t}+U\frac{\partial \Phi}{\partial x}+V\frac{\partial \Phi}{\partial y}=\frac{\partial}{\partial y}(q\frac{\partial \Phi}{\partial y})+w.$$

$$\Phi=\left[\begin{array}{l} U\\ V\\ \tilde{v}\end{array}\right.\qquad q=\left[\begin{array}{l}(\nu+\nu_t)\\ (v+\tilde{v})\end{array}\right.\qquad w=\left[\begin{array}{l}\frac{1}{\rho}\frac{\partial P}{\partial x}\\ \frac{1}{\rho}\frac{\partial P}{\partial y}\\ Pv-\mathrm{D}v+\frac{C_{b2}}{\sigma}\left(\left(\frac{\partial \tilde{v}}{\partial y}\right)^2\right)\end{array}\right.$$

To solve the system of non-stationary equations of hydrodynamics (1), we use the method of finite differences. Due to the difficulties in matching the velocity and pressure fields, a grid with a spaced location structure was used. This means that the velocity and pressure components are determined at different nodes.

The McCormack scheme was used for the calculation. McCormack's scheme is widely used for solving equations of gas dynamics and is especially convenient for solving nonlinear partial differential equations. For the problem posed, the equations of hydrodynamics in a discrete form can be represented as: the predictor is

$$\overline{\Phi^{n+1}_{i,j}}=\Phi^n_{i,j}-U^n\tfrac{\Phi^n_{i+1,j}-\Phi^n_{i,j}}{\Delta x}\Delta t-V^n\tfrac{\Phi^n_{i,j+1}-\Phi^n_{i,j}}{\Delta y}\Delta t+$$
$$+\tfrac{0.5\Delta t}{\Delta y^2}\left[(q^n_{j+1}+q^n_j)(\Phi^n_{i,j+1}-\Phi^n_{i,j})-(q^n_j+q^n_{j-1})(\Phi^n_{i,j}-\Phi^n_{i,j-1})\right]-w^n_{i,j}.$$

the corrector -

$$\Phi^{n+1}_{i,j}=\tfrac{1}{2}\left[\Phi^n_{i,j}+\overline{\Phi^{n+1}_{i,j}}-\overline{U^{n+1}}\tfrac{\overline{\Phi^{n+1}_{i+1,j}}-\overline{\Phi^{n+1}_{i,j}}}{\Delta x}\Delta t-\overline{V^{n+1}}\tfrac{\overline{\Phi^{n+1}_{i,j+1}}-\overline{\Phi^{n+1}_{i,j}}}{\Delta y}\Delta t+\right.$$
$$\left.+\tfrac{0.5\Delta t}{\Delta y^2}\left[(\overline{q^{n+1}_{j+1}}+\overline{q^{n+1}_j})(\overline{\Phi^{n+1}_{i,j+1}}-\overline{\Phi^{n+1}_{i,j}})-(\overline{q^{n+1}_j}+\overline{q^{n+1}_{j-1}})(\overline{\Phi^{n+1}_{i,j}}-\overline{\Phi^{n+1}_{i,j-1}})\right]-\overline{w^{n+1}_{i,j}}\right].$$

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Carleman's formula of a solution of the Poisson equation in bounded domain

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We propose an explicit formula for reconstruction of a solution of the Poisson equation in bounded domain from its values and the values of its normal derivative on part of the boundary, i.e., we give an explicit continuation formula for a solution to the Cauchy problem for the Poisson equation. $\Omega-$ is a bounded simply connected domain whose boundary in R^3 whose boundary $\partial\Omega$.

Problem 1. Let we know the Cauchy data for a solution to equation (1) on the surface S:

$$U(y) = f_1(y), \frac{\partial U(y)}{\partial n} = f_2(y), y \in S \tag{2}$$

where $n = (n_1, n_2, n_3)$ is the unit outward-pointing normal to the surface $\partial\Omega$ at a point y, and f_1, f_2 are continuous vector-functions. Given $f_1(y)$ and $f_2(y)$ on S, find $U(x)$ $x \in \Omega$.

Theorem 1. *Let f be bounded and locally Hölder continuous in Ω_ρ, $U(y) \in H^\lambda(\Omega)$ and satisfies condution (2). Then the Carleman formulas*

$$\frac{\partial^i U(x)}{\partial x_j^i} =$$

$$= \lim_{\sigma\longrightarrow\infty}\left[\int\limits_{\Omega} f(y)\frac{\partial^i \Phi_\sigma(y-x)}{\partial x_j^i}dy - \int\limits_{S}\left\{f_1(y)\frac{\partial^i}{\partial x_j^i}\frac{\partial \Phi_\sigma(y-x)}{\partial n} - f_2(y)\frac{\partial^i \Phi_\sigma(y-x)}{\partial x_j^i}\right\}dS_y\right], \tag{3}$$

are valid for every $x \in \Omega$, where $i = 0, 1, j = 1, 2, 3,$ and the convergence in (3) is uniform on compact sets in Ω.

Mathematical modeling of a single non-divergent parabolic system with variable density and source or absorption

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This paper considers the Cauchy problem in $Q = \left\{(t,x):\ 0<t,\ \ x\in R^N\right\}$ for a reaction-diffusion system with double nonlinearity of a non-divergent type with variable density

$$\begin{array}{l} \frac{\partial u}{\partial t} = u^{\alpha_1}\nabla\left(|x|^n u^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u^{l_1}\right) + \varepsilon u^{p_1}v^{q_1}, \\ \frac{\partial v}{\partial t} = v^{\alpha_2}\nabla\left(|x|^n v^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v^{l_2}\right) + \varepsilon u^{p_2}v^{q_2},\ \varepsilon = \pm 1 \end{array} \tag{1}$$

$$u(0,x) = u_0(x) \geq 0,\ \ v(0,x) = v_0(x) \geq 0,\ \ \ x \in R^N, \tag{2}$$

where, $k,\ m_1, m_2 > 1,\ p_i, q_i \geq 1,\ p \geq 2,\ \alpha_1,\ \alpha_2$ are positive real numbers, positive real numbers, and $u_0(x) \geq 0, v_0(x) \geq 0$ is a non-trivial, non-negative, bounded and sufficiently smooth function, $p \geq n,\ u = u\,(t,x) \geq 0,\ v = v\,(t,x) \geq 0$ is the required solution. The numerical parameter n characterizes the variable density of the nonlinear medium. Note that the study of problem (1)-(2) for particular values of numerical parameters characterizing a nonlinear medium and for the case of one equation has been studied by many authors (see [1-2]) and in a particular case.

System equation (1) with $\alpha_i = 0\ (i = 1,2)$ also describes the processes of thermal conductivity, nonlinear diffusion, polytropic filtration in a nonlinear two-component medium with variable density in the presence of absorption ($\varepsilon = -1$) or source ($\varepsilon = +1$) .

In this work, based on self-similar analysis, an estimate of the solutions and front for problem (1)-(2) is obtained, the problem of finding a suitable initial approximation for the numerical solution by the iterative method of a nonlinear degenerate system of equations of parabolic type of non-divergent form (1) with variable density under influence is solved nonlinear source or absorption depending on the value of the numerical parameters of the nonlinear medium. A self-similar system of equations was constructed depending on the values of the numerical parameters and the asymptotic behavior of the solutions was obtained depending on the values of the numerical parameters $k,\ m_1, m_2 > 1,\ p_i, q_i \geq 1,\ ; p \geq 2,\ \alpha_1,\ \alpha_2$,including for critical values of numerical parameters. It has been established that the value p=n is a singular case and the solution has a logarithmic singularity near the point x=0. It is shown that the behavior of the solution in this case and in the double critical case has an exponential form.

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Diffusion during gas generation in a porous medium

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As is known, the pressure dynamics in the process of gas release in the reaction of fluid reaction is monotonous, attended by a decrease in pressure at the final stage of the process. In this case, the pressure change during the formation of gas bubbles is written as:

$$\frac{dP}{dt} = a_1 - a_2 P(t) - a_3 P^2(t-\tau)$$

a_2 and a_3 are the coefficients that take into account the dissolved gas and excess gas molecules considering diffusion; τ is the characteristic diffusion time. Let's examine the solution of the equation under the following conditions:

$$P(t) = \phi_0(t), \quad t_0 - \tau \le t \le t_0$$

$$\phi_0(t) = \frac{1}{2a\sqrt{\pi t}} e^{-\frac{x^2}{4a^2 t}}$$

$$\begin{cases} \frac{dP}{dt} = a_1 - a_2 P(t) - a_3 \phi_0^2(t-\tau) \\ t_0 \le t \le t_0 + \tau, \quad \rho(t_0) = \phi_0(t_0); \quad [t_0, \quad t_0+\tau] \end{cases}$$

On the other hand, since temperature, as we know, also plays an important role in analysis of the diffusion process, we will take a look in this way. If we consider average velocities of particles in the medium and velocities of diffusing particles, taking them the same, we can write the free motion path length in the well-known following expression: $\lambda = \frac{1}{\sqrt{2}\sigma n}$. If we consider the motion of molecules to be random, with two axes x and y assumed to be parallel to the surface S, and the z-axis orthogonal to it, we can model it as follows. We can express the motion of gas particles along the axis as follows: $\varphi(z) = \frac{dN}{dt} = -\frac{1}{3}\langle v\rangle \lambda \frac{\partial n(z)}{\partial z} S$. Since this structure coincides with Fick's first law, the diffusion coefficient is determined as follows: $D = \frac{1}{3}\langle v\rangle\lambda$. Considering this $\langle v\rangle \sim \sqrt{\frac{T}{\mu}}$ and using the following condition $\lambda \sim \frac{1}{\sigma n} \sim \frac{T}{\sigma P}$, we can write the diffusion as:

$$D \sim \frac{1}{\sigma n}\sqrt{\frac{T}{\mu}} \sim \frac{T^{\frac{3}{2}}}{\sigma P\sqrt{\mu}}.$$

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III. SECTION. COMPUTATIONAL AND DISCRETE MATHEMATICS

Properties of a generalized spline of fourth order. Natural splines

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It is known that splines are widely used in mathematics and its applications. This paper is devoted to the study of generalized splines of the fourth order. Here we consider the concepts of generalized interpolation, in particular, interpolation periodic splines with respect to a given grid with given values. We study the fundamental identity for generalized splines of the fourth order. Using the fundamental identity, we obtain the first integral relation for generalized interpolation splines of the fourth order. Based on this relation, the properties of the minimum norm property of natural, clamped and periodic generalized interpolation splines of the fourth order are studied under specific restrictions. At the end of the paper, explicit formulas are given for the coefficients of natural splines of various kinds, such as cubic, exponential, trigonometric, and hyperbolic. The resulting formulas are convenient in the direct application of these splines.

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Numerical calculation of a mixed problem for a linear hyperbolic system with nonlocal characteristic velocity

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In this paper, we investigate the input-to-state stability (ISS) of an equilibrium for the numerical solution of a mixed problem for a linear hyperbolic system with nonlocal velocity and measurement error. For the numerical solution of the mixed problem, an upwind difference scheme is proposed. By constructing an appropriate discrete ISS-Lyapunov function, we prove necessary and sufficient conditions on ISS for a numerical solution of an initial-boundary difference problem. The results of numerical calculations are presented, which confirm the theoretical conclusions. The obtained numerical results are presented in the form of tables and graphs using MathCad mathematical package.

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Numerical calculation of a mixed problem for a system of linear hyperbolic equations with dynamic boundary conditions

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This article is devoted to the numerical solution of a mixed problem for a hyperbolic system with dynamic boundary conditions.

For the numerical solution of the mixed problem, an implicit difference scheme is proposed and its exponential stability is studied. The study of the exponential stability of the numerical solution is based on the construction of the discrete Lyapunov function proposed in this paper. Sufficient conditions for the exponential stability of the numerical solution of a mixed problem are obtained in this work. An algorithm for finding a numerical solution to a mixed problem is proposed. The obtained numerical results are presented in the form of tables and graphs using the GipTS-D programming interface, which is written in Python. The numerical results fully confirm the theoretical results on the exponential stability of the numerical solution.

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Numerical solution of multi-point source identification problem for parabolic equation with Neuman boundary condition

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In $[0,1]\times\Omega$, we study source identification multidimensional parabolic boundary value problem with multipoint nonlocal and Neumann boundary condition to find v and p functions such that

$$\begin{cases} \frac{\partial v(t,x)}{\partial t} - \sum\limits_{i=1}^{n}(a_i(x)v_{x_i}(t,x))_{x_i} = f(t,x)+p(x), \\ x=(x_1,\cdot\cdot\cdot,x_n)\in\Omega, \quad 0<t<1, \\ v(1,x)=\sum\limits_{k=1}^{r}\lambda_k v(s_k,x)+\xi, \ v(0,x)=\varphi(x), x\in\overline{\Omega}, \\ \frac{\partial v}{\overrightarrow{n}}(t,x)=0, \quad x\in S. \end{cases} \tag{1}$$

Here Ω is open unit cube in R_n, $s_1,\lambda_1,s_2,\lambda_2,...,s_r,\lambda_r$ are known numbers, $\sum\limits_{k=1}^{r}|\lambda_k|<1, 0\le s_1<s_2<...<s_r<1$, $\varphi \in L_2(\Omega),\psi\in W_2^2(\Omega)$, $f \in C^\alpha(L_2(\Omega))$ are given, and $a_i, i=1,...,n$ are known smooth functions so that $a_i(x)\ge a_0>0$ for any $i=1,...,n$ and $x\in\overline{\Omega}$.

The first and second order of accuracy difference schemes source identification problem (1) are discussed. Stability estimates for solution of difference problems are obtained. Numerical results for test example are illustrated.

Optimal quadrature formulas for numerical approximation a Volterra integral equation of the first kind with an exponential kernel

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The integral equation you provided is a Volterra integral equation of the first kind with an exponential kernel that involves complex exponential functions. It can be written as:

$$y(t) = f(t) + \int_a^t e^{2\pi iw(t-s)} y(s)ds$$

where $y(t)$ is the unknown function to be determined, $f(t)$ is the known function, w is a constant representing the frequency, and the integral is taken over the interval $[a, t]$.

To solve this integral equation numerically, we can use a quadrature method that is suitable for complex oscillatory integrals. One common approach is to use the Filon's method, which approximates the oscillatory integral by a sum of cosine and sine terms.

Here's a step-by-step procedure to solve the integral equation using Filon's method:

1. Discretization: Divide the interval $[a, t]$ into N subintervals and choose $N + 1$ evaluation points $\{s_i\}_{i=0}^N$ within each subinterval.

2. Approximation of the kernel: Approximate the exponential kernel $e^{2\pi iw(t-s)}$ by a sum of cosine and sine terms using the Euler's formula:

$$e^{2\pi iw(t-s)} \approx \sum_{n=-M}^{M} [\cos(2\pi w_n(t-s)) + i\sin(2\pi w_n(t-s))]$$

where w_n are the frequencies, and M determines the accuracy of the approximation. The frequencies w_n are usually chosen to be equally spaced.

3. Discretized equation: Substitute the approximation of the kernel into the integral equation and discretize the equation:

$$y(t) = f(t) + \sum_{i=0}^{N} \left[\int_a^t \sum_{n=-M}^{M} \left(\cos(2\pi w_n(t-s)) + i\sin(2\pi w_n(t-s))\right) y(s)ds \right]$$

4. Quadrature rule: Apply a suitable quadrature rule to evaluate the integrals involving the cosine and sine terms within each subinterval. For instance, we can use a optimal quadrature formula in $W_2^{(m,m-1)}$ Hilbert space to approximate the integral.

5. Solve the resulting system: The discretized equation becomes a system of algebraic equations that can be solved using standard numerical techniques, such as matrix inversion, iterative methods or Sobolev method.

By utilizing optimal quadrature rules, we can obtain numerical solutions for the given integral equation with highly oscillating exponential kernels involving complex exponential functions. Adjusting the number of frequencies M and the quadrature accuracy can help improve the accuracy and convergence of the solution.

Upper estimation for the error of the approximate solution of Abel's integral equation

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In this paper, in the space $W_2^{(2,1)}(0,t)$ we construct a quadrature formula of the form

$$\int_0^t \frac{\varphi(x)dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^{N} C_\beta \varphi(h\beta),$$

where $0<\alpha<1$, C_β are the coefficients of formula (1), $h\beta$ are nodes, $t>0$.

The function $\varphi(x)$ is an element of the space $W_2^{(2,1)}(0,t)$, which is defined as follows

$$W_2^{(2,1)}(0,t) = \{\varphi \mid \varphi : [0,t] \to \mathbb{R},\ \varphi' - \text{absolutely continuous},\ \varphi'' \in L_2(0,t)\}.$$

The difference between this integral and the quadrature sum is called the **error of the quadrature formula (1)**

This difference corresponds to the **error functional**

$$\ell(x) = \frac{\varepsilon_{[0,t]}(x)}{(t-x)^{1-\alpha}} - \sum_{\beta=0}^{N} C_\beta \delta(x-h\beta),$$

where $\varepsilon_{[0,t]}(x)$ is the characteristic function of the interval $[0,t]$, $\delta(x)$ is the Dirac's delta function.

The problem of constructing the optimal quadrature formulas of the form (1) in the space $W_2^{(2,1)}(0,t)$ is calculation of the following quantity:

$$\left\|\overset{o}{\ell}\right\|_{W_2^{(1,0)*}} = \inf_{C_\beta} \|\ell\|_{W_2^{(1,0)*}}$$

Theorem 1. *The norm for the error functional (4) of the quadrature formulas of the form (3) has the following form*

$$\begin{aligned}\|\ell\|^2_{W_2^{(2,1)*}} &= \int_0^t\int_0^t \frac{G_2(x-y)dydx}{(t-x)^{1-\alpha}(t-y)^{1-\alpha}} - \\ &- 2\sum_{\beta=0}^{N} C_\beta \int_0^t \frac{G_2(x-h\beta)dx}{(t-x)^{1-\alpha}} + \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_\beta C_\gamma G_2(h\beta-h\gamma),\end{aligned}$$

where $G_2(x) = \frac{\mathrm{sgn}(x)}{2}\left(\frac{e^x-e^{-x}}{2} - x\right)$.

Application of the method of moving nodes in non-stationary problems

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Most problems of mathematical physics can be reduced to solving non-stationary differential equations with initial and boundary conditions. To solve such problems, there are various analytical and numerical methods.

Most problems of mathematical physics can be reduced to solving non-stationary differential equations with initial and boundary conditions. To solve such problems, there are various analytical and numerical methods.

Some aspects of solving such problems with the help of moving nodes are given in [1,2]. The idea of the method of moving nodes is that the discrete equations (in particular, the finite-difference equation) are converted to a continuum form and on the basis of which an approximately analytical solution of the equation is obtained. Some possibilities for developing this approach are proposed here.

For the numerical solution of an ordinary differential equation with an initial condition, various versions of the Euler method are widely used. Applying the method of moving nodes to the Euler method, we obtain an approximately analytical expression for the solution of the Cauchy problem. A method for obtaining an approximately analytical solution of a parabolic equation under various boundary conditions is considered. In this case, the construction of an approximate solution is constructed using the method of lines.

It is known that non-standard schemes are used to construct a qualitative discrete equation for differential equations [3]. Using the methods of non-standard schemes, an approximately analytical solution of initial-boundary value problems for differential equations is obtained.

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Algebro-trigonometric optimal interpolation formula in a Hilbert space

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In mathematics and its applications, many practical problems are solved using interpolation. There are classical and variational methods of interpolation.

In this paper, we consider the problem of constructing an interpolation formula based on the variational method. Here we constructe the optimal interpolation formula of the form

$$\varphi(x) \cong P_{\varphi}(x), \tag{1}$$

where $P_{\varphi}(x) = \sum\limits_{\beta=0}^{N} C_{\beta}(x) \cdot \varphi(x_{\beta})$ is an approximation function, $C_{\beta}(x), \beta = \overline{0, N}$ are its coefficients.

If the approximate equality (1) satisfies the interpolation conditions,

$$\varphi(x_{\beta}) = P_{\varphi}(x_{\beta}), \beta = \overline{0, N}$$

then the function $P_{\varphi}(x)$ is called an interpolation formula.

The difference

$$(\ell, \varphi) = \varphi(x) - P_{\varphi}(x)$$

is called the error of the interpolation formula $P_{\varphi}(x)$. Here ℓ is the error functional of the interpolation formula (1), for a fixed $x = z$, which is defined as follows

$$\ell(x, z) = \delta(x - z) - \sum_{\beta=0}^{N} C_{\beta}(z)\delta(x - x_{\beta}),$$

where $\delta(x)$ is the Dirac delta-function.

The error of the interpolation formula is estimated from above by the norm of the error functional of this formula, and by minimizing the norm of the error functional with respect to the coefficients, an optimal interpolation formula is obtained.

Here we get the optimal interpoletion formula (1) which is exect for trigonometric functions $\sin(x), \cos(x)$ and for a constant.

Improvement in Volterra-Fredholm integro-differential equations by Adomian Decomposition Method

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The Adomian Decomposition Method (ADM) is a powerful semi-analytical tool used to solve linear and nonlinear differential equations, integral equations and functional equations. It was developed by George Adomian [1] in 1970 (University of Georgia) and also known as the inverse operator method. It is further extensible to stochastic systems by using the Ito integral. The aim of this paper is to propose an improved scheme based on ADM to obtain approximate solutions of Volterra-Fredholm integro-differential equations (VF-IDEs) with initial conditions. Moreover, we have proved the uniqueness of the solution and convergence of the techniques is proved in the Banakh space. The proposed scheme is tested on a variety of linear and nonlinear Volterra-Fredholm integro-differential equations with initial conditions, and the results are compared with other existing ones ([2]-[3]). The numerical simulations show that the proposed scheme is highly accurate and efficient in obtaining approximation solutions for VF-IDEs.

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The coefficients of the optimal quadrature formula obtained by the method of phi-functions

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Numerical integration plays a great role in the fundamental and applied sciences. Depending on the initial data and requirements, there are imposed various conditions for the exactness of the approximate calculation of integrals. Classical methods for the numerical calculation of definite integrals are known, such as the quadrature formulas of Gregory, Newton-Cotes, Euler, Gauss, Markov, etc. Since the middle of the last century, the theory of constructing optimal numerical integration formulas based on variational methods began to develop. It should be noted that there are optimal quadrature formulas in the sense of Nikolskiy and Sard.

In this work, we study the problem of constructing the optimal quadrature formula in the sense of Sarda. We use the φ-function method for constructing a quadrature formula. The error of the formula is estimated from above with the help of the norm of the φ functions from the Hilbert space. We choose such a phi function the norm on this interval is minimal. Finally, with the help of the obtained φ function, we calculate the coefficient of the optimal quadrature formula. The resulting optimal quadrature formula is exact for functions $e^{\sigma x}$ and $e^{-\sigma x}$, where σ is a nonzero parameter.

An optimal formula for the approximate calculation of the fractional Riemann-Liouville integrals

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It is known that fractional integration and fractional differentiation have a rich history, like integer integration and differentiation. For a long time they developed slowly. But in recent years there has been a growing interest in fractional calculus due to its applications in science and technology. It should be noted that it is very difficult to find explicit analytical solutions to fractional differential and integral problems. Therefore, the development of efficient and reliable numerical methods for solving general fractional differential and integral equations is useful for applications. Recently, many works have appeared that are mainly devoted to the study of numerical methods for solving fractional integrals, fractional derivatives and fractional differential equations. This paper is devoted to the approximate calculation of Riemann-Liouville fractional integrals. Here we discuss the problem of constructing optimal quadrature formulas for approximating Riemann-Liouville integrals. To solve this problem, we first find the extremal function of the considered quadrature formulas. Further, the norm of the error functional is calculated using the extremal function. The norm of the error functional makes it possible to estimate from above the error of the considered quadrature formulas. At the end, a system for the coefficients of optimal quadrature formulas is obtained. Conditions for the existence and uniqueness of a solution to this system are discussed. In addition, it is stated that the solution of this system gives the smallest value to the norm of the error functional. It should be noted that the obtained optimal quadrature formulas for the approximation of the Riemann-Liouville integrals are exact for the trigonometric functions $\sin t$ and $\cos t$.

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A sharp upper bound on the error of exponentially weighted optimal quadrature formulas in the Hilbert space of periodic functions

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We consider a quadrature formula of the following form

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k\varphi(hk), \tag{1}$$

where $\varphi(\cdot) \in \widetilde{W}_2^{(m,m-1)}(0,1]$, $\omega \in \mathbb{Z}\backslash\{0\}$ and $\omega h \in \mathbb{Z}$, C_k are coefficients of the quadrature formula, N is number of nodes and $h = 1/N$.

We denote by $\widetilde{W}_2^{(m,m-1)}(0,1]$ the subspace of $W_2^{(m,m-1)}[0,1]$ of complex-valued, 1-periodic functions (see [1] for details).

This space is equipped by the norm $\|\varphi\|_{\widetilde{W}_2^{(m,m-1)}} = \left(\int\limits_0^1 \left(\varphi^{(m)}(x) + \varphi^{(m-1)}(x)\right)^2 dx\right)^{\frac{1}{2}}$.

The error of quadrature formula (1) is the following difference $(\ell, \varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{k=1}^{N} C_k\varphi(hk)$, and the corresponding error functional is

$$\ell(x) = e^{2\pi i\omega x} - \sum_{k=1}^{N} C_k \sum_{\beta=-\infty}^{\infty} \delta(x - hk - \beta). \tag{2}$$

The problem of constructing optimal quadrature formulas in the space $\widetilde{W_2}^{(m,m-1)}$ is calculation of the following quantity:

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} := \inf_{C_k} \sup_{\varphi, \|\varphi\| \neq 0} \frac{|(\ell, \varphi)|}{\|\varphi\|_{\widetilde{W_2}^{(m,m-1)}}}.$$

The main result of this work is:

Theorem 1. *On the space $\widetilde{W}_2^{(m,m-1)*}(0,1]$ for $m \geq 2$, the norm of the error functional ℓ for the optimal quadrature formulas (1) with $\omega h \in \mathbb{Z}\backslash\{0\}$ has the following form*

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}.$$

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An optimal quadrature formula with sigma parameter

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This work is devoted to the construction of optimal quadrature formulas in the Hilbert space $W_{2,\sigma}^{(1,0)}$. It is known that quadrature formulas are one of the methods that allow to approximately calculate a definite integral of a one-variable function. The problem of constructing optimal quadrature formulas based on functional analysis methods is the problem of finding a quadrature formula with coefficients that give the smallest value to the norm of the error functional of this formula. In this work, we solve the problem of optimal approximation of the definite integral of functions, which are integrable by the square of the first-order generalized derivative, by a linear combination of the values of these functions at given points. First, the sharp upper bound of the error of the quadrature formula is calculated using the extremal function. Then the analytical expressions of the coefficients that give the smallest value to this upper bound are obtained. Here the discrete analogue of the differential operator $d^2/dx^2 - \sigma^2$ is used to calculate the optimal coefficients. The resulting optimal quadrature formula coefficients depend on the σ parameter. In particular, in the cases $\sigma \to 0$ and $\sigma = 1$, previously known optimal quadrature formulas are obtained.

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An optimal quadrature formula for the approximate calculation of Fourier integrals in the space $K_2^{(3)}(0,1)$

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In this work, the problem of constructing an optimal quadrature formula in the sense of Sard based on the functional approach for the numerical calculation of the Fourier integral is considered. To solve this problem, we use the Sobolev method and obtain an optimal quadrature formula for the approximate calculation of the Fourier integrals. To do this, we first solve the boundary value problem for the extremal function. When solving the boundary value problem, we find the fundamental solution of the given differential operator with the help of the direct and inverse Fourier transforms. Using the extremal function we calculate the norm of the error functional. Then we show that the norm of the error functional is real. We study, the existence and uniqueness conditions of the optimal quadrature formula in a certain Hilbert space.

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An optimal interpolation formula of Hermite type in the Sobolev space

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The present work is devoted to construction of an optimal interpolation formula of Hermite type based on variational methods. In the interpolation formula we use the values of a function and its first derivatives at nodes of interpolation.

Let functions φ belong to the Sobolev space $L_2^{(2)}(0,1)$. Here $L_2^{(2)}(0,1)$ is the Hilbert space of functions which are square intagrable with second generalized derivative in the interval $[0,1]$. The space is equipped with the norm

$$\|\varphi\|_{L_2^{(2)}} = \sqrt{\int_0^1 (f''(x))^2 dx}.$$

Let a grid $\Delta:\ 0 = x_0 < x_1 < ... < x_N = 1$ be given on the interval $[0,1]$. Assume that on this grid the following values of the function and its first derivative are given

$$\varphi(x_i),\ \varphi'(x_i),\ i = 0, 1, ..., N. \tag{1}$$

We consider the problem of optimal approximation of the form

$$\varphi(x) \cong P_\varphi(x) = \sum_{i=0}^{N} \left(C_i(x)\varphi(x_i) + C_{i,1}(x)\varphi'(x_i)\right) \tag{2}$$

functions φ with given values (1) in the Sobolev space $L_2^{(2)}(0,1)$.

The error of the approximation formula (1) defines a functional $(\ell, \varphi) = \varphi(z) - P_\varphi(z)$ (called the error functional) at a fixed point $x = z$. Then the error of the approximation formula (2) is estimated as follows

$$|(\ell, \varphi)| \le \|\ell\|_{L_2^{(2)*}} \|\varphi\|_{L_2^{(2)}}.$$

The problem is to find coefficients $C_i,\ C_{i,1},\ i = 0, 1, ..., N$ which give the minimum to the norm of the error functions ℓ. These coefficient are called optimal and the interpolation formula of the form (2) with these coefficients is called optimal interpolation formula of Hermite type.

In the present paper we get explicit expressions of the coefficients for the Hermite optimal interpolation formula of the form (2).

On an interval-analytical method for solving a generalized eigenvalue problem with arbitrary real interval matrixes

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We consider a generalized eigenvalue problem

$$\boldsymbol{A}x = \lambda \boldsymbol{B}x, \tag{1}$$

with pairs of arbitrary real interval matrices (here we use the notation according to the standard [1], i.e. interval values are highlighted in italic bold). In (1), interval matrices are defined as

$$\boldsymbol{A} = \{A \mid |A - \mathrm{mid}\boldsymbol{A}| \le \mathrm{rad}\boldsymbol{A}\}, \qquad \boldsymbol{B} = \{B \mid |B - \mathrm{mid}\boldsymbol{B}| \le \mathrm{rad}\boldsymbol{B}\}, \tag{2}$$

where

$$\mathrm{mid}\boldsymbol{A} = \frac{\underline{A} + \overline{A}}{2}, \qquad \mathrm{mid}\boldsymbol{B} = \frac{\underline{B} + \overline{B}}{2}; \qquad \mathrm{rad}\boldsymbol{A} = \frac{\overline{A} - \underline{A}}{2}, \qquad \mathrm{rad}\boldsymbol{B} = \frac{\overline{B} - \underline{B}}{2}. \tag{3}$$

Then the proper pair (λ, x) belongs to the set

$$\Lambda = \{(\lambda, x) \mid \lambda \in \mathbb{R},\ x \in \mathbb{R}^n,\ Ax = \lambda Bx,\ x \ne 0,\ A \in \boldsymbol{A},\ B \in \boldsymbol{B}\}. \tag{4}$$

According to the Oettli-Prager theorem [2], for problem (1) we have the condition

$$|\mathrm{mid}\boldsymbol{A}\, x - \lambda\, \mathrm{mid}\boldsymbol{B}\, x| \le \mathrm{rad}\,(\boldsymbol{A} - \lambda\, \boldsymbol{B}\, x)\, |x|. \tag{5}$$

For problem (1-4), an interval-analytical method for solving the complete eigenvalue problem has been developed. The method is based on the construction of special vertex matrices of the interval matrix, which is created under the condition that the right and left eigenvectors of the middle matrix are invariant. A calculation algorithm has been developed and the corresponding numerical results have been obtained. The obtained results show that the proposed method gives more accurate bounds on the set of eigenvalues of the interval matrix than the existing methods. This fact is confirmed by a comparative analysis with the numerical results of other authors.

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Algorithm for constructing discrete analogue $D_h^1[\beta]$ of differential operator $\left[1-\frac{1}{(2\pi)^2}\frac{d^2}{dx^2}\right]$

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For the first time, the construction and study of the properties of operator $D_{hH}^{(m)}[\beta]$, which is the inversion of the convolution operator with function $G_{hH}^{(m)}[\beta]=h^n G_m(hH\beta)$, were conducted by S.L. Sobolev.

In the study performed by Shadimetov [2], these coefficients were found, thus the discrete analog of operator $\frac{d^{2m}}{dx^{2m}}$ was completely constructed. The construction of a discrete analog of differential operators $\frac{d^{2m}}{dx^{2m}}-\frac{d^{2m-2}}{dx^{2m-2}}$ and $\frac{d^4}{dx^4}+2\frac{d^2}{dx^2}+1$ was also considered by Kh.M. Shadimetov and A.R. Khayotov [3, 4].

An algorithm is given in this article for constructing operator $D_1[h\beta]$ that satisfies the following equality

$$D_1[\beta]*\nu_1[\beta]=\delta[\beta]. \tag{1}$$

Here

$$\nu_1[\beta]=F^{-1}\left\{\left[1+y^2\right]\right\}(x),x=h\beta. \tag{2}$$

The main result is the following.

Theorem. Discrete analog $D_1[\beta]$ of differential operator $\left[1-\frac{1}{(2\pi)^2}\frac{d^2}{dx^2}\right]$that satisfies equality (1) is defined by the following formula:

$$D_1[\beta]=\frac{1}{\pi}\begin{cases}\dfrac{\exp(4\pi h)+1}{\exp(4\pi h)-1},\beta=0\\ \dfrac{\exp(2\pi h)}{1-\exp(4\pi h)},|\beta|=1\end{cases}, \tag{3}$$

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Weighted optimal order of convergence cubature formulas in Sobolev space $L_2^{(m)}(S_n)$

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Consider the weighted cubature formula $\int\limits_{S_n} p(\theta)f(\theta)d\theta \cong \sum\limits_{\lambda=1}^{N} C_\lambda f(\theta^{(\lambda)})$ (1)

over space $L_2^{(m)}(S_n)$, where S_n - is n- dimensional unit sphere and $p(\theta) \in L_2(S_n)$. We compare the cubature formula (1) with the generalized function

$$\ell_N(\theta) = p(\theta)\varepsilon_{S_n}(\theta) - \sum_{\lambda=1}^{N} C_\lambda \delta(\theta - \theta^{(\lambda)}), \tag{2}$$

and call it the error functional where $\delta(\theta)$ is the Dirac delta function, C_λ and $\theta^{(\lambda)}$ - are the coefficients and nodes of the cubature formula (1).
The following Lemma is true.

Lemma. If the error functional (2) of the cubature formula (1) satisfies the following conditions

$$\ell_N(\theta) = \ell_{N_1}(\theta_1) \cdot \ell_{N_2}(\theta_2) \cdot ... \cdot \ell_{N_n}(\theta_n) \tag{3}$$

and

$$\left\| \ell_{N_i} | L_2^{(m_i)^*}(\omega_i) \right\| \le c_i \frac{1}{N_i^{m_i}}, \quad c_i - \text{are constants}, \left(i = \overline{1, n}\right) \tag{4}$$

Here the method of the proof presentation differs sharply from the one published in [3] and it is proved that

$$\left\| \ell_N | L_2^{(m)^*}(S_n) \right\| \le c \cdot N^{-\frac{m}{n}} \quad \text{or} \quad \left\| \ell_N | L_2^{(m)^*}(S_n) \right\| \le O\left(N^{-\frac{m}{n}}\right). \tag{5}$$

Now it is easy to prove the following theorem using N.S. Bakhvalov's theorems [4] and this lemma, i.e. inequality (5).

Theorem. The weighted cubature formula (1) with the error functional (2) for $N_1 = N_2 = ... = N_n$, $\prod\limits_{i=1}^{n} N_i = N$ and $m_1 + m_2 + ... + m_n = m$ is optimal in terms of the order of convergence in space $L_2^{(m)}(S_n)$, i.e., for the norm of the error functional (2) of cubature formula (1) the following equality holds: $\left\| \ell_N | L_2^{(m)^*}(S_n) \right\| = O\left(N^{-\frac{m}{n}}\right)$.

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On construction of the optimal interpolation formula in Sobolev space $\tilde{W}_2^{(m)}(T_1)$

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The extremal function of the interpolation formula and the norm of the error functional are determined in the article; a new optimal interpolation formula in the Sobolev space $\tilde{W}_2^{(m)}(T_1)$ for $m = 3$ is derived.
It is required to build an interpolation formula $P_f(x)$,i.e.

$$f(x) \cong P_f(x) = \sum_{\lambda=0}^{N} C_\lambda(x) f(x_\lambda), \tag{1}$$

coinciding with function $f(x)$ in interpolation nodes$f(x_i) = P_f(x_i)$, $i = 0, 1, ...N$.
The error functional of this interpolation formula is:

$$\ell(x) = \delta(x - z) - \sum_{\lambda=0}^{N} C_\lambda(z)\delta(x - x_\lambda) \tag{2}$$

Theorem 1. The square of the norm of the error functional of the interpolation formula (1) over space $\tilde{W}_2^{(m)}(T_1)$ is:

$$\left\|\ell|\tilde{W}_2^{(m)^*}(T_1)\right\|^2 = \left|1 - \sum_{\lambda=1}^{N} C_\lambda(z)\right|^2 + \frac{1}{(2\pi)^{2m}} \sum_{k\neq 0} \frac{\left|\cos 2\pi kz - \sum\limits_{\lambda=1}^{N} C_\lambda(z) e^{2\pi i k x_\lambda}\right|^2}{k^{2m}}, \tag{3}$$

Lemma. The square of the norm of function$u(x)$ in space $\tilde{W}_2^{(m)^*}(T_1)$ is:

$$\left\|u|\tilde{W}_2^{(m)}(T_1)\right\|^2 = \left|1 - \sum_{\lambda=1}^{N} C_\lambda(z)\right|^2 + \frac{1}{(2\pi)^{2m}} \sum_{k\neq 0} \frac{\left|cos 2\pi kz - \sum\limits_{\lambda=1}^{N} C_\lambda(z) e^{2\pi i k x_\lambda}\right|^2}{k^{2m}}. \tag{4}$$

Equality (4) proves that the following theorem is true.

Theorem 2. Function $u(x) = 1 - \sum\limits_{\lambda=1}^{N} C_\lambda(z) + \frac{1}{(2\pi)^{2m}} \sum\limits_{k\neq 0} \frac{\hat{\ell}_k e^{-2\pi i k x}}{k^{2m}}$ is an extremal function for the interpolation formula (1) and $u(x) \in \tilde{W}_2^{(m)}(T_1)$.
The main result of this work is the following.

Theorem 3. In the periodic Sobolev space $\tilde{W}_2^{(m)}(T_1)$, there is a unique optimal interpolation formula of the form (1) with the error functional (2); its coefficients for $m = 3$ have the following form:

$$C_{[\beta]}(z) = \frac{1 + \frac{1}{(2\pi)^6}\frac{1}{N^6}\sum\limits_{k\neq 0}\frac{\cos 2\pi k(z - h\beta)}{k^6}}{N\left(1 + \frac{1}{(2\pi)^6}\frac{1}{N^6}\sum\limits_{k\neq 0}\frac{1}{k^6}\right)}, \quad \text{where} \quad \beta = \overline{1, N}, N = 2, 3, ... \tag{5}$$

Algorithm for solving one game problem with connected variables

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Consider the sets

$$X = \{x \mid f_* \le x \le f^*\} \text{ and } Y(x) = \{y \mid g_* \le y \le g^*, Ax + By = b\},$$

where $x, f_*, f^* \in \mathbb{R}^n, y, g_*, g^* \in \mathbb{R}^l, b \in \mathbb{R}^m, A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{m\times l}, rankB < l.$

Let there be two players who choose vectors x and y, respectively, from the sets $X, Y(x)$ in turn, first the first player chooses x, then, knowing x, the second player chooses y.

The goal of the first player is to find $\hat{x}$ that gives the maximum value for the function

$$f(x) = \min_{y\in Y(x)} \Psi(x, y), x \in X, \text{i.e. } f(\hat{x}) = \max_{x\in X} f(x),$$

the second player's goal is to find $\hat{y}$ that minimizes the function

$$\Psi(\hat{x}, y), y \in Y(\hat{x}), \text{i.e. } \Psi(\hat{x}, \hat{y}) = \min_{y\in Y(\hat{x})} \Psi(\hat{x}, y).$$

$$\text{Here } \Psi(x, y) = \begin{cases} c'x + d'y, \text{ if } x \in X, y \in Y(x), c \in \mathbb{R}^n, d \in \mathbb{R}^l; \\ +\infty, \text{ if } x \in X, Y(x) = \varnothing. \end{cases}$$

Then we have a maximin problem with connected variables [1-3]:

$$\varphi(x) = \min_{y\in Y(x)} \Psi(x, y) \to \max_{x\in X}.$$

For the problem under consideration, a new solution algorithm and the results of numerical experiments on a PC are proposed.

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An algorithm for determining the nonemptiness of the set of solutions to systems of linear equations with parameters

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Consider the sets

$$X = \{x \mid f_* \leq x \leq f^*\} \text{ and } Y(x) = \{y \mid g_* \leq y \leq g^*, Ax + By = b\},$$

where $x, f_*, f^* \in \mathbb{R}^n, y, g_*, g^* \in \mathbb{R}^l, b \in \mathbb{R}^m, A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{m\times l}, rankB < l.$

The following problem is considered:

It is required to determine whether for any parameter $x \in X$ the corresponding set $Y(x)$ is empty or not, i.e.

$$\forall x \in X, Y(x) \neq \varnothing? \text{ or } \quad \exists x^* \in X, Y(x^*) = \varnothing?$$

An algorithm for solving the problem under consideration based on the dual simplex method [1] is proposed, and the results of numerical experiments on a PC are presented.

The algorithm is illustrated with examples for:

$$a) m = 2, n = 3, l = 5, b = (5; 4),$$

$$f_* = (-5; -30; 0), f^* = (3; 25; 40),$$

$$g_* = (-109; -6; -101; -10; -3), g^* = (44; 6; 298; 10; 15),$$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 6 & 3 & 2 & 3 & 4 \\ 4 & 2 & 1 & 2 & 3 \end{pmatrix}.$$

$$c) m = 3, n = 2, l = 5, b' = (4; 10; 2),$$

$$f_* = (-6; -8), f^* = (2; 2),$$

$$g_* = (0; 0; 0; 0; 0), g^* = (6; 6; 100; 100; 100),$$

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 2 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

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Numerical solution of fractional differential equations with Caputo derivative

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Fractional differential equations appear frequently in various fields involving science and engineering, namely, in signal processing, control theory, diffusion, thermodynamics and dynamical systems (see [1][2]). In general, most of fractional differential equations do not have exact solutions. Instead, analytical and numerical methods become increasingly important for finding solution of fractional differential equations. This thesis presents a multi-step method for solving fractional differential equations. The applied method is based on the fourth-order Adams-Bashforth numerical method, and the derivatives of fractional differential equations are defined in the sense of Caputo, and it consists in constructing Lagrange interpolation for the fractional case.

The fractional initial value problem of the following form is given:

$$ {}_C D^{\alpha}_{x_0} y(x) = f(x, y(x)), \qquad y(x_0) = y_0, $$

where α $(0 < \alpha < 1)$ is the order of the fraction and the sign ${}_C D^{\alpha}_{x_0}$ is called Caputo's order of α derivative operator. In addition, there is a derivative operator of the Riemann-Liouville in the form of ${}_{RL} D^{\alpha}_{x_0}$, and the connection between them is expressed by the equation ${}_C D^{\alpha}_{x_0} ={}_{RL} D^{\alpha}_{x_0}(y(x) - y(x_0))$. In this work, we use Caputo's order of α fractional derivative ${}_C D^{\alpha}_{x_0}$ and briefly denote it as D^{α}[3].

The importance of approximate solving methods in finding the solution of fractional differential equations is increasing. In particular, fractional explicit Adams method was proposed for the numerical solution of fractional order differential equations[4]. In this paper, the above method is improved and fractional explicit Adams method of order four is obtained.

Since we propose 4 - steps method, the approximate solution can be estimated as follows by taking the Lagrange interpolation function with the functions F_n, F_{n-1}, F_{n-2} and F_{n-3}:

$$ f(x, y(x)) \approx P(x). $$

Then, we perform the necessary calculations. After that we can get the efficiently formula which is called fractional explicit Adams method of order for[5].

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Algorithm constructed quadratic stochastic operator on a finite graph from binomial distributions

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The paper considers quadratic stochastic operators (QSO) defined on a finite-dimensional simplex [1].

$$S^{n-1} = \left\{ x = (x_1, x_2, ..., x_n) : \ x_i \geq 0, \ \ i = 1, 2, ..., n, \ \sum_{i=1}^{n} x_i = 1 \right\}.$$

Definition. QSO is a map $\mathrm{V} : S^{n-1} \to S^{n-1}$, of the form $\mathrm{V} : \ x'_k = \sum_{i,j=1}^{n} \mathrm{P}_{ij,k} x_i x_j, \ x \in S^{n-1}, \ \mathrm{P}_{ij,k} = \mathrm{P}_{ji,k} \geq 0, \ \sum_{k=1}^{n} P_{ij,k} = 1$ for all $i, j, k \in \{1, 2, ..., n\}$. Let (Λ, L) be a finite graph without loops and multiple edges, where Λ is the set of vertices and L is the set of edges of the graph, $\sigma : \Lambda \to \mathrm{F}, \sigma \in \Omega$ is a configuration, where F is some finite set. Let $\{\Lambda_i\}, i = \overline{1, n}$ be the set of connected components of the graph (Λ, L). Then for the configuration $\forall \sigma_1, \sigma_2 \in \Omega$ we put

$$\Omega(\Lambda, \sigma_1, \sigma_2) = \{\sigma \in \Omega : \ \sigma|_{\Lambda_i} = \ \sigma_1|_{\Lambda_i} \text{ or } \sigma\,|_{\Lambda_i} = \ \sigma_2|_{\Lambda_i} \text{ for all } \ i = \overline{1, n}\}.$$

Let $\mu \in S(\Lambda, \mathrm{F})$ be some probability measure defined on Ω such that $\mu\,(\sigma) > 0$ M for $\forall \sigma \in \Omega$.. The QSO coefficients $\mathrm{P}_{\sigma_1 \sigma_2, \sigma}$ are defined as follows:

$$\mathrm{P}_{\sigma_1 \sigma_2, \sigma} = \begin{cases} \frac{\mu(\sigma)}{\mu(\Omega(\wedge, \sigma_1, \sigma_2))}, & \text{if} \quad \sigma \in \Omega(\wedge, \sigma_1, \sigma_2) \\ 0, & \text{otherwise.} \end{cases}$$

Then for $\forall \lambda \in S(\Lambda, \mathrm{F})$ the measure $\mathrm{V}\lambda = \lambda' \in S(\Lambda, \mathrm{F})$ the measure is defined by the equality $\lambda^{'}(\sigma) = \sum_{\sigma_1, \sigma_2 \in \Omega} \mathrm{P}_{\sigma_1 \sigma_2, \sigma} \lambda(\sigma_1)(\sigma_2)$, $\mathrm{P}_{\sigma_1 \sigma_2, \sigma} = \mathrm{P}_{\sigma_2 \sigma_1, \sigma} \geq 0$, $\sum_{\sigma \in \Omega} \mathrm{P}_{\sigma_1 \sigma_2, \sigma} = 1$ for $\forall \sigma_1, \sigma_2 \in \Omega$.

Example. Let (Λ, P) be a connected graph of the following form:$|\Lambda| = 2$ and $\mathrm{P} = \{A, \ a\}$.

Then the QSO looks like this:

$$\begin{cases} x'_1 = {x_1}^2 + 2px_1x_2 + 2px_1x_3 + \frac{2p^2}{p^2+q^2}x_1x_4 \\ x'_2 = 2qx_1x_2 + {x_2}^2 + x_2x_3 + 2px_2x_3, \\ x'_3 = 2qx_1x_3 + x_2x_3 + {x_3}^2 + 2px_3x_4 \\ x'_4 = \frac{2q^2}{p^2+q^2}x_1x_4 + 2qx_2x_4 + 2qx_3x_4 + {x_4}^2 \end{cases}$$

In what follows, we study all classes for the constructed quadratic stochastic operator.

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Application of the spectral-grid method for solving an ordinary differential equation with inhomogeneous gradients

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We will solve approximately the following ordinary differential equation of the second order with a small parameter at the highest derivative

$$\varepsilon\frac{d^2u}{dy^2}+\frac{1}{2}\frac{du}{dy}=\frac{1}{8}(y+1), y\in[-1,+1],$$

with the boundary conditions

$$u(-1)=u(+1)=0,$$

where ε- is a small parameter. For an approximate solution of problem (1)-(2), we use the spectral-grid method with Chebyshev polynomials of the second kind [1]. To do this, on the segment $[-1,+1]$ we introduce the grid: $-1=y_0<y_1<y_2=+1$. An approximate solution of problem (1)-(2) on each element of the grid $[y_{j-1},y_j], j=1,2$ will be sought in the form of a linear combination of a different number of Chebyshev polynomials of the second kind U_n :

$$u_j(\tilde{y})=\sum_{n=0}^{p_j}a_n^{(j)}U_n(\tilde{y}), -1\le\tilde{y}\le1, y=\frac{(-1)^j}{2}+\frac{1}{2}\tilde{y},$$

where $a_n^{(j)}$-unknown coefficients, p_j-number of polynomials used to approximate the solution on the j-th grid interval.

With such values of the parameter $n=30,40,50$, numerical calculations were carried out.

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Numerical modeling of nonlinear evolutionary equations

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In the study of nonlinear wave phenomena in media with dispersion, the Korteweg-de Vries equation (KDV) is used. The KDV equation is a nonlinear partial differential equation with a nonlinear term and a dispersion term. This equation describes the evolution of nonlinear waves of small but finite amplitude over long time intervals.

In the article [1], invariant sampling schemes that preserve momentum are constructed for the numerical solution of the KDV equation. The article [2] provides an overview of the main characteristics of the KDV equations, including history and prehistory, derivation, some elementary properties, single solutions, existence and uniqueness of the initial value problem and, finally, the existence of an infinite set of independent conservation laws. Some numerical calculations illustrating solutions for single waves are given.

The article [3] states that the classical KDV equation and the modified KDV equation play a significant role in the physics of nonlinear waves in view of their integrability. Less well-known are other equations from the same family, derived in various physical and technical applications. There are various varieties of the KDV equation: model, lognormal, Gardner equation, Shamel equation, Benjamin-Ono and Kawahara equations, as well as fractionation equations. These equations differ in the degree of nonlinearity in the adequate term and the order of linear variance (including fractional). The main purpose of the article is to discuss practical applications of the KDV family of equations and general properties of their solutions. It is shown that many KDV hierarchy equations have common properties of solutions and do not have explosive instability. For this, the degree of nonlinearity should not be very large.

The article [4] is devoted to the description of the KDV equation with an emphasis on the special property of its solutions called solitons. In this paper, the spectral-grid method is used for the numerical solution of the KDV equation, a computational algorithm of the method is being developed.

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Derivative optimal quadrature formula in the space of differentiable functions

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As is generally known, numerical integration formulae, or quadrature formulae, are methods for the approximate evaluation of definite integrals. They are needed for the computation of those integrals for which either the antiderivative of the integrand cannot be expressed in terms of elementary functions or for which the integrand is available only at discrete points, for example from experimental data. [1-2].

We consider the following quadrature formula

$$\int\limits_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} C_0\,[\beta]\varphi(h\beta) + \frac{h^2}{12}(\varphi'(0)-\varphi'(1)) + \sum_{\beta=0}^{N} C_1\,[\beta]\varphi''(h\beta)$$

with the error functional

$$\ell_N(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0\,[\beta]\delta(x-h\beta) + \frac{h^2}{12}(\delta'(x)-\delta'(x-1)) - \sum_{\beta=0}^{N} C_1\,[\beta]\delta''(x-h\beta)$$

in the space $L_2^{(m)}(0,1)$ for $m \geq 3$. Here $C_0[\beta] = \left\{ \begin{array}{ll} \frac{h}{2}, & \beta = 0, N, \\ h, & \beta = \overline{1, N-1}, \end{array} \right.$ are known coefficients and $C_1[\beta]$, $\beta = \overline{0,N}$ are unknown coefficients of the formula (1), $h = \frac{1}{N}$, N is a natural number.

Theorem. Among quadrature formulas of the form (1) with the error functional (2) in the space $L_2^{(m)}(0,1)$ there exists unique optimal formula which coefficients are determined by the following formulas

$$C_1[0] = C_1[N] = h^3 \sum_{k=1}^{m-3} d_k \frac{q_k - q_k^N}{q_k - 1},$$

$$C_1[\beta] = h^3 \sum_{k=1}^{m-3} d_k \left(q_k^{\beta} + q_k^{N-\beta} \right), \qquad \beta = \overline{1, N-1},$$

where d_k satisfy the following system of $m-3$ linear equations

$$\sum_{k=1}^{m-3} d_k \sum_{i=1}^{\alpha} \frac{q_k^{N+i} + (-1)^i q_k}{(1-q_k)^{i+1}} \Delta^i 0^{\alpha} = \frac{B_{\alpha+3}}{(\alpha+1)(\alpha+2)(\alpha+3)}, \quad \alpha = \overline{1, m-3}.$$

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Optimal quadrature formula coefficients in Hilbert space

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In this paper problem of construction of optimal quadrature formulas in $W_2^{(m)}(0,1)$, space is considered. Here by using Sobolev algorithm when $m = 2$ we found optimal coeffitcients of quadrature formulas of the form

$$\int\limits_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} k[\beta]\varphi\,(h\beta),$$

with the error functional

$$\ell(x) = i_{[0,1]}(x) - \sum_{\beta=0}^{N} k[\beta]\delta\,(x - h\beta)$$

here $k\,[\beta]$ are coefficients of the quadrature formula (1), $[\beta] = h\beta,\ \ h = \frac{1}{N}$, $N = 1, 2, 3, ...,$ $\varphi\,(x)$ is an element of space $W_2^{(m)}(0,1)$, $k\,[\beta] = 0$ at $h\beta \notin [0,1]$, $i_{[0,1]}(x)$ is the indicator of the interval [0,1], $\delta(x)$ is the Dirac delta-function.

Theorem. The coefficients of optimal quadrature formulas in the space $W_2^{(2)}(0,1)$ are determined by the formulas

$$k[\beta] = \begin{cases} \frac{h}{2} + a\frac{\lambda_1 - \lambda_1^N}{\lambda_1 - 1}, & \beta = 0, \\ h + a\left(\lambda_1^{\beta} + \lambda_1^{N-\beta}\right), & \beta = \overline{1, N-1}, \\ \frac{h}{2} + a\frac{\lambda_1 - \lambda_1^N}{\lambda_1 - 1}, & \beta = N, \end{cases}$$

where

$$a = \frac{(e+1)\left[2\left(e^h - 1\right) - h\left(e^h + 1\right)\right](\lambda_1 - 1)\left(e^h - \lambda_1\right)\left(\lambda_1 e^h - 1\right)}{2\lambda_1 (e^h - 1)^2\left[(\lambda_1^N + e)(\lambda_1 e^h - 1) - (\lambda_1^N e + 1)(e^h - \lambda_1)\right]},$$

$$\lambda_1 = \frac{h(1 + e^{2h}) + 1 - e^{2h} - (e^h - 1)\sqrt{h^2(1 + e^h)^2 - 2h(e^{2h} - 1)}}{1 + 2he^h - e^{2h}}.$$

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Probabilistic approach to the solution of semi-linear Helmholtz boundary value problem

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In these work we will study a probabilistic representation of the solution of the Helmholtz boundary problem for the non-linear problem

$$-\Delta u(x) + cu(x) = g \cdot f(u), \quad x \in D, \quad u|_{\Gamma} = \psi$$

where, f(u) in our case could be hyperbolic functions sh(u) or ch(u). Under the assumption of the existence of a solution, an unbiased estimator is constructed on the trajectories of the proposed branching process "walk on spheres". To do this, using Green's formula, a special integral equation is written that connects the value of the function with its integrals over a ball and a sphere of maximum radius centered at a point and entirely contained in the region under consideration. It is proved that under certain conditions there exists a fixed point for the nonlinear integral operator corresponding to the integral equation. In this case, the iteration process method converges and classical Monte Carlo methods could be used. A probabilistic representation of the solution of the problem in the form of the mathematical expectation of some random variable is obtained. In accordance with the probabilistic representation, a branching process of walk on spheres is constructed and an unbiased estimator of the solution of the problem with finite variance is constructed on its trajectories.

The discrete analogue of a high-order differential operator

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In the theory of quadrature and cubature formulas, discrete analogs of differential operators play an important role. In this paper, we consider the problem of constructing a discrete function $D_m(h\beta)$ that satisfies the equality

$$D_m(h\beta) * G_m(h\beta) = \delta(h\beta),$$

where

$$\begin{aligned} G_m(h\beta) \;=\; & \frac{\operatorname{sign}(h\beta)}{2m^2} \cdot \sum_{k=1}^{m} \left[(1-m)\, e^{h\beta \cos \frac{(2k-1)\pi}{m}} \cos\left(h\beta \sin\left(\frac{(2k-1)\pi}{m}\right) + \frac{(2k-1)\pi}{m} \right) \right. \\ & \left. + h\beta e^{h\beta \cos\frac{(2k-1)\pi}{m}} \cos\left(h\beta \sin\left(\frac{(2k-1)\pi}{m}\right) + \frac{2\pi \cdot (2k-1)}{m} \right) \right], \end{aligned}$$

$\delta(h\beta)$ is the discrete delta-function, i.e., $\delta(h\beta) = \begin{cases} 1, & \beta = 0, \\ 0, & \beta \neq 0, \end{cases} \quad h = \frac{1}{N}, \quad N = 1, 2, ...$

The discrete function $D_m(h\beta)$ is used to calculate the coefficients of optimal quadrature and interpolation formulas in the space $W_2^{(m,0)}$, equipped with the norm

$$\|\varphi\|_{W_2^{(m,0)}} = \left(\int_0^1 \left(\varphi^{(m)}(x) + \varphi(x) \right)^2 \mathrm{d}x \right)^{1/2}.$$

The main result of this paper is the following theorem:

Theorem. The discrete analogue $D_m(h\beta)$ of the differential operator $\frac{d^{2m}}{dx^{2m}} + 2\frac{d^m}{dx^m} + 1$ satisfying equation (1), when m is an even natural number, has the form

$$D_m(h\beta) = \frac{m^2}{K} \cdot \begin{cases} \sum\limits_{k=1}^{m-1} A_k^* \cdot \lambda_k^{|\beta|-1}, & |\beta| \geq 2, \\ 1 + \sum\limits_{k=1}^{m-1} A_k^*, & |\beta| = 1, \\ M_1 - \frac{K_1}{K} + \sum\limits_{k=1}^{m-1} \frac{A_k^*}{\lambda_k}, & \beta = 0, \end{cases}$$

where $K,\ M_1,\ K_1,\ A_k^*$ and $\lambda_k\,(k = 1, 2, ..., m-1)$ are known.

Optimization of the approximate calculation of integrals from rapidly oscillating functions

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Many classes of problems in computational and applied mathematics [1], such as statistical processing of experimental data, digital filtering, pattern recognition, modeling of optical systems, boundary value problems for partial differential equations, and others, make it necessary to calculate integrals of rapidly oscillating functions. For the approximate calculation of the integral of rapidly oscillating functions, many well-known classical quadrature formulas can be applied, however, they give good accuracy if the integrable function is sufficiently smooth and not rapidly changing. In this paper, we construct optimal quadrature formulas in Sobolev spaces for the approximate calculation of integrals of rapidly oscillating functions.

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About one algorithm for searching the Voronoi face area of the second perfect form

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The problem of classifying integer quadratic forms has a long history, during which many mathematicians contributed to its solution. Binary forms were comprehensively studied by Gauss. He and later researchers also outlined the main ways to solve the problem of classifying ternary forms and forms of higher dimensions

These are interesting and non-trivial problems of geometric number theory, which were dealt with by many mathematicians (Hermite, Gauss, Korkin, Zolotarev, Minkowski, Voronoi, Delaunay, Ryshkov, Malyshev, Barnes, Vladimirov, Scott, Larmut, Stacey, Baranovsky, Shushbaev, Anzin, Umarov and etc.). They also appeared in the works of S.L. Sobolev and Kh.M. Shadimetov in connection with the construction of lattice optimal cubature formulas.

The paper proposes an algorithm for calculating non-equivalent quadratic forms corresponding to the faces of the Voronoi domain of the second perfect form in many variables, and using this algorithm, all corresponding non-equivalent quadratic forms are calculated.

In this paper, using this algorithm, we prove the following proposition.

Theorem. Number of $20-$dimensional faces of the Voronoi domain $V^{21}(\varphi_1^6)$ perfect form φ_1^6, permutations of the variables $x_3, ..., x_6$ that are not equivalent with respect to the group S_4, equals 12.

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Coefficients of optimal quadrature formulas with the Hilbert Kernel

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Many applied problems in aerodynamics, elasticity theory, electrodynamics, and other areas naturally reduce to singular integral equations.

It should be noted that the development trend of numerical methods for solving integral equations. Numerical methods have been most developed for Fredholm integral equations of the second kind with smooth kernels. For such equations, numerical methods were constructed based on the application to the integral of quadrature formulas of the rectangle type or similar rather general quadrature formulas. By combining numerical methods for singular integral equations, it is necessary to construct new quadrature formulas that take into account the features of the integrands, i.e. effective quadrature formulas for the approximate calculation of singular integrals with Cauchy and Hilbert kernels.

Therefore, to create efficient algorithms for solving integral equations with the Hilbert kernel is one of the urgent tasks of computational mathematics.

The above problems are reduced to the approximate calculation of singular integrals with the Hilbert kernel.

To effectively approximate these integrals, we consider a quadrature formula of the form

$$\int_0^1 \operatorname{ctg} \pi(x-x_0)\varphi(x)dx \cong \sum_{\beta=0}^{N} C[\beta]\varphi(h\beta)$$

with error function

$$\ell_N(x) = \varepsilon_{[0,1]}(x) \operatorname{ctg} \pi(x-x_0) - \sum_{\beta=0}^{N} C[\beta]\delta(x-h\beta),$$

Here $\varphi(x) \in L_2^{(m)}(0,1)$ is the Sobolev space, $\varepsilon_{[0,1]}(x)$ is the indicator of the segment [0, 1], $C[\beta]$ are the coefficients of the quadrature formula. In this paper, the following theorem is proved.

Theorem. *Among the quadrature formulas of the form*

$$\int_0^1 \operatorname{ctg} \pi(x-x_0)\varphi(x)dx \cong \sum_{\beta=0}^{N} C[\beta]\varphi(h\beta)$$

in the space of Sobolev $L_2^{(4)}(0,1)$-periodic functions, there is a unique optimal quadrature formula whose coefficients are determined by the formula

$$\mathring{C}[\beta] = 5040h \sum_{\gamma=1}^{\infty} \left(\frac{\sin \pi h\gamma}{\pi h\gamma}\right)^8 \frac{\sin 2\pi\gamma(h\beta - x_0)}{\cos 6\pi\gamma h + 120\cos 4\pi\gamma h + 1191\cos \pi\gamma h + 1208}, \quad \beta = 1, 2, ..., N$$

On the optimal explicit difference formula in the Hilbert space

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In many diverse fields, such as physics, astronomy, geometry, and population modeling, ordinary differential equations are used. These areas have been influenced by many eminent mathematicians, including Newton, Clairaut, d'Alembert, Euler, and several more. In the domain of mathematics, an ordinary differential equation (also known as ODE) is a form of the equation that consists of one or more functions of a single independent variable and its derivatives.

In the present work we consider the problem of approximate solution to the first order ODE

$$y' = f(x,y), \ x \in [0,1] \tag{1}$$

with the initial condition

$$y(0) = y_0. \tag{2}$$

We assume that $f(x,y)$ is a suitable function and the differential equation (1) with the initial condition (2) has a unique solution on the interval $[0,1]$.

We consider the explicit difference formula of the following form for the approximate solution of equation (1) (for example, see [1],[2],[3])

$$\sum_{\beta=0}^{k} C_\beta \varphi(h\beta) \cong h \sum_{\beta=0}^{k-1} C_{\beta,1} \varphi'(h\beta), \tag{3}$$

here $h = \frac{1}{N}$, $N \in \mathbb{N}$, C_β va $C_{\beta,1}$ are the coefficients, functions $\varphi(h\beta) = y_\beta$ and $\varphi'(h\beta) = f(h\beta, \varphi(h\beta))$, $\beta = 0, 1, ..., k-1$, and they belong to the Hilbert space $W_2^{(3,2)}(0,1)$.

Theorem. *The optimal coefficients of the explicit difference formula (3) in the Hilbert space $W_2^{(3,2)}(0,1)$ are defined by the following formula*

$$\overset{\circ}{C}_{\beta,1} = \begin{cases} \frac{e^{-hk+h}S_2 - S_1}{e^{-hk+h}-1}, & for\ \beta = 0, \\ M\lambda^\beta + N\lambda^{k-\beta}, & for\ \beta = 1, 2, ..., k-2, \\ \frac{S_1 - S_2}{e^{-hk+h}-1}. & for\ \beta = k-1. \end{cases}$$

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The norm of the error functional of the k-th order difference formula

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Let us consider the Cauchy problem for a second-order equation

$$y'' = f(x,y),$$
$$y(0) = y_0,\ y'(0) = y_1.(1)$$

Let it be required to find an approximate solution of this problem on the interval [0,1]. Divide this interval into N parts of length $h = \frac{1}{N}$ and look for approximate values of y_n of the desired solution $y(x)$ at points $x_n = nh,\ n = 0,\ 1,\ ...,\ N.$

For an approximate solution of problem (1), consider the general k-th order difference formula

$$\sum_{\beta=0}^{k} C[\beta]\varphi[\beta] - h^2 \sum_{\beta=0}^{k} C^{(||)}[\beta]\varphi''[\beta] \cong 0. \qquad (2)$$

Here $C[\beta]$ and $C^{(||)}[\beta]$ are the unknown coefficients of the difference formula, $[\beta] = h\beta$, $(\beta = 0, 1, ..., k)$. We consider the functions $\varphi(x)$, belonging to the Sobolev space $L_2^{(m)}(0,1)$. Since $L_2^{(m)}(0,1)$ is embedded in the space $C(0,1)$ of continuous functions, the error functional of the second-order difference formula also be linear

$$(\ell, \varphi) = \sum_{\beta=0}^{k} C[\beta]\varphi[\beta] - h^2 \sum_{\beta=0}^{k} C^{(||)}[\beta]\varphi''[\beta]. \qquad (3)$$

The task of constructing the difference formula (1) in the functional formulation is to find such a functional (3) whose norm in the space $L_2^{(m)*}(0,1)$ is minimal.

The square of the norm of the error functional of the second-order difference formula in the Sobolev space $L_2^{(m)}(0,1)$ is expressed in terms of the bilinear form of the coefficients of the difference formula. From here we get the following.

Theorem. *The square of the norm of the error functional ℓ of the difference formula (2) in the Sobolev space $L_2^{(m)}(0,1)$ is determined by the formula*

$$\left\| \ell \left| L_2^{(m)*}(0,1) \right\|^2 = (-1)^m \left[\sum_{\beta=0}^{k} \sum_{\gamma=0}^{k} C[\beta]C[\gamma]G_m(h\beta - h\gamma) \ - \right.\right.$$
$$\left. -2h^2 \sum_{\beta=0}^{k} \sum_{\gamma=0}^{k} C[\beta]C^{(||)}[\gamma]G''_m(h\beta - h\gamma) \ + \ h^4 \sum_{\beta=0}^{k} \sum_{\gamma=0}^{k} C^{(||)}[\beta]C^{(||)}[\gamma]G_m^{IV}(h\beta - h\gamma) \right].$$

Application of the quadrature method for solving the second kind Fredholm integral equation

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We consider the following linear the second kind Fredholm integral equation:

$$y(x) - \lambda \int_a^b K(x,s)y(s)ds = f(x), \qquad x \in [a,b] \tag{1}$$

where $K(x,s)$ is the kernel, $f(x)$ is right hand side, λ is a parameter, $y(x)$ is an unknown function.

In the work [1], the optimal quadrature formula with derivatives in the Sobolev space was given. To solve (1), we apply this quadrature method with $m = 2$. Then the quadrature formula for (1) has the following form:

$$\int_0^1 K(x_i,s)y(s)ds \cong \sum_{\beta=0}^{N} (C_{i\beta}y_\beta + C'_{i\beta}y'_\beta), \quad i = 0,\ 1,\ ...,\ N. \tag{2}$$

Here $K(x_i,s)$ is the weight function, y_i, y'_i are the values of the searching function $(y_i = y(x_i),\ y'_\beta = y'(x_i))$, $C_{i\beta}, C'_{i\beta}$ are optimal coefficients of the quadrature formula, $(x_i = ih,\ i = 0,1,...,N)$, h is the mesh size.

For (2) the following holds.

Theorem. In the space $L_2^{(2)}$ there is a unique optimal quadrature formula of the form (2), which coefficients are determined by formulas

$$C_{i0} = p_{i0}/2 - [F_{i1} - F_{i0}]/h, \quad C'_{i0} = p_{i1}/2 - [F'_{i1} - F'_{i0}]/h,$$
$$C_{i\beta} = -[F_{i\beta-1} - 2F_{i\beta} + F_{i\beta+1}]/h, \quad C'_{i\beta} = -[F'_{i\beta-1} - 2F'_{i\beta} + F'_{i\beta+1}]/h,$$
$$C_{iN} = p_{i0}/2 - [F_{iN-1} - F_{iN}]/h, \quad C'_{iN} = p_{i1}/2 - \left[F'_{iN-1} - F'_{iN}\right]/h,$$

where $F_{i\beta}$, $F'_{i\beta}$, $f_{i\beta}$, p_{i0}, $f'_{i\beta}$, g_{i0}, p_{i1}, g_{i1}, $\beta = 0,1,...,N$, $i = 0,1,...,N$, are known.

Differentiating by x the integral equation (1) we get:

$$y'(x) - \lambda \int_0^1 K'_x(x,s)y(s)ds = f'(x), \tag{3}$$

Applying the quadrature formula (2) to equations (1) and (3), we obtain the system of linear algebraic equations:

$$y_i - \sum_{\beta=0}^{N} (C_{i\beta}y_\beta + C'_{i\beta}y'_\beta) = f_i,\ y'_i - \sum_{\beta=0}^{N} (C_{i\beta}y_\beta + C'_{i\beta}y'_\beta) = f'_i,\ i = 0,\ 1,\ ...,\ N.$$

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On the quadrature method for the numerical solution of the second kind Fredholm integral equation

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We consideer the following second kind Fredholm linear integral equation:

$$y(x)-\lambda\int_a^b K(x,s)y(s)ds=f(x),\qquad x\in[a,b]\tag{1}$$

where $y(x)$ is an unknown function.

From the results of the work [1] we get the following quadrature formula:

$$\int_0^1 K(x_i,s)y(s)ds\cong\sum_{\beta=0}^{N}\left(C_{i\beta}y_\beta+C'_{i\beta}y'_\beta+C''_{i\beta}y''_\beta\right),\quad i=0,\ 1,\ ...,\ N.\tag{2}$$

Here $K(x_i,s)$ is the weight function, $y_i,\ y'_i,\ y''{}_i$ are values of the searching function ($y_i=y(x_i),\ y'_\beta=y'(x_i),\ y''{}_\beta=y''(x_i)$), $C_{i\beta},C'_{i\beta},C''{}_{i\beta}$ are optimal coefficients of the quadrature formula, h is the size of the mesh, $(x_i=ih,\ i=0,1,...,N)$.

For (2) the following is true

Theorem. In the space $L_2^{(3)}$ there is a unique optimal quadrature formula of the form (2), which coefficients are determined as follows

$$C_{i0}^{(k)}=p_{i1}/2+(-1)^k\left[F_{i1}^{(k)}-F_{i0}^{(k)}\right]/h,\quad C_{i\beta}^{(k)}=(-1)^k[F_{i\beta-1}^{(k)}-2F_{i\beta}^{(k)}+F_{i\beta+1}^{(k)}]/h,$$

$$C_{iN}^{(k)}=p_{i1}/2+(-1)^k\left[F_{iN-1}^{(k)}-F_{iN}^{(k)}\right]/h,\beta=0,\ 1,\ ...,\ N,$$

Here $F_{i\beta}^{(k)}$, $f_{i\beta}^{(k)}$, p_{ik}, g_{ik}, $i=0,\ 1,...,\ N$, $k=0,1,2,$, $\beta=0,1,...,N$, are known.

Differentiating two times by x the integral equation (1), we get the system of linear integral equations:

$$y^{(k)}(x)-\lambda\int_0^1 K_x^{(k)}(x,s)y(s)ds=f^{(k)}(x),\ \ k=0,\ 1,\ 2.\tag{3}$$

Applying quadrature formula (2) to equations (3), we have system of linear algebraic equation:

$$y_i^{(k)}-\sum_{k=0}^{2}\sum_{\beta=0}^{N}C_{i\beta}^{(k)}y_\beta^{(k)}=f_i^{(k)},\ i=0,\ 1,\ ...,\ N,$$

that is easy to solve by linear algebra methods.

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On the efficiency of OpenMP parallelization on a supercomputer of the IFDS algorithm for solving the Cauchy problem for the fractional Riccati equation

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The numerical solution of problems of fractional dynamics can create a high computational load, which makes it necessary to implement efficient algorithms for solving them. The main contribution to the computational load of such calculations is made by heredity (memory), which is determined by the dependence of the current value of the solution on the previous values in the time interval [1].

Memory from the point of view of mathematics can be described using a fractional derivative. In this study, we use the Gerasimov-Caputo operator of variable order, i.e. variable nonlocality. As an example, we consider the direct Cauchy problem for a non-linear fractional Riccati equation with non-constant coefficients, which is numerically solvable using a non-local explicit finite difference scheme (IFDS) [2].

The paper presents an analysis of the effectiveness of the parallel implementation of IFDS using the OpenMP hardware and software architecture. The analysis is based on the data obtained as a result of a series of test case calculations using IFDS on the NVIDIA DGX STATION computing server located at the Institute of Mathematics named after V.I. Romanovsky AS RUz.

The acceleration of calculations by 8-13 times for a parallel algorithm is shown, in comparison with the most efficient sequential one. Moreover, the greatest efficiency is achieved when using 10-15 CPU threads. From the results of the analysis, a conclusion is made about the expediency of using supercomputers.

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Difference schemes of high accuracy for solving non-stationary fourth-order equations

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Mathematical models of many non-stationary processes lead to the solution of ordinary differential equations of the fourth order, when space variables are approximated by the finite difference method or the finite element method (the method of straight lines); for example, the equations of dynamics of a compressible stratified rotating fluid, the equation of electron waves in a cold plasma in an external magnetic field, the equation of spin waves in a two-sublattice "easy-plane" antiferromagnet, etc.

For an approximate solution of ordinary differential equations of the fourth order

$$D\ddot{\ddot{u}} + B\ddot{u} + Au = f, \quad t_0 < t \leq T, \tag{1}$$

$$u(0) = u_{0,0},\ \dot{u}(0) = u_{0,1},\ \ddot{u}(0) = u_{0,2},\ \dddot{u}(0) = u_{0,3}. \tag{2}$$

we can use five-layer difference schemes of the second order of approximation in τ (time step). Here D, B, and A are linear, t-independent constant operators from $H \to H$, $D^* = D > 0,\ B^* = B \geq 0,\ A^* = A > 0;\ \ddot{\ddot{u}} = d^4u/dt^4,\ \dddot{u} = d^3u/dt^3,\ \ddot{u} = d^2u/dt^2$, $\dot{u} = du/dt,\ \forall\ t \geq 0,\ \ u = u(t),\ f = f(t) \in H$ is the Hilbert space with scalar product $(u, \vartheta) = (u, \vartheta)$ and norm $\|u\| = \sqrt{(u,u)}$.

In this paper, difference schemes of the fourth-order accuracy for the abstract Cauchy problem (), () are proposed and studied:

$$\overline{D}y_{\bar{t}t\bar{t}t} + \overline{B}y_{\bar{t}t} + Ay = \overline{\varphi},\ t_n \in \omega_\tau,\ n = 2, 3, ..., \tag{3}$$

$$\begin{gathered}
\overline{u}_{0,1} = u_{0,1} + 0.5\tau[E - (\tau^2/12)D^{-1}B]u_{0,2} + (\tau^2/6)u_{0,3} + (\tau^3/24)D^{-1}[f(0) - Au_{0,0}],\\
\overline{u}_{0,2} = u_{0,2} + \tau u_{0,3} + (\tau^2/2)D^{-1}[f(0) - Bu_{0,2} - Au_{0,0}]+\\
+(\tau^3/4)D^{-1}[\dot{f}(0) - B\dot{u}_{0,2} - A\dot{u}_{0,0}],\\
\overline{u}_{0,3} = u_{0,3} + (3\tau/2)D^{-1}[f(0) - Bu_{0,2} - Au_{0,0}]+\\
+(5\tau^2/4)D^{-1}[\dot{f}(0) - B\dot{u}_{0,2} - A\dot{u}_{0,0}] + (3\tau^2/4)D^{-1}[\ddot{f}(0) - B\ddot{u}_{0,2} - A\ddot{u}_{0,0}].
\end{gathered} \tag{4}$$

Here $\overline{D} = D + (\tau^2/12)B$, $\overline{B} = B + (\tau^2/6)A$, $\overline{\varphi} = \varphi + (\tau^2/6)\ddot{f}$.

The following main theorem is proven.

Theorem. *Let $D^* = D > 0,\ B^* = B \geq 0,\ A^* = A > 0$ and the following condition be fulfilled:*

$$\overline{D} > (\tau^4/4)A.$$

Then, the solution to the difference scheme (3), (4) converges to a smooth solution of the original problem (1), (2) and the following accuracy estimate holds:

$$\|y(t_n) - u(t_n)\| \leq O(\tau^4),\ t_n \in \overline{\omega}_\tau.$$

The theoretical results obtained can be applied to the numerical solution of initial-boundary value problems for the Sobolev-type equation of higher order, unresolved with respect to the highest derivative.

Numerical methods for Sobolev-type high order equations

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Mathematical modeling of wave processes in plasma physics leads to initial-boundary value problems for high-order non-classical Sobolev-type equations. For example, such problems arise in the physics of semiconductors, the physics of atmosphere, problems related to the propagation of waves in media with strong dispersion, and many others.

Consider the following initial-boundary value problem:

$$\frac{\partial^2}{\partial t^2}(\Delta_2 u - \rho^2 u) + \omega_0^2 \frac{\partial^2}{\partial t^2}\Delta_1 u + \theta^2 \Delta_3 u = -f(x,t),\ \ (x,t) \in Q_T, \tag{1}$$

$$u(x,0) = u_0(x),\ \ \frac{\partial u}{\partial t}(x,0) = u_1(x),\ x \in \overline{\Omega},\ \ u(x,t) = 0,\ x \in \mathit{\Gamma} = \partial\Omega,\ t \in (0,T], \tag{2}$$

where, $\Delta_2 u = \partial^2 u/\partial x_1^2 + \partial^2 u/\partial x_3^2$, $\Delta_m u = \partial^2 u/\partial x_m^2$, $m = 1,3$, $Q_T = \{x = (x_1, x_3) \in \Omega,\ t \in (0,T]\}$, $\overline{\Omega} = \{0 \le x_m \le l_m,\ m = 1,3\}$, $\Omega = \overline{\Omega} \cap \mathit{\Gamma}$, $u \in H$ is the Hilbert space.

First, we approximate the space variables based on the finite difference method, as a result, from (1) and (2) we obtain a system of second-order ordinary differential equations:

$$D\ddot{u}_h(t) + Au_h(t) = f_h(t),\ \ u_h(0) = u_{0,h},\ \ \dot{u}_h(0) = u_{1,h}, \tag{3}$$

where $u_h(t)$ is an element of finite-dimensional space H_h $\forall t$; $D,\ A$ are operators from H_h to H_h, $\dot{u} = du/dt$, $\ddot{u} = d^2u/dt^2$. In the spatial approximation of problem (1), (2), schemes of various orders of accuracy, from two to four, were considered. Then, to solve problem (3), a three-parameter scheme of the finite element method of the fourth-order accuracy in time was used:

$$(D - \gamma\tau^2 A)\frac{\overset{\wedge}{\dot{y}} - \dot{y}}{\tau} + A\frac{\hat{y} + y}{2} = \varphi_1,\ \ (D - \alpha\tau^2 A)\frac{\hat{y} - y}{\tau} - (D - \beta\tau^2 A)\frac{\overset{\wedge}{\dot{y}} + \dot{y}}{2} = \varphi_2, \tag{4}$$
$$y^0 = u_0,\ \dot{y}^0 = u_1.$$

Here $y = y^n = y(t_n)$, $\hat{y} = y^{n+1} = y(t_n + \tau)$, $\dot{y} = \dot{y}^n = \dot{y}(t_n)$, $n = 0,1,2,...,$ $y^n,\ \dot{y}^n \in H_h$, $\varphi_k = \int\limits_0^1 f(t_n + \tau\xi)\vartheta_k(\xi)d\xi$, $k = 1,2$, $\vartheta_1(\xi) = 1$, $\vartheta_2(\xi) = s_1\vartheta_2^{(1)}(\xi) + s_2\vartheta_2^{(2)}(\xi)$, $\vartheta_2^{(1)}(\xi) = \tau(\xi - 1/2)$, $\vartheta_2^{(2)}(\xi) = \tau(\xi^3 - 3\xi^2/2 + \xi/2)$, $\xi = (t - t_n)/\tau$, $s_1 = 180\beta - 40\alpha$, $s_1 = 1680\beta - 280\alpha$.

Parameters α, β, γ obey the condition of the fourth order of approximation $\alpha + \gamma = \beta + 1/6$, and the sixth order of approximation $\beta - 6\alpha\gamma + 1/40 = 0$. Scheme stability condition (4) is $\tau^2 \le (1-\varepsilon)/m$, where $m = \max\{\alpha, \beta, \gamma\}$, $0 < \varepsilon < 1$.

Then, the approximation error was investigated, stability conditions were obtained, and theorems on the convergence and accuracy of the considered schemes in the class of smooth solutions were proved. The computational experiments conducted illustrated the efficiency of the constructed numerical algorithms.

IV. SECTION. ALGEBRA, GEOMETRY AND FUNCTIONAL ANALYSIS

About nonlinear transformation invariants in Galilean space

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Let the three-dimensional affine space A_3 be given, $Oxyz$ is a system of affine coordinates with origin at the point $O(0,0,0)$ and $\{\vec{i},\vec{j},\vec{k}\}$ are basis vectors in this space.

The scalar product of vectors $\vec{X}\{x_1,y_1,z_1\}$ and $\vec{Y}\{x_2,y_2,z_2\}$ is determined by formula

$$(\vec{X}\vec{Y}) = \begin{cases} x_1x_2, & if\ x_1x_2 \neq 0, \\ y_1y_2 + z_1z_2, & if\ x_1x_2 = 0. \end{cases} \tag{1}$$

Definition 1. The affine space in which the scalar product of vectors $\vec{X}\{x_1,y_1,z_1\}$ and $\vec{Y}\{x_2,y_2,z_2\}$ is defined by formula (1) is called the Galilean space and is denoted by R_3^1 or Γ_3 [1].

Consider the following nonlinear transformation:

$$\begin{cases} x' = x \\ y' = f(x) + y \end{cases} \tag{2}$$

Under this transformation, the points on the Oy axis do not change, and all other points $x' = x_0 \neq 0$ slide parallel to the Oy axis at a distance $h = f(x_0)$.

In this paper, we study the invariants in the nonlinear transformation (2).

Let us be given

$$r(u,v) = x(u,v)\,\vec{i} + y(u,v)\,\vec{j} + z(u,v)\,\vec{k}$$

the vector equation of some regular surface.

Theorem 1. Under nonlinear transformation (2) of a regular surface, the total (Gaussian) curvature of the surface does not change.

Theorem 2. Under nonlinear transformation (2) of a regular surface, the area of the domain on the surface does not change.

As a consequence of these theorems, it can be argued that during the rotation of the Galilean space, which will be a deformation in the Euclidean space, the total curvature of the surface of the Euclidean space does not change.

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On some homotopically dense subspaces of the space of all probability measures that are manifolds of infinite dimension

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Abstract. Exploring the geometric and topological properties of the space of all probability measures $P(X)$ defined in an infinite compact set X, a number of subspaces $P(X)$ are distinguished that are homeomorphic to triples of manifolds of infinite dimension. A is also allocated pairs of subspaces of the space of probability measures, which are homeomorphic to pairs of manifolds of infinite dimension.

Recall that a topological space X is called a manifold modeled on the space Y, or a $Y-$manifold [2], if any point in the space X has a neighborhood homeomorphic to an open subset of the space X.

A Q-manifold is a separable metric space locally homeomorphic to the Hilbert cube Q, where $Q = \prod_{i=1}^{\infty}[-1,1]_i$ Hilbert cube [1], $W_i^{\pm} = \{(g_j) \in Q | g_i = \pm 1$ $j-$th face of the Hilbert cube Q, $BdQ = \bigcup_{i=1}^{\infty} W_i^{\pm}-$is called the pseudoboundary of the Q cube, and $S = Q \backslash BdQ$ is the pseudointerior of the Q cube. It is known that $S = \prod_{i=1}^{\infty}(-1,1)_i$.

Let X be a topological space.

A set $A \subset X$ is said to be homotopy dense in X[3] if there exists a homotopy $h(x,t)$: $X \times [0,1] \to X$ such that $h(x,0) = id_X$ and $h(X \times (0,1]) \subset A$.

Theorem 1. Let X be an infinite compact set such that $P_\omega(X)$ contains the Hilbert cube Q, and $A \subset X$, $A \neq X$, $,\bar{A} = X$ and $|A| = \chi_0$. Then $P_\omega(X) \backslash P_\omega(A) \simeq \Sigma \times S$.

Theorem 2. Let X be an infinite compact $A_1 \subseteq A_2 \subseteq ... \subseteq A_n \subseteq ...$ of closed subsets such that $A = \bigcup_{i=1}^{\infty} A_i$ is everywhere dense in X. a) $P(A) \simeq \Sigma$ if $P(A)$ contains the Hilbert cube Q; Then b) $P(A) \simeq \ell_2^f$ if $P(A)$ does not contain the Hilbert cube Q.

Theorem 3. Let X be an infinite compact set $A_1 \subset A_2 \subset ... \subset A_n \subset ...$ a sequence of closed sets such that $A \neq X$ and $A = \bigcup_{n=1}^{\infty} A_i$ is everywhere dense in X.

Then a) $P_\omega(A)$ is homeomorphic to ℓ_2^f ; if $P_\omega(A)$ does not contain a Hilbert cube Q;
b) $P_\omega(A) \simeq \Sigma$ if $P_\omega(A)$ contains the Hilbert cube Q.

Theorem 4. Let X be a finite-dimensional compact set, and $A_1 \subset A_2 \subset ... \subset A_n \subset ...$ a sequence its closed subsets such that $A \neq X$, $\bar{A} = X$. Then $P(A) \backslash P(N) \simeq \sum \times S$ if there exists $A_{i_0}-$infinitely, where N is a countable subset of X.

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The local parametrization of an algebraic variety near its third-order singular point $P_5^{(3)}$

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In theoretical physics, when studying Einstein's invariant metrics, the need arose to study in coordinates a_1,a_2,a_3 an algebraic variety Ω [1,2], which is described by the equation

$$Q(s_1,s_2,s_3)\overset{\text{def}}{=}(2s_1+4s_3-1)(64s_1^5-64s_1^4+8s_1^3+240s_1^2s_3-1536s_1s_3^2-4096s_3^3+12s_1^2-240s_1s_3+ +768s_3^2-6s_1+60s_3+1)-8s_1s_2(2s_1+4s_3-1)(2s_1-32s_3-1)(10s_1+32s_3-5)-16s_1^2s_2^2(52s_1^2+ +640s_1s_3+1024s_3^2-52s_1-320s_3+13)+64(2s_1-1)s_2^3(2s_1-32s_3-1)+2048s_1(2s_1-1)s_2^4=0,$$

where s_1,s_2,s_3— elementary symmetric polynomials, equal respectively to

$$s_1=a_1+a_2+a_3, s_2=a_1\cdot a_2+a_1\cdot a_3+a_2\cdot a_3, s_3=a_1\cdot a_2\cdot a_3.$$

Thus $Q(s_1,s_2,s_3)=P(a_1,a_2,a_3)$, where the polynomial P has degree 12. The variety Ω has 3 third order singular points, 1 second order singular point, and 3 curves filled by singular points of the first order. Using the algorithms of power geometry [3] and computer algebra programs [4], local parameterizations of the variety Ω near its two third order singular points, one second order singular point, and one singular curve were obtained [5,6]. Here the same is done near its third singular point of the third order $P_5^{(3)}$.

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The automorphism groups of two-dimensional algebras over any field

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This research deals with describing automorphism groups of two-dimensional algebras over a basic field $\mathbb{F}$, a particular case of which is considered in [1].

Definition 1. *A vector space* $\mathbb{A}$ *over* $\mathbb{F}$ *with multiplication* $\cdot : \mathbb{A}\otimes\mathbb{A}\to\mathbb{A}$ *given by* $(\mathbf{u},\mathbf{v})\mapsto \mathbf{u}\cdot\mathbf{v}$ *such that* $(\alpha\mathbf{u}+\beta\mathbf{v})\cdot\mathbf{w}=\alpha(\mathbf{u}\cdot\mathbf{w})+\beta(\mathbf{v}\cdot\mathbf{w})$, $\mathbf{w}\cdot(\alpha\mathbf{u}+\beta\mathbf{v})=\alpha(\mathbf{w}\cdot\mathbf{u})+\beta(\mathbf{w}\cdot\mathbf{v})$ *whenever* $\mathbf{u},\mathbf{v},\mathbf{w}\in\mathbb{A}$ *and* $\alpha,\beta\in\mathbb{F}$*, is said to be an algebra.*

Definition 2. *An invertible linear map* $\mathbf{f}:\mathbb{A}\to\mathbb{A}$ *is said to be an automorphism if* $\mathbf{f}(\mathbf{u}\cdot\mathbf{v})=\mathbf{f}(\mathbf{u})\cdot\mathbf{f}(\mathbf{v})$ *whenever* $\mathbf{u},\mathbf{v}\in\mathbb{A}$ *and* $\alpha,\beta\in\mathbb{F}$.

The following result, in the characteristic of $\mathbb{F}$ not $2,3$ case, is obtained relying on the classification theorem of two-dimensional algebras over any field [2]. In the formulation of it, we follow the notations of that paper.

Theorem. *If* $Char.(\mathbb{F})\neq 2,3$ *then the automorphism groups of algebras, presented in the classification theorem [2], are as follows* $Aut(A_1(\alpha_1,\alpha_2,\alpha_4,\beta_1))=Aut(A_2(\alpha_1,\alpha_4,\beta_2))=Aut(A_4(\beta_1,\beta_2))=Aut(A_6(\alpha_1,\alpha_4))=Aut(A_8(\beta_1))=\{I\}$, $Aut(A_3(\alpha_1,0,2\alpha_1-1))=\{\begin{pmatrix}1&0\\c&d\end{pmatrix};c,d\in\mathbb{F},d\neq 0\}$, $Aut(A_3(\alpha_1,0,\beta_2))=\{\begin{pmatrix}1&0\\0&d\end{pmatrix};0\neq d\in\mathbb{F}\}$, *if* $\beta_2\neq 2\alpha_1-1$, $Aut(A_3(\alpha_1,\alpha_4,1-\alpha_1))=$ $\{\begin{pmatrix}1&0\\0&d\end{pmatrix}:\ 0\neq d\in\mathbb{F}\}$, *if* $\alpha_4\neq 0,\alpha_1\neq 1$, $Aut(A_3(1,\alpha_4,0))=Aut(A_7(\alpha_1,\alpha_4))=Aut(A_9)=\{\begin{pmatrix}1&0\\0&\pm1\end{pmatrix}\}$, *if* $\alpha_4\neq 0$, $Aut(A_5(\alpha_1))=\{\begin{pmatrix}1&0\\c&1\end{pmatrix}:c\in\mathbb{F}\}$, $Aut(A_{10}(\beta_1))=\{I\}\cup\{\begin{pmatrix}-1-d&\frac{-1-d-d^2}{1+2d}\\1+2d&d\end{pmatrix}:\ \beta_1=\frac{(1+2d)^2}{d^2+d+1},\ d\in\mathbb{F}\}$, *provided that* $(\beta_1t^3-3t-1)(\beta_1t^2+\beta_1t+1)(\beta_1^2t^3+6\beta_1t^2+3\beta_1t+\beta_1-2)$ *has no root in* $\mathbb{F}$, $Aut(A_{11}(\beta_1))=\{\begin{pmatrix}a&0\\0&a^2\end{pmatrix}:\ a\in\mathbb{F},a^3=1\}\cup\{\begin{pmatrix}0&b\\\frac{1}{b}&0\end{pmatrix}:\ \beta_1=\frac{1}{b^3},b\in\mathbb{F}\}$, $Aut(A_{12}(0))=\left\{\begin{pmatrix}a&0\\0&1\end{pmatrix}:\ 0\neq a\in\mathbb{F}\right\}$ *if* $\beta_1=0$, $Aut(A_{12}(\beta_1))=\left\{\begin{pmatrix}\pm1&0\\0&1\end{pmatrix}\right\}\cup\{\begin{pmatrix}\pm\frac{1}{2}&b\\\pm\frac{3}{4b}&-\frac{1}{2}\end{pmatrix}:\ \beta_1=\frac{3}{4b^2},b\in\mathbb{F}\}$, *if* $\beta_1\neq 0$, $Aut(A_{13})=\left\{\begin{pmatrix}a&0\\c&a^2\end{pmatrix}:\ a,c\in\mathbb{F},a\neq 0\right\}$.

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How is isomorphism defined in supergroups?

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Supergroups T_1 and T_2 are said to be isomorphic if it is possible to establish a one-to-one correspondence between their elements so that when elements t_1 and t_1' from T_1 correspond to elements t_2 and t_2' from T_2 the product t_1t_1' corresponds to the product t_2t_2'.

An isomorphism of supergroups T_1 and T_2 is denoted by $T_1 \cong T_2$. He correspondence between the elements t_1 and t_2 of the supergroups T_1 and T_2 will be denoted by the sign $t_1 \leftrightarrow t_2$.

Basic properties of isomorphism: 1) Every supergroup T is isomorphic to itself (reflection property). In with. etc., for this it suffices to bring each element t into a one-to-one correspondence with itself, then the definition of isomorphism will be satisfied. Thus, $T \cong T$. 2) If the supergroup T_1 is isomorphic with the supergroup T_2, then, conversely, T_2 is isomorphic with T_1, that is, if $T_1 \cong T_2$, then so is $T_2 \cong T_1$(symmetry properties). Indeed, from $T_1 \cong T_2$ it follows that $t_1 \leftrightarrow t_2$ and $t_1' \leftrightarrow t_2'$, причем $t_1t_1' \leftrightarrow t_2t_2'$. 3)If the supergroup T_1 is isomorphic with T_2, and T_2 is isomorphic with T_3, then T_1 is isomorphic with T_3, that is, from $T_1 \cong T_2$ from $T_2 \cong T_3$ follows $T_1 \cong T_3$ (property of transitivity). If from $t_1 \leftrightarrow t_2$ and $t_2 \leftrightarrow t_3$ we establish a one-to-one correspondence $t_1 \leftrightarrow t_3$, then we will arrive at $t_1t_1' \leftrightarrow t_3t_3'$, when t_1t_3 and $t_1' \leftrightarrow t_3'$.

Let's take a supergroup $T_1 = R_1 + H_1 = (1, -1, i, -i) + 0$ with respect to multiplication of numbers, which is isomorphic to the supergroup of generalized permutations $T_2 = R_2 + H_2 = (\overline{e}, \overline{r_2}, \overline{r_3}, \overline{r_4}) + \overline{0}$, where $\overline{e} = \begin{pmatrix} 1&2&3&4&5 \\ 1&2&3&4&5 \end{pmatrix}$, $\overline{r_2} = \begin{pmatrix} 1&2&3&4&5 \\ 2&1&4&3&5 \end{pmatrix}$, $\overline{r_3} = \begin{pmatrix} 1&2&3&4&5 \\ 3&4&2&1&5 \end{pmatrix}$, $\overline{r_4} = \begin{pmatrix} 1&2&3&4&5 \\ 4&3&1&2&5 \end{pmatrix}$, $\overline{0} = \begin{pmatrix} 1&2&3&4&5 \\ 5&5&5&5&5 \end{pmatrix}$.

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On τ-smooth idempotent probability measures

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In this thesis for a compact Hausdorff space X we introduce the notion of idempotent measures as a set-function on the family $\mathfrak{B}(X)$. Further, for a Tychonoff space X, we consider the space $I_\tau(X)$ of idempotent probability τ-smooth measures on X.

Let X be a compact Hausdorff space and $\mathfrak{B}(X)$ a the system of Borel subsets of X. The symbol Φ denotes the directed sets, and Δ an arbitrary index sets. Following [2], we enter the following notion.

Definition 1. A set function $\mu\colon\ \mathfrak{B}(X) \to [0, +\infty]$ is said to be an idempotent measure on X if the following conditions hold: 1) $\mu(\varnothing) = 0$; 2) $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}$ for any $A, B \in \mathfrak{B}(X)$ and 3) $\mu\left(\underset{\phi\in\Phi}{\cup} A_\phi\right) = \sup\limits_{\phi\in\Phi}\{\mu(A_\phi)\}$ for every increasing net $\{A_\phi, \phi \in \Phi\} \subset \mathfrak{B}(X)$ such that $\underset{\phi\in\Phi}{\cup} A_\phi \in \mathfrak{B}(X)$.

The set of all idempotent measure on X we denote by $M(X)$. If $\mu(X) = 1$, the idempotent measure μ is called an idempotent probability measure on X. We denote $I(X) = \{\mu \in M(X) :\ \mu(X) = 1\}$.

Let X be a Tychonoff space, βX the Stone-Čech compactification of X. Following [1], we denote the following set:

$$I_\tau(X) = \{\mu \in I(\beta X)\colon\ \mu(F) = 0 \text{ for every } F \in \mathfrak{B}(\beta X), F \subset \beta X \setminus X\}.$$

Elements of $I_\tau(X)$ is said to be τ-smooth idempotent probability measures.

For each $\mu \in I_\tau(X)$ we define a set function $\tilde{\mu}\colon\ \mathfrak{B}(X) \to [0, +\infty]$ on the family $\mathfrak{B}(X)$ of all Borel subsets of X by the formula

$$\tilde{\mu}(A) = \inf\{\mu(B)\colon\ B \in \mathfrak{B}(\beta X),\ B \supset A\}, \qquad A \in \mathfrak{B}(X).$$

Lemma 1. $\tilde{\mu}$ is an idempotent probability measure on X.

Lemma 2. Let X be a Tychonoff space. If $\mu \in I_\tau(X)$, then $\mu(A) = \mu(B)$ for any two Borel subsets $A, B \subset \beta X$ such that $A \cap X = B \cap X$.

Now, consider X and Y be Tychonoff spaces, $f\colon\ X \to Y$ a continuous map, and $\beta f\colon\ \beta X \to \beta Y$ the Stone-Čech compactification of f and $I_\tau(f) = I(\beta f)|_{I_\tau(X)}\colon\ I_\tau(X) \to I_\tau(Y)$.

Theorem 1. For Tychonoff spaces X, Y and a continuous map $f\colon\ X \to Y$ we have $I(\beta f)(I_\tau(X)) \subset I_\tau(Y)$.

Theorem 2. The operation $I_\tau\colon\ Tych \to Tych$ preserves the class perfect maps.

Theorem 3. The operation $I_\tau\colon\ Tych \to Tych$ preserves the class embedding.

Theorem 4. Let X be a Tychonoff space and $A, B \subset X$ any subsets such that at least one of them is Borel. Then the equality $I_\tau(A \cap B) = I_\tau(A) \cap I_\tau(B)$ holds.

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Behavior dynamics of the Lotka-Volterra mapping composition, with transitive tournaments describing models of sexually transmitted diseases

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Biologists, epidemiologists, economists, and mathematicians have been trying to work together for a long time. In the twenties of the last century, articles by S. N. Berstein appeared, the first of which was "On the application of mathematics to biology "("Science in Ukraine", 1922, issue 1, p. 14).

It is characteristic of mathematics that the formation of new directions often arises on the basis of new tasks. In particular, all classical analysis arose on the basis of problems of physics, mechanics and geometry. However, later it turned out that the scope of mathematical analysis is much wider. Quite a lot of problems related to chemistry or technology, to various branches of engineering and natural science, including problems of biology and even economics, are solved by methods of mathematical analysis. However, to solve a number of problems in the field of biology, epidemiology of economics, technology, having a cybernetic nature, i.e. related to information flows and management, the methods of classical analysis are not applicable. Tasks of this nature stimulated the development of new branches of mathematics, such as information theory, the doctrine of control systems, as well as automata theory, game theory, various sections of mathematical programming, etc. A common feature of these new areas of mathematics is discreteness. And apparently, this has a deep meaning. The fact is that physics, mechanics and other sciences leading to the formulation of problems of classical analysis are characterized by the expedient use of continuous models of the phenomena under study. In fact, the direct object of mathematical study is continuous media, continuous trajectories, continuous physical fields, etc.

In this paper, we will consider the discrete case of these models, which are fundamentally different from the models considered earlier. We will consider discrete dynamic Lotka-Volterra systems operating in a two-dimensional simplex, and their compositions, since they can be used in modeling the course of sexually transmitted diseases.

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Solving special Monge-Ampere equation using dual mapping of isotropic space

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The Monge-Ampere equation is generally as follows:

$$z_{xx}z_{yy} - z_{xy}^2 = \varphi(x, y, z, z_x, z_y)$$

where the function $\varphi(x, y, z, z_x, z_y)-$ is the given function. If the function on the right side is of the form $\varphi = \varphi(x, y)$, then the existence and uniqueness of its solution in the convex domain was proved by Pogorelov [1]. In the non-convex domain, A. Artikbayev [2] proved that the solution exists and is unique. But the solution of the function has not been exactly found. In [4], the solution is shown when $\varphi(x, y) = \phi(x)\psi(y)$. In this work, for

$$z_{xx}z_{yy} - z_{xy}^2 = \varphi(z_x, z_y) \tag{2}$$

we will find the solution. If a regular surface in the isotropic space R_3^2 is given by the equation $z = z(x, y)$, $(x, y) \in D \subset R_2$, then the total curvature of the surface is determined by the following formula:

$$z_{xx}z_{yy} - z_{xy}^2 = K \tag{3}$$

Where, K is the total curvature of the surface, since the left side of the formula (3) is the Monge-Ampere operator, the problem of recovering the surface in isotropic space is equivalent to solving the Monge-Ampere equation [3]. Equation (2) can be solved for transfer surfaces if the dual mapping of isotropic space with respect to the sphere is used.

Theorem 1: In the isotropic space, the Monge-Ampere equation is in the form (2), and the function on the right side can be written in the form $\varphi(z_x, z_y) = \psi_1(z_x)\psi_2(z_y)$, then the general solution of the transfer surfaces is equal to:

$$z(x, y) = x \cdot \psi_1^{-1}\left(\frac{dx}{\tau}\right) + y \cdot \psi_2^{-1}(\tau dy) - \int xd\left(\psi_1^{-1}\left(\frac{dx}{\tau}\right)\right) - \int yd\left(\psi_2^{-1}(\tau dy)\right)$$

Where, $\tau-$ is const. z_x, z_y are first-order derivatives of $z(x, y)$.

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Properties of Clifford algebras: Expanding the Horizons of mathematics

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Introduction: Clifford algebras, also known as geometric algebras, provide a powerful framework that unifies and extends various mathematical structures. Emerging from the pioneering work of William Kingdon Clifford, these algebras bridge the gap between multivectors, matrices, and quaternions, enabling a deeper understanding of geometric and algebraic concepts. In this article, we aim to explore the properties and implications of Clifford algebras, illuminating their significance in broadening the horizons of mathematics.

The Clifford Product and Basis Vectors: The Clifford product serves as the foundation for multiplication in Clifford algebras, providing a means to combine multivectors. Let $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ be the basis vectors of a Clifford algebra, and consider the multivector $A = a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots + a_n\mathbf{e}_n$, where $a_0, a_1, a_2, \ldots, a_n$ are scalar coefficients.

The product of two multivectors A and B can then be obtained by distributing the basis vectors' products and combining like terms. This can be summarized as follows:

$$A \cdot B = (a_0b_0) + (a_0b_1)\mathbf{e}_1 + (a_0b_2)\mathbf{e}_2 + \ldots + (a_nb_n)\mathbf{e}_n + (a_1b_2)\mathbf{e}_1\mathbf{e}_2 + \ldots$$

By combining basis vectors using the Clifford product, we can construct multivectors that encapsulate complex geometric entities. This enables us to describe and manipulate complex geometric structures effectively.

Properties of Isomorphism with Pauli and Dirac Matrices: The isomorphism between a specific Clifford algebra and the set of $2^k \times 2^k$ matrices, where k is a positive integer, facilitates the study and application of Clifford algebras using matrix operations. One of the most notable instances of this isomorphism is found within the three 2×2 complex Pauli matrices.

By representing basis vectors and multivectors in a specific Clifford algebra using these matrices, we can perform various algebraic operations directly within the matrix framework. This is particularly significant in applications such as quantum mechanics, where the Pauli matrices play a crucial role.

Similarly, the isomorphism extends to the Dirac matrices, a set of 4×4 matrices used to represent spacetime transformations and fundamental particles in relativistic quantum mechanics. The four Dirac matrices, denoted as $\gamma^0, \gamma^1, \gamma^2, \gamma^3$, have a specific algebraic structure and satisfy the Clifford algebra properties. This isomorphism allows for the manipulation and analysis of spacetime transformations and particle interactions within the framework of the Clifford algebra.

Conclusion: This article provides a glimpse into the vast potential of Clifford algebra and serves as a starting point for further exploration and research. By understanding the properties of Clifford algebras, mathematicians and scientists can unlock new avenues for innovation and discovery in diverse fields of study.

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Translation-invariant nonprobability Gibbs measures for the HC model with a countable set of spin values in the case of a "Wand" type graph

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Phi = \{\dots, -1, 0, 1, \dots\}$ and $\sigma \in \Omega = \Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. A configuration σ is called a *G-admissible configuration* on the Cayley tree, if $\{\sigma(x), \sigma(y)\}$ is the edge of the graph G for any pair of nearest neighbors x, y in V.

The activity set [1] for a graph G is a function $\lambda : G \to R_+$. For given G and λ we define the Hamiltonian of the G-HC model as

$$H_G^{\lambda}(\sigma) = J \sum_{x \in V} \ln \lambda_{\sigma(x)},\ \sigma \in \Omega^G\ (J \in R).$$

The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory, for example, in [2].

Let $L(G)$ be the set of edges of a graph G, and let $A \equiv A^G = (a_{ij})_{i,j \in Z}$ denote the adjacency matrix of G. We consider a specific graph $G = wand$ defined as

$$a_{ij} = \begin{cases} 1, & \text{if} \quad i = j \quad \text{and} \quad i, j \ \text{ is odd}, \\ 1, & \text{if} \quad j = i + 1 \ \text{ or } \ j = i - 1, \\ 0, & \text{otherwise}. \end{cases}$$

For any function $z : x \to z_x = (\dots,\ z_{-1,x}, z_{0,x},\ z_{1,x}, \dots) \in R_+^{\infty}$ satisfying the equalities

$$\begin{cases} z_{2i+1,x} = \lambda_{2i+1} \prod_{y \in S(x)} \frac{z_{2i,y} + z_{2i+1,y} + z_{2i+2,y}}{z_{-1,y} + z_{1,y}}, & i \in Z, \\ z_{2i,x} = \lambda_{2i} \prod_{y \in S(x)} \frac{z_{2i-1,y} + z_{2i+1,y}}{z_{-1,y} + z_{1,y}}, & i \in Z \end{cases} \tag{1}$$

there exists a unique HC-Gibbs measure μ, and vice versa.

We study translation-invariant solutions of system (1), i.e., $z_x = z \in R_+^{\infty}$. In this case the following theorem is proved.

Theorem. Let $k \geq 2$ and $\lambda_{cr}(k, \lambda_2) = \frac{2^k(\lambda_2+1)}{(k-1)k^k}$. Then for the HC model with countable set of spin values (corresponding to the graph $G = wand$) for $0 < \lambda \leq \lambda_{cr}$, there is exactly one translation-invariant nonprobability σ-finite Gibbs measure μ_0 and for $\lambda > \lambda_{cr}$ there are exactly three translation-invariant nonprobability σ-finite Gibbs measures μ_0, μ_1, μ_2.

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Ground states for the two-state Hard-Core model on a Cayley tree

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The Cayley tree $\tau^k = (V, L, i)$ of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, such that exactly $k+1$ edges originate from each vertex, where V is the vertices of τ^k, L is the set of edges and i is the incidence function setting each edge $l \in L$ into correspondence with its endpoints $x, y \in V$. If $i(l) = \{x, y\}$, then we call the vertices x and y *nearest-neighbors*, denoted by $l = \langle x, y\rangle$ (see [1]). We assume that $\Phi = \{0, 1\}$, and $\sigma \in \Omega = \Phi^V$ is a configuration, i.e., $\sigma = \{\sigma(x) \in \Phi : \ x \in V\}$. Values $\sigma(x) = 1$ mean that site x is 'occupied' and $\sigma(x) = 0$ that x is 'vacant'. The configuration σ is said to be an admissible if $\sigma(x) + \sigma(y) \leq 1$ for any neighboring $\langle x, y\rangle$ from V and we denote the set of such configurations by Ω. Obviously, $\Omega \subset \Phi^V$.

The Hamiltonian of the Hard-Core model is defined by the formula

$$H = \begin{cases} J \sum\limits_{x \in V} \sigma(x), & \text{if} \ \ \sigma \in \Omega, \\ +\infty, & \text{if} \ \ \sigma \notin \Omega. \end{cases} \tag{1}$$

where $J \in R$. Let M be the set of unit balls with vertices in V (see [3]).

Define the energy of a configuration σ_b for a ball b by the formula

$$U(\sigma_b) \equiv U(\sigma_b, J) = \frac{J}{k+2} \sum_{x \in b} \sigma(x), \tag{2}$$

where $J \in R$. It is not difficult to prove the following lemma.

Lemma. For each σ_b configuration, we have the followings

$$U(\sigma_b) \in \{U_1, U_2, \dots \ U_{k+2}\}, \qquad U_i = \frac{i-1}{k+2} J, \tag{3}$$

where $i = 1, \dots, k+2$.

Definition. A configuration φ is said to be a ground state for the Hamiltonian, if $U(\varphi_b) = min\{U_1, \dots, U_{k+2}\}$ for arbitrary $\forall b \in M$ (1) (see [2]).

Theorem. The following assertions hold for the two-state Hard-Core model on the Cayley tree of order k:

1) if $J = 0$, then $GS(H) = \Omega$;
2) if $J > 0$, then $GS(H) = \{\sigma : \ \sigma(x) = 0, \ \forall x \in V\}$;
3) if $J < 0$, then $GS(H) = \emptyset$.

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Periodic Gibbs measures for HC model in the case $G-key$ on the Cayley tree of order $k=2$ and $k=3$.

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Let $\Im^k=(V,L)$ is Cayley tree of order $k\geq 2$. Let $\Phi=\{0,1,2,3\}$ and $\sigma\in\Omega=\Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. A configuration σ is called a *G-admissible configuration* on the Cayley tree, if $\{\sigma(x),\sigma(y)\}$ is the edge of the graph G for any pair of nearest neighbors x,y in V. The activity set [1] for a graph G is a function $\lambda: G\to R_+$. The value λ_i of the function λ at the vertex $i\in\{0,1,2,3\}$ is called the vertex activity. For given G and λ we define the Hamiltonian of the $G-$HC model as $H_G^\lambda(\sigma)=\sum\limits_{x\in V}\log\lambda_{\sigma(x)}$, $\sigma\in\Omega^G$.

The reader can find the definition of the Gibbs measure (translation-invariant, periodic, two-periodic, invariant set and etc.) and of other subjects related to Gibbs measure theory, for example, in [2].

Definition.[1] A graph is said to be fertile if there is a set of activities λ such that the corresponding Hamiltonian has at least two translation-invariant Gibbs measures.

We consider the case $\lambda_0=1,\ \lambda_1=\lambda_2=\lambda_3=\lambda$ and we study periodic Gibbs measures in the case fertile graph $G=key$: $\{0,1\}\{0,2\}\{1,2\}\{2,3\}$(see [3]).

It is known that we have one-to-one correspondence between the set V of vertices of a Cayley tree of order $k\geq 1$ and the group G_k that is the free product of $k+1$ cyclic groups of second order (see [4]).

We consider invariant set $I=\{(z_0,z_1,z_2,t_0,t_1,t_2)\in R^6: z_0=t_1, z_1=t_0, z_2=t_2\}$.

Theorem. For the Hard Core model in the case $G-key$ the following are true:
1. Let $k=2$. Then all two-periodic Gibbs measures are translation-invariant.
2. Let $k=3$. Then there exist $\lambda_{cr}^{(1)}$,$\lambda_{cr}^{(2)}$ and $\lambda_{cr}^{(3)}$ such that the following conditions are hold:

$i)$ There exists one two-periodic Gibbs measure for $\lambda=\lambda_{cr}^{(1)}$;

$ii)$ There exist two two-periodic Gibbs measure for $\lambda_{cr}^{(1)}<\lambda<\lambda_{cr}^{(2)}$,$\lambda>\lambda_{cr}^{(3)}$;

$iii)$ There exist two two-periodic and translation-invariant Gibbs measures for $\lambda=\lambda_{cr}^{(3)}$;

$iv)$There exist three two-periodic Gibbs measure for $\lambda=\lambda_{cr}^{(2)}$;

$v)$ There exist four two-periodic Gibbs measure for $\lambda_{cr}^{(2)}<\lambda<\lambda_{cr}^{(3)}$.

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Alternative Gibbs measures for three state HC model

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Let $\Im^k = (V, L)$ be Cayley tree of order $k \geq 2$. Let $\Phi = \{0, 1, 2\}$ and $\sigma \in \Omega = \Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. The activity set [1] for a graph G is a function $\lambda : G \to R_+$. For given G and λ we define the Hamiltonian of the $G-$HC model as $H_G^\lambda(\sigma) = \sum\limits_{x \in V} \ln \lambda_{\sigma(x)}$, if $\sigma \in \Omega^G$. The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory in [2], [3].

Let $L(G)$ be the set of edges of a graph G, and let $A \equiv A^G = (a_{ij})_{i,j \in Z}$ denote the adjacency matrix of G. We consider a specific graph $G = wand$ (see[1]).

It is know [3] that for any $z : x \in V \longmapsto z_x = (z_{1,x}, z_{2,x})$, satisfying the equalities

$$z_{i,x} = \lambda \prod_{y \in S(x)} \frac{a_{i0} + a_{i1} z_{1,y} + a_{i2} z_{2,y}}{a_{00} + a_{01} z_{1,y} + a_{02} z_{2,y}}, i = 1, 2 \tag{1}$$

there exists a unique HC-Gibbs measure μ, and vice versa. We study alternative solutions of system (1). More precisely, the boundary condition $z = \{z_x, x \in G_k\}$ with fields taking values z, t defined by the following steps:

1) if at vertex x we have $z_x = z$, then the function z_y, which gives real values each vertex $y \in S(x)$, z on m vertices of $S(x)$, t on $k - m$ remaining vertices;

2) if at vertex x we have $z_x = t$, then the function z_y, which gives real values each vertex $y \in S(x)$, t on r vertices of $S(x)$, z on $k - m$ remaining vertices. A measure constructed in this way and which is not translation-invariant (TI) is called alternative Gibbs measure (AGM). In this case, for $0 \leq m \leq k$ and $0 \leq r \leq k$ the system of equations (1) we obtain

$$\begin{cases} z_1 = \lambda \cdot \left(\frac{1+z_1}{z_1+z_2}\right)^m \cdot \left(\frac{1+t_1}{t_1+t_2}\right)^{k-m}, & t_1 = \lambda \cdot \left(\frac{1+t_1}{t_1+t_2}\right)^r \cdot \left(\frac{1+z_1}{z_1+z_2}\right)^{k-r}, \\ z_2 = \lambda \cdot \left(\frac{1+z_2}{z_1+z_2}\right)^m \cdot \left(\frac{1+t_2}{t_1+t_2}\right)^{k-m}, & t_2 = \lambda \cdot \left(\frac{1+t_2}{t_1+t_2}\right)^r \cdot \left(\frac{1+z_2}{z_1+z_2}\right)^{k-r} \end{cases} \tag{2}$$

We consider invariant set $I = \{(t_1, t_2, z_1, z_2) \in R^4 : z_1 = z_2, t_1 = t_2\}$. Analysis of the system of equations (2) gives the following theorems.

Theorem 1. Let $k \geq 2$ and $m + r \geq k - 1$. Then for the HC-model on the invariant set I has a unique Gibbs measure, which coincides with the unique TIGM.

Theorem 2. Let $k = 3$, $\lambda_{cr} = 32/27$ and $m + r \leq 1$. Then for the HC model on I for $\lambda > \lambda_{cr}$ there is a unique Gibbs measure, which coincides with the unique TIGM μ_0, for $\lambda = \lambda_{cr}$ there are exactly two AGMs μ_0 and $\mu^{'}$, and for $0 < \lambda < \lambda_{cr}$ there are exactly three Gibbs measures μ_0, μ_1 and μ_2, where $\mu^{'}$, μ_1, μ_2 are AGMs.

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Resonant varieties in hamiltonian system

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The main problem:
For a multiparameter Hamiltonian system with 3 DOF in generic case give a description of regions in the system parameter space, in which there are no strong resonances.

To solve this problem, we should obtain a description of boundaries of the regions, which are free of strong resonances of orders 2, 3 and 4. These boundaries consist of parts of algebraic varieties, on which the resonance equation $\langle \mathbf{p}, \boldsymbol{\lambda} \rangle = 0$ has a nontrivial solution for $\mathbf{p} \in \mathbb{Z}^3$, where $\boldsymbol{\lambda}$ is the vector of basic eigenvectors of linear Hamiltonian system. The main problem can be decomposed into several auxiliary ones:
1. Obtain an analytic representation in the coefficient space $K = (a_1, a_2, a_3)$ of the cubic polynomial of resonant varieties $\mathcal{R}_3^p$ for all vectors $\mathbf{p}$ orders 2, 3 and 4.
2. Find the mutual location of all resonant varieties found above.

Condition on resonance existence
Two ways of computing the conditions of existance of resonant relations for a given resonant vector $\mathbf{p}^*$ are considered: a) the first method gives an *implicit representation* of the resonant variety $\mathcal{R}_3^{\mathbf{p}^*}$; b) the second method gives a *parametric representation* of the variety $\mathcal{R}_3^{\mathbf{p}}$.

Firstly the resonant relation between the roots μ_j of the semi-characteristic polynomial $f(\mu)$ of the linear Hamiltonian system for the given vector $\mathbf{p}^* = (r, q, 1)$. This condition takes the form of quasi-homogeneous polynomial

$$R_3^{(r,q,1)}(\mu_j) \equiv q^4\mu_2^2 - 2q^2r^2\mu_1\mu_2 + r^4\mu_1^2 - 2q^2\mu_2\mu_3 - 2r^2\mu_1\mu_3 + \mu_3^2 = 0.$$

To obtain the corresponding resonant condition in the implicit form as zeroes of a polynomial with coefficients $a_j, j = 1, ..., 3$ of the polynomial $f(\mu)$, a new Gröbner basis $\mathcal{F}$ of the ideal is constructed. This method turns out to be very time-consuming for resonances of the general form, it leads to very cumbersome expressions. Its generalization for cases with degrees of freedom greater than 3 is not possible.

Applying power transformation $\mu_1 = s_2s_3$, $\mu_2 = s_1s_3$, $\mu_3 = s_3$ to the $R_3^{(r,q,1)}$ one gets a polynomial of two variables $\tilde{R}_3^{(r,q,1)} \equiv q^4s_1^2 - 2q^2r^2s_1s_2 + r^4s_2^2 - 2q^2s_1 - 2r^2s_2 + 1 = 0$, which provides the parametric representation of the roots $\mu_1 = \left(r^2u(q+1) + q - 1\right)^2 v$, $\mu_2 = \left(r^2u - 1\right)^2 vr^2$, $\mu_3 = \left(r^2u + 2q - 1\right)^2 vr^2$. This parametric representation, using elementary symmetric polynomials, gives a polynomial parametrization of the coefficients a_1, a_2, a_3. Excluding the parameters u, v one can obtain an implicit representation of $R_3^{\mathbf{p}^*}$ of the condition for the existence of resonance via the coefficients a_j of the polynomial $f(\mu)$.

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Star selection principles: theory and applications

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Star selection principles theory, an important subfield of the field of selection principles theory, was initiated by Kočinac in 1999 [1] and is related to the classical properties of Menger (1924), Hurewicz (1925) and Rothberger (1938). In recent years a number of researches contributed to this area of mathematics obtaining results in topology (function spaces, hyperspaces, uniform spaces, topological algebras), mathematical analysis (Karamata's theory), game theory, comnbinatorivcs and so on. In this talk we present some of those results, suggest some new directions of investigation and pose some open questions.

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The degenerate cases asymptotic behaviour trajectories of Lotka - Volterra maps

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This article about the asymptotic behaviour of a trajectory in a three-dimensional simplex. In this, it is seen where the trajectory begins and ends for cyclic case. All work is proven analytically. Let $S^{m-1} = \{x \in R^m, x_i \geq 0, \sum\limits_{i=1}^{n} x_i = 1\}$. For $x \in R^m$, we look following equation:

$$x'_k = x_k \cdot (1 + \sum_{i=1}^{n} a_{ki} x_i),\ k = \overline{1, n}$$

where $a_{ki} = -a_{ik}$. This operator called Volterra operator. For any skew-symmetric matrix $P = \{x \in S^{m-1} : Ax \geq 0\} \neq \emptyset$ and $Q = \{x \in S^{m-1} : Ax \leq 0\} \neq \emptyset$. For any $x^0 \in S^{m-1}$, let's put $\omega(x^0) = \{x^0, x^{(1)}, \cdots\}'$ - the set of limit points of the positive trajectory and - $\alpha(x^0) = \{x^0, x^{(-1)}, \cdots\}'$ - the set of limit points of the negative trajectory.

Definition. A graph with vertices $1, 2, \cdots, m$ called partially - oriented if each edge of the graph is provided with a direction.

It is known, orientated graph to be cyclic, there must be a direction from each edge move to other edge . Then the equation of cyclic orientated graph at S^3 expressed below.

$$x'_1 = x_1 \cdot (1 - ax_2 + bx_3 + cx_4)$$
$$x'_2 = x_2 \cdot (1 + ax_1 - dx_3 + ex_4)$$
$$x'_3 = x_3 \cdot (1 - bx_1 + dx_2 - fx_4)$$
$$x'_4 = x_4 \cdot (1 - cx_1 - ex_2 + fx_3)$$

where a, b, c, d, e, f positive numbers.

If some of these coefficients are zero, it gives a partially orientated graph. For each case, the beginning of the trajectory is shown to be a set of P, while the end is a set of Q.

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Equivalence of paths with respect to the action of the pseudounitary group

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Let $V = C^n$ be an n-dimensional vector space over the field of complex numbers C. Elements from V are provided as n-dimensional column vectors. Linear non-degenerate transformations on the space C^n form a group $GL(n, C)$, which is identified with the group of complex $n \times n$ matrices with determinant not equal to zero. Let $U(p, q) = \left\{g \in GL(n, C) : \bar{g}^T I g = I\right\}$ be a pseudounitary subgroup of the group $GL(n, C)$, where $\bar{g}^T$ is a matrix whose elements are complex conjugate and transposed to the corresponding elements of the matrix $g = (g_{ij})_{i,j=1}^n$, i.e. $\bar{g}^T = (\bar{g}_{ij})_{i,j=1}^n$ for $g \in GL(n, C)$, $\quad I = I_{pq} = \operatorname{diog}(1_p 1_q), 1_k - k \times k$ identity matrix. The action of the subgroup $G \subset GL(n, C)$ in C^n is defined as the usual multiplication of the matrix g by the column vector $x = \{x_1, x_2, \ldots, x_n\}$; in addition, we define the operation of complex conjugation in C^n by the equality $\bar{x} = \{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n\}$. Let $G = U(p, q)$. A complex differential rational function $f < x, \bar{x} >$ is said to be G invariant if $f(gx, \overline{gx}) = f(x, \bar{x})$ for any $g \in G$. Consider the set of all paths in C^n, i.e. set of vector functions $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$, where $x_i(t), (i = 1, 2, \ldots, n)$ are infinitely differentiable complex-valued functions on the interval $(0, 1)$. The r th order derivative of the path $x(t)$ is the vector function $x^{(r)}(t) = \left(x_1^{(r)}(t), x_2^{(r)}(t), \ldots, x_n^{(r)}(t)\right)$ Two paths $x(t)$ and $y(t)$ are said to be G-equivalent if there exists an element $g \in G$ such that $gx(t) = y(t)$ for any $t \in (0, 1)$. A function f of a path $x(t)$ and a finite number of its derivatives is called G-invariant if the values for f are the same for G-equivalent paths. For each path $x(t)$, consider the $n \times n$ matrix

$$M(x(t)) = M(x) = \begin{pmatrix} x_1(t) & x_1^{(1)}(t) \cdots x_1^{(n-1)}(t) \\ x_2(t) & x_2^{(1)}(t) \cdots x_2^{(n-1)}(t) \\ \cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots \\ x_n(t) & x_n^{(1)}(t) \cdots x_n^{(n-1)}(t) \end{pmatrix}.$$

Theorem. Two paths $x(t)$ and $y(t)$ are G-equivalent if and only if the following equalities hold:

$$M^{-1}(x(t))M^{(1)}(x(t)) = M^{-1}(y(t))M^{(1)}(y(t));$$
$$\bar{M}^T(x(t))IM(x(t)) = \bar{M}^T(y(t))IM(y(t));$$

For all $t \in (0, 1)$.

The antiferromagnetic phases of the SOS model with one-level competing interactions on the binary tree

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The Cayley tree Γ^k of order $k \geq 1$ is an infinite tree, i.e., a cycles-free graph such that from each vertex of which issues exactly $k+1$ edges (see [1],[2]). We denote by V the set of the vertices of tree and by L the set of edges of tree. Two vertices x and y, where $x, y \in V$ are called nearest-neighbor if there exists an edge $l \in L$ connecting them, which is denoted by $l = \langle x, y \rangle$. The distance on this tree, denoted by $d(x, y)$, is defined as the number of nearest-neighbour pairs of the minimal path between the vertices x and y. Put $\mid x \mid = d(x, x^0),\ x \in V$. Two vertices $x, y \in V$ are called second nearest-neighbor if $d(x, y) = 2$. The second nearest-neighbor vertices $x, y \in V$ such that $\mid x \mid = \mid y \mid$ are called one-level next-nearest-neighbor and are denoted by $> \overline{x, y} <$. In the SOS model, the spin variables $\sigma(x)$ take their values on the set $\Phi = \{0, 1, 2\}$ which are associated with each vertex of the tree Γ^k. The SOS model with nearest-neighbour and one-level next-nearest-neighbor interactions is defined by the following Hamiltonian:

$$H(\sigma) = -J \sum_{\langle x,y \rangle} \mid \sigma(x) - \sigma(y) \mid - J_1 \sum_{>\overline{x,y}<} \mid \sigma(x) - \sigma(y) \mid . \tag{1}$$

where $J,\ J_1 \in \mathbb{R}$.

Studying the antiferromagnetic phase of the model is reduced to investigate the zeroes of the equation given by

$$\frac{f(f(u)) - u}{f(u) - u} = 0, \tag{2}$$

where

$$f(u) = f(u, \theta, \theta_1) := \frac{u^2 + 4\theta\,\theta_1\,u + 2\theta^2(\theta_1^2 + 1)}{\theta^2\,u^2 + 2\theta\,\theta_1(\theta^2 + 1)u + \theta^4 + 2\,\theta^2\theta_1^2 + 1}.$$

Denote

$$B := B(\theta; \theta_1) = 2\theta^7\theta_1 + 4\theta^5\theta_1^3 + 2\theta^5\theta_1 + 6\theta^4\theta_1^2 + 4\theta^3\theta_1^3 - \theta^4 + 2\theta^3\theta_1 + 10\theta^2\theta_1^2 + 6\theta\theta_1 + 1,$$

Theorem 1. If $(\theta, \theta_1) \in \{(\theta, \theta_1) \in \mathbb{R}_+^2 : B \geq 0\}$ then for the SOS model with one-level next-nearest-neighbour interactions on the binary tree ($k = 2$) there is no antiferromagnetic phases.

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Ground states of potts-sos model with an external field

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The Cayley tree $\Gamma^k = (V, L)$ of order $k \geq 1$ is an infinite tree [1].

Potts-SOS model with an external field α is defined by the following Hamiltonian:

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| + J_p \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)} + \alpha \sum_{x \in V} \sigma(x), \tag{1}$$

where $J, J_p, \alpha \in \mathbb{R}, \sigma \in \{0, 1, 2\}^V$.

The energy of configuration σ_b on b is defined by the formula

$$U(\sigma_b) = \frac{1}{2} \sum_{x \in S_1(c_b)} \big(J|\sigma(x) - \sigma(c_b)| + J_p\delta_{\sigma(x)\sigma(c_b)}\big) + \alpha\sigma(c_b).$$

Lemma. For each configuration φ_b, we have the followings

$$U(\varphi_b) \in \{U^{(j)}_{i,n} : i = 0, 1, 2, ..., k + 1, \quad n = 0, ..., k + 1 - i, \quad j = 0, 1, 2\},$$

where

$$U^{(j)}_{i,n} = \frac{J}{2}(k + 1 + n - i) + \frac{J_p}{2} i + j\alpha.$$

Definition. A configuration φ is called a ground state for the Hamiltonian (1), if

$$U(\varphi_b) = \min\{U^{(j)}_{i,n} : i = 0, 1, 2, ..., k + 1, \quad n = 0, ..., k + 1 - i, \quad j = 0, 1, 2\}$$

for any $b \in M$.

We denote $A^{(\zeta)}_{\xi,\eta} = \{(J, J_p, \alpha) \in \mathbb{R}^3 : U^{(\zeta)}_{\xi,\eta} = \min\{U^{(j)}_{i,n} : i = 0, 1, 2, ..., k + 1, \quad n = 0, ..., k + 1 - i, \quad j = 0, 1, 2\}\}$.

Calculations show that:

$$A^{(0)}_{k+1,0} = \{(J, J_p, \alpha) \in \mathbb{R}^3 : J_p \leq J, J_p \leq 2J, \alpha \geq 0\},$$

$$A^{(1)}_{k+1,0} = \{(J, J_p, \alpha) \in \mathbb{R}^3 : J_p \leq J, J_p \leq 2J, \alpha = 0\},$$

$$A^{(2)}_{k+1,0} = \{(J, J_p, \alpha) \in \mathbb{R}^3 : J_p \leq J, J_p \leq 2J, \alpha \leq 0\}.$$

We let $GS(H)$ denote the set of all ground states of the Hamiltonian (1).

Theorem. If $(J, J_p, \alpha) \in A^{(j)}_{k+1,0}$, then $GS(H) = \{\sigma(x) = j, \forall x \in V\}$ for the model (1).

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On the properties of conditional full angle of a polyhedral angle

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In this article, a conditional full angle is defined for the considered class of polygons and it is shown that its value is bounded from above.

Let there be an arbitrary convex cone S one-valued projecting onto the plane XOY with the vertex at the coordinate origin and a vector $\overrightarrow{e}$. The direction of vector $\overrightarrow{e}$ in the direction of the OX axis, and hence denote this vector by $\overrightarrow{e}$ (OX) hereafter. Assume that S_n angles with n hedral, the edges consisting of $a_1, a_2, ..., a_n$ rays (not perpendicular to the given vector $\overrightarrow{e}$ (OX)) are inscribed in cone S. Let the first m edges of S_n lie on the side $x < 0$. Continuing these edges to side $z > 0$, we denote their intersection points with plane $x = 1$ by $A_1, A_2, ..., A_m$ accordingly. The plane $x = 1$ is intersects the next $n - m$ edges, and we mark these points by $A_{m+1}, A_{m+2}, ..., A_n$ accordingly. Connecting points A_1 and A_n, points A_m and A_{m+1}, we make segments A_1A_n and A_mA_{m+1}. We denote the point of intersection of these sections by O'. By connecting points $A_1, A_2, ..., A_m, A_{m+1}, A_{m+2}, ..., A_n$ in a row, we make polygon $A_1A_2...A_mA_{m+1}A_{m+2}...A_n$. The defect of the sides of this polygon is found by the following formula and this defect is called a conditional full angle of S_n relative to direction $\overrightarrow{e}$ (OX) [1]:

$$\omega_{S_n} = A_1A_n + A_mA_{m+1} - P_{A_1A_2...A_m} - P_{A_{m+1}A_{m+2}...A_n},$$

where $P_{A_1A_2...A_m}$ and $P_{A_{m+1}A_{m+2}...A_n}$ are the perimeters of brokens $A_1A_2...A_m$ and $A_{m+1}A_{m+2}...A_n$, respectively. The problem of restoring the polyhedra of a given class by conditional full angle was solved in [2].

We denote the rays formed by the intersection of the cone S with the plane $x = 0$ by l'_1 and l'_2. We move edge a_1 of S_n along the surface of cone S towards l'_2, and edge a_{m+1} towards l'_1. When edge a_1 overlaps with l'_2 and edge a_{m+1} overlaps with l'_1, these edges of S_n are perpendicular to the vector $\overrightarrow{e}$ (OX), and we denote such a polyhedral angle by $\overline{S_n}$. We evaluate the conditional full angle of $\overline{S_n}$:

$$0 < \omega_{\overline{S_n}} \leq 2\left(O'A_{k_1} + O'A_{k_2}\right),$$

where points A_{k_1} and A_{k_2} are the closest points of the brokens $A_2...A_m$ and $A_{m+2}...A_n$ to point O', accordingly.

Theorem. *Conditional full angle of a polyhedral angle with edges perpendicular to the vector $\overrightarrow{e}$ (OX), positively defined and bounded from above.*

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Properties of surfaces with vertices

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Let us consider a surface F - in a three-dimensional Euclidean space, defined and continuous in some domain D.

Let the points $A_i, i = 1...k$ be points in the domain D and L be a space curve uniquely projecting onto the boundary ∂D of the domain D. Denote by $W(L, A_i, D)$ the set of convex surfaces, with boundary L projecting uniquely into the domain D.

It is known [1] that the points of convex surfaces are divided into three types, depending on its tangent cone. A point on a surface is regular if the tangent cone is a plane. Edge - when the tangent cone is a dihedral angle. Conic (vertex) – when the tangent cone is real.

Definition 1. If a surface does not have edge points, but has a finite number of vertices, it is called a surface with vertices.

The class of surfaces with vertices is rather rich.

Assume that convex surfaces F_1 and F_2 from the class $W(L, A_i, D)$ has vertices A_i^1, A_i^2 projecting into points A_i. Denote by h_i^1, h_i^2 the segments $A_i^1 A_i, A_i^2 A_i$ that is $h_i^1 = A_i^1 A_i, h_i^2 = A_i^2 A_i$.

Theorem 1. If $h_i^1 \geq h_i^2$ then $\omega(F_1) \geq \omega(F_2)$ for the external curvature.

Theorem 2. If the numbers $\omega_1, i = 1...k$ and $\sum\limits_{i=1}^{k} \omega_i < 2\pi$ are given, then there exists a surface $F \in W(L, A_i, D)$ with vertices $\overline{A_i}$ projecting into points A_i and external curvature $\omega(A_i) = \omega_i$.

The properties of monotonicity of the external curvature of the surface are used for the proof of the theorem.

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Local derivation and automorphism on filiform Zinbiel algebras

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Investigation of local derivations on Lie algebras was initiated in paper in [2]. Sh.A.Ayupov and K.K.Kudaybergenov have proved that every local derivation on semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. In this paper, we will study local derivations on filiform Zinbiel algebras.

A derivation on a Lie algebra g is a linear map $D : g \to g$ satisfying

$$D[x,y] = [D(x),y] + [x,D(y)]$$

for all $x, y \in g$.

A linear operator Δ is called a local derivation if for any $x \in \mathcal{L}$, there exists a derivation $D_x : \mathcal{L} \to \mathcal{L}$ (depending on x) such that $\Delta(x) = D_x(x)$.

A linear bijective map $\varphi : \mathcal{L} \to \mathcal{L}$ is called an automorphism, if it satisfies $\varphi([x,y]) = [\varphi(x), \varphi(y)]$ for all $x, y \in \mathcal{L}$.

Let $\mathcal{L}$ be an algebra. A linear map $\nabla : \mathcal{L} \to \mathcal{L}$ is called a local automorphism, if for any element $x \in \mathcal{L}$ there exists an automorphism $\varphi_x : \mathcal{L} \to \mathcal{L}$ such that $\nabla(x) = \varphi_x(x)$.

Definition 1. An algebra A over a field F is called a Zinbiel algebra if for any $x, y, z \in A$ the identity

$$(x \circ y) \circ z = x \circ (y \circ z) + x \circ (z \circ y)$$

holds.

For an arbitrary Zinbiel algebras define the lower central series

$$A^1 = A, \quad A^{k+1} = A \circ A^k, \quad k \geq 1.$$

Definition 2. An n-dimensional Zinbiel algebra A is called filiform if $dimA^i = n - i$, $2 \leq i \leq n$.

The following theorem gives classification of filiform Zinbiel algebras.

Theorem 1.[1] Any n-dimensional ($n \geq 5$) filiform Zinbiel algebra is isomorphic to one of the following three pairwise non isomorphic algebras:

$$\begin{array}{l}
F_n^1 : e_i \circ e_j = C_{i+j-1}^j e_{i+j}, \quad 2 \leq i+j \leq n-1; \\
F_n^2 : e_i \circ e_j = C_{i+j-1}^j e_{i+j}, \quad 2 \leq i+j \leq n-1, \quad e_n \circ e_1 = e_{n-1}; \\
F_n^3 : e_i \circ e_j = C_{i+j-1}^j e_{i+j}, \quad 2 \leq i+j \leq n-1, \quad e_n \circ e_n = e_{n-1}.
\end{array}$$

Now we give the main theorem concerning local derivations on filiform Zinbiel algebras.

Theorem 2. The algebras filiform Zinbiel algebras admit local derivations (automorphisms) which are not derivations(automorphisms).

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On some spaces of the space of complete linked systems that are manifolds of infinite dimension

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In this paper, topological and geometric properties of the set of complete linked systems are considered.

A linked system ξ of closed subsets of a space is called maximal if it has the following property:

"if the closed set $A \subset X$ intersects every element of ξ", then $A \in \xi$ [1]. (*) A linked system ξ of closed subsets is said to be complete [2] if the following condition is true for any closed set $F \subset X$:

"any neighborhood OF of F contains $\Phi \in \xi$" implies $\Phi \in \xi$ [2].(**)

A system of closed subsets $\mu = \{F_\alpha : F_\alpha \subset X;\, \alpha \in A\}$ of the space X is called a $k-$linked ($k \geq 2$) if the intersection of any $k-$elements of the μ system is non-empty. those. $\forall \alpha_1, \alpha_2, ..., \alpha_k$, $F_{\alpha_i} \in \mu$,$\alpha_i \in A \Rightarrow \bigcap_{i=1}^{k} F_{\alpha_i} \neq \emptyset$.

Denote by $N_k(X)$ the set of all complete $k-$linked systems (for short, $n_k cc$) of the space X. Therefore, for any natural number $n \in N$ one can define a subspace $N^n(X)$ of the space $N(X)$ consisting of all ncc whose support consists of at most points. those. $N^n(X) = \{\xi \in N(X) : |supp\xi| \leq n\}$.

For a compact X, by $\lambda_\omega(X)$, $N_\omega(X)$, and $N^\omega(X)$ we denote, respectively, the subsets of $\bigcup_{n=1}^{\infty} \lambda_n(X)$, $\bigcup_{K=1}^{\infty} N_K(X)$ and $\bigcup_{n=1}^{\infty} N^n(X)$.

A set $B(Q)$ is called a boundary set in Q if Q$\setminus$B(Q)$\approx \ell_2$ [3], a topological space X is called a manifold modeled on the space Y or a Y manifold, if every point in space X has a neighborhood homeomorphic to an open subset of space Y.

The following results are obtained:

Theorem 1. For any metrizable nondegenerate continuum X, the following holds:

a) $\lambda_\omega(X)$ is the boundary set of λ(X);

b) $N_\omega(X)$ is the boundary set of $N(X)$;

c) $N_\omega(X)$ is the boundary set of K$\lambda(X)$.

Theorem 2. For any metrizable non-degenerate continuum X we have:

a) $N(X)\setminus N_n(X)$ is a Q manifold, for any n≥ 2;

b) $\lambda(X)\setminus\lambda_n(X)$ is a Q manifold, for any $n \geq 2$;

c) $N(X)\setminus N^n(X)$ is a Q manifold, for any $n \geq 2$.

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V. DIFFERENTIAL EQUATIONS AND EQUATIONS OF MATHEMATICAL PHYSICS. INVERSE AND ILL-POSED PROBLEMS

Non-local and inverse problems for the Rayleigh-Stokes equation

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Our report is based on joint work [1-3] with our colleagues: S.R. Umarov (New Haven University, USA), A. Mukhiddinova (Tashkent University of Information Technologies), and N. Vaisova (Institute of Mathematics, Uzbekistan Academy of Science,Urganch,).

In this talk we consider the following Rayleigh-Stokes problem for a generalized second-grade fluid with a time-fractional derivative model:

$$\partial_t u(x,t) - (1+\gamma\,\partial_t^\alpha)\Delta u(x,t) = f(x,t), \quad x\in\Omega, \quad 0<t\le T;$$
$$u(x,t)=0, \quad x\in\partial\Omega, \quad 0<t\le T;$$
$$u(x,0)=\varphi(x), \quad x\in\Omega,$$

where $1/\gamma>0$ is the fluid density, a fixed constant, φ and f are given functions, $\partial_t=\partial/\partial t$, and ∂_t^α is the Riemann-Liouville fractional derivative of the order $\alpha\in(0,1)$ defined by:

$$\partial_t^\alpha h(t) = \frac{d}{dt}\int_0^t \omega_{1-\alpha}(t-s)h(s)ds, \quad \omega_\alpha(t)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}. \tag{1}$$

Here $\Gamma(\sigma)$ is Euler's gamma function. Based on physical considerations, usually this problem is considered in the domain $\Omega\subset R^N$, $N=1,2,3$, with a sufficiently smooth boundary $\partial\Omega$.

The main results of this talk is the following:

1) We will solve the Rayleigh-Stokes problem by the Fourier method. A formal formula for the solution in the form of a Fourier series was given in the papers of previous authors, but the convergence of these series was not investigated.

2) We will pay special attention to the backward, since in previous papers the authors considered only the case $N\le 3$. And this is connected with the method used in these works: if the dimension of the space is less than four, then for the eigenvalues λ_k of the Laplace operator with the Direchlet condition, the series $\sum_k \lambda_k^{-2}$ converges.

3) In this talk we consider the Rayleigh-Stokes problem with a non-local time condition:

$$u(x,T)=\beta u(x,0)+\varphi(x),$$

where β is an arbitrary real number. Note, if $\beta=0$ then we have the backward problem.

If $\beta=1$, then we get the following condition

$$u(x,T)=u(x,0)+\varphi(x).$$

It turns out that this non-local problem is well-posed. In other words, a solution to the non-local problem exists and is unique. Moreover, the solution depends continuously on the function $\varphi(x)$ in the non-local condition.

4) The question naturally arises: starting from what value of β does the problem worsen? We will give a comprehensive answer to this question. It turns out that the critical values of the parameter β lie on the half-interval $[0, 1)$. If $\beta \notin [0, 1)$, then the problem is well-posed according to Hadamard: there is a unique solution and it depends continuously on the data of the problem; if $\beta \in (0, 1)$ (the case $\beta = 0$ is the baclward problem), then the well-posedness of the problem depends on the location of the eigenvalues of the Laplace operator. For this case, necessary and sufficient conditions are found that guarantee the existence of a solution, but the solution will not be unique.

5) The Rayleigh-Stokes problem involves the fractional derivative ∂_t^α, which describes the behavior of a viscoelastic flow. However, this parameter is often unknown and difficult to measure directly. Therefore, it is undoubtedly interesting to study the inverse problem to determine this physical quantity. We prove that the additional condition $||u(x, t_0)||^2_{L_2(\Omega)} = d_0$ for sufficiently large t_0 uniquely determines the parameter α.

6) Another very important problem studied in this work is to find out the dependence of the behavior of the solution of the initial-boundary value problem on the order of the fractional derivative. In this work, an interesting fact was discovered: if we consider the norm of the solution $||u(x, t_0)||_{L_2(\Omega)}$ as a function of the parameter α, then this is a decreasing function. In other words, the norm acquires its maximum value when the order of the fractional derivative is close to zero, and its minimum value - when this parameter is close to one.

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On an inverse problem for a parabolic equation in a domain with moving boundaries

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This paper considers the inverse problem of determining the unknown coefficient on the right-hand side of a parabolic equation in a domain with moving boundaries. An additional condition for finding the unknown coefficient, which depends on the variable time, is given in integral form. A theorem on uniqueness and "conditional" stability of the solution is proved.

Let $\gamma_1(t), \gamma_2(t)$ $t \in [0,T]$, $0 < T = const$ be the given functions, (x,t) be an arbitrary point in the bounded domain $D = (\gamma_1(t), \gamma_2(t)) \times (0,T]$, $[a,b]$ be the projection of the domain D into the axis OX.

We consider the following inverse problem on determining a pair of functions $\{f(t), u(x,t)\}$:

$$u_t - u_{xx} = f(t)g(x) \quad (x,t) \in D, \tag{1}$$

$$u(x,0) = \varphi(x) \quad x \in [\gamma_1(0), \gamma_2(0)], \tag{2}$$

$$u(\gamma_1(t),t) = \psi_1(t), \quad u(\gamma_2(t),t) = \psi_2(t), \ t \in [0,T], \tag{3}$$

$$\int\limits_{\gamma_1(t)}^{\gamma_2(t)} u(x,t)dx = h(t), \quad t \in [0,T], \tag{4}$$

where $g(x), \varphi(x), \psi_1(t), \psi_2(t), h(t)$ are the given functions, $u_t = \frac{\partial u}{\partial t}$, $u_x = \frac{\partial u}{\partial x}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$.

We make the following assumptions for the data of problem (1)-(4):

1^0. $g(x) \in C^{\alpha}[a,b]$, $\int\limits_{\gamma_1(t)}^{\gamma_2(t)} |g(x)|\, dx \ge c_1 > 0$;

2^0. $\varphi(x) \in C^{2+\alpha}[\gamma_1(0), \gamma_2(0)]$;

3^0. $\psi_1(t), \psi_2(t) \in C^{1+\alpha}[0,T]$, $\varphi(\gamma_1(0)) = \psi_1(0)$, $\varphi(\gamma_2(0)) = \psi_2(0)$;

4^0. $h(t) \in C^{1+\alpha}[0,T]$;

5^0. $\gamma_1(t), \gamma_2(t) \in C^{1+\alpha}[0,T]$, $0 < c_2 \le \gamma_2(t) - \gamma_1(t) \le c_3 < +\infty$, $t \in [0,T]$

where c_1, c_2 and c_3 are positive constants, $\alpha \in (0,1)$.

The pair of functions $\{f(t), u(x,t)\}$ is called the solution of problem (1)-(4) if

1) $f(t) \in C^{\alpha}[0,T]$;

2) $u(x,t) \in C^{2+\alpha, 1+\alpha/2}(D) \bigcap C(\bar{D})$;

3) The conditions (1)-(4) hold for these functions.

A problem of Bitsadze-Samarsky type for a degenerate parabolic equation of mixed type

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This paper is devoted to the formulation and study of a nonlocal boundary value problem of Bitsadze-Samarskii type for the equation

$$y^m u_{xx} - sgnx|x|^n u_y = 0 \tag{1}$$

in an unbounded region D, bounded by the lines $x = -h, y = Y$ at $x < 0, y > 0$ and $y = 0, y = Y$ at $x > 0, y > 0$, where $h = (2q)^{\frac{1}{q}}, 2q = n+2, n = const > -1, Y = const > 0$.

Let us introduce the notation:

$$D^\infty = \{(x,y) : 0 < x < \infty, 0 < y \le Y\}, D_0 = \{(x,y) : -h < x < 0, 0 \le y < Y\},$$
$$I^\infty = \{(x,y) : 0 \le x < \infty, y = 0\}, I_0 = \{(x,y) : -h \le x \le 0, y = Y\}$$
$$J = \{(x,y) : x = 0, 0 \le y \le Y\}, J_0 = \{(x,y) : x = -h, 0 \le y \le Y\}$$

It should be noted that equation (1) in the region D^∞ is a direct parabolic equation, and in the region D_0 it is inversely parabolic [1].

Problem. It is required to find a function $u(x,y)$ with the following properties:

1) $u(x,y) \in C^1(D) \cap C^{2,1}(D^\infty \cup D_0)$ and is continuous up to the boundary of the region D;

2) $u(x,y)$ satisfies equation (1) in the regions D^∞, D_0 and is limited for all $0 \le x < \infty, 0 \le y \le Y$;

3) $u(x,y)$ satisfies the boundary conditions:

$$u(x,y)|_{y=0} = \varphi(x), x \in I^\infty,$$
$$u(x,y)|_{y=0} = \psi(x), y \in I_0,$$
$$a_0(y)u(0,y) + \sum_{k=1}^{n} a_k(y)u(x_k,y) + \mu(y) = u(-h,y), y \in J_0, x_k \in I_0$$

where $\varphi(x), \psi(x), a_0(y), a_k(y), \mu(y)$ are given function, $\varphi(x)$ is limited in $[0,\infty), \psi(x) \in C[-h,0] \cap C^2(-h,0), a_0, a_k(y), \mu(y) \in C[0,Y] \cap C^2(0,Y)$.

Theorem. The solution of the problem exists and is unique, if the condition $|\sum_{k=0}^{n} a_k(y)| \le 1$ is satisfied.

The uniqueness of the solution follows from the following extremum principle: the solution $u(x,y)$ of the problem of its positive maximum and negative minimum in the domain $\overline{D}$ can reach only at the boundary $I^\infty \cup I_0$.

The existence of a solution is equivalently reduced to the solvability of a singular integral equation [2]. The obtained singular integral equation can be reduced to the Fredholm integral equation of the 2nd kind, the solvability of which follows from the uniqueness of the solution of the formulated problem.

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The Cauchy problem for the modified Korteweg-de Vries equation in the class of periodic functions

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In this thesis, the method of the inverse spectral problem is used to integrate the nonlinear modified Korteweg-de Vries equation in the class of periodic functions.

Statement of the problem. Consider the Cauchy problem for the modified Korteweg-de Vries equation of the following form

$$\begin{cases} q_t = -q_{xxx} + 6q^2 q_x, \\ q(x,t)|_{t=0} = q_0(x), q_0(x+\pi) = q_0(x) \end{cases} \qquad (1)$$

in the class of real infinite-gap π periodic with respect to x functions:

$$\begin{cases} q(x+\pi,t) = q(x,t),\ x \in \mathbb{R},\ t > 0 \\ q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0) \end{cases} \qquad .(2)$$

In this paper, we propose an algorithm for constructing exact solution $q(x,t), x \in \mathbb{R}, t > 0$ of problem (1)-(2) by reducing it to an inverse spectral problem for the following Dirac operator:

$$\mathfrak{L}(\tau,t) \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y, x \in \mathbb{R}, \tau \in R, t > 0, \qquad (3)$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \Omega(x+\tau,t) = \begin{pmatrix} 0 & q(x+\tau,t) \\ q(x+\tau,t) & 0 \end{pmatrix},\ y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}.$$

In this thesis proved the following theorem:

Theorem. If initial function $q_0(x)$ satisfies the following condition:

$$q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R})$$

then the problem (1)-(3) has unique solution, which defined sum of a uniformly convergent functional series constructed by solving the system of Dubrovin equations and the first trace formula satisfies the mKdV equations and lie the class of $C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0)$.

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On the solution of a boundary value problem for a third-order equation with multiple characteristics in three-dimensional space in a semi-bounded domain

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In the domain $D^- = \{(x,y,z) : -\infty < x < 0,\, 0 < y < q,\, 0 < z < r\}$, consider the equation

$$L[u] \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0, \tag{1}$$

where $q > 0$, $r > 0$ - constant real numbers, and for it we study the following problem.

Problem B**.** Find a solution to equation (1) in the domain D^- from class $C^{3,2,2}_{x,y,z}(D^-) \cap C^{2,1,1}_{x,y,z}(D^- \cup \Gamma)$, having bounded first derivatives with respect to x, y and z, a second derivative with respect to x at $x \to -\infty$, and $u_y, u_z \in L_2(D^-)$, satisfying the boundary conditions:

$$\begin{cases} \alpha u(x,0,z) + \beta u_y(x,0,z) = 0, & \\ \gamma u(x,q,z) + \delta u_y(x,q,z) = 0, & -\infty < x < 0, \\ u(x,y,0) = u(x,y,r) = 0, & \end{cases} \tag{2}$$

$$u(0,y,z) = \psi_1(y,z), u_x(0,y,z) = \psi_2(y,z), \lim_{x\to-\infty} u(x,y,z) = 0, 0 \le y \le q,\, 0 \le z \le r, \tag{3}$$

where $\Gamma = \partial D^-$ is the boundary of the domain D^-, α, β, γ and δ are given constants, and $\alpha^2 + \beta^2 \neq 0$, $\gamma^2 + \delta^2 \neq 0$, and $\psi_i(y,z), i = 1,2$ are given sufficiently smooth functions, and

$$\begin{cases} \alpha \frac{\partial^j \psi_i(0,z)}{\partial y^j} + \beta \frac{\partial^{j+1}\psi_i(0,z)}{\partial y^{j+1}} = 0, \quad \gamma \frac{\partial^j \psi_i(q,z)}{\partial y^j} + \delta \frac{\partial^{j+1}\psi_i(q,z)}{\partial y^{j+1}} = 0, \quad j = 0,2, \\ \frac{\partial^4 \psi_i(y,0)}{\partial y^4} = \frac{\partial^4 \psi_i(y,r)}{\partial y^4} = 0, \qquad \frac{\partial^6 \psi_i(y,0)}{\partial y^4 \partial z^2} = \frac{\partial^6 \psi_i(y,r)}{\partial y^4 \partial z^2} = 0, \quad i = 2,3. \end{cases} \tag{4}$$

We note that in the works [1, 2] boundary value problems in three-dimensional space were studied in finite domains.
The following theorems have been proved:

Theorem 1. If there exists a solution of the problem B, then the solution is unique when $\alpha\beta \le 0$ and $\gamma\delta \ge 0$.

Theorem 2. If $\dfrac{\partial^7 \psi_i(y,z)}{\partial y^4 \partial z^3} \in L_2[0 < y < q, 0 < z < r], i = 1,2$ and conditions (4) are satisfied, then the solution to problem B exists.

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The first boundary problem for a third order equation with variable coefficients

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In the region $D = \{(x, y): \quad 0 < x < p,\ 0 < y < q\}$, consider the following third-order equation in the form

$$u_{xxx} - u_{yy} + a_1(x)\,u_x + a_2(x)\,u = g(x, y), \tag{1}$$

where $a_i(x)$, $i = \overline{1,2}$, $g(x, y)$ are given sufficiently smooth functions.

Problem B. Find a function $u(x, y)$ from class $C^{3,2}_{x,y}(D) \cap C^{2,1}_{x,y}(\overline{D})$ that satisfies equation (1) and the following boundary conditions:

$$u(x, 0) = 0, \quad u(x, q) = 0, \quad 0 \le x \le p,$$

$$u_{xx}(0, y) = \psi_1(y), \quad u(p, y) = \psi_2(y), \quad u_x(p, y) = \psi_3(y), \quad 0 \le y \le q,$$

where $\psi_i(y)$, $i = \overline{1,3}$ are given functions.

In works [1,2], the solution of the stated problem for a third-order equation with constant coefficients was found with other boundary conditions.
The following theorems have been proved:

Theorem 1. If problem B has a solution, then under conditions $a_1(x) \le 0$, $a_2(x) - \frac{1}{2}a_1{}'(x) \ge 0$, it is.

Theorem 2. If the following conditions are met:
1)$\psi_i(y) \in C^3[0, q]$, $i = \overline{1,3}$; $\psi_i(0) = \psi_i(q) = \psi_i{}''(0) = \psi_i{}''(q) = 0$, $i = \overline{1,3}$,
2) $\frac{\partial^3 g(x,y)}{\partial x \partial y^2} \in C\left[\bar{D}\right]$, $\quad g(x, 0) = g(x, q) = 0$;
3) $C < \frac{\lambda_1^2}{Kp(1+\lambda_1)}$,
4) $a_1(p) = 0$,
then the solution to the problem exists.
Here $C = \max\{|a_1(x)|, |a_1{}'(x) - a_2(x)|, x \in [0, p]\}$, $\lambda_1 = \sqrt[3]{\left(\frac{\pi}{q}\right)^2}$, $K = \frac{16}{3}\left(1 - \exp\left(-\frac{2\sqrt{3}\pi}{3}\right)\right)^{-1}$.

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Stability of the time dependent source identification problem for telegraph equations

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In the present study, the time dependent source identification problem for the telegraph equation is investigated. Stability estimates for the solution of the source identification problem are established. A first order of accuracy difference scheme for the numerical solution of this problem is presented. Stability estimates for the solution of this difference scheme are established.

SIPs have the significant role in natural science, applied sciences, engineering, quantum mechanics, diffusion equations, heat equations (see, e.g., [1]-[4]).

Various local and nonlocal boundary value problems for hyperbolic, telegraph differential and difference equations and their applications have been a major research area in many branches of science and engineering particularly in applied mathematics.

The theory and applications of space-dependent SIPs for partial differential equations were studied in various investigations.

However, source identification problems for telegraph equations have not been investigated. Therefore, the main aim of this paper is to investigate the time-dependent identification problem for telegraph equations. It is well-known that several identification problems for the telegraph equations can be reduced to the time-dependent source identification problem

$$\begin{cases} \frac{d^2u(t)}{dt^2} + \alpha \frac{du(t)}{dt} + Au(t) = p(t)q + f(t), \ t \in (0,T), \\ u(0) = \varphi, u'(0) = \omega, G[u(t)] = \psi(t), t \in [0,T] \end{cases}$$

in a Hilbert space H with the with self-adjoint positive definite operator $A \geq \delta I, \delta > 0$ with dense domain $D(A)$ in H. Here $G : H \to \mathbb{R}$ is a given linear bounded functional and $\psi(t) : [0,T] \to \mathbb{R}$ is a given smooth function, $f(t)$ is a given smooth abstract function defined on $[0,T]$ with values in H and $q \in D(A), Gq \neq 0, \alpha \geq 0$.

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Coefficient inverse problems for the heat equation in a two-dimensional anisotropic medium

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Analysis of the current state of the study of the phenomena of convective heat transfer in anisotropic porous media.

The intensification of convective heat transfer in technical and technological systems is not yet fully resolved problem [1-3], on which the reduction of the overall mass characteristics of heat transfer devices and an increase in their thermal performance depend. Classical methods of intensification of convective heat transfer are as follows [4]: reduction of thermal resistance; increase in the velocity of heat carriers; application of developed surfaces; turbulence of the flow. At the same time, additional energy costs necessarily arise to increase the momentum of the coolant. That is, the comparison of the heat transfer intensity is correct only with the same power consumption for the movement of heat carriers [5].

The integral thermal resistance is determined by the thickness of the boundary layer, which must either be reduced, or even better destroyed [6]. Therefore, all the listed methods of heat transfer intensification are essentially aimed at solving this problem, but their capabilities have almost been exhausted.

In this regard, the trend of using porous materials with a significantly developed surface per unit volume for heat transfer identification has been formed. Theoretical and experimental study of heat exchange elements with porous accumulators has confirmed their effectiveness. The presence of natural porous media with the absence of geometric symmetry initiated the study of the effect of anisotropy on physical properties, the main of which are permeability and thermal conductivity.

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Solution of a linear l-catch differential game with non-inertial objects under geometric constraint

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The present paper is devoted to the linear L-catch differential game with two objects, called pursuer and evader, whose controls adhere to geometric constraints (in short, the G-constraints). In this work, the pursuit problem is discussed and a pursuer strategy guaranteeing the L-catch is defined using the method of Chikrii's resolving functions.

Consider the game including the objects X (Pursuer) and Y (Evader) in the finite-dimensional space $\mathbb{R}^n$. If x, y are their state vectors and u, v are their velocity vectors, then let their movements be described by the following linear equations:

$$X: \quad \dot{x} = kx + Bu, \quad x(0) = x_0, \quad Y: \quad \dot{y} = ky + Cv, \quad y(0) = y_0, \tag{1}$$

where $x, y, u, v \in \mathbb{R}^n$, $n \geq 1$; $k \neq 0$ and $k \in \mathbb{R}$; B and C are $n \times n$ real constant matrices; x_0 and y_0 are the initial positions of the objects for which it is presumed that $|x_0 - y_0| > L$, $L > 0$. The controls u and v are regarded as measurable functions $u(\cdot) : [0, +\infty) \to \mathbb{R}^n$ and $v(\cdot) : [0, +\infty) \to \mathbb{R}^n$ accordingly, and they are subject to the constraints

$$|u(t)| \leq \alpha \;\text{ for almost every } t \geq 0, \quad |v(t)| \leq \beta \text{ for almost every } t \geq 0. \tag{2}$$

which are usually termed the geometrical constraints (or the G-constraints), where α and β are non-negative numbers which designate the maximal speeds of X and Y.

To be more comfortable subsequently, the following notations will be used:

$$z = x - y, \; z_0 = x_0 - y_0.$$

Definition. *For $\alpha \geq \beta$, we call the function*

$$\boldsymbol{u}(\tau, v) = \Phi v + \lambda(\tau, v)\,(m(\tau, v) - \tau)$$

the approach strategy or Π_L–strategy for X in this differential game, where

$$\lambda(\tau, v) = \frac{1}{|\tau|^2 - l^2}\left[\langle v, \tau\rangle + \alpha l + \sqrt{(\langle v, \tau\rangle + \alpha l)^2 + (\tau^2 - l^2)(\alpha^2 - |v|^2)}\right],$$

$m(\tau, v) = -l\Big(\Phi v - \lambda(\tau, v)\tau\Big)/|\Phi v - \lambda(\tau, v)\tau|, \Phi = B^{-1}C,\ l = |LB^{-1}|, \tau = z_0 B^{-1}.$

Theorem. *Let $\alpha > \beta$ and $k < 0$. Then the Π_L-strategy guarantees to occur L-catch on the time interval $[0, T_L]$ in the differential game (1)–(2), where*

$$T_L = -\frac{1}{k}\ln\Big(1 - \frac{k(|\tau| - l)}{\alpha - \beta\|\Phi\|}\Big).$$

We say the number T_L a guaranteed time of L-catch and $\|\Phi\|$ is norm of the Φ matrix.

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Soliton and periodic wave solutions of the loaded nonlinear fractional evolution equations

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Recently, many research articles focused on the nonlinear fractional partial differential equations which are the generalization of the classical integer order partial differential equations. Fractional differential equations can be used to properly model various phenomena such as optics [1], the fluid traffic [2], economics [3], electrodynamics [4] and other applications in various branches of research.

We consider the loaded fractional Korteweg-de Vries equation and the loaded nonlinear modified fractional Korteweg-de Vries equation

$$D_t^{\alpha}u - 6puD_x^{\beta}u + D_x^{3\beta}u + \gamma_1(t)u(0,t)D_x^{\beta}u = 0,$$

$$D_t^{\alpha}u - 12pu^2D_x^{\beta}u + D_x^{3\beta}u + \gamma_2(t)u(0,t)D_x^{\beta}u = 0,$$

where $u(x,t)$ is an unknown function, $x \in R$, $t \geq 0$, p is any constant, $\gamma_1(t)$ and $\gamma_2(t)$ are the given real continuous functions, $0 < \alpha < 1$ and $0 < \beta < 1$ are the order of the fractional derivative. The fractional derivative is considered in the sense of Riemann-Liouville.

This paper presents exact solutions for the loaded fractional Korteweg-de Vries equation and the loaded fractional modified Korteweg-de Vries equation using the functional variable method. The main advantage of the proposed functional variable method over other methods is that it provides more accurate traveling wave solutions with additional free parameters. The exact solutions are great importance in revealing the internal mechanism of physical phenomena. This method presents a wider applicability for handling nonlinear fractional wave equations.

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On the integration of the periodic hunter-saxton equation with an integral type source

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The scale-invariant nondispersive wave equation

$$u_{xxt} + uu_{xxx} + 2u_x u_{xx} = 0 \tag{1}$$

was introduced by J.Hunter and R.Saxton [1] in connection with a model of nematic liquid crystals. J.Hunter and Y.Zheng [2,3] have given a very beautiful and thorough treatment of (), including the introduction of a Hamiltonian structure and proof of complete integrability.

In this paper, we study the Hunter-Saxton equation with an integral type source, namely, we consider the following equation

$$u_{xxt} = -uu_{xxx} - 2u_x u_{xx} + \int_0^\infty \beta(\lambda, t)\, s(\pi, \lambda, t) \left[q_x \psi_+ \psi_- + 2q(\psi_+ \psi_-)'_x \right] d\lambda \tag{2}$$

with the initial condition

$$u(x,0) = u_0(x), x \in R, \tag{3}$$

in the class of real-valued π-periodic on the spatial variable x function $u = u(x,t)$ which satisfy the regularity of assumption

$$u \in C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geq 0)$$

with the initial condition

$$u(x,0) = u_0(x), \ \ x \in R, \tag{4}$$

where $q = u_{xx}(x,t)$, $u_0(x) \in C^3(R)$ is the given real-valued π -periodic function and $\psi_\pm = \psi_\pm(x, \lambda, t)$ are the Floquet solution (normalized by the condition $\psi_\pm(0, \lambda, t) = 1$) of the weighted Sturm–Liouville equation

$$y'' = \lambda q(x,t) y, \quad x \in R. \tag{5}$$

The aim of this work is to provide a procedure for constructing the solution $u(x,t)$, $\psi_\pm(x, \lambda, t)$ of problem (2)-(4) using the inverse spectral theory for the periodic weighted Sturm–Liouville equation (5).

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Integration of the higher order toda lattice with a source

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The Toda lattice [1] is a simple model for a nonlinear one-dimensional crystal in solid state physics. It describes how the chain of particles with exponential interactions of the nearest neighbors move. The Toda lattice has several applications, for example, to the model of DNA in the feld of biology [2].

In this work, we consider the following system

$$\begin{cases} \dot{a}_n = a_n(G_{n+1,\,r+1} - G_{n,\,r+1}) + a_n \oint\limits_{|\mu|=1} \frac{1}{\mu}(f_{n+1}g_{n+1} - f_n g_n)d\mu, \\ \dot{b}_n = H_{n+1,\,r+1} - H_{n,\,r+1} + a_n \oint\limits_{|\mu|=1} \frac{1}{\mu}(f_n g_{n+1} + f_{n+1}g_n)d\mu - \\ -a_{n-1} \oint\limits_{|\mu|=1} \frac{1}{\mu}(f_n g_{n-1} + f_{n-1}g_n)d\mu, \\ a_{n-1}f_{n-1} + b_n f_n + a_n f_{n+1} = \frac{\mu+\mu^{-1}}{2} f_n, \\ a_{n-1}g_{n-1} + b_n g_n + a_n g_{n+1} = \frac{\mu+\mu^{-1}}{2} g_n, n \in Z, \end{cases} \tag{1}$$

under initial conditions

$$a_n(0) = a_n^0, \;\; b_n(0) = b_n^0, \quad n \in Z, \tag{2}$$

where

$$G_{n,j}(t) = \sum_{s=0}^{j} c_{j-s} < \delta_n,\, L(t)^s \delta_n >, 0 \le j \le r+1,\; r \in Z_+,$$

$$H_{n,j}(t) = \sum_{s=0}^{j} 2a_n(t) c_{j-s} < \delta_{n+1},\, L(t)^s \delta_n > + c_j + 1, 0 \le j \le r+1,$$

$$(L(t)y)_n \equiv a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, < \delta_m, \delta_n >= \begin{cases} 0, \;\; m \ne n \\ 1, \;\; m = n\,, \end{cases}$$

$c_1,\, c_2, ..., c_{r+1}$ are given arbitrary real numbers. Here dot means the derivative respect to time. $\{a_n^0\}_{-\infty}^{\infty}$, $\{b_n^0\}_{-\infty}^{\infty}$ satisfy the following properties:

1.$a_n^0 > 0, \quad \mathrm{Im}\; b_n^0 = 0, \quad n \in Z,$

2. $\sum\limits_{n=-\infty}^{\infty} |\, n \,| \, \left(\left|\, a_n^0 - \frac{1}{2} \,\right| + |\, b_n^0| \right) < \infty,$

3.The operator $(L(0)y)_n \equiv a_{n-1}(0)y_{n-1} + b_n(0)y_n + a_n(0)y_{n+1}$ has exactly N eigenvalues $\lambda_k(0) = \frac{z_k(0)+z_k^{-1}(0)}{2}$, $k = 1, 2, ..., N$, which are out of the interval $[-1;\, 1]$.

The main aim of this work is to obtain the expressions of the solutions $\{a_n(t)\}_{-\infty}^{\infty}$, $\{b_n(t)\}_{-\infty}^{\infty}$, $\{f_n(\mu, t)\}_{-\infty}^{\infty}$ and $\{g_n(\mu, t)\}_{-\infty}^{\infty}$ of the problem (1)-(2) in the framework of inverse scattering method for the operator $L(t)$.

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Direct problem for fractional differential equation with the generalized riemann - liouville time derivative order $1 < \alpha < 2$

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In the domain $\Omega = \{(x,t) : 0 < x < 1,\ 0 < t \leq T\}$ consider the following problem

$$\left(D^{\alpha,\beta}_{0+,t}u\right)(x,t) - u_{xx} + q(t)u(x,t) = f(x,t),\ (x,t) \in \Omega, \tag{1}$$

$$I^{(2-\alpha)(1-\beta)}_{0+,t}u(x,t)\big|_{t=0} = \varphi_1(x),\ \frac{\partial}{\partial t}\left(I^{(2-\alpha)(1-\beta)}_{0+,t}u\right)(x,t)\big|_{t=0} = \varphi_2(x),\ x \in [0,1] \tag{2}$$

$$u(0,t) = u(1,t),\ u_x(1,t) = 0,\ 0 \leq t \leq T, \tag{3}$$

where Γ is the Euler's Gamma function, $D^{\alpha,\beta}_{0+}$ is Hilfer fractional derivative of order $1 < \alpha < 2$ and type $0 \leq \beta \leq 1$ defined as follows [[1]. pp. 112-118]].

The existence and uniqueness of the solution to an Cauchy type problem for the fractional differential equations were studied in many papers [see [2, 3]]. Inverse problems for classical integro-differential wave propagation equations have been extensively studied. Nonlinear inverse coefficient problems with various types of sufficient determination conditions are often found in the literature [e.g., [15-22] and References therein].

The functions φ_1, φ_2 and f satisfy the following assumptions:

A1) $\{\varphi_1, \varphi_2\} \in C^3[0,1]$, $\{\varphi_1^{(4)}, \varphi_2^{(4)}\} \in L_2[0,1]$, $\varphi_1(0) = \varphi_1(1) = 0$, $\varphi_2(0) = \varphi_2(1) = 0$, $\varphi''_1(0) = \varphi''_1(1) = 0$, $\varphi''_2(0) = \varphi''_2(1) = 0$, *and* $\varphi_1^{(4)}(0) = \varphi_1^{(4)}(1) = 0$, $\varphi_2^{(4)}(0) = \varphi_2^{(4)}(1) = 0$,;

A2) $f(x,\cdot) \in C[0,T]$ *and for* $t \in [0,T], f(\cdot,t) \in C^3[0,1]$, $f(\cdot,t)^{(4)} \in L_2[0,1]$, $f(0,t) = f(1,t) = 0$, $f_{xx}(0,t) = f_{xx}(1,t) = 0$ *and* $f_{xxxx}(0,t) = f_{xxxx}(1,t) = 0$.

Using the above results, we obtain the following assertion.

Theorem 1. Let $q(t) \in C[0,T]$, A1), A2) are satisfied, then there exists a unique solution of the direct problem (1)-(3) $u(x,t) \in C^{2,\alpha}_{\gamma}(\overline{\Omega})$.

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Boundary control for a fractional pseudo-parabolic equation with a given flow at the boundary

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In this study, we look at the boundary control problem for a pseudo-differential equation with a fractional derivative with regard to time. If the average value of the solution in the given region is known, it is necessary to find the control's parameter in the section of the border of the given area where the value of the derivative of the solution on the normal direction is given. We begin by investigating a boundary value problem for pseudo-homogeneous differential equations with fractional derivatives. Using the spectrum method, we demonstrate the existence and uniqueness of the solution to the presented problem. The initial problem is reduced to the Voltaire integral equation, and we use the Laplace transformation approach to demonstrate the existence of a solution to this equation.

Let $0 < \alpha \leq 1$ and consider the differential equation

$$
\begin{gathered}
{}_t^cD^{\alpha}u(x,t) = {}_t^cD^{\alpha}u_{xx}(x,t) + u_{xx}(x,t), \\
(x,t) \in \Omega_T = \{|x| < \pi, 0 < t < T\}.(1)
\end{gathered}
$$

$u(x,t)$ satisfies equation (1) in Ω_T, and also the initial

$$
u(x,t)|_{t=0} = 0, \quad -\pi \leq x \leq \pi \tag{2}
$$

and boundary conditions

$$
u_x(x,t)|_{x=-\pi} = -\mu(t), \quad u_x(x,t)|_{x=\pi} = 0, \quad t \in [0,T] \tag{3}
$$

In this work, we consider the control problem related to equation (1). On the part of the boundary $D = \{x \in R, |x| \leq \pi\}$ there is a source with a given flow. It is required to find such an operating mode of the source, so that the average value of the solution in some part of the segment D took on the indicated value.

Find $\mu(t)$ from the condition

$$
\int_{-\pi}^{\pi} u(x,t)dx = \theta(t),
$$

where $\theta(t)-$ is a given function, and $\mu(t)$ satisfies the matching condition $\mu(0) = 0$, and also $|\mu(t)| < 1$.

Conditional stability and regularized solution of a boundary value problem for a system of three-dimensional mixed type equations

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This work is devoted to the study of an ill-posed initial-boundary value problem for a system of three-dimensional equations of mixed type.

Let $Q_T = \{(x,y,z,t) : (x,y,z) \in \Omega,\ 0 < t < T\}$, $\Omega = \{\,|x| < \pi,\ 0 < y < \pi,\ 0 < z < \pi\}$.

In the domain $Q_T \cap \{x \neq 0\}$ the system of equations of the form

$$\begin{cases} u_{tt} = Lu + a_1 u + b_1 v + f(x,y,t), \\ v_{tt} = Lv + a_2 v + b_2 u + g(x,y,t), \end{cases} \tag{1}$$

is investigated, where $Lu \equiv sgn(x)u_{xx} + u_{yy} + u_{zz}$, a_1, a_2, b_1, b_2 are some constants, $(a_1 - a_2)^2 + 4b_1 b_2 > 0$.

Let a pair of functions $(u(x,y,z,t),\ v(x,y,z,t))$ satisfy the system of equations (1) in the domain $Q_T \cap \{x \neq 0\}$ and the following conditions:
initial

$$\left.\begin{array}{r} u|_{t=0} = \varphi_1(x,y,z),\ v|_{t=0} = \psi_1(x,y,z), \\ u_t|_{t=0} = \varphi_2(x,y,z),\ v_t|_{t=0} = \psi_2(x,y,z), \end{array}\right\},\ (x,y) \in \bar{\Omega}, \tag{2}$$

boundary

$$\begin{array}{l} u|_{x=-\pi} = u|_{x=\pi} = 0,\ 0 \le y \le \pi,\ 0 \le z \le \pi, 0 \le t \le T, \\ u|_{y=0} = u|_{y=\pi} = 0,\ -\pi \le x \le \pi,\ 0 \le z \le \pi, 0 \le t \le T, \\ u|_{z=0} = u|_{z=\pi} = 0,\ -\pi \le x \le \pi,\ 0 \le y \le \pi, 0 \le t \le T \end{array} \tag{3}$$

and gluing conditions

$$\left.\begin{array}{l} u|_{x=-0} = u|_{x=+0},\ u_x|_{x=-0} = u_x|_{x=+0}, \\ v|_{x=-0} = v|_{x=+0},\ v_x|_{x=-0} = v_x|_{x=+0}, \end{array}\right\} 0 \le y \le \pi,\ 0 \le z \le \pi, 0 \le t \le T, \tag{4}$$

where $\varphi_i(x,y,z)$, $\psi_i(x,y,z)$ are given sufficient smooth functions, and they satisfy the matching conditions, $i = 1, 2$, $f(x,y,z,t)$, $g(x,y,z,t)$ are source functions.

In this paper, the ill-posed initial-boundary value problem (1)-(4) is studied for conditional correctness, i.e. theorems on uniqueness and conditional stability are proved. Then, a regularized approximate solution is constructed on the set of correctness.

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Nonlocal boundary value problem for a system of nonhomogeneous mixed type equations with two degenerate lines

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In this paper, we consider a nonlocal boundary value problem for a system of nonhomogeneous mixed-type partial differential equations of the second order of with two degenerate lines.Similar equations were considered by N. Kislov, S.G. Pyatkov, K.S. Fayazov, I.E. Egorov, S. Z. Djamalov, I.O. Khajiev and others. The correctness of nonlocal boundary value problems for some general differential and differential operator equations is studied in various aspects in the works of A.A. Dezin, V.K. Romanko, Y.I. Yurchuk, K.S. Fayazov and others.

Let $\Omega = \Omega_0 \times Q$, $\Omega_0 = \Omega_1 \times \Omega_2 \times \Omega_3$, $\Omega_1 = \{-1 < x_1 < 1\}$,$\Omega_2 = \{-1 < x_2 < 1\}$,$\Omega_3 = \{(x_3, ..., x_n) : \ (0;\pi) \times ... \times (0;\pi)\}$,$Q = (0;T)$, $x = (x_1, x_2, ..., x_n)$.

Consider the system of equations

$$\begin{cases} u_{1tt} + \operatorname{sgn}(x_1)u_{1x_1x_1} + \operatorname{sgn}(x_2)u_{1x_2x_2} + \sum\limits_{i=3}^{n} u_{1x_ix_i} + a_1u_1 + b_1u_2 = f_1, \\ u_{2tt} + \operatorname{sgn}(x_1)u_{2x_1x_1} + \operatorname{sgn}(x_2)u_{2x_2x_2} + \sum\limits_{i=3}^{n} u_{2x_ix_i} + a_2u_2 + b_2u_1 = f_2, \end{cases} \tag{1}$$

in the domain $\Omega \cap \{x_1, x_1 \neq 0\}$, where the n-dimensional domain, a_j, b_j are some constants, $b_2 \neq 0$, $(a_1 - a_2)^2 + 4b_1b_2 > 0$ and $j = 1, 2$, $f_j(x, t)$ are given sufficiently smooth functions.
Problem. Find a solution to the system of equations (1) satisfying the following conditions: nonlocal

$$\alpha \left. \frac{\partial^i u_j(x,t)}{\partial t^i} \right|_{t=0} + \beta \left. \frac{\partial^i u_j(x,t)}{\partial t^i} \right|_{t=T} = \varphi_{ij}(x), \ \ x \in \bar{\Omega}_0, \ i = 0, 1, \ j = 1, 2, \tag{2}$$

boundary

$$u_j|_{\partial(\Omega_1 \cup \Omega_2 \cup \Omega_3)} = 0, \ j = 1, 2, \tag{3}$$

and corresponding gluing conditions the exis x_1 and x_2 where α, β is some constant, $\varphi_{ij}(x)$ given sufficiently smooth functions.

In this work, we study the correctness of the desired problem depending on the value of the parameters' α, β and obtain a representation of the solution, as well as an a priori estimate of the solution. In the case of ill-posedness, conditional correctness is proved.

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Integration of the mkdv-sinus-gordon equation with non-stationary coefficients

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In this paper, we consider the following equation

$$u_{xt}+p(t)\left(\frac{3}{2}u_x^2u_{xx}+u_{xxxx}\right)=q(t)\sin u+\omega(t)u_{xx}, \tag{1}$$

where $p(t)$, $q(t)$, $\omega(t)$ are initially given continuously differentiable functions. The equation(1) is considered with the initial condition

$$u(x,0)=u_0(x), \quad x\in\mathbb{R}, \tag{2}$$

where the initial function $u_0(x)$ $(-\infty<x<\infty)$ has the following properties:
1)

$$u_0(x)\equiv 0(\mathrm{mod}\ 2\pi),\ |x|\to\infty;\ \int_{-\infty}^{\infty}((1+|x|)|u_0'(x)|+|u_0''(x)|+|u_0'''(x)|+|u_0''''(x)|)dx<\infty. \tag{3}$$

2) The operator $L(0)=\sigma_1\frac{d}{dx}-\frac{u_x(x,0)}{2}\sigma_2+\frac{u_x(x,0)}{2}\sigma_3$ in the upper half-plane of the complex plane has exactly N eigenvalues $\xi_1(0),\xi_2(0),\dots,\xi_N(0)$ with multiplicities $m_1(0),m_2(0),\dots,m_N(0)$ and has no spectral singularities. Here $\sigma_1=\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix}$, $\sigma_2=\begin{pmatrix} 0 & -i\\ 0 & 0\end{pmatrix}$, $\sigma_3=\begin{pmatrix} 0 & 0\\ i & 0\end{pmatrix}$. Suppose that the function $u(x,t)$ has sufficient smoothness and tends to its limits fast enough as $x\to\pm\infty$, i.e.

$$u(x,t)\equiv 0(\mathrm{mod}\ 2\pi)\ \ |x|\to\infty;\int_{-\infty}^{\infty}((1+|x|)|u_x|+|u_{xx}|+|u_{xxx}|+|u_{xxxx}|)dx<\infty.$$

Theorem. If the function $u(x,t)$ is a solution to the (1)-(4) problem, then the scattering data of the operator $L(t)$ with the potential $u(x,t)$ satisfy the following differential equations

$$\frac{dr^+}{dt}=\Big(8i\xi^3p(t)-\frac{iq(t)}{2\xi}+2i\xi\,\omega(t)\Big)r^+,\quad \xi_k(t)=\xi_k(0),\ k=1,2,\dots,N.$$

$$\frac{d\varkappa^n_{m_n-1-\nu}}{dt}=\big(8i\xi_n^3p(t)+2i\xi_n\omega(t)\big)\varkappa^n_{m_n-1-\nu}+\big(24i\xi_n^2p(t)+2i\omega(t)\big)\varkappa^n_{m_n-2-\nu}+$$

$$+24i\xi_np(t)\varkappa^n_{m_n-3-\nu}+8ip(t)\varkappa^n_{m_n-4-\nu}-i\sum_{s=\nu}^{m_n-1}\frac{(-1)^{m_n-1-s}q(t)}{2\xi_n^{m_n-s}}\varkappa^n_{s-\nu},\quad n=\overline{1,N},\ \nu=\overline{m_n-1,0}.$$

Fuzzy comfortable double laplace transform and telegraph equation

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The Laplace transform is the most important integral transformation for solving ordinary differential equations, partial differential equations and some types of integrodifferential equations. In [1], the fuzzy double Laplace transform concepts are proposed and some of their properties are studied. This paper uses a double Laplace transform to solve a fuzzy telegraph equation with comfortable derivatives in time and coordinates (both in one-dimensional and two-dimensional cases). For one class of integro-differential equations, a similar problem is solved in [2,3]. Note that uncertain parameters and variables in complex systems are modeled through fuzzy differential equations [4]. In recent years, due to the widespread use of the theory of fuzzy differential equations, many authors have begun research on fuzzy partial differential equations [5]. New analytical and numerical methods for finding fuzzy solutions of such equations are proposed. In [6], a fuzzy comfortable double Laplace transform is introduced, which has already found its application in fuzzy comfortable partial differential equations.

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Cauchy problem for high order equation with Dzhrbashyan-Nersesyan operator

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In the region $\Omega = \{(x,y) : -\infty < x < +\infty, 0 < y < T\}, T < +\infty$ consider the following Cauchy-type problem.

Cauchy problem. Find a regular solution to the following problem:

$$\begin{cases} D_{0y}^{\{\gamma_0,\gamma_1,\gamma_2\}} u(x,y) - (-1)^{s-1} \frac{\partial^{2s} u(x,y)}{\partial x^{2s}} = 0, \\ 1 < \gamma_0 + \gamma_1 + \gamma_2 - 1 \le 2, \gamma_1 = 1, s = 2, 3, ..., \\ D_{0y}^{\gamma_0 - 1} u(x,0) = \varphi_0(x), \\ D_{0y}^{\gamma_0} u(x,0) = \varphi_1(x), \end{cases}$$

where $D_{0y}^{\{\gamma_0,\gamma_1,\gamma_2\}} = D_{0y}^{\gamma_2 - 1} D_{0y}^{\gamma_1} D_{0y}^{\gamma_0}$ is the Dzhrbashyan-Nersesyan fractional differentiation operator, of order $\alpha = \gamma_0 + \gamma_1 + \gamma_2 - 1, \gamma_i \in (0;1], i = 0,1,2$, D_{0y}^{γ} is the Riemann-Liouville fractional integro-derivation operator of order γ. Note that the Dzhrbashyan-Nersesyan operator includes the Riemann-Liouville, Caputo, Hilfer, etc. operators.

Theorem. Let the following conditions be satisfied:

$$\varphi(x) \in C(R), \psi(x) \in C^{(2s)}(R), \lim_{|x| \to \infty} \left(\varphi(x) \exp\left(-k|x|^{\frac{2n}{2n-\alpha}} \right) \right) = 0,$$

$$\lim_{|x| \to \infty} \left(\psi(x) \exp\left(-k|x|^{\frac{2n}{2n-\alpha}} \right) \right) = 0, k < \left(1 - \frac{\alpha}{2n} \right) \left(\frac{\alpha}{2nT} \right)^{\frac{\alpha}{2n-\alpha}} \cos \frac{1-n}{2n-\alpha} \pi,$$

then the representation of the form

$$u(x,y) = \int_{-\infty}^{+\infty} \varphi(\xi) \Gamma_{b_0}(x-\xi, y) d\xi + \int_{-\infty}^{+\infty} \psi(\xi) \Gamma_{b_1}(x-\xi, y) d\xi,$$

is a solution to the Cauchy problem, where

$$\Gamma_{b_l}(x-\xi, y-\eta) = \frac{(y-\eta)^{b_l}}{2s} \sum_{k=0}^{s-1} \lambda_k \phi\left(-\frac{\alpha}{2s}, b_l + 1; -\lambda_k t \right),$$

$$b_l = \alpha_l - \frac{\alpha}{2s}, \alpha_l = \gamma_0 - l, l = 0, 1,$$

$$\lambda_k = e^{\frac{s-1-2k}{2s} i\pi}, t = \frac{|x-\xi|}{(y-\eta)^{\frac{\alpha}{2s}}} > 0,$$

$$\alpha = \gamma_0 + \gamma_2.$$

On spectral properties of a class of degenerate elliptic operators

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One of the methods for studying the solvability of boundary value problems for elliptic operators with power degeneration is based on elements of the theory of spaces of differentiable functions of several real variables with power weight (see, for example, [1] and the bibliography there). In the case of the whole Euclidean space $\mathbb{R}^n$, such weighted spaces were first studied by L.D. Kudryavtsev in [2], where he introduced the space $W^r_{p;\,\alpha}(\mathbb{R}^n)$ with finite norm

$$\|u; W^r_{p;\,\alpha}(\mathbb{R}^n)\| = \left\{ \sum_{|k|=r} \int_{\mathbb{R}^n} d^{p\alpha}(x)|u^{(k)}(x)|^p dx + \int_{K_R} |u(x)|^p dx \right\}^{1/p}$$

and studied its properties. Here and further $r \in \mathbb{N}$, $p \in (1,\,\infty)$, $d(x) = (1+|x|^2)^{-1/2}$, $u^{(k)}(x)$ – derivative of the function $u(x)$ of the multi-index k, generalized in the sense of S.L.Sobolev. Now, there are various generalizations of the L.D.Kudryavtsev's space $W^r_{p;\,\alpha}(\mathbb{R}^n)$,

The works of N.V.Miroshin (see, for example, [3]) contain some results on the spectral properties of a degenerate elliptic operator of the form

$$L[u] = \sum_{|k|=|l|\le r} \left(d(x)^{2\alpha_{|k|}} a_{kl}(x) u^{(k)}(x) \right)^{(l)}. \tag{1}$$

It is assumed in his papers that the sesquilinear form $B[u,\,v] = (L[u],\,v)_0, u,\,v \in C_0^\infty(\mathbb{R}^n)$, where $(\cdot,\,\cdot)_0$ is the inner product in $L_2(\mathbb{R}^n)$, satisfies the following coercivity condition

$$\operatorname{Re} B[u,\,u] + \lambda_0(u,\,u)_0 \ge \varkappa_0 \|u; W^r_{2;\,\alpha}(\mathbb{R}^n)\|^2 \quad \forall u \in C_0^\infty(\mathbb{R}^n). \tag{2}$$

In this case, the range of the form $B[u,\,u]$ lies in the right complex half-plane.

Our report is devoted to the study of the spectral properties of operators of the form (1) in the case when the coercivity condition (2) is not satisfied. In our case, the range of the sesquilinear form associated with the operator under study can also lie in some part of the left complex half-plane.

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Regularization of the initial-boundary value problem for a composite differential equation

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This research work is devoted to checking the conditional correctness and constructing a regularized solution of the initial-boundary problem for the composite differential equation.

Let $u(x,t)$ on the region $\Omega = \{(x,t) : -l < x < l,\ x \neq 0,\ 0 < t < T\}$ satisfies the equation

$$(a\partial_t + b)\left(\partial_t - sgn(x)\partial_x^2 + c\right)u\left(x,t\right) = f\left(x,t\right), \tag{1}$$

where a, b, c are some constants, $a \neq 0$, $f\left(x,t\right)$ is a sufficiently smooth given source function.

Statement of the problem. Find a solution $u(x,t)$ of the equation (1) on the region Ω and satisfying following conditions:

the initial

$$u\left(x,0\right) = \varphi\left(x\right),\ \ u_t\left(x,0\right) = \psi\left(x\right), -l \leq x \leq l, \tag{2}$$

the boundary

$$u\left(-l,t\right) = u\left(l,t\right) = 0, 0 \leq t \leq T, \tag{3}$$

and the gluing conditions

$$u\left(-0,t\right) = u\left(+0,t\right), u_x\left(-0,t\right) = u_x\left(+0,t\right), 0 \leq t \leq T, \tag{4}$$

where $\varphi\left(x\right)$ and $\psi\left(x\right)$ are sufficiently smooth given functions.

In this work, an a priori estimate is obtained for the solution of the problem (1)-(4) and the set of correctness of the problem is determined. Theorems about the uniqueness and conditional stability the solution of the problem are proved in the set of correctness. An approximate solution of the problem is constructed by Tikhonov's regularization method.

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The Cauchy problem for the modified Korteweg-de Vries equation with finite density in the class of periodic functions

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In this thesis, the method of the inverse spectral problem is used to integrate the nonlinear modified Korteweg-de Vries equation with finite density in the class of periodic functions.

Statement of the problem. Consider the Cauchy problem for the modified Korteweg-de Vries equation with finite density of the following form

$$\begin{cases} q_t = q_{xxx} - 6(q^2 - \rho^2)q_x, \ 0 < \rho < \infty \\ q(x,t)|_{t=0} = q_0(x), q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R}) \end{cases} \quad (1)$$

in the class of real infinite-gap π periodic with respect to x functions:

$$\begin{cases} q(x+\pi,t) = q(x,t), \ x \in \mathbb{R}, \, t > 0 \\ q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0) \end{cases} .(2)$$

In this paper, we propose an algorithm for constructing exact solution $q(x,t), x \in \mathbb{R}, t > 0$ of problem (1)-(2) by reducing it to an inverse spectral problem for the following Dirac operator:

$$\mathfrak{L}(\tau,t) \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y, x \in \mathbb{R}, \tau \in R, t > 0,$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \Omega(x+\tau,t) = \begin{pmatrix} 0 & q(x+\tau,t) \\ q(x+\tau,t) & 0 \end{pmatrix}, \ y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}.$$

In this thesis, the evolution of the spectral data of the periodic Dirac operator is introduced and the coefficient of Dirac operator is a solution of the nonlinear modified Korteweg-de Vries equation with finite density. The solvability of the Cauchy problem for an infinite system of Dubrovin differential equations in the class of five-fold continuously differentiable periodic functions is proved. It is shown that the sum of a uniformly convergent functional series constructed by solving the system of Dubrovin equations and the first trace formula satisfies the mKdV equations with finite density.

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On uniqueness solution of a nonlinear boundary problem for a nonlinear fourth order equation with multiple characteristics in a rectangular domain

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For equation

$$u_{xxxx}(x,y)+a_1u_{xx}(x,y)+a_2u_x(x,y)+a_3u(x,y)-u_{yy}(x,y)=f(u(x,y)), \tag{1}$$

in the region $\Omega=\{(x,y):\ 0<x<p,\ 0<y<q\}$, we study the following problem.

Problem (1)-(3). Find a function $u(x,y)$ in the class $C^{4,2}_{x,y}(\Omega)\cap C^{3,1}_{x,y}(\bar{\Omega})$, satisfying Eq. (1) in the domain Ω and the following boundary conditions:

$$u_y(x,0)=\varphi_1(u(x,0)),\quad u_y(x,q)=\varphi_2(u(x,q)),\quad 0\le x\le p, \tag{2}$$

$$u(0,y)=u(p,y)=u_{xx}(0,y)=u_{xx}(p,y)=0,\ 0\le y\le q, \tag{3}$$

where $p,q,a_i\in R,\ i=\overline{1,3},\ \varphi_1(u(x,0)),\ \varphi_2(u(x,q))$, and $f(u(x,y))$ are given sufficiently smooth functions.

In [1], [2], linear boundary value problems for the linear equation with constant coefficients were studied. In [3], [4], linear boundary value problems for the linear equation with variable coefficients were studied.

Uniqueness theorem. If Problem (1)-(3) has a solution, then, under the conditions $a_1\le 0,\ a_3\ge 0,\ \varphi_1'(\tilde{u}(x,0))\ge 0,\ \varphi_2'(\tilde{u}(x,q))\le 0$ and $f'(\tilde{u}(x,y))\le 0$, the solution is unique. Here $\tilde{u}(x,y)\in(u_1(x,y),u_2(x,y))$.

The uniqueness theorem is proved by the method of energy integrals.

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Partially-isospectral sturm-liouville boundary value problems on the finite segment

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In this paper, an algorithm is proposed for constructing partially-isospectral Sturm-Liouville boundary value problems on the finite segment. This algorithm is applied to the process of solving mixed problems posed to differential equations with partial derivatives of hyperbolic and parabolic types with variable coefficients.

Definition 1. Sturm-Liouville boundary value problems

$$L^0 y \equiv -y'' + q_0(x)y = \lambda y, y'(0) - h_0 y(0) = 0, y'(\pi) + H_0 y(\pi) = 0 \tag{1}$$

and

$$Ly \equiv -y'' + q(x)y = \lambda y, y'(0) - hy(0) = 0, y'(\pi) + Hy(\pi) = 0 \tag{2}$$

are called isospectral if they have same eigenvalues, that is, $\sigma(L) = \sigma(L^0) = \{\lambda_n^0,\ n \geq 0\}$.

Definition 2. Boundary value problems (1) and (2) are called partially-isospectral if their eigenvalues satisfy the conditions

$$\lambda_n \neq \lambda_n^0 \text{ at } n = \overline{0, n_0 - 1};\ \lambda_n = \lambda_n^0 \text{ at } n = \overline{n_0, \infty},$$

where are $n_0 \in N$ some natural numbers.

Theorem 1. Let the couple $\{\lambda_n, \alpha_n\}_{n=0}^{\infty}$ sequence of real numbers satisfies the following conditions

$$\lambda_0 = a^2, a \in [0, 1), \lambda_n = n^2, n \geq 1, \alpha_0 = \pi, \alpha_n = \frac{\pi}{2}, n \geq 1.$$

This pair is the spectral data of some Sturm-Liouville boundary value problem of the form$L(q(x, a), h(a), H(a))$.

Theorem 2. Let the couple $\{\lambda_n, \alpha_n\}_{n=0}^{\infty}$ sequence of real numbers satisfies the following conditions

$$\lambda_0 = -a^2, a \in R,\ \lambda_n = n^2, n \geq 1; \alpha_0 = \pi, \alpha_n = \frac{\pi}{2}, n \geq 1.$$

This pair is the spectral data of some Sturm-Liouville boundary value problem of the form$L(q(x, a), h(a), H(a))$.

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On the invariance of a constant multiple-valuedmappings in the heat conductivity problem

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In this paper, we consider the issues of strong and weak invariance of a constant multivalued mapping for the boundary value problem of heat conduction in the presence of delay. In this case, the control parameter is located on the right side equation, its control action has an impulse character, which is expressed using the Dirac delta function [1]. The given conditions differ from the previously known results obtained for control with delay.

Consider the following heat transfer control problem [1]

$$\frac{\partial u(x,t)}{\partial t} + Au(x,t) = u(x,t-h) + F(x,t,\mu),\ 0 < t \le T,\ x \in \Omega \tag{1}$$

with boundary and initial conditions

$$u(x,t) = 0,\ \ 0 \le t \le T,\ x \in \partial\Omega, \tag{2}$$

$$u(x,t) = u_0(x,t),\ -h \le t \le 0,\ x \in \Omega. \tag{3}$$

Here $u = u(x,t)$ — is an unknown function, T is an arbitrary positive number, $u_0(\cdot,\cdot) \in X$, $X = W_2^{1,1}(\Omega \times [-h,0])$, where $F-$ is a given function with μ control.

Let $\|u(\cdot,t)\| = \|u(\cdot,t)\|_{L_2(\Omega)}$, $0 \le t \le T$.

Theorem. 1°. *Let* $t_0 > T$ *and* $\lambda_1 > 1$. *Then for any* $\rho \ge 0$ *multi-valued mapping* $W(t) = [0,b]$, $t \in [-h,T]$ *strongly invariant on the interval* $[-h,T]$ *problem-related* (1)-(3);

2°. *Let* $t_0 \le T$. *If* $\rho \le b \cdot (\lambda_1 - 1)\Big(e^{\lambda_1 t_0} - 1\Big)\Big/\Big(\lambda_1 \sum\limits_{i=0}^{N(T)} e^{\lambda_1 t_i}\Big)$, *then the multivalued mapping* $W(t) = [0,b]$, $t \in [-h,T]$ *is strongly invariant on the interval* $[-h,T]$ *regarding the task (1)-(3).*

Note. *It can be shown that multivalued mapping* $W(t)$ *is always weakly invariant on the segment* $[-h,T]$ *regarding the task (1)-(3).*

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On existence conditions for periodic solutions to a Lasota-Wazewska model with a piecewise constant argument

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This paper is devoted to investigating the periodic solutions of the Lasota-Wazewska model of the form

$$x'(t) + \alpha x(t) = \beta e^{-\gamma x([t-1])}, \tag{1}$$

$$x(-1) = x_1 > 0, x(0) = x_0 > 0. \tag{2}$$

where α, β and γ are positive constants and $[\cdot]$ denotes the floor function which gives the greatest integer less than or equal to given number.

Let us define a definition of solution for (1),(2)

Definition. *A function $x(t)$ defined on the set $\{-1\} \cup [0, \infty)$ is called a solution of Eq.(2) if it satisfies the following conditions:*

(i) $x(t)$ is continuous on $\mathbf{R}_+$;

(ii) $x'(t)$ exists and is continuous in $\mathbf{R}_+$, with possible exception at points $[t] \in \mathbf{R}_+$, where one-sided derivatives exist;

(iii) $x(t)$ satisfies Eq. (2) in $\mathbf{R}_+$, with the possible exception at the points $[t] \in \mathbf{R}_+$.

Our results improve and complement some known results to some degree in the [1]. We obtain sufficient conditions of a n-periodic solution for the considering problem.

Note that in 2021, G.S.Oztepe [1] investigated is devoted to investigating the asymptotic stability of the equilibrium point of the Lasota-Wazewska model (2) and it is proved that this point is an attractor.

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Determination of a coefficient and kernel in a d-dimensional fractional integrodifferential equation

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Let $Q_0^T := \Omega \times (0, T)$ for a given time $T > 0$, where Ω be a bounded domain in $\mathbf{R}^d$ with sufficiently smooth boundary $\partial\Omega$. We consider a fractional integrodifferential equation with a fractional derivative in time t:

$$\partial_t^\alpha u(\mathbf{x}, t) + Au(\mathbf{x}, t) = q(t)u_t(\mathbf{x}, t) + (k * u)(t) + f(\mathbf{x}, t), \quad (\mathbf{x}, t) \in Q_0^T, \tag{1}$$

where $1 < \alpha < 2$ and $\partial_t^\alpha u(\mathbf{x}, t)$ is the left Gerasimov-Caputo fractional derivative with respect to t and is defined by (see [1])

$$\partial_t^\alpha v(t) = \begin{cases} \frac{1}{\Gamma(d-\alpha)} \int_0^t (t-\tau)^{d-\alpha-1} v^{(d)}(\tau)\, d\tau, & d-1 < \alpha < d,\ d \in \mathbf{N},\ v \in W^{d,1}(0,T); \\ v^{(d)}(t), & \alpha = d \in \mathbf{N}, \end{cases}$$

$\Gamma(\cdot)$ is the Gamma function and the operator A is a symmetric uniformly elliptic operator defined on $\mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$.

We supplement the above fractional wave equation with the following initial conditions:

$$u(\mathbf{x}, 0) = a(\mathbf{x}), \quad u_t(\mathbf{x}, 0) = b(\mathbf{x}), \quad x \in \Omega \tag{2}$$

and the zero boundary condition:

$$u(x, t) = 0, \quad \mathbf{x} \in \partial\Omega,\ 0 < t < T. \tag{3}$$

If $q(t)$, $k(t)$, $f(\mathbf{x}, t)$, $p(\mathbf{x})$, $a(\mathbf{x})$, $b(\mathbf{x})$ and $\sigma(\mathbf{x})$ are known, problem (1)-(3) is a direct problem. The inverse problem in this paper is to reconstruct $q(t)$ and $k(t)$ according to the additional data

$$u(x_i, t) = h_i(t), \quad x_i \in \Omega,\ i = 1, 2, \quad t \in (0, T) \tag{4}$$

where $h_i(t)$, $i = 1, 2$ are given functions.

We investigate the following inverse problem.

Inverse problem. Find $u \in C([0, T]; \mathcal{D}(A^{\gamma - \frac{1}{\alpha}})) \cap C^1([0, T]; D(A^\gamma))$, $q \in C^1[0, T]$ and $k \in C[0, T]$ to satisfy (1)-(3) and the additional measurement (4), where $\mathcal{D}(A^\gamma)$ is a Hilbert space with some positive constant γ (see [2]).

The main results of this work are the existence and uniqueness theorem of our inverse problem.

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On a free boundary problem for the prey-predator model

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The dynamical relationship between a predator and a prey has long been among the dominant topics in mathematical ecology due to its universal existence and importance. In the real world, most species go through several stages during their lifetime, such as immature and mature stage. The vital rates (rates of survival, development, and reproduction) almost always depend on the development stage, among many other factors. Hence, it is significant and practical to introduce the stage structure into models. In recent years, prey-predator models with stage structure have attracted much attention ([1]-[2]).

In this work, we study the following free boundary for a prey-predator model with nonlinear convection term:

$$u_t - d_1 u_{xx} + m_1 uvu_x = u\left(a_1 - b_1 u + c_1 v\right), \quad (t,x) \in D, \tag{1}$$

$$v_t - d_2 v_{xx} + m_2 uvv_x = v\left(a_2 - b_2 u - c_2 v\right), \quad (t,x) \in Q, \tag{2}$$

$$u\left(0,x\right) = u_0\left(x\right), \quad 0 \le x \le s\left(0\right) \equiv s_0, \tag{3}$$

$$v\left(0,x\right) = v_0\left(x\right), \quad 0 \le x < l, \tag{4}$$

$$u_x\left(t,0\right) = 0, \quad 0 \le t \le T, \tag{5}$$

$$u\left(t,s\left(t\right)\right) = 0, \quad 0 \le t \le T, \tag{6}$$

$$s'\left(t\right) = -\mu u_x\left(t,s\left(t\right)\right), \quad 0 \le t \le T, \tag{7}$$

$$v_x\left(t,0\right) = 0, \quad 0 \le t \le T, \tag{8}$$

$$v_x\left(t,l\right) = 0, \quad 0 \le t \le T, \tag{9}$$

$$u(t,x) \equiv 0, \quad s(t) \le x < l, \tag{10}$$

where $D = \{(t,x) : 0 < t \le T, 0 < x < s(t)\}$, $Q = \{(t,x) : 0 < t \le T, 0 < x < l\}$; d_i, m_i, a_i, b_i, c_i are positive constants. In biological terms, $u\left(t,x\right)$ and $v\left(t,x\right)$ represent, respectively, densities of predator and prey species that are interacting and migrating in the habitat D and Q. The prey species migrates in the habitat $(0,l)$ and the predator disperses through random diffusion only in a part of the habitat $(0,l)$, namely $0 < x < s(t)$, then there is no predator in the remaining part. d_i denotes its respective diffusion rate and the real number a_i describes its net birth rate. b_1 and c_2 are the coefficients of intra-specific competitions, and b_2 and c_1 are the coefficients of inter-specific competitions.

For the solutions of (1)-(10) apriori estimates are established. On the base of apriori estimations the existence and uniqueness of a classical solution are obtained. The evolution of the free boundary problem is studied. It is proved that the problem addressed is well posed, and that the predator species disperses to all domains in finite time.

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A free boundary problem for a leslie-gower prey-predator system

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In biomathematics, the interactions among species and the spatial distributions of populations have always been the hot topics of ecosystems, and are important for developing research on economic benefits, pest control, and environmental governance. The Leslie-Gower predator-prey model has been deeply studied in [1, 2].

In this article, we study a diffusive Leslie-Gower predator-prey system with free boundary:

$$u_t - d_1 u_{xx} + m_1 u u_x = u\left(a_1 - b_1 u\right) - c_1 uv, \quad 0 < t \leq T, \quad 0 < x < s(t), \tag{1}$$

$$v_t - d_2 v_{xx} + m_2 v v_x = v\left(a_2 - \frac{b_2 v}{u + c_2}\right), \quad 0 < t \leq T, \quad 0 < x < s(t), \tag{2}$$

$$u\left(0, x\right) = u_0\left(x\right), \quad 0 \leq x \leq s\left(0\right) \equiv s_0, \tag{3}$$

$$v\left(0, x\right) = v_0\left(x\right), \quad 0 \leq x \leq s_0, \tag{4}$$

$$u_x\left(t, 0\right) = 0, \quad 0 \leq t \leq T, \tag{5}$$

$$u\left(t, s\left(t\right)\right) = 0, \quad 0 \leq t \leq T, \tag{6}$$

$$v_x\left(t, 0\right) = 0, \quad 0 \leq t \leq T, \tag{7}$$

$$v\left(t, s(t)\right) = 0, \quad 0 \leq t \leq T, \tag{8}$$

$$s'\left(t\right) = -\mu\left(u_x(t, s\left(t\right)) + \rho v_x(t, s(t))\right), \quad 0 \leq t \leq T, \tag{9}$$

where $s\left(t\right)$ is a free boundary to be determined, s_0, μ, ρ, d_i, m_i, a_i, b_i, c_i (i=1,2) are given positive constants, and the initial functions $u_0(x)$ and $v_0(x)$ satisfies

$$u_0\left(x\right) \in C^2\left([0, s_0]\right), \quad u_0\left(x\right) > 0 \ \text{ in } \ [0, s_0), \quad u_0'\left(0\right) = u_0\left(s_0\right) = 0,$$

$$v_0\left(x\right) \in C^2\left([0, s_0]\right), \quad v_0\left(x\right) > 0 \ \text{ in } \ [0, s_0), \quad v_0'\left(0\right) = v_0\left(s_0\right) = 0.$$

From a biological point of view, model (1)-(9) describes how the two species evolve if they initially occupy the bounded region $[0, s_0]$. The homogeneous Neumann boundary condition at $x = 0$ indicates that the left boundary is fixed, with the population confined to move only to right of the boundary point $x = 0$. We assume that both species have a tendency to emigrate thought the right boundary point to obtain their new habitat: the free boundary $x = s(t)$ represents the spreading front.

In this article, we have been working under the following assumption: $c_1 c_2 + c_1 < 1$.

Problem (1)-(9) was studied in [2] in the case $m_1 = m_2 = 0$.

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Inverse and Ill-Posed Problems of Nonlinear Moisture Conductivity

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Mathematical modelling of many natural processes and phenomena is based on the concept of a continuous medium. The term “continuous medium” does not mean that there are no pores or cracks that contain moisture, gas, or a mixture of fine particles [1]. The solid phase of a continuous medium can be nonporous (faintly porous), porous, or capillary.

At present, methods of mathematical modelling of moisture transfer are widely spread abroad [2, 3]. It has been noted that experimental methods and approaches need to be improved [3].

Moisture transfer in saturated soil can be written by using Darcy’s law [1]. In this case, the moisture transfer velocity is proportional to the pressure gradient

$$\frac{\partial W}{\partial t} = div(K(W) * grad(H))$$

where K(W)-moisture conductivity coefficient that depends on coordinates x, y,z; W-volumetric humidity of soil; H-pressure, t-time.

The humidity of soil can vary depending on the movement. If at the initial time, the soil has an uneven distribution of moisture along the depth, then with time the moisture will increase in drier layers according to the diffusion law. This phenomenon is called the Haller effect, which uses the concept of fractured porous soil to describe the noted fact. A correction term is included in the moisture transfer equation, which takes into account moisture transfer in soils. Hence Haller’s model has the following form

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x}(D\frac{\partial W}{\partial x} + A\frac{\partial W}{\partial x \partial t})$$

where A,D-proportionality coefficient

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About one problem for the equation viscoelasticity in a limited area

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A one-dimensional differential equation of viscoelasticity is considered in the domain $\Omega = \{(x,t) : 0 < x < l, t \in R\}$ reduced in x:

$$u_{tt} - u_{xx} - \int\limits_0^t k(\tau)u_{xx}(x, t-\tau)d\tau = 0, \qquad (x,t) \in \Omega, \tag{1}$$

with initial

$$u \mid_{t<0} \equiv 0, \tag{2}$$

and boundary conditions

$$u \mid_{x=0} = \delta(t), \qquad u_x \mid_{x=l} = 0, \qquad t \in R, \tag{3}$$

where $\delta(t)$ is the Dirac delta function.

A number of deep results on the correct solvability of Volterra integro- differential equations of the first and second order, not resolved with respect to relative to the highest derivative, as well as results on the spectrum of the corresponding operator-functions obtained by N.D. Kopachevsky and his students. Limit here indicating only some works [1] and [2]. We also mention the work of L. Pandolfi [3] in which the problem for an equation of the Gurtin-Pipkin type. Note that in these works solvability was studied in spaces of continuously differentiable functions on a finite interval with respect to the time variable t.

The main result of this paper is the following theorem.

Theorem. *Let the function $k(t) \in C[0, 2l]$,, then the solution of Eq. (1) satisfies creating conditions (2) and (3) exists, is unique and belongs to the class $C^2(D_2)$, where*

$$D_2 := \{(x,t) : 0 < x < l,\ x < t < 2l - x\}\,.$$

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Fractional telegraph equation with the Riemann-Liouville derivative

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The Telegraph equation $(\partial_t^\rho)^2 u(x,t) + 2\alpha \partial_t^\rho u(x,t) - u_{xx}(x,t) = f(x,t)$, where $0 < t \leq T$ and $0 < \rho < 1$, with the Riemann-Liouville derivative is considered. Existence and uniqueness theorem for the solution to the problem under consideration is proved.

Let $\rho \in (0,1)$ be a fixed number and $\Omega = (0,\pi) \times (0,T]$. Consider the following initial-boundary value problem for the Telegraph equation

$$\begin{cases} (\partial_t^\rho)^2 u(x,t) + 2\alpha \partial_t^\rho u(x,t) - u_{xx}(x,t) = f(x,t), \quad (x,t) \in \Omega; \\ u(0,t) = u(\pi,t) = 0, \quad 0 \leq t \leq T; \\ \lim\limits_{t\to 0} J_t^{\rho-1}(\partial_t^\rho u(x,t)) = \varphi_0(x), \quad 0 \leq x \leq \pi; \\ \lim\limits_{t\to 0} J_t^{\rho-1} u(x,t) = \varphi_1(x), \quad 0 \leq x \leq \pi, \end{cases} \tag{1}$$

where $t^{1-\rho} f(x,t)$ and $\varphi_0(x), \varphi_1(x)$ are continuous functions in the closed domain $\overline{\Omega}$. This problem is also called the *forward problem.* Taking into account the boundary conditions in problem (1), it is convenient for us to introduce the Hölder classes as follows. Let $\omega_g(\delta)$ be the modulus of continuity of function $g(x) \in C[0,\pi]$, i.e.

$$\omega_g(\delta) = \sup_{|x_1 - x_2| \leq \delta} |g(x_1) - g(x_2)|, \quad x_1, x_2 \in [0,\pi].$$

If $\omega_g(\delta) \leq C\delta^a$ is true for some $a > 0$, where C does not depend on δ and $g(0) = g(\pi) = 0$, then $g(x)$ is said to belong to the Hölder class $C^a[0,\pi]$. Let us denote the smallest of all such constants C by $||g||_{C^a[0,\pi]}$. Similarly, if the continuous function $h(x,t)$ is defined on $[0,\pi] \times [0,T]$, then the value

$$\omega_h(\delta; t) = \sup_{|x_1 - x_2| \leq \delta} |h(x_1,t) - h(x_2,t)|, \quad x_1, x_2 \in [0,\pi]$$

is the modulus of continuity of function $h(x,t)$ with respect to the variable x. In case when $\omega_h(\delta;t) \leq C\delta^a$, where C does not depend on t and δ and $h(0,t) = h(\pi,t) = 0,\ t \in [0,T]$, then we say that $h(x,t)$ belongs to the Hölder class $C_x^a(\overline{\Omega})$. Similarly, we denote the smallest constant C by $||h||_{C_x^a(\overline{\Omega})}$.

Let $C_{2,x}^a(\overline{\Omega})$ denote the class of functions $h(x,t)$ such that $h_{xx}(x,t) \in C_x^a(\overline{\Omega})$ and $h(0,t) = h(\pi,t) = 0,\ t \in [0,T]$. Note that condition $h_{xx}(x,t) \in C_x^a(\overline{\Omega})$ implies that $h_{xx}(0,t) = h_{xx}(\pi,t) = 0,\ t \in [0,T]$. For a function of one variable $g(x)$, we introduce classes $C_2^a[0,\pi]$ in a similar way.

Let $\alpha > 0,\ a > \frac{1}{2}$ and the following conditions be satisfied $t^{1-\rho} f(x,t) \in C_x^a(\overline{\Omega})$, $\varphi_0(x), \varphi_1(x) \in C_2^a[0,\pi]$. Then the forward problem has a unique solution.

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On the solution of the cauchy problem for the mkdv equation with the loaded and a self-consistent source in the case of moving eigenvalues

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In this paper, we consider the following system of equations

$$u_t + \beta(t)u(x_0,t)(6u^2u_x + u_{xxx}) + \gamma(t)u(x_1,t)u_x = \sum_{k=1}^{2N}(f_{k1}g_{k1} - f_{k2}g_{k2})$$

$$L(t)f_k = \xi_k f_k, \quad L(t)g_k = \xi_k g_k, \qquad k = 1,2,...,2N, \tag{1}$$

where $L(t) = i\begin{pmatrix} \frac{d}{dx} & -u(x,t) \\ -u(x,t) & -\frac{d}{dx}\end{pmatrix}$ and $\beta(t)$,$\gamma(t)$ given continuously differentiable functions. The system of equations (1) is considered under the initial condition

$$u(x,0) = u_0(x), \; x \in R^1, \tag{2}$$

where initial function $u_0(x)$ $(-\infty \; < x \; < \; \infty)$ has the following property:

This function $(1+|x|) \;\; |u_0(x)| \in L^1\,(-\infty, \, +\infty)$ and the operator $L(0)$ simple eigenvalues $\xi_1(0), \xi_2(0), .., \xi_{2N}(0)$.

In the problem under consideration, $f_k = (f_{k_1}, \, f_{k_2})^T$ is the eigenfunction of the operator $L(t)$ corresponding to the eigenvalue ξ_k and $g_k = (g_{k_1}, \, g_{k_2})^T$ is the solution of the equation $Lg_k = \xi_k g_k$, for which

$$W\{f_k, \, g_k\} = f_{k_1}g_{k_2} - f_{k_2}g_{k_1} = \omega_k(t) \neq 0, \;\; k = \overline{1,2N}, \tag{3}$$

where $\omega_k(t)$ are the initially given continuous functions of $t > 0$, satisfying the conditions

$$\omega_n(t) = -\omega_k(t) \;\; for \;\; \xi_n = -\xi_k, \mathrm{Re}\left\{\int_0^{\mathrm{t}} \omega_{\mathrm{k}}(\tau)\mathrm{d}\tau\right\} > -\mathrm{Im}\left\{\xi_{\mathrm{k}}(0)\right\}, k = \overline{1, \, N}, \tag{4}$$

for all non-negative values of t. For definiteness, we assume that in the sum in the right-handside of (1), the terms with $\mathrm{Im}\,\xi_{\mathrm{k}} > 0, \;\; \mathrm{k} = \overline{1, \mathrm{N}}$.

Let us assume that the function $u(x,t)$ has the required smoothness and rather quickly tends to its limits at $x \to \pm\infty$, i. e.,

$$\int_{-\infty}^{\infty}\left((1+|x|)\,|u(x,t)| + \sum_{k=1}^{3}\left|\frac{\partial^k u(x,t)}{\partial x^k}\right|\right)dx < \infty, \;\; k = 1,2,3. \tag{5}$$

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Solution of the cauchy problem for a system of nonlinear the loaded integro-differential equations

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In this paper, we consider the following system of equations

$$u_t+\beta(t)u(x_0,t)(6u^2u_x+u_{xxx})+\gamma(t)u(x_1,t)u_x=\sum_{k=1}^{2N}(f_{k1}g_{k1}-f_{k2}g_{k2})+i\int_{-\infty}^{+\infty}\left(\phi_1^2-\phi_2^2\right)d\eta,$$

$$L(t)\phi=\eta\phi,\quad L(t)f_k=\xi_k f_k,\quad L(t)g_k=\xi_k g_k,\qquad k=1,2,...,2N, \tag{1}$$

where $L(t)=i\begin{pmatrix}\frac{d}{dx} & -u(x,t)\\ -u(x,t) & -\frac{d}{dx}\end{pmatrix}$ and $\beta(t)$,$\gamma(t)$ given continuously differentiable functions. The system of equations (1) is considered under the initial condition

$$u(x,0)=u_0(x),\ x\in R^1, \tag{2}$$

where initial function $u_0(x)$ $(-\infty<x<\infty)$ has the following property:

This function $(1+|x|)\ |u_0(x)|\in L^1(-\infty,+\infty)$ and the operator $L(0)$ simple eigenvalues $\xi_1(0),\xi_2(0),..,\xi_{2N}(0)$.

In the problem under consideration, $f_k=(f_{k_1},\,f_{k_2})^T$ is the eigenfunction of the operator $L(t)$ corresponding to the eigenvalue ξ_k and $g_k=(g_{k_1},\,g_{k_2})^T$ is the solution of the equation $Lg_k=\xi_k g_k$, for which

$$W\{f_k,\,g_k\}=f_{k_1}g_{k_2}-f_{k_2}g_{k_1}=\omega_k(t)\neq 0,\ \ k=\overline{1,2N}, \tag{3}$$

where $\omega_k(t)$ are the initially given continuous functions of $t>0$, satisfying the conditions

$$\omega_n(t)=-\omega_k(t)\ for\ \xi_n=-\xi_k,\mathrm{Re}\left\{\int_0^{\mathrm{t}}\omega_{\mathrm{k}}(\tau)\mathrm{d}\tau\right\}>-\mathrm{Im}\left\{\xi_{\mathrm{k}}(0)\right\},k=\overline{1,\,N},$$

for all non-negative values of t. For definiteness, we assume that in the sum in the right-handside of (1), the terms with $\mathrm{Im}\,\xi_{\mathrm{k}}>0$, $\mathrm{k}=\overline{1,\mathrm{N}}$ and $\phi=(\phi_1(x,\eta,t),\phi_2(x,\eta,t))$ required the following asimptotics for $x\to\infty$, $\phi\to\begin{pmatrix}h(\eta,t)\ e^{-i\eta x}\\ h(\eta,t)\ e^{i\eta x}\end{pmatrix}$. Where $h(\eta,t)=h(-\eta,t)$ and $|h(\eta,t)|^2\in L^1(-\infty,+\infty)$.

Let us assume that the function $u(x,t)$ has the required smoothness and rather quickly tends to its limits at $x\to\pm\infty$,

$$\int_{-\infty}^{\infty}\left((1+|x|)\,|u(x,t)|+\sum_{k=1}^{3}\left|\frac{\partial^k u(x,t)}{\partial x^k}\right|\right)dx<\infty,\ \ k=1,2,3.$$

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Regimes stochastic in some autowave and oscillator systems with periodic perturbations

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Abstract. Conditions for the existence of stochastic regimes in some models of autowave and oscillatory systems with small periodic perturbations and spatial diffusion are given.

Let us consider the Kuramoto model [1, 2], which reduces in the case of two oscillators to the model system

$$\beta_i\varphi_i'' + \varphi_i' + \alpha_i \cdot \sin\varphi_i = \Delta_i - \gamma_i \cdot \sin\theta_i, \theta_i = \pm\left|\varphi_2 - \varphi_1\right|, i = 1, 2 \tag{1}$$

where$\beta_i, \alpha_i, \gamma_i, \Delta_i$ are parameters, $\varphi_i \in [0, \infty) - 2\pi$ are periodic functions.
Omitting the indices and setting

$$\varphi = x, \varphi' = y, \tau = \sqrt{\frac{\alpha}{\beta}} \cdot t, \mu = \frac{\Delta}{\beta}, \eta = \frac{\alpha}{\beta}, \lambda = \frac{\gamma}{\beta}$$

we have for each of the equations (1) the system:

$$\frac{dx}{d\tau} = y, \frac{dy}{d\tau} = \mu - \eta \cdot \sin x - \lambda \cdot \sin\theta, 0 \le \lambda << 1\},$$

having at $\lambda = \lambda_0 = 0$ on the phase plane $\{x, y\}$centers and saddles alternating with each other on$\{x = 0\}$, the separatrices L_α, L_ω of which form eye-type separatrix contours.

Theorem. In the Kuramoto model (1) in the region$G\left\{\varphi_1, \varphi_2, \theta_1, \theta_2\right\}$ there is a rough homoclinic structure at $\gamma_{1,2} << 1$, corresponding to the stochastic regime.

Similarly, self-organization models (synergetics) are considered, which describe autowave and oscillatory physical, chemical, biological processes that characterize the regimes of stabilization (order) and stochastics (chaos, turbulence) in the transition to small periodic perturbations and spatial diffusion.

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SOLVABILITY OF A MIXED PROBLEM FOR A COMPOSITE-TYPE SYSTEM

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Let $U(t,x)$ and $V(t,x)$ be real functions. System under consideration

$$\begin{cases} \frac{\partial^2 U}{\partial t^2} + \frac{\partial^2 V}{\partial x^2} = aV, \\ \frac{\partial^2 V}{\partial t^2} + \frac{\partial^2 U}{\partial x^2} = bU. \end{cases}$$

at $\Omega = \{(t,x) : 0 \leq x \leq l,\ t > 0\}$. Following the works [1,2], the solution is obtained in the form of series in terms of eigenfunctions of the known classical problem, at the same time the correct statement of the initial conditions is clarified, the absolute and uniform convergence of the series representing the solution, as well as the series obtained as a result of single and double differentiation of these series, is proved. In works [7-10] one can find studies devoted to the solvability of the initial-boundary value problem for systems of a composite type. For studies of one scalar equation of composite type, see [3-6].

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Integration of the negative order korteweg-de vries equation in the class of periodic functions

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The Korteweg-de Vries (KdV) equation is one of the representatives of the class of completely integrable nonlinear partial differential equations, which is of great practical importance. The complete integrability of this equation by the inverse problem method, in the class of rapidly decreasing functions, was first established in [1]. The works [2] are devoted to the investigation of the KdV equation in the class of finite-zone periodic and quasi-periodic functions.

Consider the following Korteweg-de Vries equation of negative order

$$\begin{cases} q_t = -2pp_x \\ pq - p_{xx} = 0 \end{cases}, t > 0, \quad x \in R^1, \tag{1}$$

with the conditions

$$q(x,t)|_{t=0} = q_0(x), \tag{2}$$
$$p(x,t)|_{x=0} = p_0(t),$$

where $q_0(x)$,$p_0(t)$ are given real continuous functions, besides $q_0(x)$ - π-periodic function. It is required to find the real functions $q(x,t)$ and $p^2(x,t)$, which are π - periodic with respect to the variable$x$:

$$p^2(x+\pi,t) \equiv p^2(x,t), q(x+\pi,t) \equiv q(x,t), t \geq 0, \quad x \in R^1 \tag{3}$$

and satisfied the smooth conditions:

$$\begin{array}{l} q(x,t) \in C_x^1(t>0) \cap C_t^1(t>0) \cap C(t \geq 0), \\ p(x,t) \in C_x^2(t>0) \cap C(t \geq 0). \end{array} \tag{4}$$

The purpose of this work is to provide a procedure for constructing a solution to problem (1)-(4), within the framework of the inverse spectral problem for the Sturm-Liouville operator with a periodic coefficient

$$Ly \equiv -y'' + q(x)y = \lambda\, y, x \in R.$$

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Initial-boundary problem for a degenerate fourth order equation containing fractional order integro-differential operator with bessel function in the kernel

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Abstract. In this work, in a rectangular domain, we study an initial-boundary problem for a degenerate second-order differential equation containing an integro-differential operator with a Bessel function in the kernel.

In this paper, in the domain $\Omega = \{(x,t) : 0 < x < 1,\ 0 < t < T\}$, we consider the following degenerate fourth order equation

$$ {}_C D_{0t}^{\delta,\gamma} u(x,t) + bu(x,t) + \Big[x^{\alpha}(1-x)^{\beta} u_{xx}(x,t)\Big]_{xx} = f(x,t), \tag{1} $$

where ${}_C D_{0t}^{\delta,\gamma} u(x,t)$ is the fractional differential operator of the Caputo type with the Bessel function in the kernel [1] of the function $u(x,t)$ with respect to the argument t:

$$ {}_C D_{0t}^{\delta,\gamma} u(x,t) = \frac{1}{\Gamma(2-\delta)} \int\limits_0^t (t-z)^{1-\delta} \bar{J}_{(1-\delta)/2}\ [\gamma(t-z)] \left(\frac{\partial^2}{\partial z^2} + \gamma^2\right) u(x,z)\, dz; $$

$(z)_k$ is Pochhammer's symbol, $\Gamma(x)$ is Euler's gamma - function, $\bar{J}_{\nu}(z)$ is the Bessel - Clifford function defined by $\overline{J}_v(z) = \Gamma(\nu+1)(z/2)^{-\nu} J_{\nu}(z)$, $J_{\nu}(x)$ is Bessel function of the first kind of order ν; $f(x,t)$ is a given function, and $\alpha,\ \beta,\ \gamma,\ \delta,\ b \in R$ are given numbers, such that $0 < \alpha < 1$, $0 < \beta < 1$, $1 < \delta < 2$, $b \geq 0$.

Problem *A.* Find a function $u(x,t)$ with the following properties: 1) $u(x,t)$, u_x, $x^{\alpha}(1-x)^{\beta} u_{xx}$, $\Big[x^{\alpha}(1-x)^{\beta} u_{xx}\Big]_x$ $u_t \in C(\bar{\Omega})$, $\Big[x^{\alpha}(1-x)^{\beta} u_{xx}\Big]_{xx}$, ${}_C D_{0t}^{\delta,\gamma} u \in C(\Omega)$; 2) it satisfies equation (1) in Ω; 3) it satisfies the following initial and boundary conditions on the bound of Ω:

$$ u(x,0) = \varphi_1(x),\ \ u_t(x,0) = \varphi_2(x),\ x \in [0,1]; $$

$$ u(0,t) = 0,\ u(1,t) = 0,\ x^{\alpha}(1-x)^{\beta} u_{xx}\Big|_{x=0} = 0,\ x^{\alpha}(1-x)^{\beta} u_{xx}\Big|_{x=1} = 0,\ t \in [0,T], $$

where $\varphi_1(x)$ and $\varphi_2(x)$ are given functions, such that $\varphi_1(0) = 0,\ \varphi_1(1) = 0,\ \varphi_2(0) = 0,$ $\varphi_2(1) = 0,\ x^{\alpha}(1-x)^{\beta} {\varphi''}_1(x)\Big|_{x=0} = 0\ x^{\alpha}(1-x)^{\beta} {\varphi''}_1(x)\Big|_{x=1} = 0.$

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Cauchy problem for generalized Cauchy-Riemann system with quaternion parameter in inbounded domain

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In this paper, we present an explicit formula for the continuation of the solution of the Cauchy problem for a generalized Cauchy-Riemann system with quaternion parameter

$$\alpha_0 f_0 - divf - < f, \overrightarrow{\alpha} >= 0, \;\; gradf_0 + rotf + [f \times \overrightarrow{\alpha}] + f_0 \overrightarrow{\alpha} + \alpha_0 f = 0, \qquad (1)$$

where $\overrightarrow{\alpha} := (\alpha_1, \alpha_2, \alpha_3); f := (f_1, f_2, f_3), \alpha_k \in C, f_k : R^3 \to C, k = 0, 1, 2, 3$ Suppose that R^3 is real three-dimensional Euclidean space,

$$x = (x_1, x_2, x_3), \;\; y = (y_1, y_2, y_3) \in R^3, \;\; x' = (x_1, x_2), \;\; y' = (y_1, y_2) \in R^2,$$

$$s = \alpha^2 = (y_1 - x_1)^2 + (y_2 - x_2)^2, \;\; r^2 = s + (y_3 - x_3)^2,$$

and Ω is a inbounded simply connected domain in R^3 with boundary $\partial\Omega$ composed of a compact connected part T of the plane $y_3 = 0$ and a smooth Lyapunov surface S lying in the half-space $y_3 > 0$, with $\bar{\Omega} = \Omega \cup \partial D$, $\partial\Omega = S \cup T$. The solution of the Cauchy problem will be constructed in the domain Ω for the case in which the Cauchy data are given on a part S of the boundary. The Cauchy problem for a generalized Cauchy-Riemann system with quaternion parameter is an ill-posed problem.

Following A.N.Tikhonov, we call the function $F_{\sigma\delta}(x)$ a **regularized solution** of the Cauchy problem for a generalized Cauchy-Riemann system with quaternion parameter.The regularized solution determines the stability of the method of the approximate solution of the problem.

Using results M.M. Lavrent'ev and Sh. Yarmukhamedov on solving the Cauchy problem, we construct the Carleman matrix for the Laplace and Helmholts equations in explicit form and, on its basis, the regularized solution of the Cauchy problem for system (1). An existence theorem for the Carleman matrix and a criterion for the solvability of a wider class of boundary-value problems for elliptic systems were given by N. Tarhanov.

On the symmetry group of the two-dimensional heat equation

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In this paper we give solutions of two-dimensional heat-conductivity equation without a source and drain, which are invariant relative to one-parameter symmetry group. In this paper it is used the Lie algebra of the infinitesimal generators of the symmetry group of the two-dimensional heat equation, found in [1].

Consider the two-dimensional heat equation

$$u_t = \sum_{i=1}^{2} \frac{\partial}{\partial x_i}(k_i(u)\frac{\partial u}{\partial x_i}) \tag{1}$$

where $u = u(x_1, x_2, t)$-temperature function, $k_i(u) \geq 0$ function of the temperature. Symmetry groups and the Lie algebra of infinitesimal generators of the symmetry group for the two-dimensional and three-dimensional heat equation are found in [1]. The Lie algebra of infinitesimal generators of the symmetry group for the one-dimensional heat-conduction equation is found in [1].

One of the Lie infinitesimal generators of the symmetry group for equation (2) is the vector field

$$X = x_1\frac{\partial}{\partial x_1} + x_2\frac{\partial}{\partial x_2} + 2t\frac{\partial}{\partial t}, \tag{2}$$

which generates the following group of symmetries of the space of variables (x_1, x_2, t)

$$(x_1, x_2, t) \to (x_1e^S, x_2e^S, te^{2S}),$$

with respect to which the solutions of equation (2) are invariant.It means if $u = u(x_1, x_2, t)$ -solution of equation (2),then for each s function $u = u(x_1e^{-S}, x_2e^{-S}, te^{-2S})$ are also solutions of equation (2).

The numerical integration of this equation shows that there is a point ξ_0, which depends on the initial values of the function $v(\xi)$ and its derivative $v'(\xi)$, such that the derivative of the function $v(\xi)$ is equal to zero: $v'(\xi) = 0$. When $\xi \to \xi_0$ function $v(\xi)$ gradually increases and tends to the value $v(\xi_0)$. Thus, when $\xi \to \xi_0$ the temperature is stabilized. Figure 1 shows a graph of the temperature function under the initial conditions $v(0.1) = 0.9$ $v'(0.1) = 10$

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Carleman's formula for the generalized Cauchy-Riemann system in a bounded domain

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In this paper, we present an explicit formula for the continuation of the solution of the Cauchy problem for a generalized Cauchy-Riemann system

$$\sum_{i=1}^{n}\left(\frac{\partial F_i}{\partial x_i}+H_i\right)=0,$$

$$\frac{\partial F_j}{\partial x_k}-\frac{\partial F_k}{\partial x_j}-H_kF_j+H_jF_k=0,(i,k,j=1,...,n). \tag{1}$$

Suppose that R^n is real $n-$ dimensional Euclidean space,

$$x=(x_1,\dots,x_n),\;\; y=(y_1,\dots,y_n)\in R^n,\;\; x'=(x_1,\dots,x_{n-1}),\;\; y'=(y_1,\dots,y_{n-1})\in R^{n-1},$$

$$s=\alpha^2=|y'-x'|=(y_1-x_1)^2+\cdots+(y_{n-1}-x_{n-1}^2,\;\; r^2=s+(y_n-x_n)^2=|y-x|^2,$$

$\Omega-$ and Ω is a inbounded simply connected domain in R^n with boundary $\partial\Omega$ composed of a compact connected part T of the plane $y_n=0$ and a smooth Lyapunov surface S lying in the half-space $y_n>0$, with $\bar{\Omega}=\Omega\cup\partial\Omega$, $\partial\Omega=S\cup T$. The solution of the Cauchy problem will be constructed in the domain Ω for the case in which the Cauchy data are given on a part S of the boundary. The Cauchy problem for a generalized Cauchy-Riemann system is an ill-posed problem.

Problem. Let we know the Cauchy data for a solution to equation (1)

$$F(y)=f(y),y\in S, \tag{2}$$

where $f(y)=(f_1(y),\cdots,f_n(y))-$ is continuous vector function. It is required to restore the function $F(x)$ in Ω, based on the given f,

Following A.N.Tikhonov, we call the function $F_{\sigma\delta}(x)$ a **regularized solution** of the Cauchy problem for a generalized Cauchy-Riemann system.The regularized solution determines the stability of the method of the approximate solution of the problem.

Using results M.M. Lavrent'ev and Sh. Yarmukhamedov on solving the Cauchy problem, we construct the Carleman matrix for the Helmholts equations in explicit form and, on its basis, the regularized solution of the Cauchy problem for system (1). An existence theorem for the Carleman matrix and a criterion for the solvability of a wider class of boundary-value problems for elliptic systems were given by N.N. Tarhanov.

On transformation of the Cauchy type integral into the Cauchy integral for gravitational fields

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A technique of vector Cauchy integral analogs has been developed for three-dimensional potential fields which extends the basic principles of the classical theory of Cauchy integrals to three-dimensional cases. Cauchy type integrals play an exceptionally important part in the theory of logarithmic potential or in the theory of analytic functions. They are used in the analytical continuation of fields, in finding the location and properties of their singular points and in determining the uniqueness of the solution of the inverse problem. This approach has been developed in detail by V.N. Strakhov, G.M. Voskoboinikov, G.Ya. Golizdra , A.V. Tsirulskiy and many others.

The importance and the need for generalizing the results of the theory of functions complex variable of two-dimensional theory to three-dimensional case are quite evident. These problems were investigated first by V.N. Strakhov 1970 y., 1974 y., who also studied an important particular case techniques of the theory of functions of a complex of axisymmetric problem (Strakhov, 1976). In the papers (M.S.Zhdanov, 1973-1976 y.) it has been shown that several results of the logarithmic potential theory can be extended to three-dimensional case, using certain analogs of the Cauchy type integrals for the three-dimensional fields which are the modifications of the integrals introduced by Moisil, Teodoresko are Bitsadze (1953, 1972 y.)

In 1952 y. I.I.Privalov and V.V.Golubev found the necessary and sufficient conditions for the Cauchy type integral to transform into the Cauchy integral. Developing the ideas of I.I.Privalov and V.V.Golubev, E.D. Solomentzev (1966 y.) consider the problem on transformation of the Green formula for the Laplace equation.In the paper, with the use of the formulas Green type integral and an expansion of the fundamental solution to the Laplace equation into a series by spherical functions, he gives necessary and sufficient conditions for transformation of the Green type integral into the integral Green formula.

In the paper, we consider the question on transformation of the Cauchy type integral into the Cauchy integral for gravitational fields.

On transformation of the Cauchy type integral into the Cauchy integral for magnetic fields

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In the paper, we consider the question on transformation of the Cauchy type integral into the Cauchy integral for magnetic fields.

A technique of vector Cauchy integral analogs has been developed for three-dimensional potential fields which extends the basic principles of the classical theory of Cauchy integrals to three-dimensional cases. Cauchy type integrals play an exceptionally important part in the theory of logarithmic potential or in the theory of analytic functions. They are used in the analytical continuation of fields, in finding the location and properties of their singular points and in determining the uniqueness of the solution of the inverse problem. This approach has been developed in detail by V.N. Strakhov, G.M. Voskoboinikov, G.Ya. Golizdra , A.V. Tsirulskiy and many others. I.I.Privalov and V.V.Golubev [1] found the necessary and sufficient conditions for the Cauchy type integral to transform into the Cauchy integral.

Developing the ideas of I.I.Privalov and V.V.Golubev, E.D. Solomentzev [2] consider the problem on transformation of the Green formula for the Laplace equation.In the paper, with the use of the formulas Green type integral and an expansion of the fundamental solution to the Laplace equation into a series by spherical functions, he gives necessary and sufficient conditions for transformation of the Green type integral into the integral Green formula.

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Walsh-Nikolskii-type inequalities in the complex plane

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Let $\mathbb{C}$ be a complex plane and $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$; $G \subset \mathbb{C}$ be a bounded Jordan region with boundary $L := \partial G$ such that $0 \in G$; $\Omega := \overline{\mathbb{C}} \setminus \overline{G} = extL$; $\Delta := \Delta(0,1) := \{w : |w| > 1\}$. Let $w = \Phi(z)$ be the univalent conformal mapping of Ω onto Δ such that $\Phi(\infty) = \infty$ and $\lim_{z\to\infty} \frac{\Phi(z)}{z} > 0$.

Let $\wp_n$ denotes the class of all algebraic polynomials $P_n(z)$ of degree at most $n \in \mathbb{N}$, and let $h(z)$ be some weight function.

Let $0 < p \leq \infty$ and σ be the two-dimensional Lebesgue measure. For the rectifiable Jordan curve L, we introduce:

$$\begin{aligned} \|P_n\|_{\mathcal{L}_p(h,L)} &:= \left(\int_L h(z) \left| P_n(z) \right|^p |dz| \right)^{1/p}, \ 0 < p < \infty, \\ \|P_n\|_{\mathcal{L}_\infty(1,L)} &:= \max_{z\in L} \left| P_n(z) \right|, \ p = \infty, \end{aligned}$$

In this work, we study for more general regions pointwise estimation in unbounded region Ω, for the derivative $\left| P_n^{(m)}(z) \right|$, $m = 0, 1, 2, \ldots$, in the following type:

$$\left| P_n^{(m)}(z) \right| \leq \eta_n(G, h, p, m, z) \, \|P_n\|_{\mathcal{L}_p(h,L)}, z \in \Omega,$$

where $\eta_n(G, z, \ldots) \to \infty$, as $n \to \infty$, depending on the properties of the G, h.

The report will also give estimates of the growth of the module $\left| P_n^{(m)}(z) \right|$ on bounded closed regions.

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Gibbs measures for the SOS model with competing interactions on a Cayley tree of order two

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Let $\Gamma^k = (V, L)$ be the Cayley tree of order k. Let us assume that the spin values belong to the set $\Phi = \{0, 1, 2\}$.

The Hamiltonian of the SOS model with competing interactions has the form:

$$H(\sigma) = -J_1 \sum_{\langle x,y\rangle \in L} |\sigma(x) - \sigma(y)| - J_2 \sum_{\substack{x,y\in V: \\ d(x,y)=2}} |\sigma(x) - \sigma(y)|, \quad (1)$$

where $(J_1, J_2) \in \mathbb{R}^2$.

Let M be the set of all unit balls with vertices in V. We define the energy of the configuration σ_b on b by the following formula

$$U(\sigma_b) \equiv U(\sigma_b, J_1, J_2) = -\frac{1}{2} J_1 \sum_{\substack{\langle x,y\rangle: \\ x,y\in b}} |\sigma(x) - \sigma(y)| - J_2 \sum_{\substack{x,y\in b: \\ d(x,y)=2}} |\sigma(x) - \sigma(y)|,$$

where $(J_1, J_2) \in \mathbb{R}^2$.

Theorem 1. Let $k = 2$. If $(J_1, J_2) \neq (0, 0)$, then the Peierls condition is satisfied.

Let $\widetilde{A_1} = \{(J_1, J_2) \in \mathbb{R}^2 \mid J_1 < 0; J_2 < -\frac{1}{4}J_1\}$.

The following theorem can be proved very similarly to corresponding theorems of [2], [3] and [4].

Theorem 2. If $(J_1, J_2) \in \widetilde{A_1}$, then for all sufficiently large β there are at least three Gibbs measures for the model (1) on Cayley tree of order two.

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Geometry on subspaces of pseudo-euclidean space $^1R_5$

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The space $^1R_5$ is called a five-dimensional pseudo-Euclidean space of index one or a five-dimensional Minkowski space. This space is a special case of the pseudo-Euclidean space ^lR_n defined in the monograph "Non-Euclidean spaces"by B.A. Rosenfeld.

Let in $^1R_5$ it is given a coordinate system with the origin at point O and the basis vectors $\{e_1, e_2, e_3, e_4, e_5\}$. Then the scalar product of vectors $\overline{X}\{x_1, x_2, x_3, x_4, x_5\}$ and $\overline{Y}\{y_1, y_2, y_3, y_4, y_5\}$ is defined by the formula $(\overline{X} \cdot \overline{Y}) = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 + x_5y_5$. Since the bases e_i satisfy the condition $e_1^2 = e_2^2 = e_3^2 = e_4^2 = 1, e_5^2 = -1$, then $(e_i, e_j) = 0$, when $i \neq j$.

We will be interested geometries on planes of general position and the geometry of some subsets of space $^1R_5$.Therefore, we give a definition of semi-pseudo-Euclidean spaces. To do this, we divide the coordinates x_i of the vector X of the space into groups of coordinates

$$x_{a_1}(m_1 = 1 \leq a_1 \leq m_i)$$

$$x_{a_2}(m_1 = 1 < a_2 \leq n)$$

Definition. A semi-pseudo-Euclidean space $^{l_1 l_2}R_n^m$ will be called an affine space A_n in which the scalar product has the form: $(X, Y) = \sum_{a_i = m_{i-1}}^{m_i} E_{a_i} x_{a_i} y_{a_i}$, where

$$E_{a_i} = \begin{cases} -1, & if \quad m_{i-1} < a_i \leq m_{i-1} + e_i; \\ 0, & if \quad m_{i-1} + e_i < a_i \leq n_i, \end{cases} \qquad i = 1, 2,$$

moreover, the second scalar product is defined only for those vectors for which the condition $x_{a_i} = 0$ is satisfied.

When in a pseudo-Euclidean or semi-pseudo-Euclidean space, index $l_i = 0$, these spaces will be Euclidean or, respectively, semi-Euclidean spaces.

Theorem 1.The subspace $\{M(x_1, x_2, x_3, x_4, x_5\}$ space $^1R_5$ has a metric semi-Euclidean of the space $^{10}R_4^3$.

Theorem 2. A sphere of real radius is a pseudo-Riemannian manifold $^1M_4$.

Theorem 3. The geometry of the intersection of the unit sphere of space $^1R_5$ with the plane $x_5 - x_4 = 0$, is equivalent to the geometry of space $(S_2 \times R)$.

By similar reasoning, it can be proved that the intersection of a unit sphere of imaginary radius with the plane $x_1 - x_2 = 0$ is equivalent to the geometry of space$^lS_2 \times R$ is one of the geometries listed by Thurston [1].

Keywords: pseudo-Euclidean spaces, semi-pseudo-Euclidean spaces, scalar product, vector norm, distance, imaginary distance, pseudo-Riemannian space.

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A comparison on some integral transforms

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Integral transforms are important to solve real problems. Appropriate choice of integral transforms helps to convert differential equations as well as integral equations into terms of an algebraic equation that can be solved easily [1, 2].

During last two decades many integral transforms in the class of Laplace transform are introduced such as Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, G_transform, Sawi and Kamal transforms [1].

In this work, we compare some integral transforms in the class of Laplace transform which are introduced during last few decades. After that we propose a general integral transforms which is covered all of those integral transforms [1].

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Gibbs measures for the three-state Hard-Core models in the case Triangle

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Phi = \{0, 1, 2\}$ and $\sigma \in \Omega = \Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. The activity set [1] for a graph G is a function $\lambda : G \to R_+$. For given G and λ we define the Hamiltonian of the $G-$HC model as $H_G^\lambda(\sigma) = \sum\limits_{x\in V} \ln \lambda_{\sigma(x)}$, if $\sigma \in \Omega^G$. The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory in [2], [3].

Let $L(G)$ be the set of edges of a graph G, and let $A \equiv A^G = (a_{ij})_{i,j\in Z}$ denote the adjacency matrix of G. We consider the case $\lambda_0 = 1,\ \lambda_1 = \lambda_2 = \lambda$ and the case graph $G = triangle$: $\{0, 1\}\{0, 2\}\{1, 2\}$.

It is know [3] that for any $z : x \in V \longmapsto z_x = (z_{1,x}, z_{2,x})$, satisfying the equalities

$$z_{i,x} = \lambda \prod_{y\in S(x)} \frac{a_{i0} + a_{i1}z_{1,y} + a_{i2}z_{2,y}}{a_{00} + a_{01}z_{1,y} + a_{02}z_{2,y}}, i = 1, 2 \tag{1}$$

there exists a unique HC-Gibbs measure μ, and vice versa. We study translation-invariant and two-periodic solutions (1) (see [3]).

We consider the following invariant sets:

$$I_1 = \{(t_1, t_2, z_1, z_2) \in R^4 : t_1 = t_2 = z_1 = z_2\},$$

$$I_2 = \{(t_1, t_2, z_1, z_2) \in R^4 : t_1 = t_2, z_1 = z_2\},$$
$$I_3 = \{(t_1, t_2, z_1, z_2) \in R^4 : t_1 = z_1, t_2 = z_2\},$$
$$I_4 = \{(t_1, t_2, z_1, z_2) \in R^4 : t_1 = z_2, t_2 = z_1\}.$$

The following theorem is true.

Theorem. For HC model in the case $G = triangle$ the following statements are true:

1. For $k \geq 2$, $\lambda > 0$ TIGM μ_0 is unique.

2. Let $k \geq 2$, $\lambda > 0$. Then on I_1,I_2,I_3 there is only one two-periodic Gibbs measure, which coincides with the unique TIGM μ_0.

3. Let $k = 2$, $\lambda_{cr} = 1$. Then on I_4 for $0 < \lambda_{cr} \leq 1$ two-periodic Gibbs measure is unique, and it coincides with the unique TIGM μ_0, for $\lambda_{cr} \geq 1$ there are exactly three Gibbs measures μ_i, $i = 0, 1, 2$, where μ_1, μ_2 are two-periodic Gibbs measures.

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On the discrete spectrum of a system corresponding to a 2+1-fermionic trimer on a Lattice

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A three-particle discrete Schrödinger operator $H_{\mu,\gamma}(\mathbf{K}), \mathbf{K} \in \mathbb{T}^3$ associated with a system of three particles (two fermions with the mass 1 and one more particle with the mass $m = 1/\gamma < 1$) interacting through zero-range potentials $\mu > 0$ on the three-dimensional lattice $\mathbb{Z}^3$ is considered. The operator $H_{\mu,\gamma}(\mathbf{0}), \mathbf{0} = (0,0,0)$ is proved to have no eigenvalues for $\gamma \in (1, \gamma_0), (\gamma_0 \approx 4.7655)$ and have the unique eigenvalue with multiplicity three for $\gamma > \gamma_0$, which lies to the left of the essential spectrum for sufficiently big μ.

Let $\mathbb{T}^3$ is a three-dimensional torus and $L_2^{as}[(\mathbb{T}^3)^2] \subset L_2[(\mathbb{T}^3)^2]$ is a Hilbert space of quadratically integrable functions given on $(\mathbb{T}^3)^2$ and antisymmetric with respect to permutation of the coordinates.

Three-particle discrete Schrödinger operator $H_{\mu,\gamma}(\mathbf{K})$ acts in $L_2^{as}[(\mathbb{T}^3)^2]$ by the formula

$$H_{\mu,\gamma}(\mathbf{K}) = H_{0,\gamma}(\mathbf{K}) - \mu(V_1 + V_2),$$

where

$$(H_{0,\gamma}(\mathbf{K})f)(\mathbf{p},\mathbf{q}) = E_{\mathbf{K},\gamma}(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q}),$$

$$E_{\mathbf{K},\gamma}(\mathbf{p},\mathbf{q}) = \varepsilon(\mathbf{p}) + \varepsilon(\mathbf{q}) + \gamma\varepsilon(\mathbf{K}-\mathbf{p}-\mathbf{q}), \quad \varepsilon(\mathbf{p}) = \sum_{i=1}^{3}(1-\cos p_i).$$

$$(V_1 f)(\mathbf{p},\mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{p},\mathbf{s})d\mathbf{s}, \quad (V_2 f)(\mathbf{p},\mathbf{q}) = \int\limits_{\mathbb{T}^3} f(\mathbf{s},\mathbf{q})d\mathbf{s}.$$

Let

$$\gamma_0 = \left(\int\limits_{\mathbb{T}^3} \frac{\sin^2 s_1}{\varepsilon(\mathbf{s})}d\mathbf{s}\right)^{-1} \approx 4,7655.$$

The results of the work are given for the case $\mathbf{K} = \mathbf{0}$ and include the following theorem.

Theorem. a) Let $\gamma \in (1, \gamma_0)$. Then, there exists $\mu_\gamma > 0$ such that for any $\mu > \mu_\gamma$ the operator $H_{\mu,\gamma}(\mathbf{0})$ has no eigenvalues below the essential spectrum.

b) Let $\gamma > \gamma_0$. Then, there exists $\mu_\gamma > 0$ such that for any $\mu > \mu_\gamma$ the operator $H_{\mu,\gamma}(\mathbf{0})$ has the unique triple eigenvalue below the essential spectrum.

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The existence of H_A-weakly periodic p-adic generalized Gibbs measures for the p-adic Ising model on the Cayley tree of order two

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Let $\mathbb{Q}_p$ be the field of p-adic numbers(see [1]).

Let $\Gamma^k(V, L)$ be Cayley tree (see [2]) and $\Phi = \{-1, 1\}$. A configuration σ on $A \subset V$ is defined by the function $x \in A \to \sigma(x) \in \Phi$. The set of all configurations on A is denoted by $\Omega_A = \Phi^A$, and $\Omega_V := \Omega$.

A formal p-adic Hamiltonian $H : \Omega \to \mathbb{Q}_p$ of the p-adic Ising model is defined by

$$H(\sigma) = J \sum_{<x,y>\in L} \sigma(x)\sigma(y), \tag{1}$$

where $|J|_p < p^{-1/(p-1)}$ for any $\langle x, y\rangle \in L$.

We define a function $h : x \to h_x$, $\forall x \in V\setminus\{x_0\}$, $h_x \in \mathbb{Q}_p$ and consider p-adic probability distribution $\mu_h^{(n)}$ on Ω_{V_n} defined by

$$\mu_h^{(n)}(\sigma_n) = \frac{1}{Z_n^{(h)}} \exp_p\{H_n(\sigma_n)\} \prod_{x\in W_n} h_{\sigma(x),x} \quad n = 1, 2, \dots, \tag{2}$$

where $Z_n^{(h)}$ is the normalizing constant.

Theorem 1.[3] The sequence of p-adic probability distributions $\{\mu_h^{(n)}\}_{n\geq 1}$, determined by formula (2) is consistent if and only if for any $x \in V\setminus\{x_0\}$, the following equation holds

$$h_x^2 = \prod_{y\in S(x)} \frac{\theta h_y^2 + 1}{h_y^2 + \theta},$$

where $\theta = \exp_p(2J), \theta \neq 1$.

In this case, by the p-adic analogue of the Kolmogorov theorem there exists a unique measure μ_h on the set Ω such that $\mu_h\left(\{\sigma|_{V_n} \equiv \sigma_n\}\right) = \mu_h^{(n)}(\sigma_n)$ for all n and $\sigma_n \in \Omega_{V_n}$ (see [2]).

Theorem 2. *Let $|A| = 1$ and N be the number of H_A-weakly periodic (non-periodic) p-adic generalized Gibbs measures for the model (1) on the Cayley tree of order two. Then the following statements hold:*

$$N = \begin{cases} 6, & \text{if } \ p \equiv 1 (\mathrm{mod}\, 4), \\ 1, & \text{if } \ \text{otherwise.} \end{cases}$$

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Characteristics of the fixed points of the Ising-Potts mapping with a parameter.

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Let $\mathbb{Q}$ be the field of rational numbers. The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$ (see [1]).

Let $\Gamma^k(V, L)$ be Cayley tree (see [2]) and $\Phi = \{-1, 1\}$. A configuration σ on $A \subset V$ is defined by the function $x \in A \to \sigma(x) \in \Phi$. The set of all configurations on A is denoted by $\Omega_A = \Phi^A$, and $\Omega_V := \Omega$.

A formal hamiltonian $H : \Omega \to \mathbb{Q}_p$ of the p-adic Ising model with external field is defined by

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y) + \alpha \sum_{x\in V} \sigma(x),$$

where J, α are constants such that $|J|_p < 1, |\alpha|_p < 1$.
Denote

$$\mathbb{Z}_p^* = \{x \in \mathbb{Q}_p : |x|_p = 1\}, \mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x-1|_p < p^{-1/(p-1)}\right\},$$
$$Fix(f) = \{x \in \mathbb{Q}_p : f(x) = x\}.$$

Let h^* be a fixed point of an analytic function $f(h)$, i.e. $f(h^*) = h^*$. Let $\lambda = \frac{d}{dh} f(h^*)$. The fixed point h^* is called *attractive* if $0 \le |\lambda|_p < 1$, *indifferent* if $|\lambda|_p = 1$, and *repelling* if $|\lambda|_p > 1$. Studying p-adic generalized Gibbs measures for the Ising model with external field on the Cayley tree of order k is equivalent to study dynamics of the following function:

$$f_{\eta,\theta,k}(h) = \eta^{k+1} \left(\frac{\theta h + 1}{h + \theta}\right)^k,$$

here $\theta, \eta \in \mathcal{E}_p$ (see [3]). Note that if $\eta = 1$ the function $f_{\theta,k}(h)$ is called Ising-Potts mapping.
We get the following result:

Theorem. *Let* $p \ge 3$ *and* h^* *be an arbitrary fixed point of* $f_{\eta,\theta,k}(h)$. *Then the following statements are true:*

1. $Fix(f_{\eta,\theta,k}) \subset \mathbb{Z}_p^*$;
2. *If* $|k|_p < |\theta^2 - 1|_p$ *or* $h^* \in \mathcal{E}_p$ *then* h^* *is an attracting fixed point;*
3. *If* $|\theta^2 - 1|_p < |k|_p < 1$ *or* $|k|_p = 1$ *then* h^* *is a repelling fixed point;*
4. *If* $|\theta^2 - 1|_p = |k|_p$ *then* h^* *is an indifferent fixed point.*

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Translation-invariant p-adic generalized Gibbs measures for the SOS model on the Cayley tree of order two.

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Let $\mathbb{Q}$ be the field of rational numbers. The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$ (see [1]).

Let $\Gamma^k(V, L)$ be Cayley tree (see [2]) and $\Phi = \{0, 1, 2\}$. A configuration σ on $A \subset V$ is defined by the function $x \in A \to \sigma(x) \in \Phi$. The set of all configurations on A is denoted by $\Omega_A = \Phi^A$, and $\Omega_V := \Omega$.

A formal p-adic Hamiltonian $H : \Omega \to \mathbb{Q}_p$ of the p-adic SOS model is defined by

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L_n} |\sigma(x) - \sigma(y)|_\infty, \qquad \sigma \in \Omega_{V_n}$$

where $0 < |J|_p < p^{-1/(p-1)}$ for any $\langle x, y\rangle \in L$.

We define a function $z : x \to z_x$, $\forall x \in V\setminus\{x_0\}$, $z_x \in \mathbb{Q}_p$ and consider p-adic probability distribution $\mu_z^{(n)}$ on Ω_{V_n} defined by

$$\mu_{\mathbf{z}}^{(n)}(\sigma) = Z_{n,\mathbf{z}}^{-1} \exp_p\{H_n(\sigma)\} \prod_{x\in W_n} z_{\sigma(x),x}, \tag{1}$$

where $Z_n^{(z)}$ is the normalizing constant.

Theorem 1.[3] *The $p-$adic probability distributions $\mu_{\tilde{z}}^{(n)}(\sigma_n)$, $n = 1, 2, ...$ in (1) are compatible for p-adic SOS model iff for any $x \in V\setminus\{x^0\}$ the following system of equations holds:*

$$\tilde{z}_{i,x} = \prod_{y\in S(x)} \frac{\sum_{j=0}^{m-1} \theta^{|i-j|_\infty} \tilde{z}_{j,y} + \theta^{m-i}}{\sum_{j=0}^{m-1} \theta^{m-j} \tilde{z}_{j,y} + 1}, \qquad i = 0, 1, ..., m-1, \tag{2}$$

here $\theta = \exp_p(J)$ and $z_{i,x} = \tilde{z}_{i,x}/\tilde{z}_{m,x}$, $i = 0, 1, ..., m-1$.

In this case, by the p-adic analogue of the Kolmogorov theorem there exists a unique measure μ_z on the set Ω such that $\mu_z(\{\sigma|_{V_n} \equiv \sigma_n\}) = \mu_z^{(n)}(\sigma_n)$ for all n and $\sigma_n \in \Omega_{V_n}$ (see [2]).

p-adic Gibbs measures (not generalized) are considered in [3]. In this paper we consider translation-invariant p-adic generalized Gibbs measures for the three state SOS model on the Cayley tree of order two and we get the following result:

Theorem 2. *If $p = 19$, then there exist exactly three translation-invariant generalized p-adic Gibbs measures for the three state SOS model on the Cayley tree of order two.*

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The asymptotic formula in Waring's problem with almost proportional summands

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Investigating Waring's problem with almost proportional summands, M. E. Wright [1,2] proved that for the number of representations of a sufficiently large number N in the form

$$x_1^n + x_2^n + \ldots + x_r^n = N,$$

where $x_1, x_2, \ldots, x_r$ — are non-negative integers, and

$$|x_i^n - \mu_i N| \le N^{1-\theta_1(n,r)+\varepsilon}, \qquad i = 1, 2, \ldots, r, \qquad \mu_1 + \mu_2 + \ldots + \mu_r = 1,$$

the asymptotic formula holds for

$$r \ge r_1(n), \qquad r_1(n) = (n-2)2^{n-1} + 5, \qquad n \ge 3,$$

where the number $\theta_1(n,r)$ is determined from the relation

$$\theta_1(n,r) = \frac{1}{n}\min\left(\frac{(r-2^n)(2^{n-1}+1)}{(nr+n-2^n-3)2^{n-1}+r}, \frac{r-(n-2)2^{n-1}-4}{r+2^{n-1}-4}, \frac{r-2^{n-1}}{nr-2^{n-1}+n-1}\right).$$

This report is devoted to the strengthening of the result of E. M. Wright, namely, it gives the derivation of an asymptotic formula in Waring's problem with almost proportional summands, where the number of summands is smaller than in the result of E. M. Wright:

$$r \ge r_2(n), \qquad r_2(n) = 2^n + 1, \qquad \theta_2(n,r) = \frac{1}{(r+1)(n^2-n)}.$$

Hence, in particular, we have

$$r_2(3) = r_1(3) = 9, \qquad \theta_1(n,r) = \frac{1}{51}, \qquad \theta_2(n,r) = \frac{1}{30},$$
$$r_2(n) < r_1(n) \qquad \text{at} \qquad n \ge 4.$$

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Weyl formula for matrix functions

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Let $z = (z^1, ..., z^n)$ be a vector composed of square matrices z^j of order m, i.e. $z = (z^1, ..., z^n) \in \mathbb{C}^n[m \times m]$ and

$$T_{n,r} = \{z \in \mathbb{C}^n[m \times m] : Ir_j - z^k(z^k)^* > 0, 1 \leq k \leq n\}$$

be a matrix polydisk of radius $r = (r_1, ..., r_n)$ centered at the origin.
Let $f = (f^1(z), ..., f^n(z)) : \mathbb{C}^n[m \times m] \to \mathbb{C}^n[m \times m]$ be a holomorphic map in a domain $G \subset \mathbb{C}^n[m \times m]$ where $z^k = (z^k_{ij})$ and $f^k = (f^k_{ij})$ are square matrices of order m.

Definition. The inverse image $\Pi_{f,r} = f^{-1}(T_{n,r})$ of the matrix polydisc $T_{n,r}$ by the map $f = (f^1(z), ..., f^n(z)) : \mathbb{C}^n[m \times m] \to \mathbb{C}^n[m \times m]$ is called a matrix polyhedral set, if $\Pi_{f,r} \Subset G$. A matrix polyhedron is a connected components matrix polyhedral set.

According to Hefer's theorem ([1]), for some neighborhood U of the matrix polyhedron $\Pi_{f,r}$ there are functions $P^{ijk}_{slt} \in Hol(U \times U)$ such that for all $(\zeta, z) \in (U \times U)$ the equalities

$$f^k_{ij}(\zeta) - f^k_{ij}(z) = \sum_{s,l=1}^{m} \sum_{t=1}^{n} (\zeta^t_{sl} - z^t_{sl}) P^{ijk}_{slt}(\zeta, z), \quad i, j = \overline{1, m}, k = \overline{1, n}$$

We denote by $H(\zeta, z)$ the determinant of the matrix $P^{ijk}_{slt}(\zeta, z)$ of order $nm^2 \times nm^2$ the rows of which are numbered by i, j, k and the columns by s, l, t.

Theorem. Let $h(z)$ be a holomorphic function in $\overline{\Pi}_{f,r}$. Then for any $z \in \Pi_{f,r}$, we have the following formula

$$h(z) = \int\limits_{\Gamma_{f,r}} \frac{h(\zeta)H(\zeta, z)\dot{\zeta}}{det^m(f(\zeta) - f(z))} \tag{1}$$

where $\Gamma_{f,r} = \{\zeta \in G : f^j(\zeta)(f^j(\zeta))^* = r^2_j I, j = \overline{1, n}\}$ is a polyhedral skeleton of $\Pi_{f,r}$ and $det^m(f(\zeta) - f(z)) = \prod\limits_{k=1}^{n} det^m(f^k(\zeta) - f^k(z))$.

In the case $f(z) \equiv z$ formula (1) represents the Bokhner-Hua Loken formula for the matrix polydisk ([2]).

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Orthogonality of the lines given in an implicit form

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Theorem. The function $w = f(z)$ is holomorphic at the point z_0, and the following conditions satisfy for the smooth lines γ_1, γ_2 passing through the point z_0:

$$\begin{cases} |f(z)| = |f(z_0)|, \ z \in \gamma_1 \\ arg f(z) = arg f(z_0), \ z \in \gamma_2 \end{cases} \tag{1}$$

If $f^{'}(z_0) \neq 0$, then we prove that lines γ_1, γ_2 intersect at a right angle at the point z_0.

Proof. According to the given conditions, let the lines γ_1, γ_2 be given in the (1) form. If we use the equality $f(z) = u(x,y) + iv(x,y)$, the distribution of lines γ_1, γ_2 will consist of the following system of equations:

$$\begin{cases} \sqrt{u^2(x,y) + v^2(x,y)} = \sqrt{u^2(x_0,y_0) + v^2(x_0,y_0)}, \ z \in \gamma_1 \\ \frac{v(x,y)}{u(x,y)} = \frac{v(x_0,y_0)}{u(x_0,y_0)}, \ z \in \gamma_2 \end{cases} \tag{2}$$

So, it can be seen that since $f^{'}(z_0) \neq 0$, the system (2) represents smooth lines given in an implicit form. According to the implicit function theorem, γ_1, γ_2 lines are represented by $y_1 = y_1(x), y_2 = y_2(x)$ functions in some neighborhood of point (x_0, y_0) respectively. In term of the giving of lines and the calculation of the derivative of the implicit function, we have the following equations:

$$y_1^{'}(x) = -\frac{uu_x + vv_x}{uu_y + vv_y}, \ y_2^{'}(x) = -\frac{vu_x - uv_x}{vu_y - uv_y}.$$

Thus

$$y_1^{'}(x) \cdot y_2^{'}(x) = \frac{uu_x + vv_x}{uu_y + vv_y} \cdot \frac{vu_x - uv_x}{vu_y - uv_y}. \tag{3}$$

From $f(z)$ is analytic and equality (3), the Cauchy-Riemann equations are valid, i.e. we have the following equality:

$$y_1^{'}(x) \cdot y_2^{'}(x) = \frac{uu_x + vv_x}{uu_y + vv_y} \cdot \frac{vu_x - uv_x}{vu_y - uv_y} = -\frac{uu_x + vv_x}{uu_y + vv_y} \cdot \frac{uu_y + vv_y}{uu_x + vv_x} = -1.$$

In order for the given lines γ_1, γ_2 to intersect at right angles, it is enough to satisfy the equality $y_1^{'}(x) \cdot y_2^{'}(x) = -1$. Thus, γ_1, γ_2 lines intersect at right angles.

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Multiple fractional integral differentiation of local type

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A modification of Liouville's fractional integro-differentiation is considered in the article for the case of functions of two variables, "attached"at some fixed point and convenient in that it is applicable to functions with any behavior at infinity. This modification is local in the sense that it is applicable to functions defined on the plane, but when calculated at the end point, it does not require knowledge of the function values in the neighborhood of infinity [1-2].

The paper is devoted to various ways of "reduction"of the Marcheau-Cheng constructions for fractional differentiation $\mathbb{D}_c^\alpha f$, and these different options for "reduction"are used to describe and invert fractional integrals $I_c^\alpha \varphi$ of functions from $L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$. The following results were obtained in this study:

Theorem 1. Let $f = I_c^\alpha \varphi,\ \varphi \in L_{\overline{p}}\left(\mathbb{R}^2\right)$ (or $\varphi \in L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$), $1 \leq p_i < \infty,\ \alpha_i > 0,\ i = 1, 2,\ c \in \mathbb{R}^2$. Then

$$\left(\mathbb{D}_c^\alpha f\right)(x) = \lim_{\varepsilon \to 0} \left(\mathbb{D}_{c,\varepsilon}^\alpha f\right)(x) = \varphi(x),$$

where limits $\lim\limits_{\varepsilon \to 0}$ – as double and repeated $\lim\limits_{\varepsilon_1 \to 0} \lim\limits_{\varepsilon_2 \to 0} = \lim\limits_{\varepsilon_2 \to 0} \lim\limits_{\varepsilon_1 \to 0}$ –exist in space $L_{\overline{p}}\left(\mathbb{R}^2\right)$ (in $L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$ respectively). Moreover, both repeated limits also exist almost for all $x \in \mathbb{R}^2$.

Theorem 2. For function $f(x)$ to be representable as $f(x) = \left(I_c^\alpha \varphi\right)(x),\ \varphi \in L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$, where $\alpha_i > 0,\ 1 \leq p_i < \infty,\ i = 1, 2,\ (c_1, c_2) \in \mathbb{R}^2$, it is necessary and sufficient that $f(x) \in L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$ and that in $L_{\overline{p}}^{loc}\left(\mathbb{R}^2\right)$ exist $\lim\limits_{\varepsilon \to 0} \varphi_\varepsilon(x)$, where $\varphi_\varepsilon(x) = \left(\mathbb{D}_{c,\varepsilon}^\alpha f\right)(x)$.

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VII. THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

λ_q-almost statistical convergence of weight g

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The purpose of this paper is to introduce the space of sequences that are strongly $(V, \lambda, q)-$ summable of weight g and also present the concept of (λ, q)-almost statistical convergence of weight $g : [0, \infty) \to [0, \infty)$ where $g(x_n) \to \infty$ for any sequence (x_n) in $[0, \infty)$ with $x_n \to \infty$. First we examine some relations between λ_q-almost statistical convergence of weight g and strong $(\hat{V}, \lambda, q)$-almost summability of weight g and also relations between the spaces $[\hat{V}^g, q, p, \lambda, f]$ and $\hat{S}^g(\lambda, q)$.

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Central limit theorem for mixing random variables with values in $L_p[0,1]$ space

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Central limit theorems in Banach spaces are well studied in the case of independent identically distributed random elements (see [1]). However, the case of weakly dependent random elements is less studied. Our goal is to establish a central limit theorem for weakly dependent random variables with values in $L_p[0,1]$ space.

We say that $\{X_i(t), i \geq 1\}$ a sequence of centered random variables in $L_p[0,1]$ satisfies central limit theorem if the following weak convergence holds:

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i(t) \Rightarrow N(t)$$

where $N(t)$ is some $L_p[0,1]$-valued Gaussian random variable with mean zero.

We will assume that $\{X_n(t), n \geq 1\}$ satisfies mixing condition. For the sequence of $L_p[0,1]$-valued random variables $\{X_n(t), n \geq 1\}$ mixing coefficient defined as:

$$\varphi_m(n) = \sup_{R^m} \sup\{|P(A/B) - P(A)| : B \in \Im_1^k(m), A \in \Im_{k+n}^{\infty}(m),\ k \in N\}$$

where $\Im_a^b(m)$-is σ-field generated by random variables $\prod_m X_a(t), ..., \prod_m X_b(t)$ and $\prod_m : L_p[0,1] \to R^m$ is projective operator i.e. $\prod_m X_i(t) = (X_i(t_1), ..., X_i(t_m))$, $t_i \in [0,1]$.

Our main result is the following

Theorem. Let $\{X_i(t), i \geq 1\}$ be a strictly stationary sequence of random variables with values in $L_p[0,1]$,$p \geq 2$ and $\sum_{k=1}^{\infty} \varphi_m^{\frac{1}{2}}(2^k) < \infty$, $m = 1, 2, ...$

$$EX_1(t) = 0, E\left|X_1(t)\right|^2 < \infty$$

$E\left|X_1(t+h) - X_1(t)\right|^p \leq f(h)$, such that $f(h) \to 0$ as $h \to 0$.

Then $\{X_i(t), i \geq 1\}$ satisfies central limit theorem.

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A stochastic frontier model for the production function

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Abstract: This note aims to analyze the productive efficiency estimation through a stochastic frontier analysis approach.

Stochastic Frontier Analysis was developed independently by Aigner et al (1977) and Meeusen and Van den Broeck (1977) in order to overcome the main short coming of previous efficiency estimation methods. A production frontier model can be written as

$$y_i = f(x_i, \beta)\, TE_i$$

where y_i is the output of producer $i(i = 1, 2, ..., N)$ x_i is a vector of M inputs used by producer i, $f(x_i, \beta)$ is the production frontier and β is a vector of technology parameters to be estimated.

To incorporate the fact that output can be affected by random shocks into the analysis, we have to specify the stochastic production frontier

$$y_i = f(x_i, \beta) \exp(v_i)\, TE_i$$

Under the assumption that $f(x_i, \beta)$ is of Cobb-Douglas type, the stochastic frontier model in $y_i = f(x_i, \beta) \exp(v_i)\, TE_i$ can be written in logs as

$$y_i = \alpha + x_i\beta + \varepsilon_i \quad i = 1, 2, ..., N$$

where ε_i is an error term with $\varepsilon_i = v_i - u_i$.

The main purpose of the stochastic model is to find the estimate of u_i or $u_i - \min_i u_i$ taking into account the randomness.

Our main goal is to study this model by taking various distributions as a distribution of u_i.

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About the fluctuation critical branching processes with immigration

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Let $\{\xi_{k,j}, k, j \geq 1\}$ and $\{\varepsilon_k, k \geq 1\}$ be two independent collections of independent random variables taking non-negative integer values such that random variables $\xi_{k,j}, k, j \geq 1$ are identical distributed. Let the sequence of random variables $X_k, k \geq 0$ defined by the following recursive relations:

$$X_0 = 0, \quad X_k = \sum_{j=1}^{X_{k-1}} \xi_{k,j} + \varepsilon_k, \ k = 1, 2, ... \tag{1}$$

Stochastic processes defined in this way often appear in population theory (see, for example, [1]) and are called branching processes with immigration. Such a name becomes clear if we interpret the random variable $\xi_{k,j}$ as the number of descendants of the j-th particle of a certain population of particles in $k-1$-generations, and the value ε_k as the number of the particles immigrating into the population in k-th generation. Then the value X_k will be total number of population particles in the k-th generation.

Let $m = \mathrm{E}\xi_{1,1} < \infty$. The branching process (1) is called subcritical, critical and supercritical if $m < 1$, $m = 1$ and $m > 1$ respectively.
We introduce the following notation: $\Phi(x)$-standard normal distribution,

$$m = \mathrm{E}\xi_{1,1}, \ \sigma^2 = D\xi_{1,1}, \ \lambda_k = \mathrm{E}\varepsilon_k, \ b_k^2 = var\varepsilon_k, \ \theta_k = \mathrm{E}\left|\varepsilon_k - \lambda_k\right|^3, \ A_n = \sum_{k=1}^{n} \lambda_k,$$

$$\Delta_n = \sup_{-\infty<x<\infty} \left| \mathrm{P}\left(\frac{X_n - A_n}{B_n} < x\right) - \Phi(x) \right|.$$

$C, C_1, C_2, ...$-positive absolute constants.

We assume that, random variables ε_k, $k \geq 1$ are independent, λ_k and b_k^2, $k \geq 1$, such that

$$\lambda_k = k^{\alpha} \ell_{\alpha}(k), \ \ b_k^2 = k^{\beta} \ell_{\beta}(k), \ k \geq 1 \tag{2}$$

where $\alpha, \beta \geq 0$-are fixed numbers, ℓ_α and ℓ_β-slowly varying functions at infinity.

Theorem. Let $m = 1$, $\sigma^2 < \infty$, condition (1) is stisfied, $\lambda_n \to \infty$ and

$$\lambda_n = o\left(n^{-1} b_n^2\right), \ \theta_n \sim n^{\tau} \ell_{\tau}(n) \quad \text{as} \ \ n \to \infty,$$

where $\tau \geq 0$, $\ell_\tau(n)$-is a slowly varying function at infinity and $\theta_n = o(\sqrt{n} b_n^3)$. Then for sufficiently large n we have the estimate

$$\Delta_n \leq C \frac{\theta_n}{\sqrt{n} b_n^3} + C_1 \left(\frac{n\lambda_n}{b_n^2}\right)^{1/3}.$$

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Fisher information in various models of random censoring

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R.A.Fisher [1] introduced the concept of Fisher information in 1925 as a means of comparing statistical data. He studied the loss of precision using various estimators of the unknown parameter such as median or maximum likelihood estimators. Fisher information is used in the Cramer-Rao inequality and provides a lower bound on the variance of the unbiased estimator, and is related to the asymptotic variance of the maximum likelihood estimates. In [2], the Fisher information was calculated for the Weibull distribution and the generalized exponential distribution in the case of a complete and right-censored sample, when the censoring random variable (r.v.) is constant. And in [3], the Fisher information was studied for a censored sample of types I and II, for which the censoring random variable is also constant. In [4], the Cramer-Rao lower bound and the system of Bhattacharya lower bounds were studied under random censoring. In this paper, we present the Fisher information for various models of random censoring. In this project various models of random censoring are considered and the dependence of the Fisher information on the considered parameter for the main distributions is numerically found.

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On the maximum of weakly dependent random variables

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The distributions of the maximum of random variables have been studied by many authors. In the books [1]-[3], the results and methods for the cases of independent and weakly dependent random variables are given.

We consider stationary sequences of random variables $\{X_n, n \in N\}$ which satisfy certain conditions. We assume that there exists another sequence $\{\xi_n,\ n \in Z\}$ of random variables such that

$$X_n = f\left(\{\xi_{n+i},\ i \in Z\}\right), n \in N, \tag{1}$$

where $f:\ R^Z \to R$ is a measurable function. Denote $M_n = \max\limits_{1 \le i \le n} X_i$.

Now we can formulate our main result.

Theorem. Let $\{X_n,\ n \in N\}$ be a sequence of the form (1) with a sequence $\{\xi_i, i \in Z\}$ of independent and identically distributed random variables. Assume that there exist measurable functions $X_n^{(m)} = f^{(m)}\left(\xi_{-m},\ ...,\ \xi_0, ..., \xi_m\right)$, $f^{(m)}\left(\xi_{-m},\ ...,\ \xi_0, ..., \xi_m\right):\ R^{2m+1} \to R$, $m = 1,\ 2,\ ...$ such that

$$\limsup_{n\to\infty} nP\left(\frac{1}{b_n}\left|X_1 - X_1^{(m)}\right| > \varepsilon\right) \to 0 \text{ as } m \to \infty \text{ for any } \varepsilon > 0$$

and the following holds for

$$\lim_{n\to\infty} nP\left(X_n^{(m)} > a_n + b_n x\right) = u(x),\ m = 1, 2, \ldots,$$

where $0 < u(x) < \infty$ on some interval of positive length.

Then for all $x \in R$

$$P\left(M_n < a_n + b_n x\right) \to H(x),\ \text{as } n \to \infty$$

where $H(x) = e^{-u(x)}$ and $e^{-\infty}$.

We will consider the cases of mixing sequences $\{\xi_i, i \in Z\}$ as well.

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Asymptotic behavior of generalized regenerative random processes in the M/G/1 queuing system with semi-Markov arrival of customers

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Consider a queuing system with a Poisson input flow with a parameter λ and with one server. The demand that made the server busy with the service becomes queued, the places for waiting for the service are not limited.

The system receives requests of m various types, and the first request of the type J_1 arrives at the moment $t = 0$ and is serviced over time τ_1. Let us denote τ_k by the service time and τ_k the type of k the customer received by the system, $k \geq 1$. Suppose that $\{\tau_k, k \geq 1\}$ is a sequence of independent random variables, $\{J_k, k \geq 1\}$ is a homogeneous, irreducible and ergodic Markov chain with a finite set of states $H = \{1, 2, ..., m\}$.

Let us denote by $\eta(t)$, $\nu(t)$, and $N(t) = \max\left(n \geq 1 : \sum\limits_{i=1}^{n} \omega_i \leq t\right)$, respectively, the size of the queue in the system, including the serviced request, the number of serviced requests during the time and the number of completed busy periods by the moment t.

To study the asymptotic behavior of generalized regenerating random processes $\xi(t) = \int\limits_0^t \eta(t)dt$, $\nu(t)$ and $N(t)$, $0 \leq t \leq \infty$ with an increase in the system operation time, it becomes necessary, as far as possible, to explicitly indicate the centering, normalizing and limiting constants, and these constants are expressed through the moments of increments of the above processes during the periods of system occupancy, which are periods of regeneration, since in the periods of system middle time following the periods of system busyness, the values of random processes $\xi(t)$ and $\nu(t)$ are equal to zero.

In this research equations for the moments of increments and the regeneration period of the above generalized regenerating random processes are compiled and conditions for the existence of their solutions are obtained, also limit theorems are proved in the scheme of series on the weak convergence of continuous functionals in the uniform topology from normed and centered random processes $\xi(t) = \int\limits_0^t \eta(t)dt$, $\nu(t)$ and $N(t)$, $0 \leq t \leq \infty$.

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Data independent prediction model: an analysis of the impacts of nonpharmaceutical and pharmaceutical interventions on covid-19 mitigation

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Abstract: Global tourism and leisure were suspended due to the Covid-19 pandemic. The new normal automatically truncated the ways we do things because of the spread of the virus. Community life and social mingling were postponed because of the lockdown to curtail the spread of the virus. Various forms of non-pharmaceutical approaches (NPA) were adopted in the absence of a vaccine. As time progresses, different vaccines became available (Pharmaceutical approach PA) to mitigate the spread of the virus. To reassure the safety of people, different levels of social distancing values in meters were applied to reduce the possibility of a higher infection rate due to assumed airborne transmission and close physical contact. This study tends to determine whether NPA and PA interventions could be used to reduce the spread of the virus. The proposed data independent prediction model (DIPM) was applied to predict the impact of NPA and PA in mitigating the spread of the virus. The DIPM utilized onset data from the NPA and PA interventions to predict the probability of mitigating the spread of the virus for a specific period. The results revealed that both the NPA and the PA are very effective means of mitigating the spread of the virus. The analysis demonstrated that as more people are vaccinated with time, the probability of infection reduces drastically thereby increasing the probability of social mingling. Therefore, this study concluded that the DIPM using the NPA, and PA input is a robust prediction model to predict the probability of mitigating the spread of the Covid-19 virus over time.

Comparative analysis of robust dimension reduction techniques for two groups classification problems

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Abstract: In high dimensional small sample (HDSS) classification problems, the issue of relevant and irrelevant data, curse of singularity and dimensionality, persist for the classical covariance dependent methods such as the Fisher linear classification method (FLCM). The presence of irrelevant variables has generated different problems in the classification domain such as, computational time, misclassification rate, and performance evaluation criteria. For HDSS data, the FLCM is impracticable, therefore, the independent classification rule (ICR) was developed to solve these problems. However, the training and validation of the ICR learned model still depends on the relevant and irrelevant data in the variables. To overcome these problems, we applied the principal component analysis (PCA) and the Weighted-PCA (W-PCA) to reduce the dimension of the data set, so that new variants of the FLCM and other covariance dependent methods could be applied to perform classification tasks on the HDSS data set. A benchmark extraction method (BEM) is also proposed to tackle the aforementioned HDSS classification problems of the covariance dependent methods. The study also investigates the number and percentage of relevant variables selected, computational time, and the probability of correct classification (PCC). To evaluate the performance of these methods, the performance evaluation criteria (PEC) is applied to analyze the probability of correct classifications. The results revealed that the W-PCA procedure is very sensitive to dimension reduction, which is done by selecting the vital few variables (Minimum number of vital variables) followed by the BEM procedure. The W-PCA variants have the best computational time while the BEM has the overall best PCC for the data set investigated. The findings demonstrated that the BEM approach outperformed other methods in terms of probability of correct classification, while the W-PCA has the best optimal dimension reduction capability.

The effect of contamination proportion on classical and robust fisher linear discriminant analysis

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Abstract: The classifier coefficient of the classical Fisher linear discriminant analysis (CFLDA) strictly relies on the data, the sample mean, the covariance matrix, and the pooled covariance matrix. The CFLDA was designed to perform optimally if the normality and homoscedasticity assumptions are satisfied. However, if the assumptions are violated, then the CFLDA is non-robust. The general performance of the CFLDA is dependent on the composition of the data. If the composition of the data contains influential observations (IOs), the sample means and the covariance matrix that are highly susceptible to IOs will hamper the classifier accuracy due to the training and validation (test) sizes. Different techniques to transform the IOs to obtain robust sample means and covariance matrices have been developed over several decades. In this paper, we applied Mahalanobis distance weight to detect and delete the IOs, then the inlier data is used to train the classifier coefficient. The focus of this study is to investigate the effects of the sizes of the training and validation with contamination level on the classifier performance. The problem of overfitting is also investigated in this paper. The performance evaluation based on the misclassification rate revealed that the training sample sizes (70-80) percentage and the validation sizes (30-20) percentage with moderate to high contamination proportions are suitable to determine the robust performance of any classifiers. The misclassification rates affirmed that the Robust FLDA outperformed the CFLDA for all simulations investigated. The analysis also demonstrated that the proposed performance measure is able to solve the problem of overfitting. Therefore, this paper concludes that the sizes of training and validation with the proportions of contamination affect the performance of the classifiers.

Strong laws of large numbers for weighted mixing random variables with values in Hilbert space

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Let $\{X_i, i \geq 1\}$ be a sequence of random variables with values in a separable Hilbert space H with inner product $(a, b)$and norm $\|a\| = \sqrt{(a, a)}$.

Let $\{a_n, n \geq 1\}$ and $\{b_n, n \geq 1\}$ be sequences of positive real numbers. We use notation $a_n \cong b_n$ instead of $0 \leq \liminf \frac{a_n}{b_n} \leq \limsup \frac{a_n}{b_n} < \infty$.
Denote an orthonormal basis of H by $\{e_i, i \geq 1\}$ and thus

$$X_n = \sum_{i=1}^{\infty} X_n^{(i)} e_i.$$

We assume that $\{X_n, n \geq 1\}$ is coordinatewise φ-mixing. Coefficients of φ-mixing for the sequence of coordinates $\left\{X_n^{(i)}, i \geq 1\right\}$ $n = 1, 2, ...$ defined as following

$$\varphi(k) = \sup\left\{|P(A/B) - P(A)| : A \in \Im_1^k, B \in \Im_{k+n}^{\infty}, n \geq 1\right\},$$

where $\Im_a^b$ is a σ -field generated by $X_a^{(i)}, ..., X_b^{(i)}$.
Denote $c_n = \frac{b_n}{a_n \log n}$ and $l(x)$ is a slowly varying function.

Theorem. Let $\{X_i, i \geq 1\}$ be a sequence of random variables with values in a separable Hilbert space H. Assume that the following conditions hold

$$\sum_{k=1}^{\infty} \varphi^{\frac{1}{2}}\left(2^k\right) < \infty$$

$$\sum_{j=1}^{\infty} P(\left|X_n^{(j)}\right| > x) \cong l(x)x^{-\alpha} \quad \text{for each } \alpha \in (1, 2)$$

$$\sum_{n=1}^{\infty} l(c_n) c_n^{-\alpha} < \infty.$$

Then

$$\frac{1}{b_n} \sum_{k=1}^{n} a_k X_k \to 0 \text{ a.s. as } n \to \infty.$$

For pairwise negatively quadrant dependent H-valued random variables such results were obtained in [1].

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VIII. INFORMATION SECURITY

Issues of information security in cloud computing

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The concept of cloud computing is one of the fastest growing areas of information technology, where resources such as infrastructure, platform, or software-related services are available via Internet as on-demand self-service basis which can be rapidly provisioned and released. with minimum management effort or service providers interaction.

A distributed system, or cloud, is described as a collection of interconnected, virtualized computer systems which are introduced or dynamically provisioned in accordance with a larger pool of computing resources that are dependent on service level agreements [1].

The term "information security" means protecting information and information systems from unauthorized access, use, disclosure, disruption, modification, or destruction in order to provide integrity, confidentiality and availability [2].

Information security of cloud computing is a section of cybersecurity dedicated to the protection of cloud systems and data from internal and external threats, including best practices, policies, and technologies which help companies prevent unauthorized access and data leakage.

Cloud security, also known as information security of cloud computing, is a collection of security measures designed to protect cloud-based infrastructure, applications, and data. These measures ensure user and device authentication, data and resource access control, and data privacy protection.

Cloud security technologies are aimed at ensuring the security of both software and hardware parts of the system, including: network and server equipment; physical information carriers; OS, network and other software and applications; user equipment (PCs and laptops, smartphones and gadgets, IoT devices); information itself as the main resource.

The purpose of this article is to compare existing approaches that ensure the information security of cloud computing, to classify the key cloud computing security threats which are common to all cloud services in general, and to develop recommendations for protecting cloud computing for future use.

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Minimal logical automaton representation of steganographic algorithm microcommands

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At present, when the volume and value of information is increasing, the problem of its protection is becoming more acute. There are many ways to protect information, including steganographic.

This paper discusses image steganography. Image steganography is a process in which we hide data in an image so that there are no visible changes in the original image. The traditional image steganography algorithm is the LSB embedding algorithm.

The paper proposes an optimal model for writing the LSB steganographic algorithm to a microcontroller in the following form:

$$
\begin{array}{lll}
Y_0 = 1; & Y_1 = 1; & Y_2 = X_1X_2; \\
Y_3 = \overline{X_1}; & Y_4 = \overline{X_1}; & Y_5 = \overline{X_1}X_3; \\
Y_6 = \overline{X_1X_3}; & Y_7 = \overline{X_1X_3}X_4; & Y_8 = \overline{X_1X_3X_4}; \\
Y_9 = \overline{X_1X_3X_4}; & Y_{10} = \overline{X_1X_3X_4}X_5; & Y_{11} = \overline{X_1X_3X_4X_5}X_6X_7; \\
Y_{12} = \overline{X_1X_3X_4X_5}X_6\overline{X_7}; & Y_{13} = \overline{X_1X_3X_4X_5}X_6; & Y_{14} = \overline{X_1X_3X_4X_5}X_6X_8; \\
Y_{15} = \overline{X_1X_3X_4X_5X_6}; & Y_k = \overline{X_1X_3X_4X_5X_6}; &
\end{array}
$$

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Analysis of a phishing attack on information system based on production logic

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In this era of developing cyber security issues, research on a scientific and fundamental basis is relevant. In this case, based on the information provided by cyber security subjects, making a decision to include objects in the unified register of important information infrastructure objects, establishing requirements for ensuring the cyber security of important information infrastructure objects, determining the procedure for attesting information objects and important information infrastructure objects in accordance with cyber security requirements, licensing activities related to the development, and other issues need to be resolved. For example, using production logic[1], you can apply logical functions to identify phishing attacks[2,3] in an information system:

$$
\begin{aligned}
F1 = {} & x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\wedge\overline{x_{10}}\wedge x_{11}\wedge\overline{x_{12}}\wedge\overline{x_{13}}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge \\
& x_8\wedge x_9\wedge\overline{x_{10}}\wedge x_{11}\wedge\overline{x_{12}}\wedge x_{13}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\wedge\overline{x_{10}}\wedge x_{11}\wedge x_{12}\wedge\overline{x_{13}}\vee x_1\wedge\overline{x_2}\wedge \\
& \overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\wedge\overline{x_{10}}\wedge x_{11}\wedge x_{12}\wedge x_{13}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\wedge x_{10}\wedge\overline{x_{11}}\wedge \\
& \bar{x}_{12}\wedge\bar{x}_{13}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\wedge x_{10}\wedge\overline{x_{11}}\wedge\overline{x_{12}}\wedge x_{13}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge \\
& x_8\wedge x_9\wedge x_{10}\wedge\bar{x}_{11}\wedge x_{12}\wedge\bar{x}_{13}\vee\ldots\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge\overline{x_{11}}\wedge x_{12}\wedge\overline{x_{13}}\vee x_1\wedge \\
& x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge\overline{x_{11}}\wedge x_{12}\wedge x_{13}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge \\
& x_{11}\wedge\overline{x_{12}}\wedge\overline{x_{13}}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge x_{11}\wedge\bar{x}_{12}\wedge x_{13}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge \\
& x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge x_{11}\wedge x_{12}\wedge\overline{x_{13}}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\wedge x_{10}\wedge x_{11}\wedge x_{12}\wedge x_{13};
\end{aligned}
$$

List of signs of a phishing attack on an information system:

№	Designation	Feature names
1	x_1	number of domain tokens
2	x_2	using multiple top-level domains within a domain name
3	x_2	length of the URL
...	...	...
$n-2$	x_{n-2}	path separator
$n-1$	x_{n-1}	number of characters in the domain
n	x_n	entropy domain

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Studying the information system of defacement on the basis of production logic

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In the era of development of digital and information technologies, cyber security issues are relevant. Development of regulatory legal documents and state programs in the field of cyber security, control over the implementation of legislation on cyber security, implementation of operational and search activities, pre-investigation investigations and investigative actions in connection with cyber security incidents, taking organizational and technical measures to prevent, identify and eliminate cyber security incidents and adopt appropriate measures in relation to them, including the elimination of their consequences, the organization of research and monitoring in the field of cyber security, the formation of a unified register of important information infrastructure objects, as well as the organization and maintenance of this register, and other issues should be resolved. For example, in an information system, defacement[2] can be reduced to the form of Boolean functions[1] using production logic:

$$F0 = \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge \overline{x_8} \wedge x_9 \wedge x_{10} \wedge \overline{x_{11}} \wedge x_{12} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge \overline{x_8} \wedge x_9 \wedge x_{10} \wedge x_{11} \wedge \overline{x_{12}} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge \overline{x_8} \wedge x_9 \wedge x_{10} \wedge x_{11} \wedge x_{12} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge \overline{x_{10}} \wedge \overline{x_{11}} \wedge \overline{x_{12}} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge \overline{x_{10}} \wedge \overline{x_{11}} \wedge x_{12} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge \overline{x_{10}} \wedge x_{11} \wedge \overline{x_{12}} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge \overline{x_{10}} \wedge x_{11} \wedge x_{12} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge x_{10} \wedge \overline{x_{11}} \wedge \overline{x_{12}} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge x_{10} \wedge \overline{x_{11}} \wedge x_{12} \vee \overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge \overline{x_9} \wedge x_{10} \wedge x_{11} \wedge \overline{x_{12}} \vee \quad \ldots \quad \vee x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge x_9 \wedge x_{10} \wedge \overline{x_{11}} \wedge x_{12} \vee x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge x_9 \wedge x_{10} \wedge x_{11} \wedge \overline{x_{12}} \vee x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \wedge x_9 \wedge x_{10} \wedge x_{11} \wedge x_{12};$$

List of signs for the situation of disfiguring the information system:

No	Designation	Feature names
1	x_1	number of domain tokens
2	x_2	average path token length
...	...	...
$n-2$	x_{n-2}	name of the tariffication directory
$n-1$	x_{n-1}	number of characters in the domain
n	x_n	entropic domain

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Identification of a malicious attack on information system based on production logic

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In the modern developing period, it is necessary to study and analyze cyber security issues on a scientific and practical basis. This includes the development of plans to prevent attempts at cyber attacks on important information infrastructure facilities and their direct implementation, regulation of the activities of cyber security units, independent expert services and groups, interaction with law enforcement agencies in the field of countering cyber threats, informing state and economic authorities. For example, one of the main cyber security issues is identification, using production logic, you can describe a malicious attack in the form of Boolean functions:

$$F0 = x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge\overline{x_6}\wedge x_7\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge\overline{x_6}\wedge x_7\wedge\overline{x_8}\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge\overline{x_6}\wedge x_7\wedge x_8\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge\overline{x_6}\wedge x_7\wedge x_8\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge\overline{x_8}\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge x_8\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge x_8\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge x_7\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge x_7\wedge\overline{x_8}\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge\overline{x_4}\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge x_4\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge x_4\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge\overline{x_8}\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge x_4\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge\overline{x_9}\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge x_4\wedge\overline{x_5}\wedge\overline{x_6}\wedge\overline{x_7}\wedge x_8\wedge x_9\vee x_1\wedge\overline{x_2}\wedge\overline{x_3}\wedge x_4\wedge\overline{x_5}\wedge\overline{x_6}\wedge x_7\wedge\overline{x_8}\wedge\overline{x_9}\vee\ldots\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge\overline{x_6}\wedge x_7\wedge x_8\wedge x_9\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge\overline{x_8}\wedge x_9\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge x_8\wedge\overline{x_9}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge\overline{x_7}\wedge x_8\wedge x_9\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge\overline{x_8}\wedge\overline{x_9}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge\overline{x_8}\wedge x_9\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge\overline{x_9}\vee x_1\wedge x_2\wedge x_3\wedge x_4\wedge x_5\wedge x_6\wedge x_7\wedge x_8\wedge x_9;$$

List of signs of a malicious attack on an information system:

№	Designation	Feature names
1	x_1	number of domain tokens
2	x_2	use of several top-level domains within a domain name
3	x_2	length of the URL
...	...	...
$n-2$	x_{n-2}	number of characters in the domain
$n-1$	x_{n-1}	entropy domain
n	x_n	entropy expansion

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Research and analysis of cyber attacks based on production logic

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The world of information and communication technologies has become an integral part of everyday life. The development of information and communication technologies opens up wide opportunities for managing cyber security issues with a large amount of data, which directly affects the level of security of cyberspace through their collection and processing. Therefore, research and analysis of cyber attacks is relevant. The knowledge base is checked for specific types of cyber attacks. It is assumed that the logical functions are expressed as follows:

$$(X_i \to Y) \equiv 1, \quad i = \overline{1, n}$$
$$((X_i \wedge X_j) \to Y) \equiv 1, \quad i, j = 1...n, \quad i \neq j$$
$$(X_i \wedge X_j \wedge X_k \to Y) \equiv 1, \quad i, j, k = 1...n, \quad i \neq j \neq k$$
$$(X_{i_1} \wedge X_{j_2} \wedge ... \wedge X_{i_k} \to Y) \equiv 1, \quad i_1, i_2...i_k = 1...n, \quad i_1 \neq i_2 \neq ... = i_k, \quad 1 \leq k \leq n$$
$$((X_1 \vee X_2 \vee ... \vee X_n) \to Y) \equiv 1$$
$$((X_i \vee X_j) \to Y) \equiv 1 \quad i, j = 1...n, \quad i \neq j$$
$$((X_i \vee X_j \vee X_k) \to Y) \equiv 1 \quad i, j, k = 1...n, \quad i \neq j \neq k$$
$$((X_{i_1} \vee X_{j_2} \vee ... \vee X_{i_k}) \to Y) \equiv 1, \quad i_1, i_2...i_k = 1...n, \quad i_1 \neq i_2 \neq ... = i_k, \quad 1 < k < n$$
$$((X_1 \vee X_2 \vee ... \vee X_n) \to Y)$$

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DPI based network management model and architecture

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Deep Packet Inspection (DPI) is a technology that enables the network owner to analyze internet traffic, through the network, in real-time and to differentiate them according to their payload. Since, this has to be done on real time basis at the high speeds it cannot be implemented by software running on normal processors or switches. It has only become possible in the last few years through advances in computer engineering and in pattern matching algorithms. The payload or content of the packet, which contains (all or part of) the text, images, files or applications transmitted by the user, was not considered to be a concern of the network operator. DPI allows network operators to scan the payload of IP packets as well as the header. DPI systems use expressions to define patterns of interest in network data streams. The equipment is programmed to make decisions about how to handle the packet or a stream of packets based on the recognition of a regular expression or pattern in the payload. This allows networks to classify and control traffic based on the content, applications, and subscribers [1]. Deep Packet Inspection equipment analyzes the first packets of a traffic stream or all packets passing through it. In this case, signature analysis, statistical methods for tracking the characteristics of packets, behavioral analysis, and other approaches are used. Previously, there was a division of DPI into two generations, depending on the methods of analysis and system performance. By analyzing packets, DPI checks protocol and application signatures. The signature is a fixed sequence of characters that must be located in the packet, either at a certain offset or at a random position. Signatures can be divided into 3 groups: the exact value of the hex code string, the string using regular expressions, and based on statistical characteristics [2]. A network application may initiate a data transfer using one or more packet streams. The data stream from the application is usually distinguished from the general traffic using address information of 2-4 levels of the OSI model (MAC addresses of the source and destination, IP addresses of the source, destination, transport ports of the source, and destination, and the value of the protocol type field).

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Text Steganography Methods

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The problem of protecting information from unauthorized access has been solved at all times throughout the history of mankind. Already in the ancient world, there were two main directions for solving this problem, which exist to this day: cryptography and steganography. Steganography is a field of knowledge that deals with the secret transmission of information. Steganography (from the Greek steganos (secret, mystery) and graphy (record)) literally means "cryptographic writing"or "covered writing"[1]. Unlike cryptography, the very fact of information transfer is hidden. Especially effective is the use of steganographic methods in conjunction with cryptographic ones. A common feature of steganographic methods and algorithms is that the hidden message is embedded in some innocuous object that does not attract attention, which is then openly transported to the addressee. Classical methods of steganography can be classified as follows:

- Hiding a container file in interformat spaces is the simplest of the listed ways to hide a message file.
- Hiding-masking directly uses the service areas and special blocks of the container file.

In a special group, you can also highlight methods that use special properties of file representation formats:

- fields reserved for expansion in computer file formats, which are usually filled with zeros and are not taken into account by the program;
- special data formatting (shifting words, sentences, paragraphs or selecting certain letter positions);
- removing identifying headers for a file.

Usually, such methods are characterized by a low degree of secrecy, low throughput and poor performance. In text steganography, character text is used to hide sensitive information. Storing text files requires less memory and its easier communication makes it preferred over other types of steganographic techniques. Because texts take up less memory, convey more information, and require less printing costs, as well as some other benefits. This article presents a new approach to systematization of text steganography methods by grouping existing methods into groups, with further discussion of some of them.

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IX. COMPUTATIONAL LINGUISTICS

Analytical review on word sense disambiguation for uzbek corpus

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We know that words make up the vocabulary of the language, interact with each other and perform various tasks. In conversation, words are semantically close to each other or, on the contrary, have opposite meanings.The goal of word sense determination (WSD) is to correctly determine the meaning of a word in a general sentence. All natural languages exhibit ambiguity in the meaning of words, and resolving this automatically remains a pressing problem. The motivation behind our current research stems from the need for new WSD methods and tools.[1,2,3]

In creating the national corpus of the Uzbek language, the issue of forming a base of grammatical homonyms and their linguistic modeling is considered important. In Uzbek linguistics, words form homonyms within one or more categories. Words can form homonyms within one, two and three categories. In order to develop models of homonymous words in computer linguistics, it is necessary to first determine which categories of words form homonymy and give them conditional symbols. We determined the formation of homonyms in the mutual part of speech and developed corresponding linguistic models for WSD: a) WSD between noun and verb. $V + ma = HW \to [N \leftrightarrow V]\ /V + moq = N \leftrightarrow V$ b) WSD between adjective and verb. $Adj + ish = HW \to [Adj. \leftrightarrow V]$ c) WSD between noun and adverb. $N + cha = Adj. \leftrightarrow Noun$ d) WSD between noun and verb. $HW \to [N1, N2, N3]$

In order to investigate the issue of WSD in the computer intelligence system, it is necessary to perform the following tasks: a) extract homonyms from Uzbek language works, explanatory dictionaries and homonyms dictionary; b) determining the level of distribution of homonyms; c) to reveal the systemic character of WSD; d) show symmetry/asymmetry (dissymmetry) aspects of homonyms; e) classification of homonyms according to word groups; f) sort homonyms in text processing; g) creating a matrix of homonyms; h) modeling homonyms. Using the homonym database, we created a tool that identifies WSD in the Uzbek Corpus, algorithmizing the use of homonyms in grammatical forms and parts of speech. The interface of this tool has a search box with the ability to filter by part of speech.

In this paper, a comparative study of WSD in different international and Turkish languages was conducted. In order to solve the WSD problem in the Uzbek corpus, as a first step, a WSD tool was created using the knowledge-based WSD approach.

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Research of mobile dictionaries in computer lexicography

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In recent years, we often come across the term MALL (Mobile Assisted Language Learning) in lexicographical works. A lot of research has been conducted in the world about the role of this field in linguistics. The term MALL language learning was first used in 2006 by Chinnery. At that time, MALL was part of CALL (Computer Assisted Language Learning). Later, Kukulska-Hulme and Shield in 2008 clarified the difference between MALL and CALL. MALL is one of the developing areas of m-learning (mobile learning), which is now popular.

In this field, researchers such as Chinnery and El-Husain, C. Houser, E. Klopfer, K. Squire, H. Jenkins, A. Kukulsha-Hulme, H. Lee, M. Levy, Kennedy, M. Charoles are illuminating the theoretical aspects of MALL with their work. We would like to present the following lexicographic aspects of the Uzbek language morpheme educational dictionary formed during our research: a) the dictionary has a morphemic analysis based on a synchronic approach; b)there are grammatical tasks that allow the user to check the knowledge gained from the dictionary; c) grammatical forms of words are defined in the search interface; d) the combination of grammatical additions is taken into account; e) morpheme derivative units are modeled; f) a dictionary of cognate words was formed based on the compilation of a dictionary.

The created educational dictionary will serve as the main guide for language learning in schools, as well as be directed to the educational goal. In the educational dictionary, interactive grammar tasks have been developed for the formation of language skills at a certain level. In interpreting the meanings of the words, linguistic dictionaries created in the Uzbek language in the electronic corpus of the Uzbek language, the morphological database of the language, educational literature, and samples of scientific, official, and artistic texts of various genres were used.

In 2021-2023, within the framework of the scientific-practical project "Creating New Generation Dictionaries and Their Mobile Application we managed to form a morpheme educational dictionary base. The morpheme base in it is 8380 derived words from 3760 (basic) stems, and the total number of lexemes is 12140. Vocabularies of each derived word and related educational tasks were prepared. Since the foundations directly and indirectly function as a constructive foundation, their explanation was also included in the dictionary.

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Semantic similarity of nouns in uzbek thesaurus

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A thesaurus is a type of dictionary that groups words together based on their similarity in meaning. Unlike a traditional dictionary a thesaurus is organized into categories of related words, called synonyms, antonyms, and sometimes hypernyms and hyponyms. Thesauruses are useful tools for writers and anyone looking to expand their vocabulary. They can help you to find more precise or interesting words to use in your writing.

1. Partonym: A partonym is a word that describes a part of a larger object. For example, "wheel"is a partonym of "car,"because a wheel is a component of a car. 2. Meronym: A meronym is a word that describes a smaller component of a larger object. For example, "finger"is a meronym of "hand,"because a finger is a smaller component of a hand. 3. Hyponym: A hyponym is a word that is more specific than another word. For example, "rose"is a hyponym of "flower,"because a rose is a specific type of flower. 1. Holonym: A holonym is a word that describes the larger object or entity of which a part is a component. For example, "car"is a holonym of "wheel,"because a wheel is a part of a car. 2. Hypernym: A hypernym is a word that describes a general category or superclass of another word. For example, "animal"is a hypernym of "dog,"because a dog is a specific type of animal. One important issue we mention that Uzbek words and semantics gathering in Thesaurus dictionaries. It is the fact that it is impossible to automatically translate hyponym, hypernym, meronym, partonym and holonym from English into Uzbek. It can be created on the basis of the human factors. The most important thing is the first research was carried out based on the concept of family and I am contributing to enrich by adding NOUNS and their hierarchical words in Uzbek Thesaurus. For illustration: **Uzbek word:** suv ,**English word:** water. **Russian word:** вода,**Synonym:** obihayot, xol, **Antonym:** quruq , **hypernym:** suyuqlik; **hyponym:** ichimlik; **meronym:** mineral; **partonym:** gazli suv, **holonym:** suv **Definition:** vodorod bilan kislorodning kimyoviy birikmasidan iborat, rangsiz, shaffof suyuqlik.

Zilol suvlar Eram kabi bog‘u bo‘stonlar, Mening sovg‘am koshonalar, oltin tuproqlar (G‘ayratiy).

Proverb: Ot boshiga ish tushsa, suvlig‘i bilan suv ichar, Er boshiga ish tushsa, etishi bilan suv kechar.

Idiom: Suv quygandek, og‘zining suvi keldi. Navoiyning uyi suv quygandek jimjit, hamma oyoq uchida yurar, bir- biri bilan shivirlab gaplashishar edi. (M.Osim. Sehrli so‘z).

Phrasal verb: suvga oqizmoq - U molini sotmadi, suvga oqizdi. Yo‘q suvga oqizgandan battar bo‘ldi. (Oybek. Tanlangan asarlar).

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Analysis of Syntactic Parsing for a low-resource Uzbek Language

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This paper presents a comprehensive analysis of syntactic parsing techniques for the low-resource Uzbek language. Despite the increasing significance of natural language processing (NLP) and the availability of syntactic parsers for several major languages, the absence of dedicated parsing tools for Uzbek has hindered its linguistic analysis and computational processing. In response, we undertake a systematic investigation of closely related languages to establish a framework for the development of an Uzbek syntactic parser.

Drawing inspiration from linguistic typology and cross-lingual transfer learning, we leverage the syntactic structures of languages sharing linguistic traits with Uzbek. By analyzing and comparing the syntax of these related languages, we identify common syntactic features that can serve as a foundation for building a tailored syntactic parser for Uzbek. We examine the structural similarities and differences, taking into account the language's unique characteristics and its linguistic kinship.

Furthermore, we propose a methodology for adapting existing syntactic parsers to the Uzbek language based on transfer learning techniques. We explore techniques such as multilingual pretraining and fine-tuning, enabling us to harness the wealth of syntactic parsing models available for more resource-rich languages. This approach seeks to bridge the gap between the lack of annotated data in Uzbek and the wealth of linguistic resources available for other languages. Through extensive experimentation and evaluation, we demonstrate the effectiveness of our proposed approach. We showcase the feasibility of utilizing transfer learning to construct a syntactic parser for Uzbek, achieving competitive parsing performance even in the absence of a substantial amount of native training data.

In conclusion, this paper offers a pioneering effort in addressing the scarcity of syntactic parsing resources for the Uzbek language. By capitalizing on the linguistic affinity between Uzbek and its counterparts, we present a novel approach to construct a syntactic parser that provides valuable insights into the language's grammatical structure. Our work not only contributes to the field of NLP for low-resource languages but also underscores the broader potential of cross-lingual transfer learning in linguistic research and computational linguistics.

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A pipeline for large-scale raw text preprocessing and model training of language models in uzbek.

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The meteoric rise of Transformer-based language models has revolutionized the Natural Language Processing (NLP) field. By pre-training these models on extensive, unlabeled corpora and employing transfer learning for subsequent tasks, benchmarks like BERT[1] have set a new standard for language model efficacy. While these advancements have propelled research into refining pre-training algorithms and devising novel neural architectures, the critical aspect of pre-training new corpora and efficient training on High-Performance Computing (HPC) clusters, particularly for under-resourced languages and domains, remains under-explored.

This study bridges this knowledge gap, focusing on the practical considerations in preparing new corpora for pre-training and effectively training models from scratch. The research centers on the development of a comprehensive preprocessing and training pipeline that caters to the unique challenges of under-resourced languages and domains, specifically illustrating the case of Uzbek. We propose a robust methodology for generating the necessary resources, augmenting the data quality, and providing insights into training language models from scratch, issues that have been less comprehensively explored in previous works.

The results of this research stand to significantly enhance the performance of language models for under-resourced languages and domains, outpacing their multilingual and general-domain counterparts under certain conditions. This study serves as a valuable resource for the NLP community, offering the tools and methodologies for developing more inclusive and equitable language models. By equipping researchers and practitioners with the necessary mechanisms for understanding and implementing the proposed pipeline, we hope to foster a more diverse and representative field of NLP.

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Transformer-based auto-correction of grammatical errors in uzbek language.

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While significant strides have been made in Grammatical Error Correction (GEC) for universal languages like English, Russian, and Chinese, it remains a challenge for low-resource languages. This paper delves into the creation and evaluation of an automatic grammatical error correction model specifically designed for one such language, Uzbek, using the Transformer [1] neural network architecture.

This research is crucial in extending the benefits of Natural Language Processing (NLP) to low-resource languages, particularly those that lack extensive GEC research. In this regard, we focus on Uzbek, a language that has not been adequately covered in the GEC task. We hypothesize that the application of Transformer architecture for GEC tasks, which has yielded satisfactory results in universal languages, will be as effective in Uzbek, given its ability to capture long-distance dependencies, which are crucial in grammar correction.

To substantiate our hypothesis, we develop a large Uzbek language corpus that can be used to evaluate GEC tasks. This corpus is not only extensive but also rich in different text types, which contributes to a comprehensive model training.

The paper provides a detailed evaluation of the Transformer-based Uzbek GEC model. The results of this evaluation offer insightful findings on how the model performs across various grammatical error types. The model performance is also compared against existing benchmarks to provide an unbiased view of its effectiveness.

Furthermore, this paper discusses the potential of this model to be adapted for other low-resource languages, highlighting how the lessons learned from developing the Uzbek GEC model could be applied in different linguistic contexts.

The primary objective of this research is to bridge the gap in GEC tasks for low-resource languages. By introducing a Transformer-based model for Uzbek and an associated extensive corpus, this paper brings new possibilities for enhancing grammatical error correction in less-studied languages. These efforts are a leap towards ensuring all languages, irrespective of their global reach, benefit from the technological advancements in the field of Natural Language Processing.

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Adapting mT5 for uzbek language: a smaller pre-trained model for nlp tasks

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The Uzbek language, spoken by nearly 50 million people, remains insufficiently explored in the field of Natural Language Processing (NLP). This has largely been due to the scarcity of language resources in Uzbek. The Transformer architecture, which is quickly becoming the norm in NLP, supersedes previous approaches such as convolutional and recurrent neural networks. The mT5 model by Google is a pre-trained multilingual model that can handle text from different languages [1]. This model extends the T5 (Text-to-Text Transfer Transformer)model [2], trained on a corpus of Internet text in multiple languages. mT5 is a seq2seq model that has proven successful in a range of NLP tasks, including translation. However, its application for less-resourced languages, such as Uzbek, is an area of exploration. Nevertheless, the substantial size of the mT5 model is a limitation when considering its implementation in practical applications that only necessitate a single language. In this research, we adjusted the mT5 model exclusively for Uzbek, resulting in a specialized, smaller-sized T5 model. We benchmarked this model's performance against the mT5 model using the same methodology and dataset for Automatic Text Summarization (ATS) and Named Entity Recognition (NER) tasks. A model fine-tuned based on uzT5 outperformed the mT5. Our findings validate the potential of creating a smaller, pre-trained model that achieves comparable results, with up to 57% size reduction. Additionally, the newly developed model uses less memory, initiates more rapidly, and boasts quicker inference times.

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Cluster analysis of document collections and semantic relatedness of terms

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In topic modeling, the number of topics in the analysis of collections of documents is usually a free parameter. As a rule, the formal tabular representation of documents is based on the use of the standard hypothesis bag of words. Due to the incorrectness of the problems being solved, the existing modeling systems are focused on the use of special regularizers to support a stable composition of topics.

The composition of topics is formed based on cluster analysis algorithms. A common requirement for analysing is to assume a standard Dirichlet distribution in document descriptions based on the frequency of occurrence of terms. There is a possibility of finding background themes without a clear meaning of the content.

A method of grouping documents (objects) from two stages is proposed. According to the distribution density parameters, at the first stage, the status of the object is determined: reachable, boundary, outlier. The status value is used as the target feature for classification. The problem of the curse of dimension is solved through the reduction of space through the synthesis of patterns (macroconcepts) [1]. According to the relation of connectedness of objects in classes, they are divided into non-overlapping groups. A measure of compactness is proposed for assessing the quality of partitioning into groups.

The use of the method is demonstrated on the example of the analysis of collections of abstracts of dissertations from the fund of the Higher Attestation Commission of the Republic of Uzbekistan.

A set of terms from the subject areas for which dissertations are defended are considered as data for building general document dictionaries. The size of the dictionary is considered a free parameter. No assumptions are made about the nature of the document description environment. The entry of a term into the general vocabulary is determined based on the value of its persistence. The grouping of terms by stability is used in the process of forming metaconcepts.

The semantic relatedness of terms for each group of documents is determined by their ordering by frequency of occurrence.

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Uzbekcorpora.uz: collection, digitalization and formatting of texts for the national corpus of the uzbek language

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The study of natural languages using automated technologies based on reliable material is a promising area of modern science. One of the effective means of solving many linguistic issues is the electronic body of the language [2]. The creation of such a system for the Uzbek language provides an opportunity to create new knowledge about the structure and lexical composition of the language, providing valuable material for the construction of linguistic models and the improvement of automated technologies for processing Uzbek texts.

Digital and non-digital texts can be used as text sources for the corpus. Naturally, in the second case, it is necessary to somehow enter the text into the computer: it must be rewritten or scanned [3]. For example, we do not have the docx format of the Fazil Yoldosh version of the Alpomish saga. Therefore, we will have to digitize this book. In this case, the manuscript is converted to pdf format during scanning of the source or book, and converted to docx format, which can be opened in Microsoft Office Word with the help of a converter (for example, Fine Reader) that recognizes the writings in it. Of course, it is very difficult for the converter to convert the scanned e-book to docx format according to the original state of the book. Therefore, based on manual work, the text in docx format is brought to the same level as the text in the original state. It corrects errors such as spelling errors, incorrect recognition of characters in the text.

Texts are available in various pdf, image, document and other formats. Before adding texts to the corpus, existing text files must be converted to *.docx format in Microsoft Office version 2010 and higher. Texts in other formats are converted to *.docx format using special programs, but the original state of the *.docx text may be damaged. Spelling and technical errors in this text are corrected by hand, and only then the text can be uploaded to the corpus [1]. In this study, we used the json format to store the texts in the corpus.

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UzNERD: Uzbek named entity recognition dataset

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Named entity recognition (NER) is an interesting task of natural language processing and information extraction that learns to identify given objects in unstructured text and classify them into categories such as names of persons, organizations, locations, medical terms, time expressions, monetary values, percentages, and other categories. The goal of NER is to extract structured data from unstructured text data and convert it into a machine-readable format. NER is widely used in data mining, sentiment analysis, question-answer systems, chatbots, geospatial analytics, financial analytics, biomedical, social media analytics, customer relationship management and various other fields. Developing NER datasets is a critical resource for training and testing NER AI models. The quality and size of the NER dataset have a significant impact on the performance of NER models. Therefore, it is an urgent task to develop NER datasets suitable for the specific domain or language in which NER models are applied.

NER datasets is available in various languages, such as CoNLL-2003, OntoNotes 5, WikiNER for English language; FactRuEval, NEREL, Gareev corpus for the Russian language; KazNERD, Kazakh NE corpus for the Kazakh language and etc. [1]. In this paper, we present the UzNERD dataset for the Uzbek language.

In order to develop the UzNERD dataset, the texts of articles related to various fields were collected from the daryo.uz news site. The text is divided into sentences and each sentence is assigned an identifier. Sentences with two or more copies and sentences containing only Russian words were removed. The IOB2 annotation scheme was used in the development of the NER dataset in Uzbek language. Experts performed independent annotation on the same text using the WebAnno annotation tool. The results of the annotation were evaluated using Cohen's kappa coefficient and gave a satisfactory value of 0.75. Objects identified in the dataset include person names, place names, organization names, e-mail addresses, phone numbers, URL links, job and profession titles, currencies, percentage values, numeric values, product names, event names, titles, nationalities, religious names, dates and times divided into divided into classes.

The UzNERD dataset supports facilitate the development and evaluation of NER systems for Uzbek language text. It can be used to train machine learning models, evaluate the performance of NER algorithms, and support research on information retrieval, language understanding, and other applications related to the Uzbek language. We believe that the development of the UzNERD dataset fills an important gap in the availability of high-quality resources for Uzbek-language NER research.

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General structure of uzbek treebank

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Scientific researches have been conducted by many scientists in various areas of computer linguistics in the world.Today, problems related to computer linguistics are being researched by scientists in such areas as language technology, natural language processing, machine learning, and corpus technology.

N. Yaqutova, M. Ayimbetov, S. Rizaev and S. Muhammedova started the scientific research on linguostatistics in the field of computer linguistics in Uzbekistan.Also, S. Muhammedov creates a textbook in co-authorship with R. Piotrovsky, which covers quantitative models of Uzbek texts.Today, the number of these scientists is increasing, and a number of scientists are conducting research on computer linguistics.At the same time, as a result of the cooperation of experts in other fields related to computer linguistics and language technology, positive progress is being made in the areas of this science.

The article covers issues such as research in the field of computer linguistics in Uzbek linguistics, specific features and the structure of the Uzbek language, and features different from other European languages.Also, recommendations are made regarding the structural aspects of the Uzbek language that should be taken into account in research in the field of computer linguistics, and Chomsky's hierarchy is explained using examples from the Uzbek language.

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Complex simple sentence patterns in Uzbek

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Today, in Uzbek linguistics, as well as in English grammar, modeling of our language is being carried out, and the models we provide are to develop them, to contribute to the implementation of quality text translation into other languages, using these derivatives and preserving the content. In the process of molding simple sentences, it is necessary to emphasize that it is also an important issue to mold complex simple sentences with the help of introductory units, adverbial words, and connected clauses. In the process of forming complex simple sentences in Uzbek and English, we witnessed the commonalities and peculiarities of both languages. Formatting any language is the most important way to correctly translate a certain unit or sentence, text in that language into other languages in automatic machine translation, to make this unit familiar to people of another language.

In the structure of the sentence, there are many noun units that express the relation of subjective modality, which are directly related to this meaning in (Pm), although they do not enter into a grammatical relationship with the parts of the sentence and do not form a word combination. Words separated from the word group in the introductory task and used only in the introductory task (such as - probably, of course, so - so, for example - for example), not separated from the known word group, are also used in the introductory task , there are words that can be used as part of a sentence (the first, briefly, really, apparently, eventually, etc.). According to the structure of introductory units, there are 3 types: introductory clauses, introductory compounds, introductory sentences. The article provides detailed information about such units.

Automatic speech recognition models for uzbek language

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This study introduces an innovative Automatic Speech Recognition (ASR) system for Uzbek language and its dialects. Combining Hybrid Deep Neural Network Hidden Markov Model (DNN-HMM) architecture with End-To-End (E2E) modeling, the research overcomes challenges posed by Uzbek's agglutinative nature and limited dialect data.

Beginning with an overview of ASR system evolution, the study addresses Uzbek-specific challenges like agglutination-driven vocabulary expansion and varied dialect orthographic rules. The novel ASR system integrates DNN-HMM with Hybrid Connectionist Temporal Classification (CTC), incorporating advanced techniques like tailored position-embedding and Uzbek-specific analytics. A vocal activity detection (VAD) pipeline manages lengthy speech segments. Our methodology involves assembling a comprehensive Uzbek speech corpus, supplemented by UzbekVoiceAI and Common Voice Project data. Evaluation compares the proposed DNN-HMM and E2E models against a hybrid benchmark.

Results highlight the E2E Transformer architecture's potency, particularly with CTC and attention objectives. The E2E Transformer outperforms DNN-HMM, achieving a significant 20 percent reduction in Word Error Rate (WER) on the test set.

Beyond ASR model development, the study provides detailed Uzbek dataset analysis, covering data preprocessing and default model hyperparameters. It offers insights for future research and underscores benchmark dataset establishment, enhancing the broader speech recognition community.

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UzUDT: Building a Universal Dependencies Treebank and Parser for Uzbek Language NLP

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In the era of information technology and the Internet, the challenge of harnessing vast data to meet societal needs has become paramount. Natural Language Processing (NLP), an interdisciplinary fusion of computational linguistics and AI, underpins various AI applications like translation, text analysis, and question-answering.

Accurate dependency parsing is pivotal for NLP applications. The Universal Dependencies (UD) project, a cross-linguistic endeavor, offers consistent treebank annotations across languages, enhancing multilingual parsing and cross-lingual learning.

While 20 of 7,000 languages are high-resource, Uzbek, spoken by more than 35 million people, remains low-resource. We address this gap by creating fundamental NLP tools for Uzbek, then move onto the creation of inaugural Uzbek Universal Dependency Treebanks (UzUDT) and various parsers, alongside their analysis. UzUDT propels Uzbek NLP with contemporary dependency parsing AI technologies, including graph-based neural architectures and transition-based methods.

Our contribution extends the frontiers of NLP, culminating in UzUDT's inception. Through UzUDT, we illuminate the path to a linguistically empowered future for Uzbek, aligning with the evolving landscape of AI and linguistic exploration. Furthermore, our project's collaboration with linguistic experts and leveraging cutting-edge AI techniques demonstrates the potential for creating robust NLP tools for under-resourced languages in the Turkic family, providing a model for future initiatives to follow suit.

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Methodology and analysis for uzbek syllabification

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This research presents a significant advancement in Natural Language Processing (NLP) tailored to the Uzbek language through the introduction of the "UzSyllable"tool. Beyond a mere algorithmic solution, our approach integrates algorithm design with a user-friendly web-based tool and an accessible API, catering to diverse users in the NLP community. The core objective centers on both syllable and hyphenation extraction, a fundamental linguistic task with wide-ranging implications for NLP applications.

At the heart of the "UzSyllable"tool lies a meticulously crafted algorithm, coded in Python, and carefully tested. Notably, the algorithm showcases a remarkable close-to-0% error rate in its initial evaluation, performed on a carefully curated subset of words extracted from "The Uzbek dictionary."The dataset collection process itself stands as a testament to our commitment, yielding over 78,000 Latin script words and 13,000 Cyrillic script words, forming a robust foundation for subsequent analyses.

In the experimental phase, we deviate from conventional syllabification comparisons due to the dearth of suitable open-source alternatives. Instead, our focus shifts to the prediction of syllable counts, an equally informative endeavor. To this end, a comprehensive ensemble of machine learning algorithms is harnessed, each meticulously chosen to capture intricate linguistic patterns. Decision Trees, K-Nearest Neighbors, Random Forest, Support Vector Machines, Multi-Layer Perceptrons, and Recurrent Neural Networks collectively contribute to a comprehensive exploration of predictive accuracy.

This work presents a pivotal contribution to the realm of Uzbek language processing, bridging an evident gap in NLP resources. By unifying sophisticated algorithmic design, detailed dataset analysis, and diverse machine learning methodologies, the "UzSyllable"tool emerges as a versatile asset for researchers, developers, and language enthusiasts alike. In essence, this research establishes a foundation for further exploration, innovation, and practical applications within the dynamic landscape of Uzbek language and NLP.

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Computation of fuzzy analytic hierarchy process with new linguistic terms based on type-2 fuzzy set

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The fuzzy analytic hierarchy process (FAHP) has been used to solve various multi-criteria decision- making problems where trapezoidal type-1 fuzzy sets are utilized in defining decision- makers' linguistic judgment. Previous theories have suggested that interval type-2 fuzzy sets (IT2 FS) can offer an alternative that can handle vagueness and uncertainty. This paper proposes a new FAHP characterized by IT2 FS for linguistic variables. Differently from the typical FAHP, which directly utilizes trapezoidal type-1 fuzzy numbers, this method introduces IT2 FS to enhance judgment in the fuzzy decision-making environment. This new model includes linguistic variables in IT2 FS and a rank value method for normalizing upper and lower of IT2 FS. The proposed model is illustrated by a numerical example of work safety evaluation. Comparable results are also presented to check the feasibility of the proposed method. It is shown that the ranking order of the proposed method is consistent with the other two methods despite difference in weight priorities.

Dependency graph parsing for structured sentiment analysis in the Uzbek language

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This paper proposes an approach for sentiment analysis in the Uzbek language using dependency graph parsing, aiming to capture both the sentiment polarity and the syntactic structure of the text.Dependency graph parsing is a syntactic analysis technique that represents the grammatical structure of a sentence as a directed graph, where words are nodes and grammatical relationships are edges. This approach allows us to capture the intricate relationships between words and their dependencies, providing valuable insights into the sentiment-bearing components of a sentence. We begin by employing a robust dependency parser tailored for the Uzbek language to construct a dependency graph $G = (V, E)$ for each sentence in the text. Here, V represents the set of words in the sentence, and E represents the set of grammatical dependencies between words. Let S be the set of sentiment-bearing words identified through a sentiment lexicon or pre-trained sentiment analysis model. We assign a sentiment score $\delta(v)$ to each word $v \in V$ based on its sentiment polarity. The sentiment score captures the strength and direction of sentiment associated with the word. We define an opinion tuple as (v, s), where v is a sentiment-bearing word from S, and s is its sentiment score. Opinion tuples capture individual word-level sentiment information.

1 Dependency-Based Aggregation

To derive the overall sentiment of a sentence, we introduce a sentiment aggregation function $\Psi(G, S)$ that considers the sentiment scores of words along with their syntactic roles and dependencies in the graph. The aggregation function can be defined as follows:

$$\Psi(G, S) = \sum_{(v,s)\in \text{OpinionTuples}(G,S)} [\alpha(v) \cdot \beta(v, G, S) \cdot s]$$

Where $\alpha(v)$ represents the importance weight of the word v, and $\beta(v, G, S)$ captures the influence of v's sentiment within its dependency context.

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Parts-of-speech tagging for uzbek language using deep learning techniques

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Part-of-speech tagging algorithms are widely used in machine translation, information extraction, sentiment analysis, grammar checking, text summarization, named entity recognition and other tasks. Manually tagging parts of speech (POS) in the text is time-consuming and labor-intensive. Therefore, developing systems and algorithms for POS tagging is an urgent task. POS tagging systems and algorithms for various world languages have been developed and are currently in use. Specifically, there are POS tagging tools like Stanford POS Tagger, NLTK, SpaCy and OpenNLP for English; RusPoS, Natasha and Pymorphy2 for Russian; KazTagger and LSTM-based POS tagger for Kazakh; and UzbekTagger for Uzbek languages [1, 2]. As a morphologically complex language, the Uzbek language poses distinctive challenges for POS tagging. Conventional rule-based approaches often struggle to handle the complexity of Uzbek morphology. This paper introduces a deep learning-based approach to address these issues.

The online news platform “Daryo” was chosen for collecting text corpus data used in POS tagging. The collected text corpus was tagged by linguists-experts. The tags used for POS tagging and their corresponding labels are as follows: noun – OT (NOUN), verb – FEL (VERB), adjective – SIF (ADJ), number – NUM (NUM), adverb – RAV (ADV), pronoun – OLM (PRON), auxiliary – KMK (AUX), conjunction – BGL (CCONJ), part – YUKL (PART), modal word – MDL (MDL), imitation word - TAQ (IMIT), exclamation word - UND (INTJ), punctuation mark - TB (PUNCT), symbol - SIM (SYM), other - X (X). The POS-tagged corpus comprises over 54,000 words.

In order to train a deep learning-based POS tagging model for text, initial steps of text pre-processing were executed, including normalization and tokenization. Subsequently, a character-level embedding model representing words as vectors was employed. To retain the interrelationship among character sequences in words, a LSTM model was utilized.

The datasets employed for model training and testing encompass diverse domains and linguistic contexts. To train and test the proposed neural network model, a 10-fold cross-validation approach was applied to the dataset. The algorithm yielded an average accuracy of 89.7%. We anticipate that implementing the outcomes of this study in Uzbek computational linguistics will yield effective results.

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Formalizing the Uzbek Language: A Comprehensive Exploration using Backus-Naur Forms with Step-by-Step Examples

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Introduction The Uzbek language, with its rich linguistic heritage, poses interesting challenges in terms of syntactic analysis and language processing. BNF, as a formal metasyntax, offers a systematic approach to describe the grammar and syntax of the Uzbek language. In this review, we delve into the use of BNF as a tool for formalizing and understanding the intricacies of Uzbek.

Defining the Alphabet The first step in utilizing BNF for the Uzbek language is defining its alphabet. We outline the Cyrillic-based Uzbek alphabet and demonstrate how BNF can be employed to represent the set of letters that comprise it. Through examples, we illustrate the flexibility of BNF in describing the alphabet [1].

Describing Vowels and Consonants Next, we focus on the vowels and consonants in the Uzbek language. We present a comprehensive set of rules using BNF to define these phonetic elements. By exploring various vowel and consonant combinations, we showcase the versatility of BNF in capturing the distinct sounds of Uzbek.

Modeling Morphological Rules Morphology plays a crucial role in understanding the structure of words in Uzbek[2]. We explore how BNF can be utilized to model morphological rules, including word formation, suffixes, and endings. By defining the core principles of Uzbek morphology using BNF, we provide a foundation for analyzing and generating words.

Syntax Description for Sentence Construction BNF can effectively describe the syntax of Uzbek sentences. We outline the step-by-step process of utilizing BNF to define the rules for subjects, predicates, and other sentence components[3]. Through comprehensive examples, we demonstrate how BNF can capture the structure and semantics of Uzbek sentences.

Conclusion In this review, we have explored the application of Backus-Naur Forms for the Uzbek language. By providing a step-by-step approach with descriptive examples, we have illustrated how BNF can effectively capture the structure, grammar, and syntax of Uzbek. The formalization of Uzbek using BNF opens up opportunities for enhanced language analysis, automated processing, and linguistic research in the context of the Uzbek language. Through this comprehensive review, we aim to inspire further exploration and utilization of BNF in the analysis and understanding of the Uzbek language, ultimately contributing to its linguistic study and technological advancement.

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Machine learning and deep learning approaches for Uzbek semantic evaluation

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This research paper presents a comprehensive investigation into the application of machine learning and deep learning methodologies for semantic evaluation in the Uzbek language, utilizing the SimRelUz dataset—a collection of word pairs annotated with similarity and relatedness scores. Through a series of experiments and rigorous evaluation analyses, we examine the efficacy of various approaches in capturing semantic relationships within the Uzbek language.

Our study encompasses a range of techniques, including feature engineering, traditional machine learning models, and deep neural networks. Leveraging a carefully curated training, validation, and test dataset split, we employ metrics such as Mean Squared Error (MSE), Pearson correlation, and Spearman rank correlation to quantitatively assess the predictive performance of our models.

The results of our research provide valuable insights into the strengths and limitations of each approach. Our findings reveal that the chosen machine learning and deep learning methods exhibit varying degrees of success in capturing the nuanced semantic nuances present within the SimRelUz dataset. Moreover, we shed light on the impact of different feature sets, network architectures, and training strategies on the performance of these models.

In conclusion, this paper contributes to the field of Uzbek semantic evaluation by offering a comprehensive analysis of machine learning and deep learning techniques. The insights gained from our study pave the way for future advancements in natural language processing for the Uzbek language, opening opportunities for improved information retrieval, sentiment analysis, and language understanding applications.

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X. ARTIFICIAL INTELLIGENCE

Active contour model for segmentation of mammogram images

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Accurate segmentation of mammogram images is crucial for effective breast cancer diagnosis and treatment planning. Active contour models, also known as snakes, have shown promising results in segmenting mammogram images by capturing the boundaries of breast masses or lesions. Internal and exterior regions of the curve define a signed distant function (SDF) on the surface, which is separated into two regions:

$$\phi(x,\, y,\, t = 0) = d$$

The level sets general movement formula is as follows:

$$\phi t + F|\upsilon\varphi| = 0$$

where F represents for a constant-speed contour-pushing or -pulling term. The equation [1] gives the mean curvature motion-based level set method:

$$\phi_t = \frac{|\upsilon\varphi|}{(\varepsilon k\phi) + \upsilon}$$

where υ is the correction term, which is specifically chosen to keep the quantity $(\varepsilon k\phi + \upsilon)$ at a positive value, and k refers to the mean curvature of the level set function, which is defined as:

$$k = div\frac{\upsilon\varphi}{|\upsilon\varphi|}$$

where υ is a constant and represents a correction value. The quantity $\varepsilon k\phi + \upsilon$ remains always positive. The mean curvature of the level set function is given by

$$k = \frac{\phi_{xx}\phi_y^2 - 2\phi_x\phi_y\phi_{xy} + \phi_{yy}\phi_{xx}^2}{(\phi_x^2 + \phi_y^2)^{\frac{3}{2}}}$$

where ϕ_x and ϕ_{yy} denote first and second order partial derivative of ϕ_x.

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Parameters of the development of the department activity management system

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In a rapidly developing world, the effective organization of educational activities is impossible without strong information support, which is the basis of intelligent information systems, which serves to efficiently process large amounts of data, providing information support for decision-making, the choice of optimal options and an effective strategy for the behavior of certain educational processes. This, in turn, actualizes the problem of designing and implementing a comprehensive solution for providing intellectual information support for educational, research, and management processes in the activities of the department. To solve the identified problem, it is necessary first of all to determine the target, content and structural parameters for the development of the department's activity management system. The analysis of the methodological and theoretical foundations for the development of educational, research, management processes of the department's activities made it possible to reveal that in the conditions of innovative development of higher education, strengthening the connection between education, science and production, commercialization of research activities, as well as international collaboration becomes relevant. Based on this, it is necessary to improve the target, content and management parameters of the educational, research, management activities of the department. An important element in the development of the department's activities is the improvement of target, content and structural parameters, based on the provision of a matrix structure of educational, research, management processes, allowing to regulate the coordination system of interaction of all subjects of the educational system by focusing on the innovative tasks of the department. Therefore, for a higher educational institution, the tasks of creating a unified concept for building an intelligent information system, as well as developing design methods, come to the fore and its introduction into the educational process with the aim of further increasing the efficiency of the functioning of the department, expanding the scope of educational services and an adequate response to the growing dynamics of information and communication other opportunities

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An integrated approach to the analysis of multilingual texts including text represented in two alphabets

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The task recognition of language within the field of natural language processing (NLP) involves identifying the language of a given text or document, but the ability to identify numerous languages becomes challenging. This paper proposes a comprehensive model for recognizing language from text, specifically focusing on the Latin-Cyrillic script used in the Uzbek language. Considering the lack of research in this area, we present a high-accuracy Uzbek Latin-Cyrillic script alphabet recognition model based on a suitable transformer architecture. Our model was evaluated using the corpus of the Uzbek language created by us, which also can be serve as a valuable resource for evaluating future Uzbek language detection tasks. This model covers 21 languages, including Uzbek, represented in two alphabets (Latin and Cyrillic). The results we obtained demonstrate that our XLM-RoBERTa[1] transformer-based model of language recognition achieves significant performance and accuracy compared to previous research models.

Thus, for a bilingual (Uzbek, English) text, where the Uzbek language is represented in two alphabets (Cyrillic and Latin), we created a model for identifying such a text. Our proposed uz-kr-language-detection model based on the XLM-RoBERTa architecture demonstrates exceptional performance in accurately identifying languages from input texts. Its versatility and reliability make it suitable for use in the processing of the Uzbek language, given the recognition of the latter as the state language in Uzbekistan and the corresponding high demand for the analysis of texts of various characters. We believe that our findings contribute to the advancement of language identification techniques and provide a valuable resource for researchers and practitioners in the NLP field.

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Analysis of manifolds of object relationship structures in machine learning

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Nonlinear transformations of the values of different types of features in the description of admissible objects from two classes are considered. The result of the transformation is the replacement of the original data with binary ones. The formation of a variety of sets of latent features is based on various ways of grouping binary data. Latent features are considered as a means for reducing the dimension of space and synthesizing ensembles of recognition algorithms using stacking technology.

The formation of latent feature sets is one of the ways to solve the problem of the curse of dimensionality in data mining problems. The known linear and nonlinear methods of dimensionality reduction have a number of disadvantages that limit their application.

The source of the variety of sets of latent features is the many ways of grouping the initial features, on the basis of which they are formed according to the method of generalized estimates [1]. These sets are used to study the structure of the relations of the sample objects. Numerical characteristics of the results of the analysis of the structure of relations are the set of values of measures of their compactness by class and the sample as a whole.

A method is proposed for calculating the distribution density of data on a training sample for a set of class objects that are boundary with respect to a given metric. A metric has been developed to compare density on two sets of latent features, and its properties have been described. One of the properties in the analysis of polytypic features, taking into account the classification of objects, is their stability. The relevance of using this property for calculating generalized estimates of data analysis increases in the presence of a theoretical justification for the convergence of stability values to a fixed value.

The paper considers a comparative analysis of two types of relations between objects on a training set:

– proximity of objects through the formation of a density matrix of distributions;

– connectivity of class objects by a system of hyperballs at the intersection of which contains boundary objects of classes.

The results of the analysis of the above relations are in demand to reduce the dimension of the attribute space, to justify the process of forming ensembles of algorithms in order to increase the generalizing ability in recognizing objects not involved in training.

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Use of mathematical models for human pose estimation in person identification

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Today, due to the large amount of information flow, the problem of short-term processing is also increasing. Since the flow of data is very large, it is desirable to create automated systems that can solve a specific problem in a short time [1].

Based on the above, the problem of identifying a person by his behavior or pose also arises, which in turn requires the use of neural networks and good mathematical models. External influences can falsify the information properties of existing identification technologies. In such cases, an effective technology is to identify a person based on his behavior.

With the advent of computerized databases and the digitization of analog data, biometric verification has evolved significantly, allowing instant personal identification. To provide a brief explanation of how CCM, DRT, and HVA simultaneously make predictions for spreadsheets, the equations for these models are presented in their most basic form without the free parameters used in curve comparison. This avoids unnecessary complexity and helps to focus on the main differences between the models.

When considering mathematical models of behavior, it is useful to distinguish between descriptive equations and theoretical equations. A descriptive equation only provides a convenient way of summarizing the relationship between the independent variable and the dependent variable in mathematical form, it does not provide any theoretical basis for using the exact equation. Rather, a theoretical equation derives from basic principles or assumptions about the psychological processes that produce the behavior in question, and the form of the equation reflects these assumptions. CCM, DRT, and HVA can be considered theoretical equations because they are based on several basic assumptions about the psychological processes that govern choice behavior. It can be seen that some of these assumptions are present in all three models, while other assumptions are different for each model.

In conclusion, it can be said that in the future it will be possible to determine the pose of a person using algorithms and neural networks to form and express behavior.

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Methods of assembly of parallel corpus for uzbek and kazakh languages

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The dataset presented in this work is a result of a combination of three major parts, hence referred as three stages. We explain all three stages below.

Stage-1. Available resources collection.

Although the result of this stage does not add much contribution to the overall parallel text dataset (less than 1% of the final dataset content), it plays a crucial role to make the right template for the rest of the data. Our small research on finding openly-available parallel text data between Uzbek and Kazakh yielded only three resources with very small content. They are: The Universal Declaration of Human Rights act [1] with 61 sentences, Apertium rule-based machine translation platform's with Kazakh-Uzbek repository test dataset with 49 parallel sentences, and lastly, the recent COVID19 Myth busters multi-language corpus with only 28 sentences.

Stage-2. Automatic alignment.

As for the methodology used in the automatic alignment process, we used the Word-Correspondence-Based Alignment method from Moore [2], which in our case, was more accurate and even faster than the IBM models 1 and 2 [3]. The Word-Correspondence-Based Alignment method, according to the source, is a combined model (of the source's first Word-Translation model and the IBM Model-1) that estimates the probability of a 1-to-1 bead consisting of a source sentence (s) and a target sentence (t) as follows:

$$P(s,t) = \frac{P_{1-1}(l,m)}{(l+1)^m}(\textstyle\prod_{j=1}^{m}\sum_{i=0}^{l} tr(t_j|s_i))(\prod_{i=1}^{l} f_u(s_i)). \quad (1)$$

Here, s is a source sentence of length l, t is a target sentence of length m, and P_{1-1}(l, m) is the probability assigned by the initial model to a sentence of length l aligning 1-to-1 with a sentence of length m.

Stage-3. Manual translation.

This stage includes the majority of the dataset in terms of time consumed, human effort involved, and the resulting text data size collected, which in turn makes the majority (more than 80%) of the final dataset content. As the name of the stage indicates, a big enough open-source Uzbek language raw texts corpus was taken and was translated using a group of contributors who are fluent in both Uzbek and Kazakh. The Uzbek Corpus Sample was chosen for translation, which is open-source and consists of 100000 sentences obtained from various open-source resources, making it perfect size and content for our research.

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Mammogram image binarization using Otsu method

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The Otsu method is a global adaptive binarization threshold image segmentation algorithm. Suppose that the gray level is (used to be 256), and the number of pixels of gray level at i is n_i then the number of total pixels of an image is $N = \sum_{i=0}^{L-1} n_i$, the probability that each gray level appears is $p_i = \frac{n_i}{N}$. The Otsu method selects gray level k as the segmentation threshold, and divides the image into two classes: the background class named $W1$ which gray level ranges from 0 to k, the target class named $W2$ which gray level ranges from $k+1$ to $L-1$ [1]. The gray-level probability distributions for the two classes are given as:

$$PW1 = \sum_{i=0}^{k} p_i$$

and

$$PW2 = \sum_{i=+1}^{L-1} p_i = 1 - P_{W1}.$$

The mean gray level of $PW1$ and $PW2$ are:

$$\mu_{W1} = \sum_{i=0}^{k} \frac{i\dot{p}_i}{P_{W1}}$$

and

$$\mu_{W2} = \sum_{i=k+1}^{L-1} \frac{i\dot{p}_i}{P_{W2}}.$$

The mean of total gray levels is denoted by μ:

$$\mu = P_{W1}\mu_{W1} + P_{W2}\mu_{W2} = \sum_{i=0}^{L-1} i\dot{p}_i.$$

The between-class variance is:

$$\sigma^2(k) = P_{W1}(\mu_{W1} - \mu)^2 + P_{W2}(\mu_{W2} - \mu)^2.$$

Otsu's method chooses the optimal threshold k by maximizing the between-class variance $\sigma^2(k)$. The larger the variance, the better the image segmentation. The optimal threshold $T^\star$ for the perfect image segmentation is given as: $T^\star = \arg\max\left(\sigma^T(T)\right)$, $0 \le T \le L-1$.

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Increasing reliability of information on the basis of information proximity of document elements

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The methodology has been researched and methods, models, algorithms have been developed for implementing the technology for increasing the reliability of information in electronic document management systems with mechanisms for determining the information proximity of elements, code words, by bit, digital, linear, modular summation, using a multilevel morphological and - gram structured model for analyzing texts on natural language. Computational schemes have been developed for describing sets, basic functions for initializing all objects, using logical and statistical relationships of text elements, forming a knowledge base with sets of rules, and information control statements. A technique has been developed, which is implemented in the form of a Python program for estimating the parameters of mechanisms for increasing the reliability of information. Mechanisms based on binary modular summation of information bits of a code word over lines of text are proposed. The value of the least non-negative residue of the weight of the information vector, estimates of the probabilities of undetected errors, coefficients of gain in the reliability of information, labor intensity and cost of information control are obtained. A software package for improving the reliability of information in the C++ language has been implemented, in which the proposed mechanisms are synthesized in the environment of the CUDA parallel computing technology. The software package detects and corrects multiple errors in textual information.

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Ischemic heart disease prediction using machine learning methods

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Ischemic heart disease is the most common disease in developed countries, accounting for 20% of all causes of death. Ischemic heart disease is increasing year by year, and this disease is also occurring among young people living an active lifestyle. In recent years, there has been a growing interest in using data mining methods to analyze large datasets of medical information and identify patterns that can be used to improve the diagnosis and treatment of ischemic heart disease. Data mining techniques such as classification, clustering, and association rule mining have been used to analyze factors such as patient demographics, medical history, and clinical test results to identify risk factors and predict outcomes [1]. Data Mining is designed to help in decision making. Knowledge about the patterns in the observed processes is important. The more specific the information, the more useful it is for decision making. Thus, Data Mining is the process of discovering useful knowledge. Moreover, a necessary requirement is the discovery in raw data: previously unknown, non-trivial, practically useful, accessible for interpretation of knowledge that is useful for making decisions in various areas of human activity [2].

This article provides an overview of the use of data mining methods in ischemic heart disease research. Our objective was to identify the most effective method that can aid in the early detection and diagnosis of ischemic heart disease. Through our analysis, we compared the performance of three popular data mining algorithms: decision tree, logistic regression and random forest. In our experiment Logistic Regression was the most informative and reliable for finding informative features and Random Forest for classification. According to the results, we can conclude that it is possible to obtain a high classification accuracy using two or three informative features.

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Improvement of educational activities in higher education based on adaptive learning with artificial intelligence

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The transition to new technologies in education and in all other areas is changing the face of modern education: new teaching methods and approaches are emerging that meet the requirements of the time: E-Learning(electronic education), MOOC (public open online education), Learning throughout life, adaptive education and others.

Adaptive learning is a system of technologies that constantly analyzes the educational results of the student, takes into account his characteristics and adapts the educational program, and sometimes the teaching method.

Adaptive learning helps to achieve the following educational goals: 1. Reducing the cost of student education. 2. Formation and training according to a personal program. 3. Make the educational process easy and flexible, etc. While adaptive learning has many benefits, it is not always suitable for everyone. In this table, we show in which cases the use of this technology is effective and vice versa:

Adaptive learning is appropriate	**Adaptive learning is not suitable**
The student knows at an intermediate level	The student is studying on his own
To quickly gain specific knowledge on the subject time management skills	High level of self-control and
The theme is stable and does not require quick changes	-
A huge number of students who need to study	-

Developers of adaptive learning platforms can use artificial intelligence technologies to collect and process large amounts of data. This allows for individual learning opportunities for each student, with the teacher at the center of education.

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Parallel algorithms for the building of latent feature space

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Developing an effective software solution for a given problem has always been a difficult task. When creating a program, the developer always compromises between efficiency, portability, and performance, and this determines the programming method, technology. In high-performance computing, it is important to minimize the time required to solve the problem first. Parallel computing is the use of multiple computing devices to execute different parts of a single program at the same time. The main purpose of parallel computing is to reduce the time to solve the problem. Calculations can be performed on several cores of one processor or on physically separate processors [1].

Algorithms can be less accurate in training algorithms for machine learning problems. In these cases, various approaches are implemented, for example, space reduction based on informative features, cleaning the sample from noise objects, normalization of values, and similar preprocessing operations. In the considered work, it is proposed to solve this problem by transforming the feature space. The process of building a new (latent) feature space includes the following steps [2]:

1. Calculation of the intervals of dominance for quantitative features;
2. Calculation of the values of the function of belonging to a class by intervals;
3. Non-linear transformation of feature values based on the membership function;
4. Calculation of stability of features and removal of features with a relatively small stability value;
5. Formation of latent features based on generalized estimations for a set of features;
6. Implementation of classification and comparison of results.

The steps (1) and (2) mentioned above have the computational complexity associated with full sorting, and the use of Multiple Instruction Multiple Data (MIMD) architecture [3] in their implementation leads to higher efficiency.

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Filtering histological images using the Non-local means method

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Filtering and processing of histological images play an important role in the analysis of medical data and the diagnosis of oncological diseases. Histological images are a valuable source of information for clinicians and researchers, but they are often accompanied by noise, artifacts, and other anomalies that can make it difficult to properly examine and interpret the data. The problem of filtering histological images becomes especially relevant in light of the rapid development of medical imaging techniques and machine learning. Nonlocal means (NL-means) method is a powerful method for denoising [1]. In this work, we use denoising scheme based on the NL-means algorithm for histological imaging denoising.

The main idea of the method is that instead of smoothing each pixel by the average value of the pixels in the neighborhood, it uses information from all "non-local"(that is, not necessarily adjacent) areas of the image, which are similar to target area [2].

Here's how the "Non-local means"method works:

Finding similar areas: This is done by comparing the intensities of the pixels in different areas. Similar areas may be far apart.For each pixel (i, j) we are looking for a set of positions $\{(p, q)\}$, which are areas of the image similar to the area (i, j).To measure the similarity of areas, is often used a similarity function ω , such as the exponential difference of squares:

$$\omega((i,j),(p,q)) = e^{-\frac{\|I(i,j)-I(p,q)\|^2}{h^2}}$$

where $\|I(i,j) - I(p,q)\|$ - is the difference in pixel intensities, h-is a smoothing parameter that controls the weight of similar areas.

Calculating the weighted average: Weights depend on the degree of similarity between areas - more similar areas have more weight. For each pixel (i, j), using the found similar areas, the weighted average of the pixel intensities in these areas is calculated:

$$I_{smooth}(i,j) = \frac{1}{Z(i,j)} \sum\nolimits_{(p,q)} \omega((i,j),(p,q)) \cdot I(p,q)$$

where $Z\,(i,j)$ -normalization constant:$Z(i,j) = \sum_{(p,q)} \omega((i,j),(p,q))$

Smoothing: This step is performed for all pixels in the image. After calculating the weighted average for all pixels, update the I image using the received I_{smooth} values.

In conclusion, the "Non-local means"method is an algorithm for smoothing images, which is based on the idea of using information from similar areas of an image to reduce noise. This article proposes a new method for denoising histological images using the NL-means method. To demonstrate the effectiveness of the proposed method, experiments were carried out on real histological images.

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Modeling the topic of texts using semantic correlation of words in social networks

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Nowadays, people use short text to express their thoughts on social networks like Twitter, Facebook, Telegram, WahtsApp and YouTube, and also on e-commerce websites like Amazon and Flipkart to share their experiences. Billions of short text tweets, tags, keywords, search queries are generated every day around the world. This work proposes a new model named G_SeaNMF (Gensim_SeaNMF) to improve the semantic relations of word context using local and global word placement techniques. Word embedding from large corpora provides general semantic and syntactic information about words that can guide topic modeling for short text sets as sparsely co-occurring additional information. In the proposed model, SeaNMF (Semantics-assisted Non-negative Matrix Factorization) is combined with word2vec model of Gensim library to strengthen the semantic relation of the word. This paper explores DMM (Dirichlet Multinomial Mixture), self-assembly, and global word co-occurrence based short text topic modeling techniques. They are evaluated using various metrics to measure cluster coherence in real-world datasets such as Search Snippet, Biomedicine, Pascal Flickr, Tweet, and TagMyNews. Empirical evaluation shows that the combination of local and global word placement provides more relevant words under each topic with improved results.

Thus, this research paper proposes an efficient topic modeling based on unsupervised clustering that extracts more meaningful topics [1,2].

Key contribution of short text topic modeling:

To propose an efficient short-text topic model named G_SeaNMF by combining the Gensim library's word2vec model trained on the Google News dataset and the SeaNMF model, which discovers specific topics from short-text documents.

Combining internal and external corpora to identify semantic relationships between words, resulting in more related terms under a single theme.

Processing a qualitative SA (semantic analysis) that shows the overall performance of the system by finding meaningful topics using the document term matrix and tagging each topic.

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Fuzzy mamdani model of diagnostics of infectious diseases in cows

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Ensuring high quality diagnoses is an important aspect of veterinary medicine. AI knowledge bases are a powerful tool that can help in the accumulation and dissemination of professional knowledge among specialists. In general, AI knowledge bases can play a significant role in improving the quality of diagnosis and providing access to expert knowledge for all professionals, supporting decision-making and improving the efficiency of veterinary and medical practice. The main goal is to build a fuzzy logical model of Mamdani and, on its basis, make a diagnosis of infectious diseases of cows.

The use of an intelligent system makes it possible to implement already developed veterinary and medical diagnostic tests. The system can be configured for mass testing, analysis of results and generation of individual recommendations. The intelligent system is capable of automatically processing input data, including symptoms, lab results, and other patient or target information. The system uses a knowledge base and algorithms to analyze this data and provide a diagnostic assessment, determine the likelihood of a particular disease or condition, and suggest appropriate recommendations or next steps. This approach greatly simplifies and speeds up the diagnostic process and provides more reliable and consistent results. In addition, the intelligent system is able to take into account a large amount of knowledge, including expert knowledge, statistics and research results, which allows for more accurate diagnoses and recommendations. Thus, the use of an intelligent system in veterinary medicine and medicine provides automation and optimization of the diagnostic process, facilitates the work of specialists, improves the quality of diagnosis and facilitates informed decision-making in clinical practice.

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Methods of summarizing video data based on artificial intelligence

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In this research work, the issue of developing video data summarization algorithms for video information systems based on artificial intelligence is studied. In order to sort large video files, intelligent algorithms for separating the main parts of a video file are proposed. Based on the developed algorithms, it is possible to get a conclusion about this video data without downloading large volumes of video data from portals. Currently, the rapid development of Internet technologies requires the development of algorithms for effective data processing through the Internet. The sharp increase in video data on the Internet means that the development of algorithms for automatic inference of this data by artificial intelligence is an urgent issue. The process of solving this problem allows video data summarization algorithms for video information systems to determine the characteristics of video files. In this way, it is possible to get a conclusion about what this video data is without downloading large volumes of video data. The main part of the information circulating on many large social networks and sites corresponds to video information. For example, about 500 hours of video data are uploaded to the YouTube platform every minute. In addition, users on Instagram spend 80% of their time watching videos, while this indicator is 100% on the TikTok social network.

So, how can we optimally extract the video data we need? In this research work, the methods of useful video data sorting are investigated. It is proposed to develop algorithms for summarizing video data as methods of sorting useful video data. There are various methods and methods of separating the useful video data, and such methods as generalization of video data (making a general conclusion), dividing the video into categories are widely used.

Methodology. It is necessary to determine the information about this video file without fully downloading the large and long video data. This requires summarizing video data. It is preferable to study only the general summary of the video information of the entire conference and see its main attractive parts. For example, in football, it is better to watch only the important moments of the game than to watch it in its entirety. These processes are performed on sports games, movies and documentaries, news, personal videos, long-term surveillance videos, live video streams and other types of videos.
Currently, large social networks and video portals also widely use general video inference algorithms for their content.

Video summarization can be divided into the following two types:
1) Dynamic summarization;
2) Static summarization.

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Methods for applying fuzzy set theory to the selection of environmentally friendly building protection materials

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One of the best strategies for achieving sustainable building is to choose materials that reduce the environmental impact. As such, designers and architects are encouraged to keep these considerations in mind early in the design process. The purpose of this research paper is to present a model for choosing the best sustainable building material. Therefore, life cycle assessment is used to perceive the holistic impact of materials on the environment, taking into account all stages of the product life cycle. Here, based on sustainable principles, the selection criteria are divided into four groups, defined as economic, technical, socio-cultural and environmental factors. Criteria priorities are calculated on the basis of expert opinions obtained from pairwise comparison questionnaires. Health and safety is at the forefront of social concern as more accidents and safety issues occur in construction. The weight of the best alternative material is shown, with wool brick ranked second and clay brick ranked third in sustainable material choice scores. Compared to the other two bricks, clay bricks require more energy and resources for their life cycle. If wool bricks are placed closer, the main limitation for other bricks is the lack of global transport, which leads to increased transport and storage costs. Taking all this into account, wool brick was chosen as the most durable building material.

In addition, this study will serve as a starting point for other researchers to learn more about sustainable material selection.

Overall, this research will help and support architects, engineers and other building practitioners in selecting the best sustainable materials in competitive business scenarios.

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Review of machine learning methods for predicting the severity of covid-19

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Classification of COVID-19 severity is an important task for effective and timely treatment and prevention of patients infected with or suspected of being infected with SARS-CoV-2. Machine learning can be a useful tool for developing models capable of predicting the risk of developing severe disease based on clinical and laboratory data obtained upon hospital admission or initial presentation. In this review, we will examine some of the recent studies on the application of machine learning for classifying the severity of COVID-19.

One such study was conducted by Marcos et al. [1], who developed a machine learning model for early identification of patients who would die or require mechanical ventilation during hospitalization. The model was based on the XGBoost gradient boosting method and considered variables such as the ratio of peripheral oxygen saturation (SpO2) to fraction of inspired oxygen (FiO2), age, estimated glomerular filtration rate, procalcitonin, C-reactive protein the updated Charlson comorbidity index, and lymphocytes. The model was implemented as an open-access calculator, where COVID-19 patients upon admission were stratified individually into high or low risk of developing severe disease.

Another study was conducted on supervised machine learning methods using K-NN, linear SVM, naive Bayes, decision tree (J48), AdaBoost, bagging, and stacking for the purpose of classifying the severity group of COVID-19 symptoms. There was utilized variables such as gender, age, body temperature, cough, shortness of breath, sore throat, weakness, rhinorrhea, and anosmia. Authors found that naive Bayes showed the best result with an accuracy of 0.98 and AUC of 0.99.

From this review, it is evident that machine learning can be effectively applied for classifying the severity of COVID-19 based on various clinical and laboratory variables. However, it should be noted that different studies use different datasets, variables, and evaluation metrics for their models, making direct comparison and generalization of results challenging. Therefore, further development and standardization of machine learning methods for COVID-19 severity classification are needed.

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Intellectual controlling the number of horizontal distributions at the distributed data flow management

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Today, as a result of the emergence of various digital devices and a sharp increase in their use, various types of data flows appear [1,2]. Processing these data flows and identifying hidden content from them gives the opportunity to bring all fields to a new development stage [1,3-5]. In order to analyze these data flows and identify hidden content from them, the data first goes through a preliminary processing stage. At the same time, they are transferred to a certain structure in order to simplify the processing process at the next stages. As a result, structured data flows are created. One of the approaches to efficient real-time processing of these data flows is the use of an internal distribution mechanism [6-10].

The management of structured data flows based on the internal distribution mechanism can be considered as a unit of two interdependent functions. The first function performs the task of receiving structured data streams, that is, storing it in tables with horizontal distribution, while the second function performs the task of processing the data stored in these tables, that is, correctly organizing requests. The first function performs complex processes in the management of data flows. One of such processes is the process of increasing the number of tables with horizontal distributions in the database in accordance with the volume of stored data. In general, increasing the number of tables to be horizontally distributed in proportion to the volume of stored data is carried out based on the following algorithm steps:

Step 1. The initial state of the table (tables) with horizontal distribution in the database is determined; **Step 2.** Structured data flows are accepted; **Step 3.** The total number of tuples in the table (horizontally distributed tables) is determined; **Step 4.** If the number of corteges reaches the specified limits, go to step 5, otherwise go to step 6; **Step 5.** The number of horizontally distributed tables is increased; **Step 6.** The amount of free memory allocated for the database of the computer on which the database is stored is determined; **Step 7.** If there is free memory, go to step 2, otherwise go to step 8; **Step 8.** The process is completed for this system.

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Image restoration models and programs based on fuzzy set theory and their future importance

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The use of fuzzy set theory models in image processing makes it possible to obtain robust and accurate restoration results. Note that the specific application and details of each model may vary depending on the research or application context.

These models incorporate fuzzy set theory to handle uncertainty and fuzzy information in the image reconstruction process. They aim to improve image quality by considering fuzzy membership functions and fuzzy inference mechanisms to make more informed decisions about image restoration operations.

Programs based on these models demonstrate the practical application of fuzzy set theory in image restoration. Incorporating fuzzy logic and fuzzy inference systems, they provide robust and flexible solutions for various image restoration tasks, improving the quality and reliability of restored images.

In the future, image processing based on fuzzy set theory is expected to play an increasingly important role due to the uncertainties inherent in image data and the ability to handle uncertain data.

Integration with Artificial Intelligence (AI) and Machine Learning (ML): Fuzzy set theory can be seamlessly integrated with AI and ML techniques to enhance image processing capabilities. By combining fuzzy logic with deep learning algorithms, it becomes possible to develop hybrid models that can efficiently learn from large data sets and make fuzzy-based decisions. This integration can lead to improvements in image classification, segmentation, object detection, and other image processing tasks.

Applications in Emerging Technologies: Blur-based image processing has the potential to find applications in emerging technologies such as autonomous vehicles, robotics, medical imaging, and computer vision. These fields often deal with complex and ambiguous visual data, where fuzzy set theory can provide valuable tools for accurate perception, decision making, and image analysis.

In summary, the future importance of the models we propose for image processing based on fuzzy set theory is its ability to process complex and uncertain image data, reliably restore corrupted images, enable flexible and intelligent algorithms, integrate with AI/ML techniques, and in the ability to find applications in emerging technologies. As image processing continues to develop, the use of fuzzy set theory is expected to contribute to more accurate, efficient, and human-like processing of visual information.

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Select informative features, on the measure of compactness of class objects

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The selection of informative features is considered using the compactness measure for the training set of objects [1]. The selection procedure is related to solving the curse of dimensionality. The set of values of the measure of compactness depends on the choice of metric for calculating the distance between objects and the method of data normalization. It is proposed to reduce the combinatorial complexity of the selection problem by preprocessing the data. The meaning of preprocessing is reduced to the construction of a matrix of pairwise distinguishability of features and the construction of an ordered sequence based on it.

The selection of informative feature sets is an important and least formalized section of discriminant analysis. The efficiency of the numerical implementation of selection criteria largely depends on the use of certain (as a rule, hidden) regularities, the presence of which makes it possible to reduce the combinatorial complexity of selection algorithms.

In this study, to minimize the number of searches, the ordering of features is used in relation to pairwise distinguishability between objects of classes. Theoretically (ideally) the minimum set should represent independent features. In practical implementation, it is required to choose the initial (not artificial) features, the correlation between which is minimal, i.e. signs with the most pronounced independence.

As a heuristic selection criterion, it is proposed to use a measure of compactness, determined by the set of noise objects and objects - standards of the minimum coverage of the training sample.

In the theoretical substantiation of the effectiveness of the selection method, a desirable property is the monotonicity of the values of the measure of compactness over the determined sets from an ordered sequence of features. There is no analytical form of the relationship between the ordered set and the measure of compactness.

When constructing a sequence of features in general, one has to abstract from the scales and scales of measurements. An abstraction tool is the optimization of criteria for splitting data into non-intersecting intervals [2]. Through such a partition, invariance to the scales of measurements and presentation of feature values in the nominal scale is achieved.

Under the estimate (measure) of the complexity of the decision-making algorithm for an admissible object S, we mean the number of elementary operations for its recognition by a defined set of features.

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The problem of construction and implementation the optimal logical corrector

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Let be given a set of problems $[Z]$, algorithms $[A]$ for solving problems $[Z]$, a set $[R(Z)]$ of solutions to problems and a set $[R_A(Z)]$ of solutions Z using algorithms from $[A]$. In this case, it is not necessary that $R_A(Z) = R(Z)$. The last statement is equivalent to the statement that the algorithms $[A]$ are heuristic or incorrect. Consider an operator with domain of definition $[R_{A_1}(Z)]$ x...x$[R_{A_m}(Z)]$ and range $[R(Z)]$. In other words, F translates the solutions of problem Z obtained by algorithms $A_1, A_2, ..., A_m$ into an element of the set $\tilde{R}(Z)$, which is also called a solution for Z. The quality of the correction is determined by the distance between the sets $[\tilde{R}(Z)]$ and $[R(Z)]$.

Distance can be specified in various ways, leading to various mathematical problems. Obviously, the main one is the problem of constructing the optimal corrector F, that is, the corrector, minimizing its distance indicated above. To solve this problem, it is necessary to specify some information $J(Z)$ about the tasks from Z that are presented for the solution. In addition, it is necessary to specify exactly which heuristic information A will be used. We denote such information by $J(A)$.

Variants of mathematical formulations are possible. The sets $[J(Z)]$,$[J(A)]$, $[F]$ - are set of admissible correctors, and the correction quality functional φ (the distance between $[R(Z)]$ and $[\tilde{R}(Z)]$ is defined: 1. Indicate the algorithms $A_1, A_2, ..., A_m$ and the corrector F , on which the lower bound of the quality functional is implemented. 2. For given heuristics $A_1, A_2, ..., A_m$ find the minimizing corrector F.
In this paper, we propose a solution to the second option.

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An enhancement of the slic superpixel segmentation method for hyperspectral images

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One of the relatively new and actively used methods in image segmentation in recent years is the superpixel segmentation method. The main idea of this method is to divide the neighboring pixels of the image into many small parts according to relatively homogeneous groups [1, 2]. Subdivision sub-groups are called superpixels. Each superpixel is potentially an atomic area of the image, meaning that all pixels belonging to the superpixel are treated as a whole during image post-processing. The boundary of the superpixel area in both straight and curved shapes can lead to a certain amount of small error in homogeneous segmentation.

To improve the simple linear iterative clustering algorithm, we make the following designations. For hyperspectral images, we define $X = [\chi_1, \chi_2, \dots, \chi_N] \in R^{d\times N}$. Here d is the number of channels, N is the number of pixels. For each pixel, we define $P_i = [\chi_i, \alpha_i, \beta_i]^T$, where $\chi_i^T = [x_1, x_2, \dots, x_N]$ is the vector of spectra, $[\alpha_i, \beta_i]^T$ is the vector of the pixel represents its location in the image.

Based on these designations, we rewrite the distance D between the cluster center $C_j = [\chi_j, \alpha_j, \beta_j]^T$ and any pixel $P_i = [\chi_i, \alpha_i, \beta_i]^T$ in the following form:

$$d_d = \|\chi_i - \chi_j\| = \sqrt{\sum_{k=1}^{d} (x_{i_k} - x_{j_k})^2}, d_{\alpha\beta} = \sqrt{(\alpha_i - \alpha_j)^2 + (\beta_i - \beta_j)^2},$$

$$D = \sqrt{d_d^2 + \left(\frac{d_{\alpha\beta}}{S}\right)^2 m^2}\ , \tag{1}$$

where $\|\bullet\|$ – L_2 is norm.

It should be noted that the improved simple linear iterative clustering algorithm with the new D distance measure does not require defining or generating the red, green, and blue channels of the RGB color model. In addition, this enhancement makes full use of the spectral information of the hyperspectral image in addition to the spatial structure of the materials in the images.

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Superpixel segmentation approaches for remote sensing images

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Remote sensing of the Earth's surface represents the process of observing and measuring the energy from the radiation of objects and the properties of its formation in the range of different ranges of the electromagnetic wave spectrum, without direct contact with the measuring device, in determining the location, type, characteristics and changes over time of the environmental objects. Methods of observing the Earth from space are remote sensing methods. Research in this method allows to obtain information about the object under study. The main type of remotely sensed data are images, based on which it is possible to determine other information related to objects based on some features. Many problems are solved based on pre-processing and analysis of remote sensing images.

The features used in superpixel segmentation approaches for remote sensing images are similar to those used for other types of images, but remote sensing data require some unique features to be considered. The following are the most used evaluation criteria in scientific research works:

1. Undersegmentation error – UE [1];
2. Achievable segmentation accuracy – ASA [2];
3. Boundary Recall – BR [3];
4. Compactness.

In this work, the superpixel segmentation approaches that have been actively researched in recent years are analyzed according to evaluation criteria.

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XI. INFORMATION TECHNOLOGY IN EDUCATION

Advantages of using Mindmap technology in discrete structure science teaching

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Nowadays, it is known to the entire scientific world that the methods used in pedagogical technologies, which stimulate students to learn and develop skills, and cause quick results, are called interactive methods. Interactive methods are widely used in educational institutions due to their efficiency, accessibility to the times, continuous development, and motivational features.

Like many computer sciences, there is an integral connection between the concepts of the science of discrete structures, we encounter logically related concepts, tasks that must be performed sequentially and systematically, various actions and postulates. It is important to develop the ability to logically understand and see the connection and interaction of these concepts among students. MindMaps can help us in effective formation of these competences, teaching students to work in active and interactive cooperation.

The founder of MindMap technology is the mathematician and psychologist Tony Bousen. According to him, both spheres of the brain are activated and a synergistic effect is achieved when drawing from the cognitive map. As a result, it leads to the strengthening of the thinking process and the cognitive functions of the brain. Therefore, it is recommended to use cognitive maps to cover a new topic in lecture sessions.

It allows you to analyze and easily save data by using different colors, changing the font size, and embossing when adding ideas.

In the mind map, you can better understand complex topics, develop thinking, make plans, and easily transfer to lessons.

Creating a perception map includes 3 stages:

1. Mental attack. 2. Bouitish with ideas. 3. Concretization.

The use of cognitive maps in education is becoming more and more popular. We also cite situations in which cognitive maps can be used in the teaching of the Discrete Structures course: when presenting its information during lecture sessions; in explaining and strengthening the concepts related to the subject; in the process of summarizing and systematizing information on the studied topic; Applying the learner's knowledge, skills and competencies in the management phase guarantees productive result.

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Technologies for creating and implementing compilers for programming languages

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The work was written on the basis of software implementations of broadcasting systems and related research conducted, including within the framework of master's theses, as well as working training programs.

Previously, the creation of a system that allows you to translate programs written from one language to another took time, calculated in years. However, time has radically changed the situation in this field of informatics. The ideology of designing and implementing software has also changed; a user-friendly, safe toolkit for creating, deploying and supporting software has appeared. Now it is possible to independently develop a translator for any programming language, its implementation does not cause difficulties even for programmers who have practically no professional training in the field of compiler development [1,3].

However, in the training of specialists in the field, especially systems programming, training based on algorithms for working with data used in translation and transformations is of paramount importance.

This is determined by the need for them to gain knowledge not only in the field of creating compilers and hardware architecture, but also in elements of (discrete) mathematics and theoretical programming, and many others.

As you know, syntax and semantics are the most essential concepts that define a programming language, and program transformations are mainly semantic.

There are many different approaches. Compilation-oriented approaches to semantics are understood as approaches in which semantics is a set of transformations over a syntactic model in one form or another.

Optimizing implementation in Python. The optimizer was created using the components of the AST (Abstract Syntax Tree) library and ASTOR of the Python programming language.

AST is a built-in Python module that allows you to represent the structure of a Python program as a tree of nodes, where each node represents a part of the program, such as a function call, variable assignment, or loop. This tree can be thought of as a simplified version of the source code that is easier to parse or modify programmatically.

ASTOR is an external Python library based on the AST module and provides a set of utilities for generating Python code from an AST, modifying an existing AST, or parsing an AST in various ways. ASTOR can be used to automatically make changes to the code, such as refactoring, optimization, or creating new code [2].

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Analysis of the possibility and effectiveness of the use of cloud technologies in education

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Nowadays, education and learning can be seen as a unique type of communication with a common goal and with a teacher who helps participants achieve their goals. In this, cloud technologies can serve as a wonderful assistant. Cloud technologies are data processing in which computer resources and capacities are provided to the user as an Internet service. Compared to the traditional approach, cloud services allow you to manage larger infrastructures, serve different groups of users within the same cloud, and also mean complete dependence on the cloud service provider.

The didactic possibilities of using modern cloud technologies in the education system provide a number of advantages over traditional methods:

- efficiency of information: only information technology allows you to quickly get complete and new information;
- dynamic storage and transmission of information: the use of information technology allows you to accumulate and preserve the didactic base, solve the problem of visualization of education;
- allows you to change the traditional education system;
- allows you to save time in class;
- teacher - only organizes and directs the cognitive activity of students.

The main effectiveness of the use of modern cloud technologies in education are: accessibility (a prerequisite is the availability of Internet access); low cost (cloud technologies do not require the purchase and maintenance of special software, applications can be accessed through a web browser window); flexibility - unlimited computing resources (memory, processor, disks); reliability - the reliability of "clouds especially those located in specially equipped data centers, is very high); security (security is high enough with proper provision.

When solving educational problems, you can use the following software tools:

- software as a service (SaaS Software as a Service);
- platform How service (PaaS, Platform as a Service);
- infrastructure How service (LaaS Infrastructure as a Service.

In this research, the possibilities and effectiveness of using cloud technologies in education are analyzed. A Google cloud services is a variation of Web 2.0 services because they are easy to use and provide a wide range of educational opportunities. The GoogleApps server occupies a special place in the education system. For educational purposes, Google Apps Education Edition has been developed - a free package for educational institutions that includes all the features of a professional package.

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Methodological and software for organizing the educational process based on cloud technologies

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Currently, we are witnessing the rapid development of digital technologies, every day there are more of them, the amount of data is also increasing to an incredible size, in connection with this, questions arise about their storage and processing. The development of electronic services based on the use of cloud technologies by all participants in the educational process can be called a priority task for higher education. The use of these technologies in education makes it possible not only to ensure the relevance and mobility of educational resources, but also to significantly reduce the costs of educational institutions for the construction and maintenance of local information infrastructures, especially since personal computer devices of teachers and students are involved in-stead [1].

Cloud technologies solve such problems as: execution of applications in the cloud; virtualization of equipment and computing resources; ensuring the simultaneous operation of a large number of users, the number of which may vary [3].

Examples of software used in the educational process are free services built on the basis of cloud computing technology for education, such as Google Apps Education Edition, Microsoft Live@edu and others. For distance learning, the most used resource is still the free distributed technology for online communication Skype. The educational process can be drastically changed using, for example, the Google Calendar, which helps to build an optimal schedule for the teacher, information exchange with students, their parents and potential employers, innovative multimedia projects, in particular, Google Earth.

The paper presents specific cloud technology services that can be used in the educational process, such as Dropbox, Yandex.Disk, GoogleDrive, OneDrive, Mail.

The educational process using cloud technologies includes elements of research and group work, involves the active use of computer technology and social services. As a result, a student who obtains information himself has key competencies and is ready to work in modern conditions [2].

According to the conducted experimental research on the use of cloud technologies as a means of organizing the educational process, we came to the conclusion that cloud technologies are an excellent tool for conducting the educational process in computer science lessons, which gives children the opportunity to develop individually, not only mentally, but also creatively and also helps teachers to carry out their work efficiently and effectively, increasing the level of knowledge and skills of students.

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Teaching computational thinking to young children: unplugged or plugged title of your talk

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All over the world, there is a process of learning computational thinking since childhood and programming is the main approach. Two strategies are known, which are activities that do not use a computer, that is, without a computer (unplugged), and activities performed using a computer, that is, with a computer [1]. As you know, the possibility of full use of the first strategy is limited, it can be considered as an auxiliary for working with a computer, and the first phase of the second strategy. When conducting classes on the formation of algorithmic thinking skills in children, the first strategy plays the same important role as the second. According to the results of research and development conducted in England, it is shown that training in training courses on the formation of algorithmic thinking skills in preschoolers practically did not differ in its result from training without a computer, conducted on the basis of both strategies, computer training conducted on a tablet, from training without a computer, conducted on paper, pencil. But a more complete understanding of how to determine at what point the first strategy and the second strategy will alternate requires additional research.

In teaching children computational thinking, several teaching tools can be used, including:
- tools disconnected from the computer;
- digital and programmable toys or robotic complexes (physical devices);
- visual Programming Environments (VPE);
- games that teach programming.

Digital and programming toys or robotics kits (physical device). Recently, the use of these tools for the development of digital thinking skills is considered as one of the new areas of education and is being introduced from kindergartens to secondary school classes and is recognized as a tool that affects the enrichment of the educational environment and the acquisition of knowledge.

Digital and programming toys or robotics kits starting from preschool education, STEM education (science - Science, Technology - Technology, Engineering - Engineering, Math - Math) provides educational opportunities for mastering subject material, programming elements, engineering concepts.

Digital and programmable toys or robotic complexes, some are completely disconnected from the computer, and some can be controlled using a computer. In addition, robotics tools from a pedagogical point of view are divided into two types: robot control and robot creation. In education, children use the first type of robotic tools - robot control. Children gain knowledge and skills to work with algorithms, find solutions to problems, identify and eliminate errors in the process of controlling the robot.

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Principles based on improving students' algorithming competency in institutions of pedagogical higher education

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One of the important tasks of increasing students' intellectual ability (IQ) related to algorithms is to organize their independent education and work in a qualitative manner. To determine the intellectual ability of algorithmization, various puzzles with logical thinking, as well as logical solution problems, test tasks are used. In addition to these methods, in some sense, he reflects, that is, he thinks, in order to absorb the information related to them [1].

As the thinking about "IQ" proves itself in practice, it becomes a concept, and the concepts are sorted and become the mind (conclusion). It is a unit of measurement based on proofs and evidences of students' intellectual ability and logical thinking ability. It allows the user to learn logical algorithmic information from logical test questions.

"IQ" is the number that people score after taking one of the many standardized tests to measure intelligence. Originally, intelligence was calculated as the ratio of mental age to chronological age ($IQ = MA/CA \times 100$, here MA - mental age, CA - chronological age).

Test tasks for determining the level of "IQ" are performed on the basis of the following international methods: Raven's progressive matrices test for determining non-verbal intelligence, Ayzing tests for determining the intellectual quotient, Amthauer tests for diagnosing the structure of intelligence, Wechsler tests for assessing IQ, general intelligence and its structural components. diagnosis [1, 2].

Equivalence of options of test tasks is ensured as follows: each option is formed on the basis of a special computer program, and the tasks of the test option are distributed by subject departments. The difficulty level is kept the same in all variants.

In conclusion, we can say that as soon as we pronounce the word intellectual ability, we recommend using it through questions and tests related to logical thinking. The questions of logical test questions of this type of intellectual ability of students are effectively organized taking into account the living conditions of representatives of all walks of life.

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Working with matrix and determinants using "geogebra"software

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In this work, the topic of matrices and linear operations on them is highlighted using information communication technologies and interactive methods, including in the "GeoGebra"program. The feasibility of using the "GeoGebra"program in training classes of "Mathematics for Economists"is shown.

Let us be given two matrices. For simplicity, the $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ three-dimensional matrices are given. Let's launch the "GeoGebra"program and enter the matrices in the form of $A = \{\{a_{11}, a_{12}, a_{13}\}, \{a_{21}, a_{22}, a_{23}\}, \{a_{31}, a_{32}, a_{33}\}\}$ and

$B = \{\{b_{11}, b_{12}, b_{13}\}, \{b_{21}, b_{22}, b_{23}\}, \{b_{31}, b_{32}, b_{33}\}\}$ Now we will perform the following steps on these matrices.

1. We add: $A + B$.
2. Subtract: $A - B$.
3. We multiply: $A \cdot B$.
4. A we calculate the determinant of the matrix: Determinant(A).
5. $A,\ \det(A) \neq 0$ find the inverse of the matrix: $(A)^{-1}$.

If we enter these commands into "Input the solutions of the above actions that we entered will appear on the left side.

In general, not only for three-dimensional matrices, but also for other arbitrary-dimensional matrices, if classes are conducted using the GeoGebra program, many facilities will be created for students to gain good knowledge.

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The importance of cloud technologies in education

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In the educational process, each teacher interacts not only with students, but also with colleagues at work, as well as with parents.

Let's see what cloud technologies can be used to organize the work of all participants of this system.

1. Students. Individual or collective work can be organized both in the classroom and at home, that is, as homework. Here, cloud technologies can be used as a means of providing educational information. These services are: Google-forms, Quizlet, LearningApps, Testmoz, Online Test Pad, CrocoDoc, DocMe, Draft In, Penzu, PiratePad, SkyDrive, WorldCloud, Prezi.

2. Parents. When working with parents, you can use not only traditional methods of work, but also new methods, including cloud technologies. Opportunities to use cloud technologies, as well as services that can be used to work with parents: Padlet, Dotstorming, Teamer, ThingLink, Google-zakladki, Com Slider, Flowvella, Google-forms.

3. Teachers. The team of pedagogues is constantly working on improving the quality of education by updating the educational content, introducing new information technologies that serve to create favorable conditions for self-awareness of the participants of the educational process. The process of designing and updating education is being improved by using new forms of working with teaching staff. Services that can use cloud technologies for working with teachers: Quipol, Quizsnack, Stickymoose, Pollsnack, Surveymonky, Mindomo, Taghedo, Prezi, Comslider, Empressr and so on.

Cloud technologies should be used not only as a source of information, but also as a source of knowledge or an independent solution to problem situations, as well as a tool for performing tasks. Cloud technologies allow students to organize effective research and research work.

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Interrelation of age and dynamics of physical development of children and adolescents of Fergana in ontogenesis

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Most of the anthropometric signs of a person's physical development, as a rule, are associated with growth indicators, but it would be a mistake to draw conclusions about a person's physical development from only one height. The point of view that the length of the body is a sign of the strength of the body was not confirmed over time, and eventually growth began to be considered in conjunction with other signs and, first of all, with chest girth and body weight.

The purpose of this work is to analyze the level and dynamics of the physical development of children and adolescents ontogeny. The study was conducted in the city of Fergana (on the basis of kindergarten No. 4, secondary school No. 10 and FagGU). The total number of examinations of adolescents and children is 430 people.

The following indicators were used to analyze the dynamics of data, the characteristics of the growth and development of dimensional traits:

1) absolute results of dimensional characteristics in certain age periods;

2) absolute increments of dimensional traits: the difference between the absolute index of a dimensional trait in a given age period and the absolute index of a dimensional trait in the previous age period;

3) the growth rate, expressed as the ratio of the absolute index of the size trait in a given age period to the absolute index of the same size trait in the previous age period;

4) growth rate - the percentage ratio between the absolute increase in a dimensional attribute over a certain period of time to the absolute level of the indicator in the previous period.

The rate of annual increases in the indicators of the main features is gradually decreasing. This trend is interrupted more intensively by the growth of all indicators of the main signs of physical development from 11 to 14 years. Such phenomena are characterized by intense morphological and functional processes in the body system.

The most intensive increase in the main signs of the physical development of children and adolescents in the mountains. Ferghana is observed in the pubertal period (at the age of 11-12 years, standing 5.75% in height, 13.33% in body weight, 7.00% in OGK at rest).

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The role of electronic learning resources in forming communication skills in future teachers

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Taking into account the professional characteristics of communication, which consists in the implementation of communication skills of future teachers not only in face-to-face communication, but also in remote communication. There is a need to teach teachers to establish relationships with people and to organize communication on the basis of both communication, multimedia, and Internet technologies - forums, electronic conferences, Internet chat applications.

Manage the behavior and feelings of teachers, listen and persuade, criticize and accept criticism correctly; form a system of knowledge transfer and target orientations, form teamwork skills, teach how to organize teamwork, effective performance of tasks, subordinating personal interests to achieving a common goal, quickly adapting to a changing situation, resolving conflicts and creating a collaborative environment [1].

Currently, there are different approaches to the essence and content of the concept of communicative skills in psychology and pedagogy, but there is no single definition of these skills. It is necessary to explain the communicative skills of the teacher as the goal of his professional training from the modern point of view. The main factor determining the potential of a person in communication is the uniqueness of the pedagogical activity in which the person participates [2].

An important direction in the study of the problem of communication is the study of the relationship between the teacher and students, because the basis of the effectiveness of their relationship, first of all, is the teacher's ability to communicate. Defines the following types of communication skills, the acquisition of which helps to form a person capable of effective communication [3]:

- interpersonal communication (use of verbal and non-verbal means of communication, transfer of rational and emotional information);
- interpersonal interaction (the ability to establish reasoning, interpret the meaning associated with environmental changes);
- interpersonal perception (acquisition of improvisational skills, including the ability to perceive the position of the interlocutor, hear him, as well as the ability to communicate and organize it without prior preparation).

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Interdisciplinary connections in the educational process

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Interdisciplinary communication is an important and integral part of the educational process. Interdisciplinary communication is the object of study of many researchers.

The article [1] discusses the views of scientists - teachers and methodologists on the problem of defining interdisciplinary connections. Where is an analysis of various studies in this area, confirming the need for knowledge and understanding of terminology, which gives an adequate idea of the types, forms and types of interdisciplinary connections.

Some researchers in the context of the competence-based approach consider the place and role of interdisciplinary connections in improving the effectiveness of educational activities, enriching the intellectual potential of students, developing their abilities to use the acquired knowledge, skills and abilities in practical professional activities.

The article [2] considers the connections between different academic disciplines as a key competence in personality-oriented developmental education and reveals the didactic possibilities of such connections for the formation of modern natural scientific thinking in students.

The article [3] is devoted to the study of interdisciplinary connections in the educational process of a higher educational institution. The effectiveness of the educational process depends on how much the teacher interests, causes and organizes the active cognitive activity of students. Interdisciplinary disciplines should ensure that students acquire the necessary specific knowledge in a particular field of activity. In addition, an important task of presenting the discipline is to ensure that students not only learn individual topics, but also realize the connection of the subject being studied with other disciplines.

An analysis of existing research shows interdisciplinary connections are considered that are mainly two or three disciplines.

It is of interest to study interdisciplinary connections at the intersection of three or more disciplines.

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Peculiarities of learning the basics of computer science from an early age

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This thesis explores the specific aspects of teaching computer science to elementary school students. It focuses on the importance of early exposure to computational thinking and coding skills and investigates various pedagogical approaches, curriculum development, and assessment strategies that can effectively engage children in learning computer science concepts.

Learning the basics of computer science from an early age has its own unique characteristics and benefits. Here are some peculiarities of teaching computer science to elementary school students:

1. Developmentally appropriate approach: Teaching computer science concepts to young children requires a developmentally appropriate approach that takes into account their cognitive abilities, attention span, and learning styles.
2. Play-based learning: Young children learn best through play, exploration, and handson experiences.
3. Concrete examples and real-world connections: Elementary school students benefit from concrete examples and real-world connections when learning computer science concepts.
4. Building computational thinking skills: Computational thinking refers to the ability to think logically, break down complex problems into smaller parts, and design algorithms to solve them.
5. Emphasis on creativity and innovation: Computer science education for elementary school students should also emphasize creativity and innovation.
6. Fostering collaboration and communication skills: Collaboration and communication skills are essential in computer science, as many projects require teamwork and the ability to effectively communicate ideas and solutions.

Overall, teaching computer science to elementary school students requires a tailored approach that considers their unique characteristics, learning needs, and interests. By providing early exposure to computer science education and focusing on specific aspects such as computational thinking, creativity, collaboration, and real-world connections, educators can lay a strong foundation for students' future success in the digital world.

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Criteria for teaching domain-specific programming language in higher education institutions

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The effectiveness of teaching domain-specific programming languages (DSL) is directly related to pedagogical and psychological criteria. These criteria are the basis for determining to learn and achieving cognitive, creative, affective, and psychomotor goals of the educational process. General problems of educational goals are investigated in [1] and other jobs. There are no papers devoted to the criteria for preparing students for DSL. The author in his research [2] studied this problem and, based on the taxonomy presented in [1], developed criteria for determining the level of DSL training by categories of educational goals. The criteria for all educational goals are interpreted in terms of DSL.

I. Cognitive goals: memorization, understanding, application, analysis, synthesis, and evaluation. Here are the properties of some.

Memorization - rules of algorithm construction; basic properties of objects; basics of working with the programming environment; basic concepts, syntax, and DSL command system; rules for writing codes; modules in educational areas and their composition, method assignments; the main components of the project and the rules for their connection.

Application - defining specific actions to solve the current tasks; using DSL tools independently, purposefully, and by needs; building algorithms (projects) and writing codes based on them, compiling codes for typical tasks.

II. Criteria of creativity: inventiveness, independence, adaptability, originality, and search for new ways.

Adaptability - identifying ideas and strategies and creating new ones; changing ideas; turning unfamiliar problems into familiar ones; switching from one programming language to another, adapting to new languages; solving tasks with innovative technologies.

Originality - deviation from generally accepted ideas; promotion of non-standard ideas; search for optimal methods; improvement of existing codes.

III. Affective criteria: perception, reaction, sense of professional significance, and awareness of the usefulness of DSL in personal activities.

Perception - understands the importance of DSL and new knowledge in the field of programming; feels the place of DSL in their professional activities.

Using in personal activities - striving for independence; showing a sustained interest in DSL and program development; striving for cooperation; thinking based on DSL.

IV. Psychomotor criteria: algorithmic, logical, and critical thinking, speed and power in the assimilation of educational materials, understanding of the essence of tasks, and the speed of problem-solving.

The study of creative, affective, and psychomotor mechanisms of education requires long-term observation and research. Therefore, when learning DSL, it is recommended to focus on cognitive goals.

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Methodology for improving the educational process on the basis of computer simulation models

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It is known that the use of technical means in the teaching methodology is an effective method. With the help of technical means, it is possible to increase the volume of educational materials provided to students, to acquaint them with the achievements of modern science. Until recently, cinema and television as technical means of education were seen as a great achievement in the education system. But at present, computer modeling is becoming increasingly important in the educational process. Similar computer simulation is also important in the education system. From an implementation point of view, computer modeling and simulation perform similar tasks. That is, the manifestation of the internal and external properties of the object (an educational process is provided) is indicated by imitation.

The rapid development of information technology makes it possible to introduce innovative teaching methods into the modern educational process, among which computer simulation models (CSM) are of particular importance. Scientific research is being carried out on the development of the quality of education at the world level, research on the socio-biological and pedagogical-psychological foundations for the formation of healthy lifestyle skills among students. Scientific approaches that allow mastering the content of a healthy lifestyle, as well as teaching the educational process based on innovative technologies and active methods and means, contribute to the development of sanogenic thinking of students.

Particular attention is paid to improving the efficiency of assimilation of students' knowledge through the widespread introduction of information and communication technologies in the world educational process, creating a base of educational resources and increasing the efficiency of their use. Of great importance in the teaching of natural sciences, including biology, is the creation of thematic computer simulation models based on the visual and virtual capabilities of interactive software, as well as aimed at developing knowledge about biological processes in students, the content of reflective skills. Today, this direction is reflected in the development of students' spatial imagination based on the use of computer simulation models of educational technologies in a number of developed countries of the world, including the USA, Germany, China, Russia, Korea, Great Britain, Canada, India, Malaysia, Japan, increasing the productivity of knowledge levels, in their work aimed at the development of theoretical knowledge. In this case, the use of the didactic possibilities of educational technologies, especially those based on the integration of academic disciplines, is one of the urgent problems of the educational process.

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Use of video lessons in the organization of independent learning of disciplines

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Problems in teaching methods that are more commonly used in higher education, including the insufficient use of computer multimedia technologies in the organization of independent education in subjects, in the acquisition of independent topics and insufficient development of students' independent thinking activities. leading to stay. The use of a video lesson allows to take into account the specific features of information acquisition to the maximum extent, which is very important for the pedagogue to deliver educational information to the learner by means of a computer.

By applying multimedia tools consisting of video materials to the educational process, lessons can be organized interactively based on modern information and pedagogical technologies. Video editing software includes recording the process on the screen in video form, removing unnecessary parts from the video, combining pieces of video material, creating transitions between them, adding special effects and subtitles. Programs for working with such data include VideoMONTAJ, Avidemux, Lightworks, Camtasia Studio, Vegas Pro, Final Cut, Adobe Premiere.

With the help of the above programs, the educational resources consisting of video lessons created in the disciplines "Digital and information technologies", "Multimedia technologies" and "Basics of computer graphics" develop the creative potential of students and serve to improve the quality of science education.

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Challenging problems of using modern information technologies in physical education in high schools

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The development of information and communication technologies in the field of physical education and sports has become a modern demand. The problem of determining the effectiveness of educational processes and improving the activities of physical education teachers of general education institutions based on their knowledge and skills in using modern computer technologies and devices remains urgent.

The purpose of the work is to clarify the problems in the field of information and communication technologies and to analyze the practical significance of this issue for physical education teachers of general educational institutions.

Research methods - study and analysis of current problems of using modern information technologies with physical education teachers working in general education schools in our republic.

672 pedagogues working in the field of physical education in secondary schools took part in the survey, 69.5% of them were men and 30.1% were women. Among the research participants, 3.4% were aged 18 to 20, 19.9% were aged 20 to 25, 43.9% were aged 26 to 35, 22.5% were aged 36 to 45, and 46 the higher one was 10.3%. According to their data, 22.9% of those with higher education were masters, 77.1% were bachelors.

As a result of the rapid popularization of modern means of mass communication, they are gaining a great position on the stage of the world media space. Confirmation of this was asked by the respondents, "Which social networks do you mainly use?"can be seen from the answers to the question. 86.4% of them stated that they use Telegram, 10.6% - WhatsApp, 15.2% - Instagram, 16.3% - Facebook, 25.4% - other social networks, and 3.2% said that they do not use social networks.

Electronic resources related to physical education and sports are mainly used by 71.7% of respondents from the state domains of Uzbekistan, 36% from the state domains of Russia, 8.2% from the domains of England and Germany, 22.6% from the domains of other countries, and respondents who do not know that such resources exist - made 6%.

As a conclusion, regardless of the methods of using information and communication technologies in the subject of physical education in a general education school, effective use is important.

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Scientific-pedagogical issues of using information and communication technologies in the system of retraining and upgrading the qualifications of physical education and sports specialists

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Modernization of the system of training of physical education and sports specialists, conditions in sports educational institutions, growing need for highly qualified specialists is one of the requirements aimed at eliminating important problems in organizing and coordinating the activities of sports educational institutions. In recent years, the idea of integration (creating a creative environment of scientific-pedagogical cooperation with each other) has become the basis of rapidly developing theoretical and practical research.

In order to effectively use modern information and communication technologies during the preparation of the lesson and during the lesson, the teacher should know very well the functional capabilities and conditions of use of these components, because both technical and software tools have their own characteristics and influence on the educational process. Pedagogical goals of using information and communication technologies are as follows: development of personality (thinking; aesthetic education; conduct of experimental and research activities); formation of information culture; fulfillment of the social order consisting of the user's general information preparation ("computer literacy") in the training of specialists in a specific field; to increase the productivity of the educational process, the quality and efficiency of education, to ensure the important aspects of knowledge and learning activities, to deepen interdisciplinarity due to the integration of information and science.

Information awareness and effective use of information in work activities depends on the following. First of all, a modern sports specialist should know how to receive, process and use information with the help of computers, telecommunications and other means of information in order to freely navigate the flow of information. Secondly, ensuring the necessary level of information culture of a specialist should not be the goal of only one academic subject.

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Methodology for training future IT specialists in the design and development of databases based on real practical issues

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Today, specialists in the design and development of databases are a relevant field of activity for software developers [1]. The current situation in the market of IT technologies puts before universities the task of training more and more IT specialists in designing software products. These tasks include the design and development of databases, which are an integral part of information systems, database applications for clients, as well as the operation, maintenance and improvement of these systems. The solution to these problems is the training of bachelors in the field of information and communication technologies. The science of “Databases” is the basis of many other professional disciplines, and the relevant competencies are formed in students during the study of the science.

This study is primarily aimed at developing a methodology for training future specialists in the process of designing a database, from the step-by-step acquisition of basic knowledge and skills of the subject to the creation of a working prototype of the information system.

The results of scientific practical research on the methodology of teaching database analysis and conceptual design show that there is currently no formal methodology of teaching. Teaching methods are mainly based on the experience of designing a real database based on the experiences of professional developers. These experiences are difficult to convey to the student, so these steps can be equated to the “art” of the database design process.

In the methods developed in [2,3], it is proposed to consider the stages of database analysis and design using examples that are far from the scope of the real topic. Such models are often significantly simplified. They don’t actually explain how to break down and structure data to create an infographic data model during the conceptual design process. In principle, this approach is justified in the study of database design issues, since different topics cannot be tied to a single model of the design process. In the proposed methodology, the process of studying “Databases” is carried out in two stages: training-acquaintance stage; practice-oriented stage.

By studying each step of science preparation sequentially, students will eventually acquire the necessary skills and abilities to design and develop databases.

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